14.581 MIT PhD International Trade
— Lecture 17: Trade and Growth (Theory) —

Dave Donaldson

Spring 2011

Today's Plan

Open economy versions of canonical growth models:

1. Neoclassical growth model
2. Learning-by-doing models
3. Endogenous growth models

Overview

- We will consider three types of growth models:
  1. Neoclassical growth model [Factor accumulation]
  2. Learning-by-doing models [Accidental technological progress]
  3. Endogenous growth models [Profit-motivated technological progress]

- **Questions:**
  1. How does openness to trade affect predictions of closed-economy growth models?
  2. Does openness to trade have positive or negative effects on growth?

- **Theoretical Answer:**

  It depends on the details of the model...

Today's Plan

Open economy versions of canonical growth models:

1. **Neoclassical growth model**
2. Learning-by-doing models
3. Endogenous growth models

- In a closed economy, neoclassical growth model predicts that:
    1. If there are diminishing marginal returns to capital, then different capital labor ratios across countries lead to different growth rates along transition path.
    2. If there are constant marginal returns to capital (AK model), then different discount factors across countries lead to different growth rates in steady state.
- In an open economy, both predictions can be overturned.

Neoclassical Growth Model

Preferences and technology

- For simplicity, we will assume throughout this lecture that:
  - No population growth: $l(t) = 1$ for all t.
  - No depreciation of capital.
- Representative household at $t = 0$ has log-preferences

$$U = \int_0^{+\infty} \exp\left(-\rho t\right) \ln c\left(t\right) dt \tag{1}$$

- Final consumption good is produced according to

$$y(t) = aF\left(k\left(t\right), l\left(t\right)\right) = af\left(k\left(t\right)\right)$$

where output (per capita) f satisfies:

$$f' > 0 \text{ and } f'' \leq 0$$

Neoclassical Growth Model

Perfect competition, law of motion for capital, and no Ponzi condition

- Firms maximize profits taking factor prices $w(t)$ and $r(t)$ as given:

$$\begin{aligned} r(t) &= af'(k(t)) && (2) \\ w(t) &= af(k(t)) - k(t)af'(k(t)) && (3) \end{aligned}$$

- Law of motion for capital is given by

$$\dot{k}(t) = r(t)\,k(t) + w(t) - c(t) \tag{4}$$

- No Ponzi-condition:

$$\lim_{t \to +\infty} \left[k(t) \exp\left(-\int_0^t r(s)ds \right) \right] \geq 0 \tag{5}$$

- **Definition** *Competitive equilibrium of neoclassical growth model consists in* (c, k, r, w) *such that representative household maximizes* (1) *subject to* (4) *and* (5) *and factor prices satisfy* (2) *and* (3).

- **Proposition 1** *In any competitive equilibrium, consumption and capital follow the laws of motion given by*

$$
\begin{aligned}
\frac{\dot{c}(t)}{c(t)} &= af'(k(t)) - \rho \\
\dot{k}(t) &= f(k(t)) - c(t)
\end{aligned}
$$

Neoclassical Growth Model

Case (I): diminishing marginal product of capital

- Suppose first that $f'' < 0$.
- In this case, Proposition 1 implies that:
  1. Growth rates of consumption is decreasing with k.
  2. There is no long-run growth without exogenous technological progress.
  3. Starting from $k(0) > 0$, there exists a unique equilibrium converging monotonically to (c^*, k^*) such that

$$\begin{aligned} af'(k^*) &= \rho \\ c^* &= f(k^*) \end{aligned}$$

Neoclassical Growth Model

Case (II): constant marginal product of capital (AK model)

- Now suppose that $f'' = 0$. This corresponds to

$$af(k) = ak$$

- In this case, Proposition 1 implies the existence of a unique equilibrium path in which c and k all grow at the same rate

$$g^* = a - \rho$$

- We will now illustrate how trade integration—through its effects on factor prices—may transform a model with diminishing marginal returns into an AK model and vice versa

Ventura (1997)

Assumptions

- Neoclassical growth model with multiple countries indexed by j
  - No differences in population size: $l_j(t) = 1$ for all j
  - No differences in discount rates: $\rho_j = \rho$ for all j
  - *Diminishing marginal returns:* $f'' < 0$
- Capital and labor *services* are freely traded across countries
  - No trade in assets, so trade is balanced period by period.
- **Notation:**
  - $x_j^l(t)$, $x_j^k(t) \equiv$ labor and capital services used in production of final good in country j

$$y_j(t) = aF\left(x_j^k(t), x_j^l(t)\right) = ax_j^l(t)\, f\left(x_j^k(t) / x_j^l(t)\right)$$

  - $l_j(t) - x_j^l(t)$ and $k_j(t) - x_j^l(t) \equiv$ net exports of factor services

Ventura (1997)

Free trade equilibrium

- Free trade equilibrium reproduces the integrated equilibrium.
- In each period:
  1. Free trade in factor services implies FPE:

$$\begin{aligned} r_j(t) &= r(t) \\ w_j(t) &= w(t) \end{aligned}$$

  2. FPE further implies identical capital-labor ratios:

$$\frac{x_j^k(t)}{x_j^l(t)} = \frac{x^k(t)}{x^l(t)} = \frac{\sum_j k_j(t)}{\sum_j l_j(t)} = \frac{k^w(t)}{l^w(t)}$$

- Like in static HO model, countries with $k_j(t)/l_j(t) > k^w(t)/l^w(t)$ export capital and import labor services.

- Let $c(t) \equiv \sum_j c_j(t) / L^w(t)$ and $k(t) \equiv \sum_j k_j(t) / L^w(t)$
- Not surprisingly, world consumption and capital per capita satisfy

$$\begin{aligned} \frac{\dot{c}(t)}{c(t)} &= af'(k(t)) - \rho \\ \dot{k}(t) &= f(k(t)) - c(t) \end{aligned}$$

- For each country, however, we have

$$\begin{aligned} \frac{\dot{c}_j(t)}{c_j(t)} &= af'(k(t)) - \rho \qquad (6) \\ \dot{k}_j(t) &= f'(k(t))\, k_j(t) - c_j(t) \qquad (7) \end{aligned}$$

- If $k(t)$ is fixed, Equations (6) and (7) imply that everything is *as if* countries were facing an AK technology.

Ventura (1997)

Summary and Implications

- Ventura (1997) hence shows that trade may help countries avoid the curse of diminishing marginal returns:
    - As long as country j is "small" relative to the rest of the world, $k_j(t) \ll k(t)$, the return to capital is independent of $k_j(t)$.
    - This is really just an application of the 'factor price insensitivity' result we saw when we studied the small open economy (or partial equilibrium version of a large economy) H-O model.
- This insight may help explain growth miracles in East Asia:
    - Asian economies, which were more open than many developing countries, accumulated capital more rapidly but without rising interest rates or diminishing returns.
    - These economies were also heavily industrializing along their development path. H-O mechanism requires this. Country accumulates capital and shifts into capital-intensive goods, exporting that which is in excess supply.

- Now we go in the opposite direction.
- AK model with multiple countries indexed by j.
  - No differences in population size: $l_j(t) = 1$ for all j.
  - Constant marginal returns: $f'' = 0$.
- Like in an "Armington" model, capital services are differentiated by country of origin.
- Capital services are freely traded and combined into a unique final good—either for consumption or investment—according to:

$$
\begin{aligned}
c_j(t) &= \left[\sum_{j'} x^c_{jj'}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\
i_j(t) &= \left[\sum_{j'} x^i_{jj'}(t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

- **Lemma** *In each period,* $c_j(t) = \rho_j k_j(t)$.
- **Proof:**
  1. Euler equation implies:
  $$\frac{\dot{c}_j(t)}{c_j(t)} = r_j(t) - \rho_j.$$
  2. Budget constraint at time t requires:
  $$\dot{k}_j(t) = r_j(t) k_j(t) - c_j(t).$$
  3. Combining these two expressions, we obtain:
  $$\dot{\left[k_j(t)/c_j(t)\right]} = \rho_j \left[k_j(t)/c_j(t)\right] - 1.$$
  4. 3 + no-Ponzi condition implies:
  $$k_j(t)/c_j(t) = 1/\rho_j.$$

- **Proposition 2** *In steady-state equilibrium, we must have:*

$$\frac{\dot{k}_j(t)}{k_j(t)} = \frac{\dot{c}_j(t)}{c_j(t)} = g^*.$$

- **Proof:**

1. In steady state, by definition, we have $r_j(t) = r_j^*$.
2. Lemma + Euler equation $\Rightarrow \frac{\dot{k}_j(t)}{k_j(t)} = r_j(t) - \rho_j$.
3. 1 + 2 $\Rightarrow \frac{\dot{k}_j(t)}{k_j(t)} = g_j^*$.
4. Market clearing implies:

$$r_j(t)\, k_j(t) = r_j^{1-\sigma}(t) \sum_{j'} r_{j'}(t)\, k_{j'}(t), \text{ for all } j.$$

5. Differentiating the previous expression, we get $g_j^* = g^*$.
6. 5 + Lemma $\Rightarrow \frac{\dot{c}_j(t)}{c_j(t)} = g^*$.

- Under autarky, AK model predicts that countries with different discount rates ρ_j should grow at different rates.
- Under free trade, Proposition 2 shows that all countries grow at the same rate.
- Because of terms of trade effects, everything is *as if* we were back to a model with diminishing marginal returns.
- From a theoretical standpoint, Acemoglu and Ventura (2002) is the mirror image of Ventura (1997)

Today's Plan

Open economy versions of canonical growth models:

1. Neoclassical growth model
2. **Learning-by-doing models**
3. Endogenous growth models

Learning-by-Doing Models

Basic Idea

- In neoclassical growth models, technology is exogenously given.
  - So trade may only affect growth rates through factor accumulation.
- **Question:**
  How may trade affect growth rates through technological changes?
- **Learning-by-doing models:**
  - Technological progress $\equiv$ 'accidental' by-product of production activities.
  - So, patterns of specialization also affect TFP growth.

Learning-by-Doing Models

Assumptions

- Consider an economy with two intermediate goods, $i = 1, 2$, and one factor of production, labor ($l_j = 1$).
- Intermediate goods are aggregated into a unique final good:

$$y_j\left(t\right) = \left[y_j^1\left(t\right)^{\frac{\sigma-1}{\sigma}} + y_j^2\left(t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \sigma > 1.$$

- Intermediate goods are produced according to:

$$y_j^i\left(t\right) = a_j^i\left(t\right) l_j^i\left(t\right).$$

- Knowledge spillovers are sector-and-country specific:

$$\frac{\dot{a}_j^i\left(t\right)}{a_j^i\left(t\right)} = \eta^i l_j^i\left(t\right). \tag{8}$$

- For simplicity, there are no knowledge spillovers in sector 2: $\eta^2 = 0$.

Learning-by-Doing Models

Autarky equilibrium

- Incomplete specialization (which we assume under autarky) requires:

$$\frac{p_j^1(t)}{p_j^2(t)} = \frac{a_j^2(t)}{a_j^1(t)} \tag{9}$$

- Profit maximization by final good producers requires:

$$\frac{y_j^1(t)}{y_j^2(t)} = \left(\frac{p_j^1(t)}{p_j^2(t)}\right)^{-\sigma} \tag{10}$$

- Finally, labor market clearing implies:

$$\frac{y_j^1(t)}{y_j^2(t)} = \frac{a_j^1(t)\, l_j^1(t)}{a_j^2(t)\left(1 - l_j^1(t)\right)} \tag{11}$$

Learning-by-Doing Models

Autarky equilibrium

- **Proposition** *Under autarky, the allocation of labor and growth rates satisfy* $\lim_{t\to+\infty} l_j^1(t) = 1$ *and* $\lim_{t\to+\infty} \frac{\dot{y}_j(t)}{y_j(t)} = \eta^1$.
- **Proof:**
  1. Equations (9)-(11) imply:
  $$\frac{l_j^1(t)}{1-l_j^1(t)} = \left(\frac{a_j^2(t)}{a_j^1(t)}\right)^{1-\sigma}.$$
  2. With incomplete specialization at every date, Equation (8) implies:
  $$\lim_{t\to+\infty}\left(\frac{a_j^2(t)}{a_j^1(t)}\right) = 0.$$
  3. $1+2 \Rightarrow \lim_{t\to+\infty} l_j^1(t) = 1.$
  4. $3 \Rightarrow \lim_{t\to+\infty} y_j(t) = a_j^1(t) \Rightarrow \lim_{t\to+\infty} \frac{\dot{y}_j(t)}{y_j(t)} = \eta^1.$

Learning-by-Doing Models

Free trade equilibrium

- Suppose that country 1 has CA in good 1 at date 0:

$$\frac{a_1^1(0)}{a_1^2(0)} > \frac{a_2^1(0)}{a_2^2(0)}. \tag{12}$$

- **Proposition** *Under free trade,* $\lim_{t \to +\infty} y_1(t) / y_2(t) = +\infty$.
- **Proof:**
  1. Equation (8) and Inequality (12) imply:

$$\frac{a_1^1(t)}{a_1^2(t)} > \frac{a_2^1(t)}{a_2^2(t)} \text{ for all } t.$$

  2. $1 \Rightarrow l_1^1(t) = 1$ and $l_2^1(t) = 0$ for all t.
  3. $2 \Rightarrow y_1(t) / y_2(t) = a_1^1(t) / a_2^2(t)$.
  4. $3 + \lim_{t \to +\infty} a_j^1(t) = +\infty \Rightarrow \lim_{t \to +\infty} y_1(t) / y_2(t) = +\infty$.

Learning-by-Doing Models

Comments

- World still grows at rate η^1, but small country does not.
- Learning-by-doing models illustrate how trade may hinder growth if you specialize in the "wrong" sector.
  - This is an old argument in favor of trade protection (see e.g. Graham 1923, Ethier 1982).
- Country-specific spillovers tend to generate "locked in" effects.
  - If a country has CA in good 1 at some date t, then it has CA in this good at all subsequent dates.
- History matters in learning-by-doing models:
  - Short-run policy may have long-run effects (Krugman 1987).

Today's Plan

Open economy versions of canonical growth models:

1. Neoclassical growth model
2. Learning-by-doing models
3. **Endogenous growth models**

Endogenous Growth Models

Basic Idea

- In endogenous growth models, technological progress results from deliberate investment in R&D.
- In this case, economic integration may affect growth rates by changing incentives to invest in R&D through:
  1. Knowledge spillovers.
  2. Market size effect.
  3. Competition effect.
- Two canonical endogenous growth models are:
  1. Expanding Variety Model: Romer (1990).
  2. Quality-Ladder Model: Grossman and Helpman (1991) and Aghion and Howitt (1992).
- We will focus on expanding variety model

Expanding Variety Model

Assumptions

- Labor is the only factor of production $(l = 1)$.
- Final good is produced under perfect competition according to:

$$c(t) = \left(\int_0^{n(t)} x(\omega, t)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}, \ \sigma > 1.$$

- Inputs ω are produced under monopolistic competition according to:

$$x(\omega, t) = l(\omega, t).$$

- New inputs can be invented with the production function given by:

$$\frac{\dot{n}(t)}{n(t)} = \eta l^r(t). \tag{13}$$

- Similar to learning-by-doing model, but applied to innovation.

- Euler equation implies:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho. \tag{14}$$

- Monopolistic competition implies:

$$p(\omega, t) = \frac{\sigma w(t)}{\sigma - 1}.$$

- Accordingly, instantaneous profits are equal to:

$$\pi(\omega, t) = [p(\omega, t) - w(t)]\, l(\omega, t) = \frac{1}{\sigma - 1} \frac{w(t) l^e(t)}{n(t)}. \tag{15}$$

where $l^e(t) \equiv \int_0^{n(t)} l(\omega, t)\, d\omega$ is total employment in production

- Because of symmetry, we drop index ω from now on.

- The value of a typical input producer at date t is:

$$v(t) = \int_t^{+\infty} \exp\left(-\int_t^s r(s')ds'\right) \pi(s)\, ds.$$

- Asset market equilibrium requires:

$$r(t)\, v(t) = \pi(t) + \dot{v}(t). \tag{16}$$

- Free entry of input producers requires:

$$\eta n(t) v(t) = w(t). \tag{17}$$

- Finally, labor market clearing requires:

$$l^r(t) + l^e(t) = 1. \tag{18}$$

- **Proposition** *In BGP equilibrium, aggregate consumption grows at a constant rate* $g^* \equiv \frac{\eta-(\sigma-1)\rho}{\sigma(\sigma-1)}$.
- **Proof:**

1. In BGP equilibrium: $r(t) = r^*$, $l^e(t) = l^{e*}$, and $l^r(t) = l^{r*}$.
2. From Euler equation, (14), we know that $g^* \equiv \frac{\dot{c}(t)}{c(t)} = r^* - \rho$.
3. From asset market clearing, (16), we also know that

$$r^* = \frac{\pi(t)}{v(t)} + \frac{\dot{v}(t)}{v(t)} = \frac{\eta\left(1 - l^{r*}\right)}{\sigma - 1} + \frac{\dot{w}(t)}{w(t)} - \frac{\dot{n}(t)}{n(t)}$$

   where the second equality derives from (15), (17), and (18).
4. By our choice of numeraire, $\frac{\dot{w}(t)}{w(t)} = \frac{\dot{c}(t)}{c(t)} = g^*$. Thus 3 + (13) imply:

$$r^* = \frac{\eta\left(1 - l^{r*}\right)}{\sigma - 1} + g^* - \eta l^{r*}.$$

5. Using 2 and 4, we can solve for l^{r*}, and in turn, r^* and g^*.

- In expanding variety model, aggregate consumption is given by:

$$c\left(t\right)=n^{\frac{\sigma}{\sigma-1}}\left(t\right)x\left(t\right)=n^{\frac{1}{\sigma-1}}\left(t\right)l^{e}\left(t\right).$$

- In BGP equilibrium, we therefore have:

$$\frac{\dot{c}(t)}{c\left(t\right)}=\left(\frac{1}{\sigma-1}\right)\times\left(\frac{\dot{n}(t)}{n\left(t\right)}\right).$$

- Predictions regarding $\dot{n}(t)/n\left(t\right)$, of course, rely heavily on innovation PPF. If $\dot{n}(t)/n\left(t\right)=\eta\phi\left(n\left(t\right)\right)l^{r}\left(t\right)$, then:
  - $\lim_{n\rightarrow+\infty}\phi\left(n\right)=+\infty\Rightarrow$ unbounded long-run growth.
  - $\lim_{n\rightarrow+\infty}\phi\left(n\right)=0\Rightarrow$ no long-run growth.

Expanding Variety Model

Open economy

- Now suppose that there are two countries indexed by $j = 1, 2$.
- In order to distinguish the effects of trade from those of technological diffusion, we start from a situation in which:
  1. There is no trade in intermediate inputs.
  2. There are knowledge spillovers across countries:

$$\frac{\dot{n}_j(t)}{n_j(t) + \Psi n_{-j}(t)} = \eta l_j^r(t)$$

where $1 - \Psi \in [0, 1] \equiv$ share of inputs produced in both countries.

- Because of knowledge spillovers across countries, it is easy to show that growth rate is now given by

$$g_j^* = \frac{\eta(1 + \Psi) - (\sigma - 1)\rho}{\sigma(\sigma - 1)} > g_{autarky}^*$$

Expanding Variety Model
Open economy

- **Question:**
  What happens when two countries start trading intermediate inputs?

- **Answer:**
  1. Trade eliminates redundancy in R&D ($\Psi \to 1$), which $\nearrow$ growth rates. Producers now have incentive to not duplicate effort.
  2. However, trade has *no further effect* on growth rates.

- Intuitively, when the two countries start trading:
  1. Spending $\nearrow$, which $\nearrow$ profits, and so, incentives to invest in R&D.
  2. But competition from Foreign suppliers $\searrow$ CES price index, which $\searrow$ profits, and so, incentives to invest in R&D.
  3. With CES preferences, 1 and 2 exactly cancel out.

Expanding Variety Model

Comments

- This neutrality result heavily relies on CES (related to predictions on number of varieties per country in Krugman 1980).

- Not hard to design endogenous growth models in which trade has a positive impact on growth rates (beyond R&D redundancy):

  1. Start from same expanding variety model, but drop CES, and assume

     $$c(t) = n^{\alpha} \left(\int_0^{n(t)} x(\omega, t)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

     If $\alpha > 0$, market size effect dominates. (If $\alpha < 0$, it's the contrary.)

  2. Start from a lab-equipment model in which final good rather than labor is used to produce new inputs.

Concluding Remarks

- Previous models suggest that trade integration may have a profound impact on the predictions of closed-economy growth models.
  - But they do not suggest a systematic relationship between trade integration and growth.
- *Ultimately, whether trade has positive or negative effects on growth is an empirical question.*
- In this lecture, we have abstracted from issues related to firm-level heterogeneity and growth (e.g. learning by exporting, technology adoption at the firm-level).
  - For more on these issues, see, eg, Atkeson and Burstein (2010), Bustos (2010), and Constantini and Melitz (2007).