14.581 MIT PhD International Trade — Lecture 5: The Ricardo-Viner and Heckscher-Ohlin Models (Theory I)—

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Today's Plan

1. Introduction to "Factor Proportions" Theory
2. The Ricardo-Viner model
    1. Basic environment
    2. Comparative statics
3. The Two-by-Two Heckscher-Ohlin model
    1. Basic environment
    2. Classical results:
        1. Factor Price Equalization Theorem
        2. Stolper-Samuelson (1941) Theorem
        3. Rybczynski (1965) Theorem

Plan of Next Lecture

1. Two-by-two-**by-two (ie two countries)** Heckscher-Ohlin model
    1. Integrated equilibrium
    2. Heckscher-Ohlin Theorem
2. Higher Dimensional issues (ie more than 2-by-2)
    1. Classical theorems revisited
    2. Heckscher-Ohlin-Vanek Theorem

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Factor Proportion Theory

- The law of comparative advantage establishes the relationship between relative autarky prices and trade flows.
  - But where do relative autarky prices come from?
- Factor proportion theory emphasizes **factor endowment differences.**
- **Key elements:**
  1. Countries differ in terms of factor abundance [i.e *relative* factor supply].
  2. And: Goods differ in terms of factor intensity [i.e *relative* factor demand].
- Interaction between 1 and 2 will determine differences in relative autarky prices, and in turn, the pattern of trade.

Factor Proportion Theory

- In order to shed light on factor endowments as a source of CA, we will assume that:
  1. Production functions are identical around the world.
  2. Households have identical homothetic preferences around the world. With free trade (ie same prices around world) this neutralizes demand-driven forces for trade.
- We will first focus on two special models:
  - **Ricardo-Viner (aka Specific Factors):** with 2 goods, 1 "mobile" factor (labor) and 2 "immobile" factors (sector-specific capital). NB: this is just a special case of a general 2-by-3 HO model.
  - **Heckscher-Ohlin:** with 2 goods and 2 "mobile" factors (labor and capital).
- The second model is often thought of as a long-run version of the first (Neary 1978).
  - In the case of Heckscher-Ohlin, what it is the time horizon such that one can think of total capital as fixed in each country, though freely mobile across sectors?

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Ricardo-Viner Model

Basic environment

- Consider an economy with:
  - Two goods, $g = 1, 2$.
  - Three factors with endowments l, k_1, and k_2.
- Output of good g is given by

$$y_g = f^g\left(l_g, k_g\right),$$

  where:
  - l_g is the (endogenous) amount of labor in sector g.
  - f^g is homogeneous of degree 1 in $\left(l_g, k_g\right)$.
- **Comments:**
  - l is a "mobile" factor in the sense that it can be employed in all sectors.
  - k_1 and k_2 are "immobile" factors in the sense that they can only be employed in one of them.
  - Model is isomorphic to DRS model: $y_g = f^g\left(l_g\right)$ with $f_{ll}^g < 0$.
  - Payments to specific factors under CRS $\equiv$ profits under DRS.

- We denote by:
  - p_1 and p_2 the prices of goods 1 and 2.
  - w, r_1, and r_2 the prices of l, k_1, and k_2.
- For now, (p_1, p_2) is exogenously given: **"small open economy."**
  - So no need to look at goods market clearing (or demand-side at all).
- **Profit maximization:**

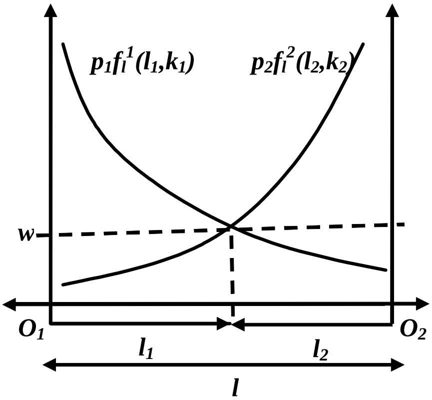
$$p_g f_l^g (l_g, k_g) = w \quad (1)$$
$$p_g f_k^g (l_g, k_g) = r_g \quad (2)$$

- **Labor market clearing:**

$$l = l_1 + l_2 \quad (3)$$

Ricardo-Viner Model

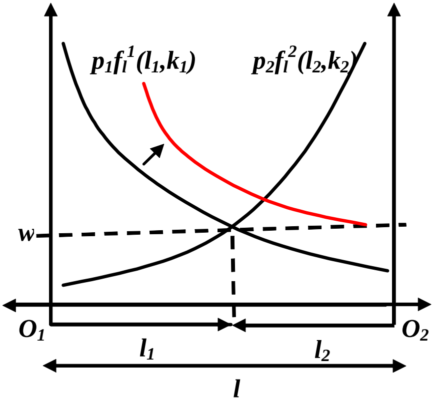
Graphical analysis



- Equations (1) and (3) jointly determine labor allocation and wage
- How do we recover payments to the specific factors from this graph?

Ricardo-Viner Model

Comparative statics



- Consider a TOT shock such that p_1 increases:
  - $w \nearrow$, $l_1 \nearrow$, and $l_2 \searrow$
  - Condition (2) $\Rightarrow r_1/p_1 \nearrow$ whereas r_2 (and a fortiori r_2/p_1) $\searrow$

- One can use the same type of arguments to analyze consequences of:
  - Productivity shocks
  - Changes in factor endowments
- In all cases, results are intuitive:
  - eg "Dutch disease" (Boom in export sectors, Bids up wages, which leads to a contraction in the other sectors).
- Easy to extend the analysis to more than 2 sectors:
  - Plot labor demand in one sector vs. rest of the economy.
  - Multi-sector model has useful political economy applications (Grossman and Helpman 1994).

- Now consider dropping the SOE assumption.
- Predictions on the pattern of trade in a two-country world depend on whether differences in factor endowments come from:
    - Differences in the relative supply of specific factors.
    - Differences in the relative supply of mobile factors.
- Accordingly, any change in factor prices is possible as we move from autarky to free trade (see Feenstra Problem 3.1 p. 98)

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      3. **Rybczynski (1965) Theorem**

Two-by-Two Heckscher-Ohlin Model

Basic environment

- Consider an economy with:
  - Two goods, $g = 1, 2$.
  - Two factors with endowments l and k.

- Output of good g is given by

$$y_g = f^g\left(l_g, k_g\right),$$

  where:

  - l_g, k_g are the (endogenous) amounts of labor and capital in sector g.
  - f^g is homogeneous of degree 1 in $\left(l_g, k_g\right)$.

Two-by-Two Heckscher-Ohlin Model

Back to the dual approach

- $c_g(w, r) \equiv$ unit cost function in sector g

$$c_g(w, r) = \min_{l,k} \left\{ wl + rk \middle| f^g(l, k) \geq 1 \right\},$$

  where w and r the price of labor and capital.

- $a_{fg}(w, r) \equiv$ unit demand for factor f in the production of good g.
- Using the Envelope Theorem, it is easy to check that:

$$a_{lg}(w, r) = \frac{dc_g(w, r)}{dw} \text{ and } a_{kg}(w, r) = \frac{dc_g(w, r)}{dr}$$

- $A(w, r) \equiv [a_{fg}(w, r)]$ denotes the matrix of total factor requirements.

Two-by-Two Heckscher-Ohlin Model

Equilibrium conditions (I): small open economy

- Like in RV model, we first look at the case of a **"small open economy"**.
  - So no need to look at good market clearing.
- **Profit-maximization:**

$$\begin{aligned} p_g &\leq wa_{lg}(w,r) + ra_{kg}(w,r) \text{ for all } g = 1,2 & (4) \\ p_g &= wa_{lg}(w,r) + ra_{kg}(w,r) \text{ if } g \text{ is produced in equilibrium} & (5) \end{aligned}$$

- **Factor market-clearing**:

$$\begin{aligned} l &= y_1 a_{l1}(w,r) + y_2 a_{l2}(w,r) & (6) \\ k &= y_1 a_{k1}(w,r) + y_2 a_{k2}(w,r) & (7) \end{aligned}$$

Classical 2-by-2 Heckscher-Ohlin Results

- We will derive 4 ‘classical’ H-O results in this 2-by-2, SOE environment:
  1. *Factor Price Insensitivity (FPI) Theorem:* What is the effect of a change in relative endowments on relative factor prices?
  2. *Factor Price Equalization (FPE) Theorem:* Will two countries with different relative endowments have different relative factor prices?
  3. *Stolper-Samuelson Theorem:* What happens to relative factor prices when relative goods prices change?
  4. *Rybczinski Theorem:* What happens to relative quantities produced when relative endowments change?
- A fifth result (the H-O Theorem) will follow in the next lecture (when we depart from the SOE environment). But it basically follows from the Rybczinski theorem and our assumptions about preferences.

- **Question:** What is the effect of a change in relative endowments on relative factor prices?

- To study this question we will need the following definition:

- **Definition**. *Factor Intensity Reversal (FIR) does not occur if:* (i) $a_{l1}(w,r)/\,a_{k1}(w,r) > a_{l2}(w,r)/\,a_{k2}(w,r)$ *for all* (w,r)*; or* (ii) $a_{l1}(w,r)/\,a_{k1}(w,r) < a_{l2}(w,r)/\,a_{k2}(w,r)$ *for all* (w,r).

- Warning: This is often described as if FIR is unlikely to occur. But if the 2 sectors both have CES technologies, but with different CES parameters, then FIR **will** occur once (ie at one unique set of relative factor prices).

- **Lemma (FPI):** *If both goods are produced in equilibrium and FIR does not occur, then factor prices $\omega \equiv (w, r)$ are uniquely determined by good prices $p \equiv (p_1, p_2)$.*

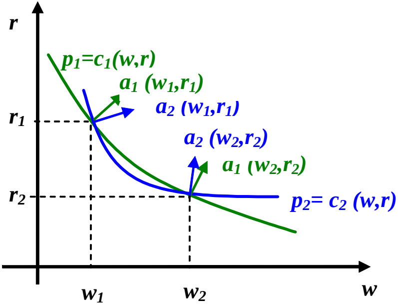
- **Proof:** If both goods are produced in equilibrium, then $p = A'(\omega)\omega$. By Gale and Nikaido (1965), this equation admits a unique solution if $a_{fg}(\omega) > 0$ for all f,g and $\det [A(\omega)] \neq 0$ for all ω, which is guaranteed by no FIR.

- **Comments:**
  - Good prices rather than factor endowments determine factor prices.
  - In a closed economy, good prices and factor endowments are, of course, related, but not for a small open economy. This has implications for, eg, the study of the effect of immigration on natives' wages.
  - All economic intuition can be gained by simply looking at Leontieff case.
  - Proof already suggests that "dimensionality" will be an issue for FIR.

Two-by-Two Heckscher-Ohlin Model

Factor Price Insensitivity (FPI): graphical analysis

- Link between no FIR and FPI can be seen graphically:



- If iso-cost curves cross more than once, then FIR must occur.

- The previous lemma directly implies (Samuelson 1949) that:
- **FPE Theorem** *If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices.*

- **Comments:**
  - This means that trade in goods can be a "perfect substitute" for trade in factors. We assumed that factors cannot move. But factors would not want to move in this world!
  - In an open economy, countries with different factor endowments can sustain the same factor prices through changing allocations of factors across sectors. Ventura (1997) argues that this is how the East Asian 'miracle' economies grew so quickly and for so long—they accumulated capital without the usual closed-economy diminishing returns to capial kicking in.
  - Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries...
  - But for the next results, we'll maintain the assumption that both goods are produced in equilibrium, but we won't need free trade and identical technologies.

- **Stolper-Samuelson Theorem** *An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor.*
- **Proof:** W.l.o.g. suppose that (i) $a_{l1}(\omega) / a_{k1}(\omega) > a_{l2}(\omega) / a_{k2}(\omega)$ and (ii) $\widehat{p}_2 > \widehat{p}_1$. Differentiating the zero-profit condition (5), we get

$$\widehat{p}_g = \theta_{lg}\widehat{w} + (1 - \theta_{lg})\widehat{r}, \tag{8}$$

where $\widehat{x} = d \ln x$ and $\theta_{lg} \equiv wa_{lg}(\omega) / c_g(\omega)$. Equation (8) implies

$$\widehat{w} \geq \widehat{p}_1, \widehat{p}_2 \geq \widehat{r} \text{ or } \widehat{r} \geq \widehat{p}_1, \widehat{p}_2 \geq \widehat{w}$$

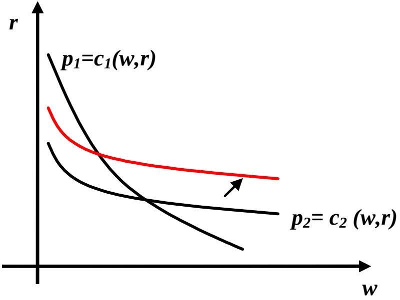
By (i), $\theta_{l2} < \theta_{l1}$. So (i) requires $\widehat{r} > \widehat{w}$. Combining the previous inequalities, we get

$$\widehat{r} > \widehat{p}_2 > \widehat{p}_1 > \widehat{w}$$

- **Comments:**
  - Previous "hat" algebra is often referred to "Jones' (1965) algebra".
  - The chain of inequalities $\widehat{r} > \widehat{p}_2 > \widehat{p}_1 > \widehat{w}$ is referred as a "magnification effect".
  - SS predict both winners and losers from change in relative prices.
  - Like FPI and FPE, SS entirely comes from zero-profit conditions (+ no joint production).
  - Like FPI and FPE, sharpness of the result hinges on "dimensionality".
  - In the empirical literature, people often talk about "Stolper-Samuelson effects" whenever looking at changes in relative factor prices as the result of tariff changes (though changes in relative good prices are rarely observed).

Heckscher-Ohlin Model

Stolper-Samuelson (1941) Theorem: graphical analysis



- Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontieff case:
  - In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies.

- Previous results have focused on the implications of the *zero profit conditions*, Equation (5), for *factor prices.*
- We now turn our attention to the implications of *factor market clearing*, Equations (6) and (7), for *factor allocation.*
- **Rybczynski Theorem** *An increase in the endowment of one factor will increase the output of the industry using that factor intensively, and* **decrease** *the output of the other industry.*

- **Proof:** W.l.o.g. suppose that (i) $a_{l1}(\omega)/a_{k1}(\omega) > a_{l2}(\omega)/a_{k2}(\omega)$ and (ii) $\widehat{k} > \widehat{l}$. Differentiating factor market clearing conditions (6) and (7), we get

$$\widehat{l} = \lambda_{l1}\widehat{y}_1 + (1-\lambda_{l1})\,\widehat{y}_2 \tag{9}$$

$$\widehat{k} = \lambda_{k1}\widehat{y}_1 + (1-\lambda_{k1})\,\widehat{y}_2 \tag{10}$$

where $\lambda_{l1} \equiv a_{l1}(\omega)\,y_1/l$ and $\lambda_{k1} \equiv a_{k1}(\omega)\,y_1/k$. Equations (8) implies

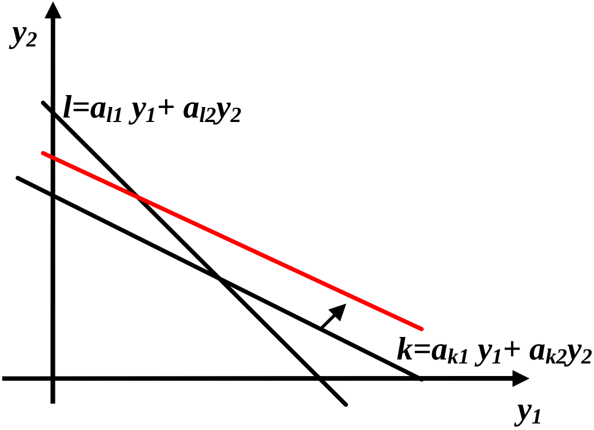
$$\widehat{y}_1 \geq \widehat{l}, \widehat{k} \geq \widehat{y}_2 \text{ or } \widehat{y}_2 \geq \widehat{l}, \widehat{k} \geq \widehat{y}_1$$

By (i), $\lambda_{k1} < \lambda_{l1}$. So (ii) requires $\widehat{y}_2 > \widehat{y}_1$. Combining the previous inequalities, we get

$$\widehat{y}_2 > \widehat{k} > \widehat{l} > \widehat{y}_1$$

- Like for FPI and FPE Theorems:
  - (p_1, p_2) is exogenously given $\Rightarrow$ factor prices and factor requirements are not affected by changes factor endowments.
  - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy.
- Like for SS Theorem, we have a "magnification effect".
- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on "dimensionality".

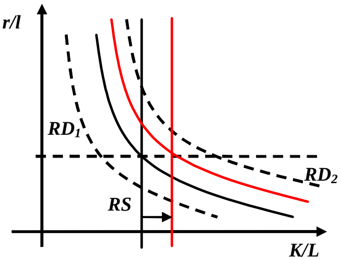
- Since good prices are fixed, it is as if we were in Leontieff case



Two-by-Two Heckscher-Ohlin Model

Rybczynski (1965) Theorem: graphical analysis (II)

- Rybczynski effect can also be illustrated using relative factor supply and relative factor demand:



- *Cross-sectoral reallocations* are at the core of HO predictions:
  - For relative factor prices to remain constant, *aggregate* relative demand must go up, which requires expansion capital intensive sector.