Chapter 10: Trade and Endogenous Growth

The link between trade and growth has long been a question of interest from both a theoretical and policy point of view. David Ricardo developed a dynamic model of corn and velvet production, with corn produced by land and labor and velvet produced with labor alone.[1] The need to pay labor from a "wage fund" established prior to production prevents all labor from being employed initially (i.e. there is a liquidity constraint on firms). Ricardo showed that in autarky the gradual expansion of the wage fund and growth of velvet production would lower its relative price, until a long-run equilibrium was reached. Opening trade, however, would allow the relative price of velvet to be maintained at world levels, thereby benefiting capitalists at the expense of landowners. This remarkable model has many of the issues that are of interest in modern discussions of trade and growth: the possibility that growth will be associated with continual changes in prices, and conversely, the impact of trade on prices and growth rates themselves.

In the first of these issues we will be primarily concerned with the effects of growth on the *terms of trade,* i.e. the price of exports relative to imports. This brings us to the famous case of "immiserizing growth," due to Bhagwati (1958), where we ask whether growth can actually lower a country's welfare due to a fall in the terms of trade. The idea that developing countries might be subject to a *decline* in their terms of trade, particularly for primary commodities, is associated with the Latin American economist Raul Prebisch (1950). While there is little evidence to support that hypothesis in general, it is still the case that terms of trade decline due to growth have been observed (Acemoglu and Ventura, 2002, Debaere, 2001).

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[1] See Findlay (1984, pp. 187-191), who cites this model to Ricardo's *Essay on the Influence of a Low Price of Corn upon the Profits of Stock* (in Volume IV, pp. 1-42, of Ricardo, 1951).

In the second part of the chapter, we model the underlying determinants of growth through research and development. This class of models goes by the name "endogenous growth," and their application to trade has been most thoroughly studied by Grossman and Helpman (1991). We will describe just one of the models described by those authors, which is a dynamic version of the CES monopolistic competition model presented in chapter 5.[2] Even in this case, we will find that trade can have a fairly wide range of results on product innovation within a country and therefore on growth. When there is free trade in *all goods,* including *free international flows of knowledge,* then trade raises growth rates and brings gains to all countries, but this is not necessarily the case when there is free trade in only a subset of the goods without flows of knowledge. Empirically, then, the beneficial effect of trade on growth depend very strongly on whether there are international spillovers of knowledge, so we shall review the empirical evidence on that and other topics.

We begin with a discussion of how to measure productivity growth. This is quite easy in the case of perfect competition, but not under imperfect competition. In the latter case, it turns out that conventional measures of productivity growth are biased due to increasing returns to scale and the markup of price over marginal cost. Hall (1988) has proposed a method to correct for this, and at the same time, estimate the markups. In the context of trade reform it allows us to estimate the extent to which trade liberalization has led to a reduction in markups by firms facing import competition. This approach has been applied to Turkey by Levinsohn (1993) and to the Ivory Coast by Harrison (1994), as we shall discuss.

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[2] This model was introduced by Judd (1985), Grossman and Helpman (1989, 1990) and Romer (1990), and also applied to trade by Rivera-Batiz and Romer (1991a,b). The alternative to the CES model is the "quality ladders" model of product improvement, as described by Segerstrom, et al (1990), Grossman and Helpman (1991) and Taylor (1993). There are many other writings on endogenous growth and trade, including Taylor (1994) on the gains from trade and Dinopoulos and Segerstrom (1999a,b) on the dynamic effects of tariffs.

Measurement of Productivity

Growth in the output of a firm or industry can occur due to growth in inputs (labor, capital, and human capital), or due to an increase in output that is *not explained* by inputs: the latter is called productivity growth. Specifically, suppose that output y_i is produced using labor L_i and capital K_i, with the production function $y_i=A_if(L_i,K_i)$, where A_i is a measure of Hick's neutral technological progress. Totally differentiating this, we have $dy_i = dA_i f(L_i, K_i) + A_i f_{iL} dL_i + A_i f_{iK} dK_i$. Dividing this by $y_i = A_if(L_i,K_i)$, and letting $\hat{z} = dz/z$ denote the percentage change in any variable, we see that,

$$\hat{y}_i = \hat{A}_i + \left[\left(\frac{f_{iL}L_i}{f(L_i,K_i)}\right)\hat{L}_i + \left(\frac{f_{iK}K_i}{f(L_i,K_i)}\right)\hat{K}_i\right]. \qquad (10.1)$$

In general, then, productivity growth can be measured by the growth in output minus a weighted average of the growth in inputs, where the weights are the *elasticities of output with respect to each factor.* The difficulty is that these weights are not directly observable. With the added assumption of perfect competition, however, the marginal products equal $A_i f_{iL} = w/p_i$ and $A_i f_{iK} = r/p_i$, so in that case the elasticities appearing in (10.1) are measured by $\theta_{iL} = wL_i/p_i y_i$ and $\theta_{iK} = rK_i/p_i y_i$, which are the *revenue shares* of labor and capital. Under the further assumption of constant returns to scale, these shares sum to unity so that $\theta_{iK} = 1 - \theta_{iL}$. Then we see from (10.1) that technological progress $\hat{A}_i$ can be measured by the difference between the growth of output and a share-weighted average of the growth in inputs, which is the definition of total factor productivity (TFP) due to Solow (1957):[3]

$$\mathrm{TFP}_i \equiv \hat{y}_i - [\theta_{iL}\hat{L}_i + (1-\theta_{iL})\hat{K}_i] \ . \tag{10.1'}$$

Notice that this measure of productivity growth is indeed a "residual" – the portion of output growth that is not explained by inputs – or a "measure of our ignorance," in the memorable phrase of Robert Solow.

Let us now weaken our assumptions, and allow for imperfect competition and increasing returns to scale. To introduce increasing returns to scale, we assume that the production function is homogeneous of degree $\mu_i > 1$, so it follows that $\mu_i f(L_i,K_i) = (f_{iL}L_i + f_{iK}K_i)$.[4] Substituting this into (10.1), we obtain,

$$\hat{y}_i = \hat{A}_i + \mu_i\left[\left(\frac{f_{iL}L_i}{f_{iL}L_i + f_{iK}K_i}\right)\hat{L}_i + \left(\frac{f_{iK}K_i}{f_{iL}L_i + f_{iK}K_i}\right)\hat{K}_i\right].$$

$$= \hat{A}_i + \mu_i\left[\left(\frac{f_{iL}L_i}{f_{iL}L_i + f_{iK}K_i}\right)\hat{L}_i + \left(1-\frac{f_{iL}L_i}{f_{iL}L_i + f_{iK}K_i}\right)\hat{K}_i\right]$$

$$= \hat{A}_i + \left(\frac{f_{iL}L_i}{f(L_i,K_i)}\right)(\hat{L}_i - \hat{K}_i) + \mu_i\hat{K}_i \, , \tag{10.2}$$

where the last line follows from $\mu_i f(L_i,K_i) = (f_{iL}L_i + f_{iK}K_i)$ and simple arithmetic.

With imperfect competition in the product market, the equilibrium condition for the

[3] In practice, total factor productivity in (10.1') would be measured using first-differences rather than infinitesimal changes, as $\mathrm{TFP}_i \equiv \Delta\ln y_i - [\theta_{iL}\Delta\ln L_i + (1-\theta_{iL})\Delta\ln K_i]$. We have already seen this definition of TFP in our discussion of chapter 4, as well as the dual definition, $\mathrm{TFP}_i \equiv (\theta_{iL}\Delta\ln w_i - \theta_{iK}\Delta\ln r_i) - \Delta\ln p_i$, where w_i is the wage, r_i is the rental on capital, and p_i is the price in industry i. Note that with the log change in input quantities or prices measured between two points in time, the factor cost-shares θ_{ij} should be measured as the *average* between these two periods; this method is called the Tornqvist index of total factor productivity (see Appendix A).

hiring of labor is,

$$p_i\left(1-\frac{1}{\eta_i}\right)A_i f_{iL} = w\,. \tag{10.3}$$

where η_i is the (positive) elasticity of demand. It follows that the real wage equals $(w/p_i) = [(\eta_i - 1)/\eta_i]A_i f_{iL}$. The share of labor in total revenue is then,

$$\theta_{iL} = \frac{wL_i}{p_i y_i} = \frac{(\eta_i - 1)A_i f_{iL} L_i}{\eta_i y_i} = \left(\frac{\eta_i - 1}{\eta_i}\right)\left(\frac{f_{iL} L_i}{f(L_i, K_i)}\right), \tag{10.4}$$

where the final equality follows from $y_i = A_i f(L_i,K_i)$. Thus, the labor share *understates* the elasticity of output with respect to labor, since $(\eta_i - 1)/\eta_i < 1$.

Substituting (10.4) into (10.2), we see that output growth is related to true productivity change by,[5]

$$\hat{y}_i = \hat{A}_i + [\eta_i/(\eta_i - 1)]\theta_{iL}(\hat{L}_i - \hat{K}_i) + \mu_i K_i\;. \tag{10.5}$$

Thus, we see that output growth is composed of true productivity change $\hat{A}_i$, plus the change in labor/capital growth times the coefficient $\eta_i/(\eta_i - 1)$, plus capital growth times the returns to scale parameters μ_i. The coefficient $\eta_i/(\eta_i - 1)$ equals the ratio of price to marginal cost, so any *change* in this coefficient are interpreted as evidence of changes in the price-cost markup. Running (10.5) as a regression therefore allows us to measure the impact of trade liberalization on the markups charged, as well as estimate the returns to scale.

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[4] See problem 1.2 in chapter 1.

[5] Alternatively, we can express (10.5) in first-differences and deduct $[\theta_{iL}\Delta \ln L_i + (1-\theta_{iL})\Delta \ln K_i]$ from both sides, obtaining $TFP_i = \Delta \ln A_i + [1/(\eta_i - 1)]\theta_{iL}(\Delta \ln L_i - \Delta \ln K_i) + (\mu_i - 1)\Delta \ln K_i$, using the definition of TFP from note 4.

Harrison (1994) applies this technique to firm-level data for the Ivory Coast, which had an import liberalization beginning in 1985. The estimating equation is (10.5) but written with discrete rather than infinitesimal changes:

$$\begin{aligned}\Delta \ln y_{it} &= \Delta \ln A_{it} + \beta_t \theta_{iL}(\Delta \ln L_{it} - \Delta \ln K_{it}) + \mu \Delta \ln K_{it} \\ &= \alpha_i + \beta_t \theta_{iL}(\Delta \ln L_{it} - \Delta \ln K_{it}) + \mu \Delta \ln K_{it} + \varepsilon_{it}, \end{aligned} \qquad (10.5')$$

where in the second line we replace the true productivity change $\Delta \ln A_{it}$ by $\alpha_i + \varepsilon_{it}$, composed of a firm fixed-effect α_i and a random component ε_{it}. The estimated ratio of price to marginal cost, $\beta_t \equiv \eta_t/(\eta_t - 1)$, is treated as common across firms i = 1,…,N within each industry but is allowed to change in the pre- and post-liberalization sample. Thus, by examining the change in β_t we can determine the effect of liberalization on markups.

Estimation is performed with both OLS and instrumental variables.[6] Initial estimates generally provided only insignificant changes in the estimated price-cost ratios. So rather than just letting the markups β_t change over time, Harrison instead interacts this variable with industry-level estimates of import penetration. Specifically, let $\beta_t = \beta_0 + \beta_1 M_t$, where M_t is the ratio of imports to consumption in each industry. Substituting this in (10.5'), we obtain an interaction term between M_t and $\theta_{iL}(\Delta \ln L_{it} - \Delta \ln K_{it})$, with the coefficient β_1. We expect that $\beta_1 < 0$, so that higher imports lead to lower markups. Harrison obtains an estimate of –0.25 for

[6] The error ε_{it} in (10.5') is likely correlated with factor inputs on the right: a firm that experiences a boost to productivity will hire more inputs. Appropriate instruments should be correlated with factor usage by the firm, but not with productivity, and Harrison uses the nominal exchange rate, the price of energy, sectoral real wages, and the firms' reported debt. The endogeneity of inputs, and appropriate methods to correct for this, is also addressed by Hall (1988), Olley and Pakes (1996) and Levinsohn and Petrin (2001).

β_1 over the entire sample (standard error of 0.05). This indicates, for example, that an increase in import penetration from 0 to 50% would reduce the ratio of price to marginal cost by 12%. She also finds that reduced tariffs are associated with lower price-cost ratios, but that result is only significant at the 15% level. So the connection between import liberalization and reduced markups is weakly confirmed in her study.

Notice that in our derivation of (10.5) we *did not* assume that the rental on capital is observable, but only used the labor share. If instead the rental is observable, then the capital share is $\theta_{iK} = rK_i/p_iy_i = [(\eta_i - 1)/\eta_i][f_{iK}K_i/f(L_i,K_i)]$, analogous to (10.4). Substituting these two equations into (10.1), we obtain an alternative relationship between output growth and true productivity change,[7]

$$\hat{y}_i = \hat{A}_i + \left(\frac{\eta_i}{\eta_i - 1}\right)\left(\theta_{iL}\hat{L}_i + \theta_{iK}\hat{K}_i\right). \tag{10.6}$$

Levinsohn (1993) applies equation (10.6), written using discrete changes, to firm-level data for Turkey, which embarked on an ambitious liberalization program in 1980. He replaces the true productivity change $\Delta \ln A_{it}$ by $\alpha_t + \varepsilon_{it}$, composed of a year fixed-effect α_t that is common across firms and a random component ε_{it}. Once again, the estimated ratio of price to marginal cost, $\beta_t \equiv \eta_t/(\eta_t - 1)$, is treated as common across firms $i = 1,\ldots,N$ within each industry but is allowed to vary over time, as liberalization proceeds. In his results, Levinsohn finds a strong connection between the industries that experience liberalization, and those where the

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7 Notice that (10.5) and (10.6) are identical if there are zero profits in the industry, so that price equals average cost. In that case, the ratio of average cost to marginal cost, which equals the returns to scale μ_i, also equals the ratio of price to marginal cost, which is $\eta_i/(\eta_i - 1)$. Also, the labor and capital shares sum to unity, so that $\theta_{iK} = 1 - \theta_{iL}$.

estimated price-cost ratios are reduced: in three industries experiencing significant liberalization, the estimated ratios fell; whereas in two other industries experiencing increased protection, the ratios rose; and in the remaining five industries where the ratios where insignificantly different from unity (i.e. the industries are perfectly competitive), there was no clear pattern to the changes in the point estimates of β_t. So for both the Ivory Coast and Turkey, the evidence supports the hypothesis that import liberalization reduces markups.

Immiserizing Growth

We now turn to the link between growth and the terms of trade, and the case of "immiserizing growth" due to Bhagwati (1958). This case is illustrated in Figure 10.2 where good 1 is the exportable and good 2 is the importable. With the initial production possibility frontier (PPF), the economy produces at point B and consumes at point C. Due to growth the PPF shifts out. If the terms of trade were unchanged, then consumption would change to a point like C′ and the representative consumer would be better off. With a fall in the relative price of the export good, however, it is possible that consumption could instead be at point C″ with production at B″. This illustrates the borderline case where utility is unchanged due to growth, and with any further fall in the terms of trade, the representative consumer would be worse off.

We are interest in solving for the conditions under which immiserizing growth can occur. To this end, it will be convenient to first solve for the conditions such that utility is *constant,* as illustrated in Figure 10.1. For simplicity we suppose that there are just two goods, and choose the importable y_2 as the numeraire, so its price is unity. Let $G(p,\alpha) = py_1+y_2$ denote the GDP function for the economy, where α is a scalar that represents a shift parameter for the PPF. For

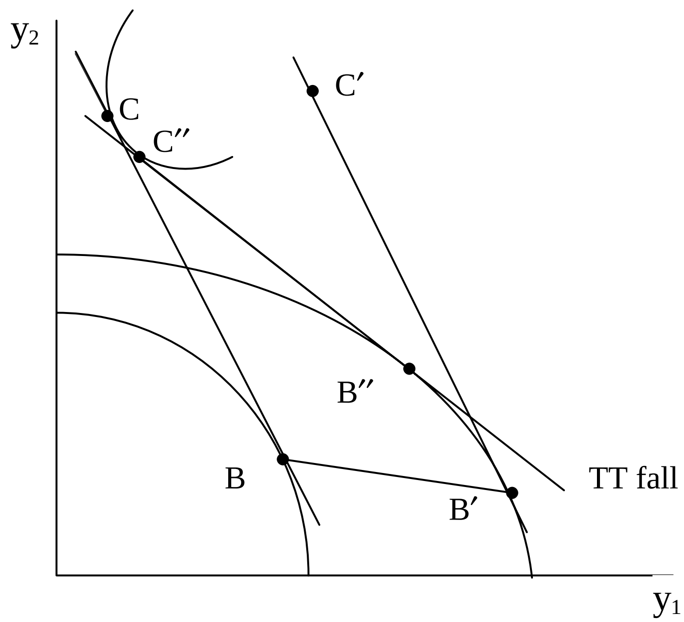

Figure 10.1

example, α can represent a factor endowment or technological progress in some industry. The total change in GDP is measured by:

$$dG = \frac{\partial G}{\partial p}dp + \frac{\partial G}{\partial \alpha}d\alpha = y_1 dp + \left(p\frac{dy_1}{d\alpha} + \frac{dy_2}{d\alpha}\right)d\alpha \ . \tag{10.7}$$

We presume that $\frac{\partial G}{\partial \alpha} = \left(p\frac{dy_1}{d\alpha} + \frac{dy_2}{d\alpha}\right)$ and $\frac{dy_1}{d\alpha}$ are both positive, meaning that growth at constant prices increases both GDP and production of the exportable.

As in chapter 7, we will allow for different consumers but suppose that the numeraire good is additively separable in consumption, so that consumer utilities can be summed. Total social welfare is then $W[p, G(p,\alpha)]$, which serves as an indirect utility function for the economy, where $\partial W/\partial p = -c_1$ is the (negative of) consumption of good 1 and $\partial W/\partial G \equiv 1$. Then social welfare is constant due to growth if and only if,

$$dW = \frac{\partial W}{\partial p}dp + \left(\frac{\partial G}{\partial p}dp + \frac{\partial G}{\partial \alpha}d\alpha\right) = (y_1 - c_1)dp + \frac{\partial G}{\partial \alpha}d\alpha = 0 \ . \tag{10.8}$$

Thus, the drop in the export price that will just keep welfare constant is,

$$dp = \frac{\partial G}{\partial \alpha}d\alpha \Big/ (c_1 - y_1) \ . \tag{10.9}$$

Next, we solve for the equilibrium change in the relative price of exports and compare that to (10.9). Equilibrium in the export market means that $(y_1 - c_1) = m_1^*$, where m_1^* is import demand from the rest of the world. Totally differentiating this, we obtain,

$$\frac{dy_1}{d\alpha}d\alpha + \left(\frac{\partial y_1}{\partial p} - \frac{\partial c_1}{\partial p}\right)dp = \frac{dm_1^*}{dp}dp \, .$$

Thus, the equilibrium change in the price of exports is,

$$dp = \frac{\dfrac{dy_1}{d\alpha}d\alpha}{\left[\dfrac{dm_1^*}{dp} - \left(\dfrac{\partial y_1}{\partial p} - \dfrac{\partial c_1}{\partial p}\right)\right]} \quad . \tag{10.10}$$

The denominator of (10.10) is negative while the numerator is positive, so this expression indicates the drop in the export price due to growth. Welfare is constant if (10.9)=(10.10), and welfare falls if (10.10) < (10.9). This will occur if and only if,

$$\frac{dy_1}{d\alpha}(y_1 - c_1)\Big/\frac{\partial G}{\partial \alpha} \; > \; \left[\left(\frac{\partial y_1}{\partial p} - \frac{\partial c_1}{\partial p}\right) - \frac{dm_1^*}{dp}\right] \quad .$$

Dividing this equation through by $(y_1 - c_1) = m_1^*$, and making use of $\frac{\partial G}{\partial \alpha} = \left(p\frac{dy_1}{d\alpha} + \frac{dy_2}{d\alpha}\right)$, we can express this necessary and sufficient condition for immiserizing growth as,

$$p\frac{dy_1}{d\alpha}\Big/\left(p\frac{dy_1}{d\alpha} + \frac{dy_2}{d\alpha}\right) > \left[\left(\frac{\partial y_1}{\partial p} - \frac{\partial c_1}{\partial p}\right)\frac{p}{m_1^*} - \frac{dm_1^*}{dp}\frac{p}{m_1^*}\right] \quad . \tag{10.11}$$

To interpret this expression, note that the first term on the right is the elasticity of export supply and is positive. For immiserizing growth to occur, it is therefore necessary that the term on the left *exceed* the amount $-\left(\frac{dm_1^*}{dp}\frac{p}{m_1^*}\right)$, which is the elasticity of foreign demand for imports. If this elasticity is less than unity, i.e. foreign demand is inelastic, then immiserizing growth can occur even when the expression on the left of (10.11) is also less than unity. However, if foreign demand for imports is elastic, then for (10.11) to be satisfied it must also be the case that the expression on the left exceeds unity. By inspection, this occurs if and only if $dy_2/d\alpha < 0$, that is, when growth reduces the output of good 2 (at constant prices). Summarizing these results, we have:

Theorem (Bhagwati, 1958)

Immiserizing growth occurs if and only if (10.11) holds. Necessary conditions for this are that either: (a) the foreign demand for imports is inelastic, or (b) growth reduces the output of the importable good (at constant prices).

Condition (a) is not surprising, since having inelastic demand for a product is the same condition that would permit farmers to be worse off after a bumper crop, as in the example given in introductory economics textbooks. What is more surprising is that immiserizing growth can occur even when foreign demand is *not* inelastic, but this requires that condition (b) hold, i.e. that growth reduces the output of the importable good. We know from the Rybczynski Theorem that this condition would hold in the two-by-two model if the factor used intensively in exports has grown. So this is one scenario when immiserizing growth can occur. Are there others?

Findlay and Grubert (1959) answer this question in the affirmative. They considered Hick's neutral technological progress in good 1. To demonstrate the effects of this, we work with the GDP function, defined as:

$$G(Ap, V) \equiv \max_{v_i \geq 0} \; pAf(v_1) + f(v_2) \text{ subject to } v_1 + v_2 = V. \tag{10.12}$$

That is, the parameter α that appeared in the earlier GDP function now equals A, which is the Hick's neutral productivity parameter on good 1. Notice that Ap enters in a multiplicative form as an argument of the GDP function, since that is how it appears in the objective function.

We have demonstrated in chapter 1 that the derivative of the GDP function with respect to price equals the output of that good, or $\partial G / \partial p = y_1$. Letting $G_1(Ap,V)$ denote the partial derivative of G with respect to its first argument, we therefore have:

$$y_1 = \frac{\partial G}{\partial p} = \frac{\partial G}{\partial (Ap)} \frac{d(Ap)}{dp} = G_1(Ap, V)A. \tag{10.13}$$

Differentiating this expression again, it follows that:

$$\frac{\partial y_1}{\partial p} = \frac{\partial}{\partial p}[G_1(Ap, V)A] = G_{11}(Ap, V)A^2 \quad , \tag{10.14a}$$

and,

$$\frac{\partial y_1}{\partial A} = \frac{\partial}{\partial A}[G_1(Ap, V)A] = G_1(Ap, V) + G_{11}(Ap, V)Ap \;. \tag{10.14b}$$

Converting these expressions into elasticity form, we readily obtain,

$$\frac{\partial y_1}{\partial A}\frac{A}{y_1} = \frac{G_1(Ap, V)A}{y_1} + \frac{G_{11}A^2 p}{y_1} = 1 + \frac{\partial y_1}{\partial p}\frac{p}{y_1}, \tag{10.15}$$

where the first equality is obtained from (10.14b), and the second from (10.13) and (10.14a).

We know that $\partial y_1 / \partial p \geq 0$, and will presume that $\partial y_1 / \partial p > 0$. It follows immediately that $(\partial y_1 / \partial A)(A / y_1) > 1$. That is, in a two-by-two economy, Hick's neutral technological progress in a good will raise its output *by more than the increase in productivity.* This result is obtained because the rise in productivity will increase factor prices in an industry, thereby attracting resources from the rest of the economy. This is illustrated in Figure 10.2, where with *unchanged* inputs industry 1 would expand by $\hat{A}$ due to the technological progress, but after attracting resources from the rest of the economy it expands by $\hat{y}_1 > \hat{A}$. Moreover, as illustrated in Figure 10.2, this will imply that *output in industry 2 must contract,* because it is losing factor inputs. Thus, there is a "Rybczynski-like" effect at work even when the source of growth is neutral technological progress in one industry, leading to a contraction of the other industry (at constant prices). This type of growth therefore satisfies condition (b) of the above theorem, making immiserizing growth possible.

The result in (10.12) that the GDP function should have price multiplied by the Hick's neutral productivity parameter in each industry was already used in chapter 3 (see equation 3.24), where we reviewed the work of Harrigan (1997). He estimated a translog GDP function over the OECD countries while allowing for technology to differ across countries. Industry prices were treated as equal across countries (due to free trade), so it was *only* the technology parameters and factor endowments that appeared in the GDP function. The result in (10.15) that the elasticity of output with respect to each industry's productivity exceeds unity then becomes a testable hypothesis: it will hold empirically provided that certain parameters of the translog GDP function satisfy appropriate conditions.[8] Thus, by estimating the GDP function we are able to test this result of Findlay and Grubert (1959).

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[8] See problem 10.2.

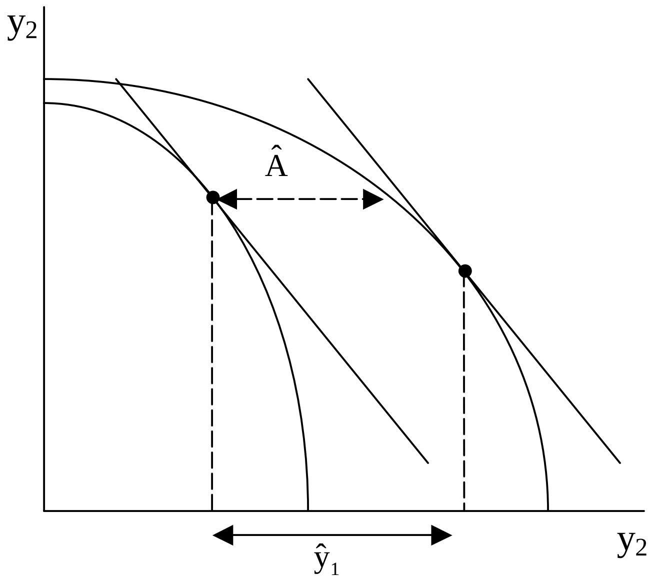

Figure 10.2

Endogenous Growth

Our discussion in the previous section did not model growth in any meaningful way, but simply treated it as a comparative statics increase in productivity or a factor endowment. Obviously, this is unsatisfactory. The celebrated model of Solow (1956) specified that capital accumulation should depend on investment, which equals to savings in the autarky one-sector economy, so that capital would gradually rise to its steady-state level. But that model still assumed that technological progress was *exogenous,* and a positive rate of technological progress or growth in the labor force was needed to obtain steady-state growth. The same assumptions were used in two-sector, open economy versions of this model, such as Smith (1977). Ultimately, interest in the profession has turned to models that instead allow for an *endogenous* rate of technological innovation, and we shall outline one of these models, taken from Grossman and Helpman (1990; 1991, chapters 3, 8 and 9).

Many of the "endogenous growth" models build upon the monopolistic competition framework we introduced in chapter 5, but rather than thinking of differentiated final products, we instead consider *differentiated intermediate inputs*. The idea is that an increase in the variety (N) of differentiated inputs will allow for an increase in output, much like an increase in variety of final goods allowed for higher consumer utility in chapter 5.[9] To make this precise, suppose that there is a single final good, with output y, produced with the CES production function:

$$y = \left[\sum_{i=1}^{N} x_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{10.16}$$

where x_i is the quantity of input variety i = 1,…,N. As usual, we assume $\sigma > 1$, so that it is

[9] See problem 5.1. The use of differentiated inputs to generate economy-wide returns to scale is often attributed to Ethier (1979).

meaningful to think of changes in the number of inputs.[10]

If inputs are all priced equally in equilibrium, then their quantities are also equal, $x_i = x$, and so (10.16) can be rewritten as,

$$y = N^{\sigma/(\sigma-1)}x = N^{1/(\sigma-1)}X \ , \tag{10.17}$$

with $X \equiv Nx$. We can think of X as the "aggregate" amount of the intermediate input, and holding this magnitude fixed, (10.17) shows that *increases in N raise output y* (since $\sigma > 1$). This amounts to a productivity gain in the final good industry, due to the proliferation of differentiated intermediate inputs. Indeed, we could re-label $N^{1/(\sigma-1)}$ on the right of (10.17) as "A" to obtain the simple production function y = AX, where technological progress A will depend on the number of new inputs.[11] By carefully modeling the dynamics of how N evolves due to the entry of new firms, we therefore have a story of endogenous technological progress, or endogenous growth.

Autarky Equilibrium

We begin by describing the autarky equilibrium. The price of the final good at each point in time is P(t). If the consumer spends E(t) on this good, then the quantity Y(t) = E(t)/P(t) will be purchased, and we assume that this provides instantaneous utility of lnY(t) = lnE(t) – lnP(t). The consumer's problem is then:

$$\max_{E(\tau)} \int_t^{\infty} e^{-\rho(\tau-t)}[\ln E(\tau) - \ln P(\tau)]d\tau, \tag{10.18}$$

[10] If $\sigma < 1$, then output is zero whenever any input is zero. Thus, all inputs are essential to production, and we cannot think of introducing new inputs. In contrast, when $\sigma > 1$, then output will increase by a finite amount when new inputs are introduced, as shown in (10.17).

[11] This is like the so-called "y = AK" production function used in some macroeconomic models.

subject to the budget constraint:

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]}E(\tau)d\tau \leq \int_t^{\infty} e^{-[R(\tau)-R(t)]}w(\tau)Ld\tau + B(t)\,, \tag{10.19}$$

where R(t) is the cumulative interest rate from time 0 to time t, so that $\dot{R}(t) \equiv dR/dt$ is the instantaneous interest rate. This budget constraint states that the discounted value of labor income w(t)L, plus initial assets B(t), cannot exceed the discounted value of expenditure. The solution to this optimization problem is,[12]

$$\frac{\dot{E}}{E} = \dot{R} - \rho\,, \tag{10.20}$$

where we omit the time index on variables when this will not cause confusion.

Condition (10.20) states that when the nominal interest rate $\dot{R}$ exceeds the consumer's discount rate then expenditure is rising. This is because with a high interest rate it is optimal to put a large amount of assets into savings initially, and gradually increase consumption thereafter. Generally, the nominal interest $\dot{R}(t)$ will depend on the normalization made on prices. Grossman and Helpman (1990, 1991) choose the normalization $E(t) \equiv 1$, which implies from (20) that $\dot{R}(t) \equiv \rho$, as we shall also use.

On the production side, the final good y is manufactured under perfect competition with the production function (10.17), where x_i is the quantity of each of N intermediate inputs (all these variables depend on t). These inputs are produced under monopolistic competition, and each unit of x_i requires one unit of labor, which is the only input. It follows that the price of each

[11] As solved for in problem 10.3.

input is,

$$p_i\left(1-\frac{1}{\sigma}\right)=w, \quad \text{or} \quad p_i = w\left(\frac{\sigma}{\sigma-1}\right), \tag{10.21}$$

where w is the wage (which depends on t).

Input-producing firms have a fixed cost only when they begin production. The key question is: how should this fixed cost be modeled? If we presume that the fixed cost is a fixed amount of labor α, as in the static monopolistic competition model, then there will be a limit to the number of products that could be profitably developed. In other words, growth of inputs N(t) will cease at some point in time. The first dynamic monopolistic competition models had this feature (Judd, 1985, Grossman and Helpman, 1989), while the work of Romer (1990) and Grossman and Helpman (1990, 1991) avoided this outcome by presuming that fixed costs are *inversely proportional* to the number of products already developed, so that fixed labor costs are $\alpha/N(t)$. The idea is that the creation of products contributes to a "public stock of knowledge," which makes the invention of new products even easier. Notice that in a two-country model, this will allow for the potential spillover of public knowledge across borders, which will turn out to be very important.

With fixed labor costs of $\alpha/N(t)$, the nominal fixed costs are $\alpha w(t)/N(t)$. These are financed by consumers purchasing equity in the firms, which provides both dividends and capital gains. The instantaneous profits of the firms are:

$$\pi_i = (p_i - w)x_i = \left(\frac{1}{\sigma-1}\right)wx, \tag{10.22}$$

using (10.21) and symmetry of the equilibrium, so that $x_i = x$. The zero profit conditions for the firms are that the present discounted value of instantaneous profits in (10.22) must equal the fixed costs $\alpha w(t)/N(t)$,

$$V(t) \equiv \int_t^{\infty} e^{-\rho(\tau - t)} \left(\frac{1}{\sigma - 1}\right) w(\tau) x(\tau) d\tau = \frac{\alpha w(t)}{N(t)}, \quad \text{for all t.} \tag{10.23}$$

Since (10.23) holds for all t, we can differentiate it with respect to t, obtaining,

$$\dot{V} = -\left(\frac{1}{\sigma - 1}\right) w(t) x(t) + \rho \int_t^{\infty} e^{-\rho(\tau - t)} \left(\frac{1}{\sigma - 1}\right) w(\tau) x(\tau) d\tau = \frac{\alpha \dot{w}}{N} - \frac{\alpha w \dot{N}}{N^2}. \tag{10.24}$$

To simplify (10.24), we can divide by the fixed costs $\alpha w/N$, obtaining,

$$\underbrace{\underbrace{\left(\frac{1}{\sigma - 1}\right)\frac{Nx}{\alpha}}_{\pi/\text{Fixed cost}}}_{\text{Dividends}} + \underbrace{\underbrace{\left(\frac{\dot{w}}{w} - \frac{\dot{N}}{N}\right)}_{\dot{V}/V}}_{\text{Capital gains}} = \underbrace{\rho}_{\text{interest rate}} . \tag{10.25}$$

This is the key arbitrage condition of the endogenous growth model with expanding input variety, as derived by Grossman and Helpman (1990, 1991). The left-hand side of (10.25) is the sum of dividends plus capital gains, which should equal the discount rate on the right, that we interpret as the real interest rate.

With (10.25) in hand we can complete the model quite easily by using the full-employment condition for the economy. This can be written as,

$$L = Nx + \left(\frac{\alpha}{N}\right)\dot{N} . \tag{10.26}$$

The left-hand side of (10.26) is the fixed endowment of labor, and the right-hand side equals the sum of labor used in production, $Nx = X$, and labor used in R&D, which is the fixed cost (α/N)

times the growth of new products. Let us define the *growth rate* of inputs as $g \equiv \dot{N}/N$. Then we can re-write (10.26) as L = X + αg, or X = L – αg. Substituting this into (10.25) we obtain,

$$\left(\frac{1}{\sigma-1}\right)\left(\frac{L-\alpha g}{\alpha}\right)+\left(\frac{\dot{w}}{w}-g\right)=\rho. \tag{10.27}$$

Now consider a steady-state solution with $\dot{w}=0$. Substituting this into (10.27), we can readily solve for the autarky growth rate as,

$$g^a = \left(\frac{1}{\sigma}\right)\left[\frac{L}{\alpha}-(\sigma-1)\rho\right]. \tag{10.28}$$

Thus, the economy achieves steady-state growth even with a fixed labor supply, provided that $(L/\alpha) > (\sigma-1)\rho$. Notice that with the growth rate fixed at g^a, the labor devoted to production is also fixed at $X = L - \alpha g^a$. Then from (10.17) we can solve for the growth rate of GDP as $\dot{y}/y = g^a/(\sigma-1)$. So along with the continual growth of new products, there is also continual growth of GDP and utility.[13]

Trade Equilibrium with Knowledge Spillovers

Let us compare the autarky growth rate in (10.28) with that achieved under free trade between the two countries. Suppose that the foreign country has the same production function as at home and differs only in its labor endowment L*. We will suppose that the final goods produced in the two countries are imperfect substitutes, with the instantaneous utility function in each country,

[13] While the economy achieves a continual growth of utility, it is not actually the case that welfare is maximized. That would require a choice of the growth rate different from g^a in (10.28). See problem 10.4.

$$U(y, y^*) = \ln\left[y^{(\eta-1)/\eta} + (y^*)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}. \tag{10.29}$$

This is the log of a CES function with elasticity of substitution $\eta > 1$. We suppose that the final goods from home and abroad are freely traded, at the prices p_y and p_y^*, respectively.

Given expenditure E in the home country, it follows that the demand for the final goods of each country is,[14]

$$y = (p_y/P)^{-\eta}\,(E/P)\,,\quad \text{and}\quad y^* = (p_y^*/P)^{-\eta}\,(E/P)\;, \tag{10.30}$$

where P refers to the overall price index of the final goods, defined as:

$$P(p_y, p_y^*) = \left[p_y^{1-\eta} + (p_y^*)^{1-\eta}\right]^{1/(1-\eta)}. \tag{10.31}$$

Demand for the final goods from the foreign county is similar, and is obtained by just replacing home expenditure E by foreign expenditure E* in (10.30)

Using (10.30), it is readily confirmed that utility in (10.29) equals U(y,y*) = ln(E/P), or the log of real expenditures. We will suppose that the capital markets are fully integrated so there is a single, cumulative world interest rate R(t). Maximizing the presented discounted sum of home utility gives us to the same objective function as in (10.18), so the first-order condition (10.20) still applies. If we again use the normalization $E(t) \equiv 1$, we will have that the real interest rate equals ρ, the discount rate in both countries.

With this setup, we want to contrast the effects of trade in two scenarios. In the first, we will suppose that in addition to free trade in the final goods and integrated capital markets, there is *also* free trade in the intermediate inputs and complete transfer of knowledge across countries.

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[14] See (5.24) and (5.25), and also problem 5.2 in chapter 5.

The latter assumption means that the fixed costs of creating a new product in either country is $\alpha/[N(t)+N^*(t)]$. Under this set of assumptions, the integrated world equilibrium with the two countries is simply a "blown up" version of either country in autarky. The effect of this increase in size is to *raise* the growth rate from that shown in (10.28) to:

$$g^w = \left(\frac{1}{\sigma}\right)\left[\frac{(L+L^*)}{\alpha} - (\sigma-1)\rho\right] . \tag{10.32}$$

The result that the growth rate increases in proportion to the size of the world economy is referred to as a "scale effect," and is the dynamic analogue to the static gains from trade that we saw in Krugman's model in the beginning of chapter 5. This result has come under some criticism in the growth literature, not so much because of the result that trade increases growth, but rather, because of another result that subsidies to R&D permanently raise the growth rate. Both of these propositions depend very strongly on our specification that fixed costs are *inversely proportional* to the number of products already developed. If this strict inversely proportionality does not hold, e.g. if we specified fixed costs as α/N^{β}, with $0 < \beta < 1$, then new product development in the absence of population growth will eventually stop. Jones (1995a,b) has referred to models of this type as "semi-endogenous" growth, and they imply that free trade or subsidies to R&D have only temporary effects on the growth rate. It follows that there is an empirical question as to which class of models is most realistic, as we shall discuss later.

Trade Equilibrium without Knowledge Spillovers

Let us now contrast the results from a fully integrated world economy to the case where there are *zero* spillovers of knowledge across borders, as in Grossman and Helpman (1991, chapter 8) and Feenstra (1996). This means that the fixed costs in each country are $\alpha/N(t)$ and

α/N*(t), respectively. Initially, we suppose that there is no trade in intermediate inputs, so that only trade in final goods and financial capital are permitted; later in the section, we will re-introduce free trade in intermediate inputs. We shall demonstrate that in the absence of knowledge spillovers across borders, free trade *will not* increase the growth rate for both countries, but rather, will slow down product development in the smaller country.

With the countries growing at different rates, we cannot simultaneously use the normalizations E(t) ≡ 1 and E*(t) ≡ 1, so it is no longer the case that $\dot{R}(t) = \rho$. Accordingly, the profits of input-producing firms in (10.23) are discounted using the cumulative interest rate R(t), and the arbitrage condition in each country is re-written from (10.25) as,

$$\left(\frac{1}{\sigma-1}\right)\frac{Nx}{\alpha} + \left(\frac{\dot{w}}{w} - \frac{\dot{N}}{N}\right) = \dot{R} \quad , \tag{10.33}$$

and,

$$\left(\frac{1}{\sigma-1}\right)\frac{N^*x^*}{\alpha} + \left(\frac{\dot{w}^*}{w^*} - \frac{\dot{N}^*}{N^*}\right) = \dot{R} \quad . \tag{10.34}$$

As before, we will use the notation X ≡ Nx and X* ≡ N*x* to denote the amount of labor devoted to production of intermediate inputs in each country. In addition, let $g \equiv (\dot{N}/N)$ and $g^* \equiv (\dot{N}^*/N^*)$ denote the home and foreign growth rates of new inputs. The full-employment condition in each country implies that X = L – αg and X* = L* – αg*, and substituting these into (10.33) and (10.34) , we can derive the growth rates:

$$g = \left(\frac{L}{\alpha\sigma}\right) - \left(\frac{\sigma-1}{\sigma}\right)\left[\dot{R} - \left(\frac{\dot{w}}{w}\right)\right], \quad \text{and,} \quad g^* = \left(\frac{L^*}{\alpha\sigma}\right) - \left(\frac{\sigma-1}{\sigma}\right)\left[\dot{R} - \left(\frac{\dot{w}^*}{w^*}\right)\right] . \tag{10.35}$$

These equations show that the growth rates are *inversely related* to the real interest rates $[\dot{R}-(\dot{w}/w)]$ and $[\dot{R}-(\dot{w}^*/w^*)]$: having a higher nominal interest rate $\dot{R}$ lowers the discounted value of profits and therefore expenditure on R&D; conversely, having a rising path of wages and prices leads to rising profits and a lower real interest rate, with higher R&D. Taking the difference between the two growth rates in (10.35), we obtain,

$$g-g^*=\left(\frac{L-L^*}{\alpha\sigma}\right)+\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\dot{w}}{w}-\frac{\dot{w}^*}{w^*}\right). \tag{10.36}$$

Let us assume henceforth that the home country is larger, L > L*. From (10.36) we see that this difference in size will tend to be associated with a faster growth rate at home, g > g*. The growth rates are also affected, however, by the change in wages on the right. In the autarky equilibrium discussed above, we had focused on the steady-state solution with $\dot{w}=0$. But now it is impossible to assume this for both countries: assuming $\dot{w}=\dot{w}^*=0$ implies that $g>g^*$ from (10.36), and we will argue below that this implies rising relative wages for the home country, $\dot{w}/w>\dot{w}^*/w^*$, which is a contradiction. Thus, the assumption of a steady-state equilibrium is inconsistent with the dynamic equations for the two countries. Accordingly, we need to solve for the growth rates outside of the steady state.

We begin by solving for the change in wages, from the production function $y=N^{1/(\sigma-1)}X$ in (10.17), where X is the amount of intermediate inputs purchased at the price of $p_i = w\sigma/(\sigma-1)$. It follows that the marginal cost of producing the final good is $p_y=p_i/N^{1/(\sigma-1)}=w\sigma/[(\sigma-1)N^{1/(\sigma-1)}]$ at home, and $p_y^*=w^*\sigma/[(\sigma-1)(N^*)^{1/(\sigma-1)}]$ abroad. Differentiating these and taking the difference, we obtain,

$$\left(\frac{\dot{p}_y}{\dot{p}_y}\right) - \left(\frac{\dot{p}_y^*}{p_y^*}\right) = \left(\frac{\dot{w}}{w} - \frac{\dot{w}^*}{w^*}\right) - \left(\frac{g - g^*}{\sigma - 1}\right). \tag{10.37}$$

This equation states that a higher growth rate at home will tend to be associated with rising relative wages, but this also depends on the changes in the prices of the final goods in each country.

To determine these prices, we make use of the CES demands in (10.30), which imply that $(p_y/p_y^*) = (y/y^*)^{-1/\eta}$. Differentiating this using the production functions $y = N^{1/(\sigma-1)}X$ and $y^* = (N^*)^{1/(\sigma-1)}X^*$, we obtain,

$$\left(\frac{\dot{p}_y}{\dot{p}_y}\right) - \left(\frac{\dot{p}_y^*}{p_y^*}\right) = -\frac{1}{\eta}\left[\left(\frac{g - g^*}{\sigma - 1}\right) + \frac{\dot{X}}{X} - \frac{\dot{X}^*}{X^*}\right]. \tag{10.38}$$

Then combining (10.36) – (10.38), we can derive the fundamental relation,

$$g - g^* = \Delta\left(\frac{L - L^*}{\alpha\sigma}\right) - \Delta\left(\frac{\sigma - 1}{\eta\sigma}\right)\left(\frac{\dot{X}}{X} - \frac{\dot{X}^*}{X^*}\right), \tag{10.39}$$

where,

$$\Delta \equiv \left[1 - \frac{(\eta - 1)}{\eta\sigma}\right]^{-1} > 1. \tag{10.40}$$

Equation (10.39) shows that the difference in the growth of new inputs across the countries depends on the difference in their size. There is also another term on the right of (10.39), depending on $\dot{X}$ and $\dot{X}^*$, but we can safely presume that this term approaches zero as $t \to \infty$. Notice that the difference in the *autarky* growth rates of the two countries is exactly

$(L - L^*)/\alpha\sigma$, but in (10.39), the limiting growth rates of the countries will differ by this *times* the amount $\Delta>1$. Therefore, in the absence of knowledge spillovers the effect of trade is to *magnify* the initial difference in the growth rates of the two countries.

We have not yet determined whether trade will increase the growth rate of the large country, or slow down the growth rate of the small country. Some additional calculations show that the first case is not possible: the growth rate of the large country must approach its autarky rate of g^a as $t \to \infty$.[15] Our results are then summarized by:

Theorem (Feenstra, 1996)

(a) $\lim_{t\to\infty} g = g^a$, which equals the autarky growth rate g^a of the large country;

(b) $\lim_{t\to\infty} g^* = g^{a*} - (\Delta - 1)\left(\frac{L - L^*}{\alpha\sigma}\right)$, which is less than the autarky growth rate g^{a*} of the small country, $g^{a*} = [(L^*/\alpha) - (\sigma - 1)\rho]/\sigma$;

(c) $\lim_{t\to\infty}\left(\frac{\dot{w}}{w} - \frac{\dot{w}^*}{w^*}\right) > 0$, and $\lim_{t\to\infty}\left(\frac{\dot{p}_y}{p_y} - \frac{\dot{p}_y^*}{p_y^*}\right) < 0$.

Parts (a) and (b) of this theorem have already been discussed above. The results in (c) follow from substituting the limiting growth rates into (10.37) and (10.38). Relative wages move in favor of the larger country, which reflects the rising productivity of that country due to the faster-growing number of intermediate inputs. But this same efficiency gain leads to a fall in the relative price of final goods. These different wage and price movements emphasize how difficult

[15] See problem 10.5.

it is to make a general prediction of the terms of trade movement in this model: it depends on which goods/factors are used to measure the terms of trade.

The finding that the small country has its rate of product development slowed down does not necessarily translate into a welfare loss, though it might. That will depend on whether intermediate inputs are traded or not. Initially continue with our assumption that inputs are not traded. The welfare obtained by consumers in the small country will depend, of course, on the price index $P(p_y, p_y^*)$ for purchases of the final goods. Let us choose the foreign wage as the numeraire, $w^* \equiv 1$. Then since the number of intermediate inputs produced abroad is slowed down by trade, the price of its final good p_y^* is higher than in autarky. This will create a potential welfare loss. But the foreign consumers *also* have the final good from the home country available, which creates a potential welfare gain. Which of these effects dominate will depend on a comparison of elasticities: if $\eta > (\sigma+1)$, then there is an overall welfare gain; but if $\eta \leq (\sigma+1)$, then there is a potential welfare loss.[16]

What about when the intermediate inputs are traded? Then the argument is quite different. Despite the fact that product development has been slowed in the small country, it can still purchase intermediate inputs from abroad. For t sufficiently large, it will definitely be the case that the efficiency gain from the additional imported inputs more than offsets the loss from the reduced number of local inputs, so that the price of the final good p_y^* is *lower* than in autarky. Thus, consumers in the small country benefit from a reduction in the price of their own

[16] Specifically, Feenstra (1996, Proposition 2) finds that the price index under free trade exceeds that under autarky for t sufficiently high when $\eta \leq (\sigma+1)$, and a weak additional condition is satisfied. The finding that welfare in the smaller country can potentially fall is related to the fact that R&D is being undertaken at less than the socially optimal rate in autarky, as demonstrated in problem 10.4. Then trade in final goods but not in intermediate inputs worsens that distortion in that country. Markusen (1989) also discusses how trade in final goods, but not in intermediate inputs, can lead to a welfare loss even in a static model.

final good as well as the imported final good, so there are welfare gains for both reasons.

It is worth emphasizing that our finding that trade can slow down the rate of product development occurs only when there are *zero* spillovers of knowledge. Provided that there are some positive spillovers, then Grossman and Helpman (1991, chapter 9) and Rivera-Batiz and Romer (1991a,b) argue that product innovation in both countries will increase. Indeed, this occurs even if knowledge spillovers are the *only* international flows, without any trade in final goods or inputs. So the assumption of zero international spillovers we have used above is an extreme case, but it serves to highlight how important these spillovers are to growth.

Empirical Evidence

Importance of Size

The first implication of the endogenous growth models is that "size matters" for growth. This follows directly from the equation for autarky growth rates in (10.28), where a higher labor stock (measured in units of effective R&D workers, or L/α) leads to a higher growth rate. This implication is analogous to what we found in Krugman's static model reviewed at the beginning of chapter 5: doubling a country's size would lead to efficiency gains through economies of scale, and additional consumer gains through increased variety. In the dynamic model outlined above, increasing a country's size leads to increased variety of intermediate inputs, which results in efficiency gains in producing the final good and also reduces the fixed costs $\alpha/N(t)$ of inventing new inputs. The latter reduction in fixed costs is what leads to a permanently higher growth rate due to larger country size.

How realistic is this result? Over very long time periods, it is perhaps reasonable to think that economies will growth in proportion to their size. Kremer (1993) considers the period from one million B.C. to the present, and finds that the growth rate of population is proportional to its level. On the other hand, over shorter time spans, the implications of the "scale effect" in the endogenous growth model do not find support. Jones (1995a,b) proposes a direct test of endogenous growth, whereby changes in policy should have permanent effects on the growth rate, and this hypothesis is decisively rejected on data for the U.S. and other advanced countries. As we discussed earlier, these negative results led Jones to propose the "semi-endogenous" growth model, and there is now a class of growth models that operate *without* the scale effect.[17]

International Trade and Convergence

The second implication of the endogenous growth model was that, with international spillovers of knowledge, trade should increase growth rates. There is an active debate over whether this hypothesis holds empirically. Advocates of this view includes Dollar (1992), Sachs and Warner (1995), Edwards (1998), Ben-David (1993, 1996, 2001), and Frankel and Romer (1999). But these empirical results are all dismissed by Rodriguez and Rodrik (2000), and more specific criticisms on individual papers are made by Harrison (1996) and Slaughter (2001). In order to evaluate these papers, it is useful to first relate them to another line of empirical research dealing with the *convergence* of countries to their steady-state growth rates.

The convergence literature is not motivated by the endogenous growth models at all, but rather, by the *exogenous* growth model such as Solow (1956) and extensions thereof. In those models, all countries converge to the same steady-state growth rate, but this still allows for different levels of GDP per-capita (depending on characteristics of the countries). So countries that are below their steady-state level of GDP per-capita should grow faster, and conversely, countries that are above their steady-state GDP per-capita should grow slower, so as to approach

[17] See Dinopoulos and Segerstrom (1999a) and Dinopoulos and Syropoulos (2001) for applications to trade.

the steady state. This property follows directly from the diminishing marginal product of capital. There is both convergence in growth rates, and some degree of convergence in GDP per-capita (though not necessarily to the same level across countries), and these two criterion tend to be used interchangably. The same is true for the papers dealing with trade, which examine either the impact on growth rates or on GDP per-capita across countries.

From the initial work of Barro (1991) and Barro and Salai-i-Martin (1991, 1992) the convergence hypothesis found strong empirical support. This was also demonstrated by Mankiw, Romer and Weil (1992), and led them to conclude that the original Solow model, suitably extended to allow for human capital, is "good enough" at explaining cross-country differences in growth rates: the endogenous growth models are apparently not needed! These findings have been questioned in subsequent work, however, that consider a broader range of countries. For example, Easterly and Levine (2000) emphasize that there has been *divergence* in the absolute levels of income per-capita across countries: the rich have grown richer and some of the poorest countries have become even poorer.[18]

A similar approach to assessing the effects of trade is taken by the authors cited above. Sachs and Warner (1995) group their countries into two groups, one of which has an "open" trade regime and the other of which is judged to be "closed." Within the open group, they find evidence of convergence, with the poorer countries growing faster, but this is not true for those countries with closed trade regimes. By this argument, openness leads to convergence of incomes across countries. Ben-David (1993, 1996, 1998) does a grouping of countries based on their membership in regional trade agreements, or by the strength of their bilateral trade ties. He also finds significant evidence of income convergence *within* these groups, but not necessarily

[18] Bernanke and Gürkaynak (2001) also question the conclusions of Mankiw, Romer and Weil.

across the groups. Dollar and Kraay (2001a,b) contrast the experience of a group of developing countries that have become more open (including China and India) with other developing countries. They argue that growth in the former group is explained by openness and has particularly benefited the poor. As noted above, all these papers are subject to some empirical criticisms: among other difficulties, openness itself is fundamentally endogenous. Frankel and Romer (1999) deal with this endogeneity by first estimating a gravity equation, and then demonstrating that the *predicted* trade shares from this equation are indeed significant in explaining income per-capita.

The finding that convergence occurs within groups, but not across groups, presents a challenge to the exogenous growth models. But this finding is in the spirit of the endogenous growth model without knowledge spillovers, discussed above, where the two countries converged to different growth rates. In a more general multi-country growth model that incorporates trade, Ventura (1997) also finds this result. In his model, capital *does not* have a diminishing marginal product due to Rybczynski effects: increases in capital raises the output of the capital-intensive good, and at fixed product prices, does not result in a reduced rental. Nevertheless, Ventura shows that in the global equilibrium, there is convergence to steady-state growth rates that differ across countries. The reason for convergence is that countries with expanding capital will have falling equilibrium prices for their capital-intensive goods, and therefore, a falling rental due to the Stolper-Samuelson theorem. So there is a general-equilibrium linkage between capital accumulation and the rental, even though there are no diminishing returns with fixed prices.

We might think of the model in Ventura (1997) as offering a third type of growth model, in which there is neither dimishing returns to capital as in the Solow model; nor endogenous growth due to increased input variety; but growth *without* dimishing returns due to Rybczynski effects. Findlay (1996) argues that this framework may well be the most relevant one to analyze the growth experience of developing countries, and these articles are recommended for further reading.

Spillovers of Knowledge

A third area of empirical investigation is international spillovers of knowledge. It is clear from our discussion above that these are crucial: gains from trade are much more likely when knowledge flows across borders. Coe and Helpman (1995) test for the presence of such spillovers using data for the OECD countries.[19] Their hypothesis is that if international spillovers occur, then the growth rates of countries should be correlated with both their *own* R&D expenditures and also the R&D expenditures of their trading partners. Thus, they construct measures of each country's R&D expenditures, and weight these using bilateral *import* shares to obtain an estimate of partner countries' R&D expenditures. They find that TFP growth rates at the country level are indeed correlated with own and partner-country R&D expenditures, lending support to the idea that research and development carried out in one country "spills over" to its trading partners.

An econometric difficulty with the Coe and Helpman estimates is that the TFP growth rates and R&D expenditures are very likely to exhibit unit-roots in their time series behavior. In that case, regressions of TFP on R&D could lead to estimates that are significantly different from zero using conventional t-tests, even though the relationship might be spurious. To investigate this, Keller (1998) re-constructs the weighted R&D expenditures of the trading partners used by

[19] Coe, Helpman and Hoffmaister (1997) expand the sample to include developing countries.

Coe and Helpman, but using *random* rather than actual import weights. He shows that this measure still has a positive correlation with TFP, suggesting that the relationship has little to do with trade, and may be spurious. The unit-roots problem can be corrected using recent panel co-integration techniques, as done by Funk (2001). He finds that there is no significant relationship between TFP growth and *import-weighted* R&D expenditures of partner countries. However, he does find a significant relationship between TFP and *export-weighted* R&D expenditures of partner countries, supporting the idea that exporters learn from their customers.

Keller (2002) has extended the results of Coe and Helpman using sectoral rather than aggregate country data. He again constructs a weighted average of R&D expenditures in other countries, but now uses the *distance* to each partner as the weight. Denoting the distance from country i to j by d^{ij}, Keller weights the other countries' R&D expenditures by $e^{-\delta d^{ij}}$. The parameter δ measures how quickly the impact of other countries' R&D expenditures on TFP diminishes with distance. In fact, this parameter turns out to be positive and highly significant, implying that spillovers are quite highly localized. For the two closest countries in his sample, Germany and the Netherlands, one dollar of R&D expenditure in Germany has 31% of the impact on Dutch productivity as does one dollar R&D expenditure in the Netherlands. The effects of foreign R&D are lower for all other countries; for example, the relative effect of American R&D on Canada is only 4% of the value of Canadian R&D on its own productivity. So Keller's results provide evidence of R&D spillovers across borders that are precisely estimated, but small.

Another way that spillovers of knowledge have been assessed is through the use of patent data. There is a large literature that applies this technique to firms within a country, and Branstetter (2001) has extended it to consider firms in both the U.S. and Japan. When firms apply for a patent in the U.S., they must classify it in one or more product areas. Treat each product area as a component of the vector $B_i = (b_{i1},\ldots,b_{iK})$, where b_{ik} is the number of patents taken out by firm i in product area k. For two firms i and j, we can use the correlation between B_i and B_j as a measure of their *proximity* in technology space. Then we can construct a weighted average of the R&D expenditures of other firms, using the proximity with firm i as the weights. This gives us a measure of the *potential* spillovers from other firms. Branstetter constructs these measures separately for spillovers from U.S. firms, and spillovers from Japanese firms.[20] Together with each firm's own R&D expenditures, these are regressed on a measure of each firm's performance (either number of patents taken out annually, or TFP growth), to determine the importance of the various spillovers.

In his results, Branstetter (2001, p. 75) finds that spillovers are primarily *intra-national* in scope. There are some regressions where Japanese firms benefit from the weighted R&D activity of American companies, but this effect does not remain significant when the domestic spillovers are included in the regression. Conversely, there is no evidence that American firms benefit from the R&D activities of the Japanese. This raises the idea that knowledge spillovers may be *asymmetric,* and presumably flow from the most technologically advanced country outwards. In other work, Branstetter (2000) has explored whether Japanese firms that have foreign affiliates in the U.S. act as a "conduit" for knowledge flows, i.e. whether there is a greater spillover between these foreign affiliates and other American firms. It appears that this is indeed the case, so that foreign investment increases the flow of knowledge spillovers both to and from the investing Japanese firms.

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[20] Both measures are constructed using the patents that Japanese and American firms take out in the U.S.

Measuring Product Variety

All of the empirical applications we have discussed so far have been *indirect* tests of the endogenous growth model, relying on hypotheses arising from it but not actually measuring the variety of inputs. Our fourth and final empirical method will be to develop a *direct* test of endogenous growth, by constructing a measure of input variety that is "exact" for the CES production function, as in Feenstra (1994) and Feenstra and Markusen (1994). We show that this measure of product variety is correlated with productivity growth.

Let us begin with the CES production function in (10.16). The problem with using this function is that it is symmetric*:* at equal prices, every input would have the same demand. That assumption is made for convenience in our theoretical models, but is unacceptable empirically: we need to let demand be whatever the data indicates. So instead we will work with the non-symmetric CES function,

$$y_t = f(x_t, I_t) = \left[\sum_{i \in I_t} a_i x_{it}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \ \sigma > 1, \tag{10.41}$$

where $a_i > 0$ are parameters and I_t denotes the set of inputs available in period t, at the prices p_{it}. The CES unit-cost function dual to (10.41) is,[21]

$$c(p_t, I_t) = \left[\sum_{i \in I_t} b_i p_{it}^{1-\sigma} \right]^{1/(1-\sigma)}, \ \sigma > 1, \ b_i \equiv a_i^{\sigma}. \tag{10.42}$$

We are interested in determining how much unit-costs are reduced when the set of product varieties expands. To this end, let us first consider the case where $I_{t-1} = I_t = I$, so there is

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[21] This is analogous to the CES price index in (5.25), and see also problem 8.3 of chapter 8 to derive the CES demand system in the non-symmetric case.

no change in the set of goods. Let us also assume that the observed input purchases x_{it} are cost minimizing for the prices and output, that is, $x_{it} = y_t(\partial c/\partial p_{it})$. In that case, the ratio of unit-costs can be measured by the price index due to Sato (1976) and Vartia (1976):[22]

Theorem (Sato, 1976; Vartia, 1976)

If the set of inputs available is fixed at $I_{t-1} = I_t = I$ and inputs are cost-minimizing, then:

$$\frac{c(p_t, I)}{c(p_{t-1}, I)} = P_{SV}(p_{t-1}, p_t, x_{t-1}, x_t, I) \equiv \prod_{i \in I} \left(\frac{p_{it}}{p_{it-1}} \right)^{w_i(I)}, \tag{10.43}$$

where the weights $w_i(I)$ are constructed from the expenditure shares $s_{it}(I) \equiv p_{it}x_{it} / \sum_{i \in I} p_{it}x_{it}$ as,

$$w_i(I) \equiv \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right) \Bigg/ \sum_{i \in I} \left(\frac{s_{it}(I) - s_{it-1}(I)}{\ln s_{it}(I) - \ln s_{it-1}(I)} \right) . \tag{10.44}$$

To interpret this result, the numerator on the right of (10.44) is a logarithmic mean of the expenditure shares $s_{it}(I)$ and $s_{it-1}(I)$, and lies between these two values. The denominator ensures that the weights $w_i(I)$ sum to unity, so that the Sato-Vartia index P_{SV} defined on the right of (10.43) is simply a geometric mean of the price ratios (p_{it}/p_{it-1}). The theorem states that this index *exactly* equals the ratio of the CES unit-cost functions, provided that the observed input quantities used to construct the weight are cost minimizing.

Now consider the case where the set of inputs is changing over time, but some of the inputs are available in both periods, so that $I_{t-1} \cap I_t \neq \varnothing$. We again let c(p,I) denote the unit-

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[22] The proof of this theorem and the next can be found in Feenstra (1994).

cost function defined over the inputs within the set I. Then the ratio $c(p_t,I)/c(p_{t-1},I)$ is still measured by the Sato-Vartia index in the above theorem. Our interest is in the ratio $c(p_t,I_t)/c(p_{t-1},I_{t-1})$, which can be measured as follows:

Theorem (Feenstra, 1994)

Assume that $I = I_{t-1} \cap I_t \neq \varnothing$, and that the inputs are cost-minimizing. Then for $\sigma > 1$:

$$\frac{c(p_t, I_t)}{c(p_{t-1}, I_{t-1})} = \left(\frac{\lambda_t(I)}{\lambda_{t-1}(I)}\right)^{1/(\sigma-1)} \prod_{i \in I} \left(\frac{p_{it}}{p_{it-1}}\right)^{w_i(I)} \tag{10.45}$$

where the weights $w_i(I)$ are constructed from the expenditure shares $s_{it}(I) \equiv p_{it}x_{it} / \sum_{i \in I} p_{it}x_{it}$ as in (10.44), and the values $\lambda_t(I)$ and $\lambda_{t-1}(I)$ are constructed as:

$$\lambda_\tau(I) = \left(\frac{\sum_{i \in I} p_{i\tau}x_{i\tau}}{\sum_{i \in I_\tau} p_{i\tau}x_{i\tau}}\right) = 1 - \left(\frac{\sum_{i \in I_\tau, i \notin I} p_{i\tau}x_{i\tau}}{\sum_{i \in I_\tau} p_{i\tau}x_{i\tau}}\right), \quad \tau = t\text{-}1, t. \tag{10.46}$$

To interpret this result, the product on the far right of (10.45) is simply the Sato-Vartia index, constructed over the set of inputs I that are common to both periods. This measures the ratio of unit-costs $c(p_t,I)/c(p_{t-1},I)$, for the inputs available in both periods. The first ratio of the right of (10.45) shows how the Sato-Vartia index must be adjusted to account for the new inputs (in the set I_t but not I) or disappearing inputs (in the set I_{t-1} but not I). From (10.46), each of the terms $\lambda_\tau(I) \leq 1$ can be interpreted as the *period τ expenditure on the inputs in the set I, relative to the period τ total expenditure.* Alternatively, this can be interpreted as *one minus the period τ expenditure on "new" inputs (not in the set I), relative to the period τ total expenditure.* When there is a greater number of new inputs in period t, this will tend to lower the value of $\lambda_t(I)$. Notice that the ratio $[\lambda_t(I)/ \lambda_{t-1}(I)]$ on the right of (10.45) is raised to the power $1/(\sigma\text{-}1) > 0$, so that a lower value of $\lambda_t(I)$ due to new inputs will reduce the unit-cost ratio in (10.45) by more when the elasticity of substitution is lower.

To see the usefulness of this theorem, let us measure "dual" factor productivity as the log difference between the index of input prices and the ratio of unit-costs:

$$\mathrm{TFP} \equiv \ln P_{SV}(p_{t-1},p_t,x_{t-1},x_t,I) - \ln[c(p_t,I_t)/c(p_{t-1},I_{t-1})]. \tag{10.47}$$

Then using the above theorems, we immediately have:

$$\mathrm{TFP} = \frac{1}{(\sigma-1)} \ln\left(\frac{\lambda_{t-1}(I)}{\lambda_t(I)}\right), \tag{10.48}$$

where the terms $\lambda_t(I)$ are defined in (10.46). Thus, the growth in new inputs, as reflected in a falling value of $\lambda_t(I)$ will be directly reflected in total factor productivity of the firm or industry using the inputs. This provides us with a direct test of the endogenous growth model with expanding input variety.

Feenstra et al (1999) provide an application of this method to industry productivity growth in South Korea and Taiwan. The data used to measure product variety are the disaggregate *exports* from these countries to the United States. While is would be preferable to use national data on the *production of intermediate inputs* from these countries, this data is not available at a sufficiently disaggregate level. Despite the limitations of using *exports* to measure product variety, it has the incidental benefit of focusing on the link between trade and growth.

These authors analyze the relationship between changes in export variety and the growth in TFP across South Korea and Taiwan, in sixteen sectors over 1975-1991. The results lend support to the endogenous growth model. They find that changes in relative export variety (entered as either a lag or a lead) have a positive and significant effect on TFP in nine of the sixteen sectors. Seven of the sectors are classified as secondary industries, in that they rely on as well as produce differentiated manufactures, and therefore seem to fit the idea of endogenous growth. Among the primary industries, which rely more heavily on natural resources, the authors find mixed evidence: the correlation between export variety and productivity can be positive, negative, or insignificant. In addition, the authors also find evidence of a positive and significant correlation between *upstream* export variety and productivity in six sectors, five of which are secondary industries.

Funke and Ruhwedel (2001) have applied the same measure of product variety to analyze economic growth across the OECD countries. Using a panel dataset of 19 countries over 1989-1996, they find that a country's export variety relative to the U.S. is a significant determinant of its GDP per-capita. Notice that these measures of product variety, which are constructed from highly disaggregate trade data, are unlikely to suffer from the endogeneity problem that plagues aggregate trade flows (as addressed by Frankel and Romer, 1999). Therefore, the construction of product variety indexes and their correlation with TFP offers an alternative way to assess the importance of trade in economic growth.

Conclusions

This chapter began with a discussion of productivity growth for a firm or industry. The measure of total factor productivity (TFP) due to Solow (1957) is the growth of output minus a weighted average of the growth of inputs. The weights equal the elasticity of output with respect to the inputs. Under perfect competition and constant returns to scale, these weights are easily measured by the revenue-shares of the inputs. Without these assumptions, however, the measurement of productivity growth becomes more difficult. Using the techniques of Hall (1988), we showed how "true" productivity growth can be identified, along with estimating the price-cost margins and returns to scale in an industry. This technique has been applied by Levinsohn (1993) and Harrison (1994) to measure the reduction in markups following liberalization in Turkey and the Ivory Coast, respectively.

Dropping the assumptions of perfect competition and constant returns to scale is also the starting point of the literature on endogenous growth. While there are several types of endogenous growth models, the one we considered in this chapter relied on expanding varieties of intermediate inputs. An essential assumption was that the fixed cost of inventing a new input is *inversely proportional* to the number of inputs already created. In the case where free trade in goods and inputs also brings international spillovers of knowledge, then trade leads to an increase in the growth rates of both countries (Grossman and Helpman, 1990, 1991). But if there are no knowledge spillovers across borders, then free trade in goods has the effect of slowing down product development in the smaller country, which may also bring losses due to trade (Feenstra, 1996). This leads to several important empirical questions: (i) does free trade increase growth rates; (ii) are knowledge spillovers global or local; (iii) is there any direct evidence on the link between product variety and productivity? There is recent empirical work on all these questions, but still some unanswered questions.

In particular, it seems that the differences in per-capita GDP across countries, which are as large as ever, must be due to *technology differences* across countries (Easterly and Levine, 2000). This brings us right back to chapter 2, where we found that the HO model can account for trade patterns only if it is extended to incorporate such technology differences. What is the source of these? The endogenous growth model in this chapter has economy-wide increasing returns to scale, whereby larger countries create more inputs and are thereby more productive. It seems to me that we have not yet related this potential explanation for country productivity differences to the implied productivity differences arising from the HO model. Could it be the case that large countries (measured inclusive of proximity to neighbors) are the most productive due to their input variety, and that this productivity also accounts for their trade patterns? Addressing this question is one area for further research.

Problems

10.1 In the two-good, two-factor model with *constant* prices, suppose that good 1 experiences Hicks-neutral technological progress, i.e. $y_1 = A_1 f_1(L_1, K_1)$, with A_1 rising. Also assume that good 1 is labor intensive. Then write down the zero-profit and full-employment conditions for the economy, and use the Jone's algebra to show the following:

(a) $\hat{w} > \hat{A}_1 > 0 > \hat{r}$

(b) $\hat{y}_1 > \hat{A}_1 > 0 > \hat{y}_2$

Note: Part (b) is rather tricky, so you may not get all of it. If you understand why it is tricky, that is good enough.

10.2 Let us adopt a translog functional form for the GDP function in (10.12), extended to include many outputs:

$$\ln G = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln(A_i p_i) + \sum_{k=1}^{M} \beta_k \ln V_k + \tfrac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln(A_i p_i) \ln(A_j p_j)$$
$$+ \tfrac{1}{2} \sum_{k=1}^{M} \sum_{\ell=1}^{M} \delta_{k\ell} \ln V_k \ln V_\ell + \sum_{i=1}^{N} \sum_{k=1}^{M} \phi_{ik} \ln(A_i p_i) \ln V_k \ .$$

Differentiating this with respect to $\ln p_i$, we obtain the output share equations,

$$s_i = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln(A_j p_j) + \sum_{k=1}^{M} \phi_{ik} \ln V_k, \quad i=1,\ldots,N.$$

where $s_i = p_i y_i / G$ is the share of each output in GDP.

(a) Write the quantity of each output as $\ln y_i = \ln(s_i G / p_i)$. Differentiate this with respect to $\ln p_i$ and obtain an expression for the output elasticity $(\partial \ln y_i / \partial \ln p_i)$. What restriction on the translog parameters must hold for this elasticity to be positive?

(b) Write the quantity of each output as $\ln y_i = \ln(s_i G / p_i)$. Differentiate this with respect to $\ln A_i$ and obtain an expression for the "Rybczynski-like" elasticity $(\partial \ln y_i / \partial \ln A_i)$. If the restrictions in (a) are satisfied, then what can we say about the magnitude of this elasticity?

10.3 Consider the problem of maximizing (10.18) subject to (10.19), where for convenience we set t = 0. Write this as the Lagrangian,

$$\int_0^\infty e^{-\rho\tau}[\ln E(\tau) - \ln P(\tau)]d\tau + \lambda\left\{\int_0^\infty e^{-R(\tau)} w(\tau)L d\tau + A(t) - \int_0^\infty e^{-R(\tau)} E(\tau)d\tau\right\}$$

$$= \int_t^\infty \left\{e^{-\rho\tau}[\ln E(\tau) - \ln P(\tau)]d\tau + e^{-R(\tau)}\lambda[w(\tau)L - E(\tau)]\right\}d\tau + \lambda B(t),$$

where in the second line we bring the Lagrange multiplier inside the integral. The expression inside the integral must be maximized at every point in time. So differentiate this expression with respect to $E(\tau)$ to obtain the first-order condition (10.20).

10.4 Consider a central planner who chooses the time-path of output to maximize utility. This problem can be written as,

$$\max_{Y(\tau)} \int_t^\infty e^{-\rho(\tau - t)} \ln Y(\tau) d\tau,$$

subject to,

$$Y = N^{\sigma/(\sigma-1)} x = N^{1/(\sigma-1)} X \quad \text{and} \quad L = X + \alpha(\dot{N}/N),$$

where $X \equiv Nx$ and the constraints are the production function (10.17) along with the full-employment condition (10.26). Let us restrict our attention to steady-state solutions where $N(\tau) = N(t)e^{g(\tau - t)}$. Substituting this equation along with the constraints into the objective function, and differentiate it with respect to g to compute the socially optimal growth rate g* of new products. Show that $g^* > g^a$, and interpret this result.

10.5 Let us solve for the limiting values of the growth rates in (10.35),

$$g = \left(\frac{L}{\alpha\sigma}\right) - \left(\frac{\sigma - 1}{\sigma}\right)\left[\dot{R} - \left(\frac{\dot{w}}{w}\right)\right], \quad \text{and,} \quad g^* = \left(\frac{L^*}{\alpha\sigma}\right) - \left(\frac{\sigma - 1}{\sigma}\right)\left[\dot{R} - \left(\frac{\dot{w}^*}{w^*}\right)\right].$$

To do so, we use the share of world expenditure devoted to the products of the home country, which is $s = p_y y/(E + E^*)$, and the share devoted to products of the foreign country, which is $s^* = p_y^* y^*/(E + E^*)$, with $s + s^* = 1$. In the absence of trade in intermediate inputs, each final good is assembled entirely from inputs produced in the same country. The price of home inputs is $p_i = w\sigma/(\sigma - 1)$, so it follows that $p_y y = p_i X = w\sigma X/(\sigma - 1) = w\sigma(L - \alpha g)/(\sigma - 1)$, where in the last equality we make use of the home full-employment condition, $L = X + \alpha g$. Therefore, the expenditure share on home products can be written as, $s = w\sigma(L - \alpha g)/[(E + E^*)(\sigma - 1)]$. Taking logs and differentiating, we obtain:

$$\frac{\dot{s}}{s} = \frac{\dot{w}}{w} - \frac{\dot{g}}{(L - \alpha g)} - \left[\frac{\dot{E}}{E}\left(\frac{E}{E + E^*}\right) + \frac{\dot{E}^*}{E^*}\left(\frac{E^*}{E + E^*}\right)\right].$$

Using (10.20), the expression in brackets on the right equals $\dot{R} - \rho$. It follows that the home and foreign real interest rates are,

$$\dot{R} - \frac{\dot{w}}{w} = \rho - \frac{\dot{s}}{s} - \frac{\dot{g}}{(L - \alpha g)}, \quad \text{and,} \quad \dot{R} - \frac{\dot{w}^*}{w^*} = \rho - \frac{\dot{s}^*}{s^*} - \frac{\dot{g}^*}{(L^* - \alpha g^*)}.$$

(a) The fact that the home country is developing more products than abroad means that

$\lim_{t \to \infty} s = 1$. What does this imply about the limiting value of the home real interest rate?

Therefore, what is the limiting value of the home growth rate?

(b) Since $\lim_{t \to \infty} s = 1$, then $\lim_{t \to \infty} s^* = 0$. What can we say about the limiting values of the foreign real interest rate, and the foreign growth rate?