One-period Portfolio Management Stochastic Optimization Model

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Abstract This paper studies portfolio management One-Period Stochastic Optimization model, under the unknown economic environment, obtains a simplex model of maximal utility function of final wealth.

Key words Portfolio management, Optimization model

1 Introduction

The applications of the portfolio management model were widespread in finance industry since the 80's.There are a great deal of feasible mathematical models in fixed income derivative and balance sheet managements, which provide scientific decision-making of the portfolio management for the investment administration. However, in the results of citation [1] and [2], the studies on stochastic behavior of portfolio management have not been deeply discussed. The paper is concerned with a portfolio management problem in a simple situation. A certain value portfolio at time t_0 will have uncertainty at time t_1 for the uncertainty of the economic environment. We assume that there are l states of the economic circumstance. We use $s_1,\ldots,s_l$ to denote the state variables and $p_1,\ldots,p_l$ to denote the occurrence probability where $\sum_{i=1}^{l} p_i = 1$, $p_i \geq 0$ and i=1,…,l. Thus,a portfolio z_0 at time t_0 will have l likelihoods at time t_l which are denoted with $z^1,\ldots,z^l$.The uncertainty is induced by the uncertainty price on economic environment. We further assume that there are m securities in the market.

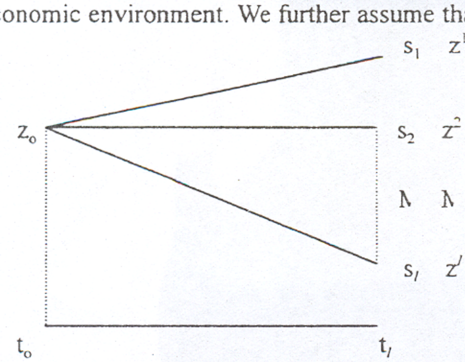

Figure 1

C_0 denotes the cash asset of the portfolio at time t_0,and b_0 denotes a m dimension vector where each component denotes the amount of each security at time t_0 in the portfolio. We seek to reconstruct a decision-making: cash and the portfolio are hold to time t_l, so that they can adapt to developing a fixed cash payment and maximizing utility function of final wealth.

2 Model

In this problem , there are 4 elements at time t_0:

(1) Vector x_0 denotes the amount of buying each security in the portfolio M.

(2) Vector y_0 denotes the amount of selling each security in the portfolio b_0.

(3) Vector z_0 denotes the amount of the securities under the primal decision.

(4) Scaler quantity v_0 denotes the amount of cash hold to time t_l.

In the study of portfolio management, there are 2 relations: one is to invest current asset which will provide standby capital for the future cash flow p_l; the other is to adjust original portfolio to buy new assets.

When we make investment decision, cash asset is composed of original cash C_0 and the return $\zeta_0 y_0$ from selling y_0 of the original portfolio b_0, where vector ζ_0 denotes the price of security which has been sold at time t_0. The outflow of cash is composed of cash-holding v_0 and the cost of buy $\xi_0 x_0$ where vector ξ_0 denotes the price of security which has been bought at time t_0. The portfolio at time t_0 has the following equilibrium constraints.

Cash $\quad C_o+\zeta_o y_o=\xi_o x_o+v_o$

Security quantity $\quad b_o+x_o=y_o+z_o$

The value of asset consists of cash $k_{\text{终}} v_0$ and the value of portfolio $\zeta^s z_0$ at time t_l where $k_{\text{终}}$ denotes continuous compounding.

The outflow of cash is composed of the payment p_l and final value of the portfolio v^s.

Thus The portfolio at time t_l also has the following equilibrium constraint.

$$k_{\text{终}} v_o+\zeta^s z_o=P_l+V^s \quad s\in S$$

If we use U(*) to denote the utility function of investor, maximal investment utility function model is as follows:

$$\max EU(x_o,y_o,z_o,v_o,V^s)$$

$$s.t\begin{cases} C_o+\zeta_o y_o=\varepsilon_o x_o+v_o \\ b_o+x_o=y_o+z_o \\ k_{\text{终}} v_o+\zeta z_o=P_l+V^s \\ x_o\geq o, y_o\geq o,\ z_o\geq o, v_o\geq o, V^s\geq o \\ s\in S \end{cases}$$

There are m+1+l constraints and (3m+1)+l variables in the model.
Where

$$u_o=(x_o,y_o,z_o,v_o)^T, \qquad u^s=(z_o,V^s)$$

$$A_0=\begin{pmatrix} -\xi_0 & \zeta_0 & 0 & -1 \\ -1 & 1 & 1 & 0 \end{pmatrix} \qquad T^s=(0,0,0,k_{\text{终}})$$

$$f_o=(-c_o,b_o)^T \qquad R^s=\begin{pmatrix} \zeta^s & -1 \\ 1 & 0 \end{pmatrix}$$

$$f_s=(P_l,0)^T$$

These constraints can be denoted

$$\begin{bmatrix} A_0 & & & \\ T^1 & R^1 & & \\ M & & O & \\ T^l & & & R^l \end{bmatrix} \begin{bmatrix} u_o \\ u^1 \\ M \\ u^l \end{bmatrix} = \begin{bmatrix} f_o \\ f_1 \\ M \\ f_l \end{bmatrix}$$

We partition the matrix, then the model is as follows:

$$\begin{cases} A_o u_o & = f_o \\ T^s u_o + R^s u^s & = f_l \qquad \forall s \in S \\ u_o \geqslant o, u^s \geqslant o & s \in S \end{cases}$$

3 General Model

Generally, two-stage stochastic programming model E is as follows.
S is a strategy set, defined as following:

u_0 -the first period investment-decision vector based on known information at time t=0.

u_l^s -the next term decision according each strategy $s \in S$.

π_s -the probability of strategy $s \in S$.

U(u_0, u_l^s)-utility of return which is the function of decisive variables.

Certainly $\forall\ s \in S$, u_l^s must be decided before observing the realization of strategy.

Decisive variables u_0, u_l^s must maximize beginning utility and satisfy all accounts, strategies, diversification of investment as well.

Matrix A_0, T_l^s, R_l^s denote general constraints, then we have stochastic optimization model E:

$$\max EU(u_o, u_l^s)$$

$$s,t \begin{cases} A_o u_o = b_o \\ R_l^s u_l^s + T_l^s u_o = b_s \qquad \forall s \in S \\ u_o \geqslant 0 \\ u_l^s \geqslant 0 \forall s \in S \end{cases}$$

Note: because all constraint sets are used, we have a large *N-LP* resolution. u_0 independs from future information set in each stage, so it is identical.

Last we can extend Stochastic programming model E to multi-term or multi-stage investment decision.

References

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