14.581 MIT PhD International Trade — Lecture 6: Ricardo-Viner and Heckscher-Ohlin Models (Theory II) —

Dave Donaldson

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Today's Plan

1. Two-by-two-by-two Heckscher-Ohlin model
   1. Integrated equilibrium
   2. Heckscher-Ohlin Theorem
2. High-dimensional issues
   1. Classical theorems revisited
   2. Heckscher-Ohlin-Vanek Theorem
3. Assignment models (briefly)

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    1. **Integrated equilibrium**
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Two-by-two-by-two Heckscher-Ohlin model

Basic environment

- Results derived in previous lecture hold for small open economies.
  - Relative good prices were taken as exogenously given.
- We now turn world economy with two countries, North and South.
- We maintain the two-by-two HO assumptions:
  - There are two goods, $g = 1{,}2$, and two factors, k and l.
  - Identical technology around the world, $y_g = f_g(k_g, l_g)$.
  - Identical homothetic preferences around the world, $d_g^c = \alpha_g(p) I^c$.
- **Question**
  What is the pattern of trade in this environment?

Two-by-two-by-two Heckscher-Ohlin model

Strategy ("Samuelson's Angel")

- Start from **Integrated Equilibrium** $\equiv$ competitive equilibrium that would prevail if *both* goods and factors were freely traded.
- Consider **Free Trade Equilibrium** $\equiv$ competitive equilibrium that prevails if goods are freely traded, but factors are not.
- Ask: Can free trade equilibrium reproduce integrated equilibrium?
  - Answer turns out to be yes, *if factor prices are equalized through trade*.
- In this situation, one can then use homotheticity to go from differences in factor endowments to the pattern of trade

- **Integrated equilibrium** corresponds to (p, ω, y) such that:

$$(ZP) \quad : \quad p = A'(\omega)\,\omega \tag{1}$$
$$(GM) \quad : \quad y = \alpha(p)\left(\omega' v\right) \tag{2}$$
$$(FM) \quad : \quad v = A(\omega)\,y \tag{3}$$

  where:

  - $p \equiv (p_1, p_2)$, $\omega \equiv (w, r)$, $A(\omega) \equiv \left[a_{fg}(\omega)\right]$, $y \equiv (y_1, y_2)$ is the vector of total world output, $v \equiv (l, k)$ is the vector of total world endowments, and $\alpha(p) \equiv [\alpha_1(p), \alpha_2(p)]$.
  - $A(\omega)$ derives from cost-minimization.
  - $\alpha(p)$ derives from utility-maximization.

- So this is the equilibrium of the world economy if factors were allowed to be mobile.

- **Free trade equilibrium** corresponds to $(p^t, \omega^n, \omega^s, y^n, y^s)$ such that:

$$\begin{aligned}
(ZP) \quad &: \quad p^t \leq A'(\omega^c)\,\omega^c \text{ for } c = n, s && (4)\\
(GM) \quad &: \quad y^n + y^s = \alpha\left(p^t\right)\left(\omega^{n\prime} v^n + \omega^{s\prime} v^s\right) && (5)\\
(FM) \quad &: \quad v^c = A(\omega^c)\, y^c \text{ for } c = n, s && (6)
\end{aligned}$$

where (4) holds with equality if good is produced in country c.

- **Definition:** *Free trade equilibrium replicates integrated equilibrium if* $\exists\ (y^n, y^s) \geq 0$ *such that* $(p, \omega, \omega, y^n, y^s)$ *satisfy conditions* (4)-(6)

Two-by-two-by-two Heckscher-Ohlin model

Factor Price Equalization (FPE) Set

- **Definition** *(v^n, v^s) are in the FPE set if $\exists$ $(y^n, y^s) \geq 0$ such that condition (6) holds for $\omega^n = \omega^s = \omega$.*
- **Lemma** *If (v^n, v^s) is in the FPE set, then the free trade equilibrium replicates the integrated equilibrium*
- **Proof:** By definition of the FPE set, $\exists$ $(y^n, y^s) \geq 0$ such that

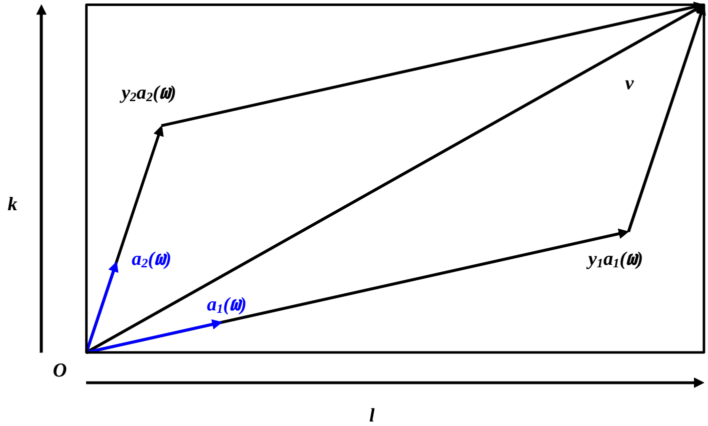
$$v^c = A(\omega) y^c.$$

So Condition (6) holds. Since $v = v^n + v^s$, this implies

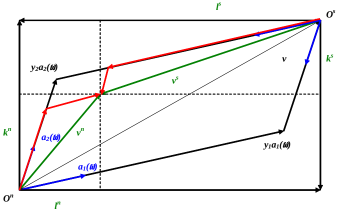
$$v = A(\omega)(y^n + y^s).$$

Combining this expression with condition (3), we obtain $y^n + y^s = y$. Since $\omega^{n\prime} v^n + \omega^{s\prime} v^s = \omega' v$, Condition (5) holds as well. Finally, Condition (1) directly implies (4) holds.

- Factor market clearing in the integrated equilibrium:



- **FPE set** $\equiv (v^n, v^s)$ inside the parallelogram

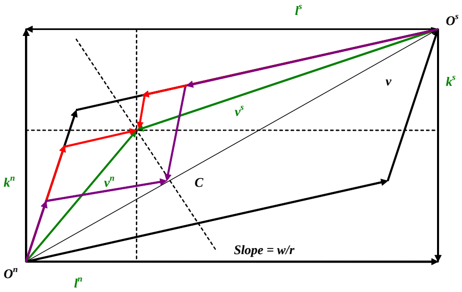


- When v^n and v^s are inside the parallelogram, we say that they belong to the same **diversification cone.**
- This is a very different way of approaching FPE than FPE Theorem.
  - Here, we have shown that there can be FPE iff factor endowments are not too dissimilar, whether or not there are no FIR.
  - Instead of taking prices as given—whether or not they are consistent with integrated equilibrium—we take factor endowments as primitives.

Two-by-two-by-two Heckscher-Ohlin model

Heckscher-Ohlin Theorem: graphical analysis

- Suppose that (v^n, v^s) is in the FPE set.
- **HO Theorem** *In the free trade equilibrium, each country will export the good that uses its abundant factor intensively.*



- Outside the FPE set, additional technological and demand considerations matter (e.g. FIR or no FIR).

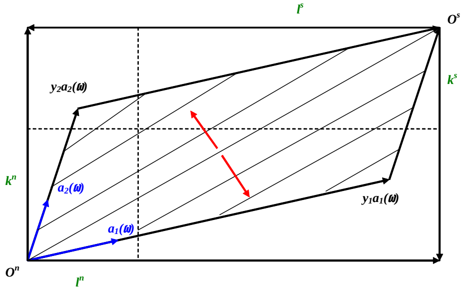
- The HO Theorem can also be derived using the Rybczynski effect:

1. Rybczynski theorem $\Rightarrow y_2^n / y_1^n > y_2^s / y_1^s$ for any p.
2. Homotheticity $\Rightarrow c_2^n / c_1^n = c_2^s / c_1^s$ for any p.
3. This implies $p_2^n / p_1^n < p_2^s / p_1^s$ under autarky.
4. Law of comparative advantage $\Rightarrow$ HO Theorem.

- Predictions of HO and SS Theorems are often combined:
  - HO Theorem $\Rightarrow p_2^n / p_1^n < p_2 / p_1 < p_2^s / p_1^s$.
  - SS Theorem $\Rightarrow$ *Moving from autarky to free trade, real return of abundant factor increases, whereas real return of scarce factor decreases.*
  - If North is skill-abundant relative to South, inequality increases in the North and decreases in the South.
- So why may we observe a rise in inequality in the South in practice? Perhaps:
  - Southern countries are not moving from autarky to free trade.
  - Technology is not identical around the world.
  - Preferences are not homothetic and identical around the world.
  - There are more than two goods and two countries in the world.

Two-by-two-by-two Heckscher-Ohlin model
Trade volumes

- Let us define trade volumes as the sum of exports plus imports.
- Inside FPE set, iso-volume lines are parallel to diagonal (HKa p.23).
  - The further away from the diagonal, the larger the trade volumes.
  - Factor abundance rather than country size determines trade volume.



- If country size affects trade volumes in practice, what should we infer?

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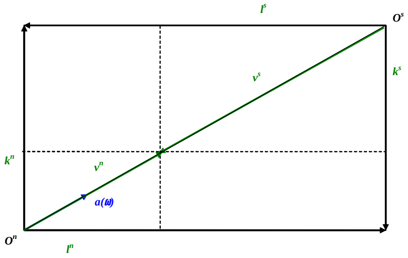
High-Dimensional Predictions

FPE (I): More factors than goods

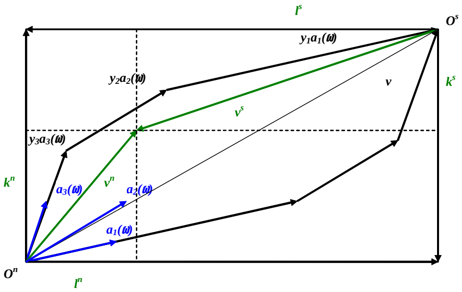
- Suppose now that there are F factors and G goods.
- By definition, (v^n, v^s) is in the FPE set if $\exists\,(y^n, y^s) \geq 0$ s.t. $v^c = A(\omega)\, y^c$ for $c = n, s$.
- If $F = G$ ("even case"), the situation is qualitatively similar.
- If $F > G$, the FPE set will be "measure zero": $\{v | v = A(\omega)\, y^c \text{ for } y^c \geq 0\}$ is a G-dimensional cone in F-dimensional space.
- **Example:** "Standard Macro" model with 1 good and 2 factors.



High-Dimensional Predictions

FPE (II): More goods than factors

- If $F < G$, there will be indeterminacies in production, (y^n, y^s), and so, trade patterns, but FPE set will still have positive measure.
- **Example:** 3 goods and 2 factors:



- By the way, are there more goods than factors in the world?

High-Dimensional Predictions

Stolper-Samuelson-type results (I): "Friends and Enemies"

- SS Theorem was derived by differentiating the zero-profit conditions.
- With an arbitrary number of goods and factors, we still have

$$\widehat{p}_g = \sum_f \theta_{fg} \widehat{w}_f , \tag{7}$$

where w_f is the price of factor f and $\theta_{fg} \equiv w_f a_{fg}(\omega) / c_g(\omega)$.

- Now suppose that $\widehat{p}_{g_0} > 0$, whereas $\widehat{p}_g = 0$ for all $g \neq g_0$.
- Equation (7) immediately implies the existence of f_1 and f_2 s.t.

$$\begin{aligned} \widehat{w}_{f_1} &\geq \widehat{p}_{g_0} > \widehat{p}_g = 0 \text{ for all } g \neq g_0, \\ \widehat{w}_{f_2} &< \widehat{p}_g = 0 < \widehat{p}_{g_0} \text{ for all } g \neq g_0. \end{aligned}$$

- So every good is "friend" to some factor and "enemy" to some other (Jones and Scheinkman 1977)

- Ethier (1984) also provides the following variation of SS Theorem.
- If good prices change from p to p', then the associated change in factor prices, $\omega' - \omega$, must satisfy

$$\left(\omega' - \omega\right) A\left(\omega_0\right) \left(p' - p\right) > 0, \text{ for some } \omega_0 \text{ between } \omega \text{ and } \omega'.$$

- **Proof:**
  Define $f\left(\omega\right) = \omega A\left(\omega\right)\left(p' - p\right)$. Mean value theorem implies

$$f\left(\omega'\right) = \omega A\left(\omega\right)\left(p' - p\right) + \left(\omega' - \omega\right)\left[A\left(\omega_0\right) + \omega_0 dA\left(\omega_0\right)\right]\left(p' - p\right)$$

for some ω_0 between ω and ω'. Cost-minimization at ω_0 requires

$$\omega_0 dA\left(\omega_0\right) = 0.$$

- **Proof (Cont.):**
  Combining the two previous expressions, we obtain

$$f\left(\omega'\right) - f\left(\omega\right) = \left(\omega' - \omega\right) A\left(\omega_0\right) \left(p' - p\right).$$

  From the zero profit conditions, we know that $p = \omega A\left(\omega\right)$ and $p' = \omega' A\left(\omega'\right)$. Thus

$$f\left(\omega'\right) - f\left(\omega\right) = \left(p' - p\right) \left(p' - p\right) > 0.$$

  The last two expressions imply

$$\left(\omega' - \omega\right) A\left(\omega_0\right) \left(p' - p\right) > 0.$$

- **Interpretation:**
  Tendency for changes in good prices to be accompanied by raises in prices of factors used intensively in goods whose prices have gone up.
- But what is ω_0?

High-Dimensional Predictions

Rybczynski-type results

- Rybczynski Theorem was derived by differentiating the factor market clearing conditions.
- If $G = F > 2$, same logic implies that increase in endowment of one factor decreases output of one good and increases output of another (Jones and Scheinkman 1977).
- If $G < F$, increase in endowment of one factor may increase output of all goods (Ricardo-Viner).
- In this case, we still have the following correlation (Ethier 1984)

$$\left(v' - v\right) A\left(\omega_0\right)\left(y' - y\right) = \left(v' - v\right)\left(v' - v\right) > 0.$$

- If $G > F$, inderteminacies in production imply that we cannot predict changes in output vectors.

High-Dimensional Predictions

Heckscher-Ohlin-type results

- Since HO Theorem derives from Rybczynski effect + homotheticity, problems of generalization in the case $G < F$ and $F > G$ carry over to the Heckscher-Ohlin Theorem.
- If $G = F > 2$, we can invert the factor market clearing condition

$$y^c = A^{-1}(\omega) v^c.$$

- By homotheticity, the vector of consumption in country c satisfies

$$d^c = s^c d$$

where $s^c \equiv c$'s share of world income, and $d \equiv$ world consumption.

- Good and factor market clearing requires

$$d = y = A^{-1}(\omega) v.$$

- Combining the previous expressions, we get net exports

$$t^c \equiv y^c - d^c = A^{-1}(\omega)(v^c - s^c v).$$

High-Dimensional Predictions

Heckscher-Ohlin-Vanek Theorem

- Without assuming that $G = F$, we can derive sharp predictions if we focus on the $G \geqq F$ case and on the *factor content of trade* rather than *commodity trade.*
- We define the *net exports of factor* f by country c as

$$\tau_f^c = \sum_g a_{fg}(\omega)\, t_g^c.$$

- In matrix terms, this can be rearranged as

$$\tau^c = A(\omega)\, t^c.$$

- **HOV Theorem** *In any country* c*, net exports of factors satisfy*

$$\tau^c = v^c - s^c v.$$

- So countries should export the factors in which they are abundant compared to the world: $v_f^c > s^c v_f$.

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- With 2 goods and 2 factors, neoclassical trade models lead to sharp comparative static predictions.
- With more than 2 goods and 2 factors, however, their predictions become weak and unintuitive.
- "Assignment approach" consists in imposing more structure on technology in order to transform analysis into an assignment problem (which has more success in high dimensions).
- **Main assumption:**
  Constant marginal product for all factors (as in a Ricardian model).
- **Main benefit:**
  Side-step many mathematical difficulties to derive strong and intuitive predictions in high-dimensional environments.

Example: Costinot and Vogel (2009)

Factor endowments

- Consider a world economy with two countries, Home and Foreign.
- There is a continuum of goods with skill-intensity $\sigma \in \Sigma \equiv [\underline{\sigma}, \overline{\sigma}]$.
- There is a continuum of workers with skill $s \in S \equiv [\underline{s}, \overline{s}]$.
- $V(s), V^*(s) > 0$ is the inelastic supply of workers with skill s.
- Home is skill-abundant relative to Foreign:

$$\frac{V(s')}{V(s)} > \frac{V(s')}{V(s)} \text{ for any } s' \geq s.$$

Example: Costinot and Vogel (2009)

Technology and preferences

- Technology is the same around the world.
- Workers are perfect substitutes in the production of each task:

$$Y(\sigma) = \int_{s \in S} A(s,\sigma) L(s,\sigma)\, ds.$$

- $A(s,\sigma) > 0$ is strictly log-supermodular:

$$\frac{A(s,\sigma)}{A(s,\sigma')} > \frac{A(s',\sigma)}{A(s',\sigma')}, \text{ for all } s > s' \text{ and } \sigma > \sigma'.$$

- Consumers have identical CES preferences around the world:

$$U = \left\{ \int_{\sigma \in \Sigma} \left[C(\sigma) \right]^{\frac{\varepsilon - 1}{\varepsilon}} d\sigma \right\}^{\frac{\varepsilon}{\varepsilon - 1}}.$$

Example: Costinot and Vogel (2009)

Results

- Generalizations of all two-by-two results: FPE, Stolper-Samuelson, Rybczynski, Heckscher-Ohlin.

- More importantly, model can be used to look at new phenomena.

- Example: North-North trade

$$
\begin{aligned}
\frac{V(s')}{V(s)} &> \frac{V^*(s')}{V^*(s)} \text{ for any } s' \geq s \geq \widehat{s}, \\
\frac{V(s')}{V(s)} &< \frac{V^*(s')}{V^*(s)} \text{ for any } \widehat{s} \geq s' \geq s.
\end{aligned}
$$

- One can show that trade integration leads to *wage polarization* in the more "diverse" country and *wage convergence* in the other country.