## Chapter 6 Mixers

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## Chapter Outline



## General Considerations

> Mixers perform frequency translation by multiplying two waveforms (and possibly their harmonics).

The LO port of this mixer is very nonlinear. The RF port, of course, must remain sufficiently linear to satisfy the compression and/or intermodulation requirements.






## Performance Parameters: Noise and Linearity, Gain

> Noise and Linearity: The design of downconversion mixers entails a compromise between the noise figure and the $\mathrm{IP}_{3}\left(\right.$ or $\left.\mathrm{P}_{1 \mathrm{~dB}}\right)$.

In a receive chain, the input noise of the mixer following the LNA is divided by the LNA gain when referred to the $R X$ input.

Similarly, the $\mathrm{IP}_{3}$ of the mixer is scaled down by the LNA gain.
> Gain: mixer gain is critical in suppression of noise while retaining linearity.

Downconversion mixers must provide sufficient gain to adequately suppress the noise contributed by subsequent stages.

However, low supply voltages make it difficult to achieve a gain of more than roughly 10 dB while retaining linearity.

## Performance Parameters: Port-to-Port Feedthrough

$>$ Owing to device capacitances, mixers suffer from unwanted coupling (feedthrough) from one port to another.


In figure above, the gate-source and gate-drain capacitances create feedthrough from the LO port to the RF and IF ports.

In the direct-conversion receiver: LO-RF feedthrough is entirely determined by the symmetry of the mixer circuit and LO waveforms.

The LO-IF feedthrough is heavily suppressed by the baseband low-pass filter(s).


## Example of LO-RF Feedthrough in Mixer

Consider the mixer shown below, where $V_{L O}=V_{1} \cos \omega_{L O} t+V_{0}$ and $C_{G S}$ denotes the gate-source overlap capacitance of $M_{1}$. Neglecting the on-resistance of $M_{1}$ and assuming abrupt switching, determine the dc offset at the output for $R_{S}=0$ and $R_{S}$ $>0$. Assume $R_{L} \gg R_{S}$.
The LO leakage to node $X$ is expressed as

$$
V_{X}=\frac{R_{S} C_{G S} s}{R_{S} C_{G S} s+1} V_{L O}
$$

Exhibiting a magnitude of $2 \sin (\pi / 2) / \pi=2 / \pi$, this harmonic can be expressed as $(2 / \pi) \cos \omega_{L O} t$, yielding

$$
\begin{aligned}
V_{\text {out }}(t) & =V_{X}(t) \times \frac{2}{\pi} \cos \omega_{L O} t+\cdots \\
& =\frac{R_{S} C_{G S} \omega_{L O}}{\sqrt{R_{S}^{2} C_{G S}^{2} \omega_{L O}^{2}+1}} V_{1} \cos \left(\omega_{L O} t+\phi\right) \times \frac{2}{\pi} \cos \omega_{L O} t+\cdots
\end{aligned}
$$

The dc component is therefore equal to

$$
V_{d c}=\frac{V_{1}}{\pi} \frac{R_{S} C_{G S} \omega_{L O} \cos \phi}{\sqrt{R_{S}^{2} C_{G S}^{2} \omega_{L O}^{2}+1}}
$$

The As expected, the output dc offset vanishes if $R_{S}=0$.

## Generation of DC Offset: an Intuitive Perspective

Suppose, as shown below, the RF input is a sinusoid having the same frequency as the LO.

$>$ Each time the switch turns on, the same portion of the input waveform appears at the output, producing a certain average.

## Effect of Feedthrough in Direct-Conversion and Heterodyne RX


$>$ A large in-band interferer can couple to the LO and injection-pull it, thereby corrupting the $L O$ spectrum.
$>$ The RF-IF feedthrough corrupts the baseband signal by the beat component resulting from even-order distortion in the RF path.

Heterodyne RX: $\square$
$>$ Here, the LO-RF feedthrough is relatively unimportant
$>$ The LO-IF feedthrough, becomes serious if $\omega_{\text {IF }}$ and $\omega_{L O}$ are too close to allow filtering of the latter.


## Port-to-Port Feedthrough in Half-RF RX

Shown below is a receiver architecture wherein $\omega_{L O}=\omega_{R F} / 2$ so that the RF channel is translated to an IF of $\omega_{R F}-\omega_{L O}=\omega_{L O}$ and subsequently to zero. Study the effect of port-to-port feedthroughs in this architecture.

For the RF mixer, the LO-RF feedthrough is unimportant as it lies at $\omega_{R} \boldsymbol{H} \mathbf{2}$ and is suppressed. Also, the RF-LO feedthrough is not critical because in-band interferers are far from the LO frequency, creating little injection pulling. The RF-IF feedthrough proves benign because low-frequency beat components appearing at the RF port can be removed by high-pass filtering.

The most critical feedthrough in this architecture is that from the LO port to the IF port of the RF mixer. Since $\omega_{I F}=\omega_{L O}$, this leakage lies in the center of the IF channel, potentially desensitizing the IF mixers (and producing dc offsets in the baseband.).

The IF mixers also suffer from port-to-port feedthroughs.


## Mixer Noise Figures: SSB Noise Figure

For simplicity, let us consider a noiseless mixer with unity gain.

> The mixer exhibits a flat frequency response at its input from the image band to the signal band.
$>$ The noise figure of a noiseless mixer is 3 dB . This quantity is called the "single-sideband" (SSB) noise.

## Mixer Noise Figures: DSB Noise Figure

Now, consider the direct-conversion mixer shown below.

> In this case, only the noise in the signal band is translated to the baseband, thereby yielding equal input and output SNRs if the mixer is noiseless.
$>$ The noise figure is thus equal to 0 dB . This quantity is called the "doublesideband" (DSB) noise figure

## Noise Behavior in Heterodyne Receiver ( I )

A student designs the heterodyne receiver shown below for two cases: (1) $\omega_{\text {LO1 }}$ is far from $\omega_{R F}$; (2) $\omega_{L O 1}$ lies inside the band and so does the image. Study the noise behavior of the receiver in the two cases.

## Solution:

In the first case, the selectivity of the antenna, the BPF, and the LNA suppresses the thermal noise in the image band. Of course, the RF mixer still folds its own noise. The overall behavior is illustrated below, where $S_{A}$ denotes the noise spectrum at the output of the LNA and $S_{m i x}$ the noise in the input network of the mixer itself. Thus, the mixer downconverts three significant noise components to IF: the amplified noise of the antenna and the LNA around $\omega_{R F}$, its own noise around $\omega_{R F}$, and its image noise around $\omega_{i m}$.


## Noise Behavior in Heterodyne Receiver (II)

A student designs the heterodyne receiver shown below for two cases: (1) $\omega_{L O 1}$ is far from $\omega_{R F}$; (2) $\omega_{L O 1}$ lies inside the band and so does the image. Study the noise behavior of the receiver in the two cases.

## Solution:

In the second case, the noise produced by the antenna, the BPF, and the LNA exhibits a flat spectrum from the image frequency to the signal frequency. As shown on the right, the RF mixer now downconverts four significant noise components to IF: the output noise of the LNA around $\omega_{R F}$ and $\omega_{i m}$, and the input noise of the mixer around $\omega_{R F}$ and $\omega_{i m}$. We therefore conclude that the noise figure of the second frequency plan is substantially higher than that of the first. In fact, if the noise contributed by the mixer is much less than that contributed by the LNA, the noise figure penalty reaches 3 dB . The low-IF receivers of Chapter 4, on the other hand, do not suffer from this drawback because they employ image rejection.


## NF of Direct-Conversion Receivers

It is difficult to define a noise figure for receivers that translate the signal to a zero IF.


$$
\mathrm{NF}=\frac{\mathrm{SNR}_{i n}}{\mathrm{SNR}_{I}}=\frac{\mathrm{SNR}_{i n}}{\mathrm{SNR}_{Q}}
$$

$>$ This is the most common NF definition for direct-conversion receivers.
$>$ The SNR in the final combined output would serve as a more accurate measure of the noise performance, but it depends on the modulation scheme.

## Example of Noise Spectrum of a Simple Mixer (I)

Consider the simple mixer shown below. Assuming $R_{L} \gg R_{S}$ and the LO has a $50 \%$ duty cycle, determine the output noise spectrum due to $R_{S}$, i.e., assume $R_{L}$ is noiseless.

## Solution:



Since $V_{\text {out }}$ is equal to the noise of $R_{S}$ for half of the LO cycle and equal to zero for the other half, we expect the output power density to be simply equal to half of that of the input, i.e., $\mathbf{2 k T} R_{s .}$. To prove this conjecture, we view $V_{n, \text { out }}(t)$ as the product of $V_{n, R s}(t)$ and a square wave toggling between 0 and 1 . The output spectrum is thus obtained by convolving the spectra of the two. (shown in next slide)

## Example of Noise Spectrum of a Simple Mixer (II)




The output spectrum consists of (a) $2 k T R_{S} \times 0.5^{2}$, (b) $2 k T R_{S}$ shifted to the right and to the left by $\pm f_{L O}$ and multiplied by (1/m) ${ }^{2}$, (c) $2 k T R_{S}$ shifted to the right and to the left by $\pm 3 f_{L O}$ and multiplied by $[1 /(3 \pi)]^{2}$, etc. We therefore write

$$
\begin{aligned}
\overline{V_{n, \text { out }}^{2}} & =2 k T R_{S}\left[\frac{1}{2^{2}}+\frac{2}{\pi^{2}}+\frac{2}{(3 \pi)^{2}}+\frac{2}{(5 \pi)^{2}}+\cdots\right] \\
& =2 k T R_{S}\left[\frac{1}{2^{2}}+\frac{2}{\pi^{2}}\left(1+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\cdots\right)\right] .
\end{aligned}
$$

It follows that the two-sided output spectrum is equal to $k T R_{S}$ and hence the one-sided spectrum is given by

$$
\overline{V_{n, \text { out }}^{2}}=2 k T R_{S}
$$

## Single-Balanced Mixers

The simple mixer previously discussed operate with a single-ended RF input and a singleended LO. Discarding the RF signal for half of the LO period.

$>$ Figure above (left) depicts a more efficient approach whereby two switches are driven by differential LO phases, thus "commutating" the RF input to the two outputs. Called a "single-balanced" mixer.
$>$ As seen in figure above (right), the LO-RF feedthrough at $\omega_{\text {LO }}$ vanishes if the circuit is symmetric

## Double-Balanced Mixers


> We connect two single-balanced mixers such that their output LO feedthroughs cancel but their output signals do not.
> Called a "double-balanced" mixer, the circuit above operates with both balanced LO waveforms and balanced RF inputs.

## Ideal LO Waveform

$>$ The LO waveform must ideally be a square wave to ensure abrupt switching and hence maximum conversion gain.
$>$ At very high frequencies, the LO waveforms inevitably resemble sinusoids.
$>$ Downconversion of interferers located at the LO harmonics is a serious issue in broadband receiver.


## Passive Downconversion Mixers: Gain

$>$ The conversion gain in figure below is equal to $1 / \pi$ for abrupt $L O$ switching.
$>$ We call this topology a "return-to-zero" (RZ) mixer because the output falls to zero when the switch turns off.

(a)


Explain why the mixer above is ill-suited to direct-conversion receivers.
Since the square wave toggling between 0 and 1 carries an average of $0.5, V_{R F}$ itself also appears at the output with a conversion gain of 0.5 . Thus, low-frequency beat components resulting from even-order distortion in the preceding stage directly go to the output, yielding a low $\mathrm{IP}_{2}$.

## Example of Downconversion Gain of SingleBalanced Topology

Determine the conversion gain if the circuit of figure above is converted to a single-balanced topology.

## Solution:





As illustrated in figure above, the second output is similar to the first but shifted by $180{ }^{\circ}$. Thus, the differential output contains twice the amplitude of each single-ended output. The conversion gain is therefore equal to $2 / \pi(\approx-4 \mathrm{~dB}$ ). Providing differential outputs and twice the gain, this circuit is superior to the single-ended topology above.

## Example of Downconversion Gain of DoubleBalanced Topology

Determine the voltage conversion gain of a double-balanced version of the above topology. (Decompose the differential output to return-to-zero waveforms.)


In this case, $V_{\text {out } 1}$ is equal to $V_{R F^{+}}$for one half of the LO cycle and equal to $V_{R F}$ for the other half, i.e, $R_{1}$ and $R_{2}$ can be omitted because the outputs do not "float." We observe that $V_{\text {out } 1}$ $V_{\text {out } 2}$ can be decomposed into two return-to-zero waveforms, each having a peak amplitude of $2 V_{0}$. Since each of these waveforms generates an IF amplitude of $(1 / \pi) 2 V_{o}$ and since the outputs are $180^{\circ}$ out of phase, we conclude that $V_{\text {out } 1}-V_{\text {out }}$ contains an IF amplitude of $(1 / \pi)\left(4 V_{0}\right)$. Noting that the peak differential input is equal to $2 V_{0}$, we conclude that the circuit provides a voltage conversion gain of $2 / \pi$, equal to that of the single-balanced counterpart.

## Sampling Mixer: the Idea

> If the resistor is replaced with a capacitor, such an arrangement operates as a sample-and-hold circuit and exhibits a higher gain because the output is held-rather than reset-when the switch turns off.

The output waveform of figure on the right (top) can be decomposed into two as figure at bottom.

(a)


## Sampling Mixer: Conversion Gain ( I )

We first recall the following Fourier transform pairs:

$$
\begin{aligned}
\sum_{k=-\infty}^{+\infty} \delta(t-k T) & \leftrightarrow \frac{1}{T} \sum_{k=-\infty}^{+\infty} \delta\left(f-\frac{k}{T}\right) \\
x(t-T) & \leftrightarrow e^{-j \omega T} X(f) \\
\prod\left(\frac{t}{T / 2}-\frac{1}{2}\right) & \leftrightarrow \frac{1}{j \omega}\left(1-e^{-j \omega T / 2}\right)
\end{aligned}
$$

Since $y_{1}(t)$ is equal to $x(t)$ multiplied by a square wave toggling between zero and 1 , and since such a square wave is equal to the convolution of a square pulse and a train of impulses shown below,

$$
y_{1}(t)=x(t)\left[\Pi\left(\frac{t}{T_{L O} / 2}-\frac{1}{2}\right) * \sum_{k=-\infty}^{+\infty} \delta\left(t-k T_{L O}\right)\right]
$$





## Sampling Mixer: Conversion Gain (II)

$$
Y_{1}(f)=X(f) *\left[\frac{1}{j \omega}\left(1-e^{-j \omega T_{L O} / 2}\right) \frac{1}{T_{L O}} \sum_{k=-\infty}^{+\infty} \delta\left(f-\frac{k}{T_{L O}}\right)\right]
$$



The component of interest in $Y_{1}(f)$ lies at the IF and is obtained by setting $k$ to $\pm 1$

$$
\begin{gathered}
\left.Y_{1}(f)\right|_{I F}=X(f) *\left[\frac{1}{j \omega}\left(1-e^{-j \omega T_{L O} / 2}\right) \frac{1}{T_{L O}} \delta\left(f \pm \frac{1}{T_{L O}}\right)\right] \\
\left.Y_{1}(f)\right|_{I F}=\frac{X\left(f-f_{L O}\right)}{j \pi}-\frac{X\left(f+f_{L O}\right)}{j \pi}
\end{gathered}
$$

As expected, the conversion gain from $X(f)$ to $Y_{1}(f)$ is equal to $1 / \pi$, but with a phase shift of $90^{\circ}$.

## Sampling Mixer: Conversion Gain (III)

The second output, $y_{2}(t)$, can be viewed as a train of impulses that sample the input and are subsequently convolved with a square pulse:

$$
\begin{aligned}
& y_{2}(t)=\left[x(t) \sum_{k=-\infty}^{+\infty} \delta\left(t-k T_{L O}-\frac{T_{L O}}{2}\right)\right] * \prod\left(\frac{t}{T_{L O} / 2}-\frac{1}{2}\right) \\
& \frac{\square}{y_{2}(t)} \equiv \frac{\Delta}{\frac{T_{\mathrm{LO}}}{2}} T_{\mathrm{LO}} \frac{3 T_{\mathrm{LO}}}{2} 2 T_{\mathrm{LO}} \frac{5 T_{\mathrm{LO}}}{2} t \\
& \frac{T_{\mathrm{LO}}}{2} \quad \frac{3 T_{\mathrm{LO}}}{2} \\
& Y_{2}(f)=\left[X(f) * \frac{1}{T_{L O}} \sum_{k=-\infty}^{+\infty} e^{-j \omega T_{L O} / 2} \delta\left(f-\frac{k}{T_{L O}}\right)\right] \cdot \frac{1}{0}\left(1-e^{-j \omega T_{L O} / 2}\right)
\end{aligned}
$$

Figure below depicts the spectrum, revealing that shifted replicas of $X(f)$ are multiplied by a sinc envelope.
in $Y_{2}(f)$


## Sampling Mixer: Conversion Gain (IV)

The component of interest in $Y_{2}(f)$ is obtained by setting $k$ to $\pm 1$

$$
\left.Y_{2}(f)\right|_{I F}=\frac{1}{T_{L O}}\left[-X\left(f-f_{L O}\right)-X\left(f+f_{L O}\right)\right]\left[\frac{1}{j \omega}\left(1-e^{-j \omega T_{L O} / 2}\right)\right]
$$

If the IF is much lower than $\mathbf{2 f} f_{\text {LO }}$

$$
\left.Y_{2}(f)\right|_{I F} \approx \frac{-X\left(f-f_{L O}\right)-X\left(f+f_{L O}\right)}{2}
$$

The total IF output is therefore equal to

$$
\begin{aligned}
\left|Y_{1}(f)+Y_{2}(f)\right|_{I F} & =\sqrt{\frac{1}{\pi^{2}}+\frac{1}{4}}\left[\left|X\left(f-f_{L O}\right)\right|+\left|X\left(f+f_{L O}\right)\right|\right] \\
& =0.593\left[\left|X\left(f-f_{L O}\right)\right|+\left|X\left(f+f_{L O}\right)\right|\right] .
\end{aligned}
$$

> If realized as a single-balanced topology, the circuit provides a gain twice this value.
$>$ Though a passive circuit, the single-ended sampling mixer actually has a voltage conversion gain greater than unity.


## Example of the Voltage Conversion Gain of DoubleBalanced Sampling Mixer

Determine the voltage conversion gain of a double-balanced sampling mixer.

## Solution:

The capacitors play no role here because each output is equal to one of the inputs at any given point in time. The conversion gain is therefore equal to $2 / \pi$, about 5.5 dB lower than that of the single-balanced topology discussed above.


## Output Current Combining of Two Single-Band Mixers


$>$ If necessary, double-balanced operation can be realized through the use of two single-balanced mixers whose outputs are summed in the current domain.
$>$ In this case, the mixer conversion gain is still equal to 1.48 dB .

## LO Self-Mixing


$>$ Due to the nonlinearity of $C_{G S 1}$ and $C_{G S 2}$ arising from large LO amplitudes, $V_{P}$ does change with time but only at twice the LO frequency.
$>$ Upon mixing with the LO signal, this component is translated to $f_{L O}$ and $3 f_{L O}$ but not to dc.
$>$ In practice, however, mismatches between $M_{1}$ and $M_{2}$ and within the oscillator circuit give rise to a finite LO leakage to node $P$.

## Noise

The output noise is given by $4 k T\left(R_{\text {on }} \| R_{L}\right)$ when $S_{1}$ is on and by $4 k T R_{L}$ when it is off. On the average,

$$
\overline{V_{n, \text { out }}^{2}}=2 k T\left[\left(R_{\text {on }} \| R_{L}\right)+R_{L}\right]
$$

If we select $R_{\text {on }} \ll R_{L}$ to minimize the conversion loss,

$$
\overline{V_{n, \text { out }}^{2}} \approx 2 k T R_{L}
$$



Dividing the result by $1 / \pi^{2}$

$$
\begin{aligned}
\overline{V_{n, i n}^{2}} & \approx 2 \pi^{2} k T R_{L} \\
& \approx 20 k T R_{L}
\end{aligned}
$$

If $R_{\text {on }}=100 \Omega$ and $R_{L}=1 \mathrm{k} \Omega$, determine the input-referred noise of the above RZ mixer.

Solution:

$$
\sqrt{\overline{V_{n, i n}^{2}}}=8.14 \mathrm{nV} / \sqrt{\mathrm{Hz}}
$$

This noise would correspond to a noise figure of $10 \log \left[1+(8.14 / 0.91)^{2}\right]=19 \mathrm{~dB}$ in a $50-\Omega$ system.

## Noise Spectrum of Sampling Mixer: Three Observations

$>$ First, in the simple circuit on the right, if $V_{i n}=0$,

$$
\overline{V_{n, L P F}^{2}}=\overline{V_{n R 1}^{2}} \frac{1}{1+\left(R_{1} C_{1} \omega\right)^{2}}
$$

We say the noise is "shaped" by the filter.

$>$ Second, in the switching circuit on the right, the output is equal to the shaped noise of $R_{1}$ when $S_{1}$ is on and a sampled, constant value when it is off.

$>$ Third, we can decompose the output into two waveforms $V_{n 1}$ and $V_{n 2}$ as shown on the right.

## Noise Spectrum of Sampling Mixer: Spectrum of $V_{n 1}$

We view this waveform as the product of $V_{n, \text { LPF }}(t)$ and a square wave toggling between 0 and 1.


In practice, the sampling bandwidth of the mixer, $1 /\left(R_{1} C_{1}\right)$, rarely exceeds $3 \omega_{L O}$, and hence

$$
\overline{V_{n 1}^{2}}(f)=2 \times\left(\frac{1}{\pi^{2}}+\frac{1}{9 \pi^{2}}\right) \frac{2 k T R_{1}}{1+\left(2 \pi R_{1} C_{1} f\right)^{2}}
$$

At low output frequencies, this expression reduces to:

$$
\overline{V_{n 1}^{2}}=0.226\left(2 k T R_{1}\right)
$$

Note that this is the two-sided spectrum.

## Noise Spectrum of Sampling Mixer: Spectrum of $V_{n 2}$



The sum of these aliased components is given by

$$
\begin{aligned}
\overline{V_{n, \text { alias }}^{2}} & =2 \times \frac{2 k T R_{1}}{T_{L O}^{2}}\left[\frac{1}{1+4 \pi^{2} R_{1}^{2} C_{1}^{2} f_{L O}^{2}}+\frac{1}{1+4 \pi^{2} R_{1}^{2} C_{1}^{2}\left(2 f_{L O}^{2}\right)}+\cdots\right] \\
& =2 \times \frac{2 k T R_{1}}{T_{L O}^{2}} \sum_{n=1}^{\infty} \frac{1}{1+a^{2} n^{2}},
\end{aligned}
$$

For the summation in equation above, we have

$$
\begin{aligned}
& \qquad \sum_{n=1}^{\infty} \frac{1}{1+a^{2} n^{2}}=\frac{1}{2}\left(\frac{\pi}{a} \operatorname{coth} \frac{\pi}{a}-1\right) \\
& \quad \overline{V_{n, \text { alias }}^{2}}=\frac{k T}{T_{L O}^{2}}\left(\frac{1}{C_{1} f_{L O}}-2 R_{1}\right) \\
& \text { Chapter } 6 \text { Mixers } \\
& \square\rangle \overline{V_{n 2}^{2}}=k T\left(\frac{1}{4 C_{1} f_{L O}}-\frac{R_{1}}{2}\right)
\end{aligned}
$$

## Noise Spectrum of Sampling Mixer: Correlation Between $V_{n 1}$ and $V_{n 2}$ (I)

$>$ The correlation arises from two mechanisms:
(1) as the circuit enters the track mode, the previous sampled value takes a finite time to vanish
(2) when the circuit enters the hold mode, the frozen noise value, $V_{n 2}$, is partially correlated with $V_{n 1}$.

$>$ The former mechanism is typically negligible.
$>$ For the latter, we recognize that the noise frequency components far below $f_{\text {LO }}$ remain relatively constant during the track and hold modes.

## Noise Spectrum of Sampling Mixer: Correlation Between $V_{n 1}$ and $V_{n 2}$ (II)

Summing the one-sided spectra of $V_{n 1}$ and $V_{n 2}$ and the low-frequency contribution, $4 k T R_{1}$, gives the total (one-sided) output noise at the IF:

$$
\overline{V_{n, \text { out }, I F}^{2}}=k T\left(3.9 R_{1}+\frac{1}{2 C_{1} f_{L O}}\right)
$$

The input-referred noise is obtained by dividing this result by $1 / \boldsymbol{\pi}^{\mathbf{2}}+\mathbf{1} / 4$ :

$$
\overline{V_{n, i n}^{2}}=2.85 k T\left(3.9 R_{1}+\frac{1}{2 C_{1} f_{L O}}\right)
$$

The input-referred noise of a single-balanced passive (sampling) mixer is equal to

$$
\begin{aligned}
\overline{V_{n, i n, S B}^{2}} & =\frac{k T}{2\left(\frac{1}{\pi^{2}}+\frac{1}{4}\right)}\left(3.9 R_{1}+\frac{1}{2 C_{1} f_{L O}}\right) \\
& =1.42 k T\left(3.9 R_{1}+\frac{1}{2 C_{1} f_{L O}}\right) .
\end{aligned}
$$

For the double-balanced passive mixer

$$
\overline{V_{n, i n}^{2}}=2 \pi^{2} k T R_{1}
$$



## Noise of the Subsequent Stage

The low gain of passive mixers makes the noise of the subsequent stage critical.


As shown above (left), each common-source stage exhibits an input-referred noise voltage of

$$
\overline{V_{n, C S}^{2}}=\frac{4 k T \gamma}{g_{m}}+\frac{4 k T}{g_{m}^{2} R_{D}}
$$

Shown above (middle), the network consisting of $R_{\text {REF }}, M_{R E F}$, and $I_{\text {REF }}$ defines the bias current of $M_{1}$ and $M_{2}$.

Can the circuit be arranged as above (right) so that the bias resistors provide a path to remove the dc offset?

## Example of DC Offset

A student considers the arrangement shown in figure below (left), where $V_{\text {in }}$ models the LO leakage to the input. The student then decides that the arrangement below (middle) is free from dc offsets, reasoning that a positive dc voltage, $V_{d c}$, at the output would lead to a dc current, $V_{d c} / R_{L}$, through $R_{L}$ and hence an equal current through $R_{S}$. This is impossible because it gives rise to a negative voltage at node $X$. Does the student deserve an $A$ ?

## Solution:






The average voltage at node $X$ can be negative. As shown above (right), $V_{X}$ is an attenuated version of $V_{i n}$ when $S_{1}$ is on and equal to $V_{i n}$ when $S_{1}$ is off. Thus, the average value of $V_{X}$ is negative while $R_{L}$ carries a finite average current as well. That is, the circuit above (middle) still suffers from a dc offset.

## Input Impedance ( I )



The current drawn by $C_{1}$ is equal to:

$$
i_{o u t}(t)=C_{1} \frac{d y}{d t}
$$

Taking the Fourier transform, we thus have

$$
I_{i n}(f)=C_{1} j \omega Y(f)
$$

We set $\boldsymbol{k}$ in previous discussion to zero so that $X(f)$ is simply convolved with $\delta(f)$
expression for the input admittance:

$$
\begin{aligned}
\frac{I_{i n}(f)}{C_{1} j \omega} & =X(f) *\left[\frac{1}{j \omega}\left(1-e^{-j \omega T_{L O} / 2}\right) \frac{1}{T_{L O}} \delta(f)\right] \\
& +\left\{X(f) *\left[\frac{1}{T_{L O}} e^{-j \omega T_{L O} / 2} \delta(f)\right]\right\} \frac{1}{j \omega}\left(1-e^{-j \omega T_{L O} / 2}\right)
\end{aligned}
$$

$$
\frac{I_{i n}(f)}{X(f)}=j C_{1} \omega\left[\frac{1}{2}+\frac{1}{j \omega T_{L O}}\left(1-e^{-j \omega T_{L O} / 2}\right)\right]
$$

## Input Impedance ( II )

If $\boldsymbol{\omega}$ (the input frequency) is much less than $\omega_{\text {LO }}$, then the second term in the square brackets reduces to $1 / 2$ and

$$
\frac{I_{i n}(f)}{X(f)}=j C_{1} \omega
$$



If $\omega \approx \mathbf{2 \pi} f_{L O}$ (as in direct-conversion receivers), then the second term is equal to $1 /(j \pi)$ and

$$
\frac{I_{i n}(f)}{X(f)}=\frac{j C_{1} \omega}{2}+2 f C_{1}
$$



Finally, if $\boldsymbol{\omega} \gg \mathbf{2} \boldsymbol{\pi} f_{L O}$, the second term is much less than the first, yielding

$$
\frac{I_{i n}(f)}{X(f)}=\frac{j C_{1} \omega}{2}
$$

For the input impedance of a For the input impedance of a
single-balanced mixer, If $\boldsymbol{\omega} \approx \boldsymbol{\omega}_{\mathbf{L o}}$, then: $\quad Z_{\text {in,SB }}=\frac{1}{2}\left[R_{1}+\frac{1}{\frac{j C_{1} \omega}{2}+2 f C_{1}}\right]$

## Flicker Noise and LO Swing

$>$ An important advantage of passive mixers over their active counterparts is their much lower output flicker noise.


MOSFETs produce little flicker noise if they carry a small current, a condition satisfied in a passive sampling mixer if the load capacitance is relatively small.

However, the low gain of passive mixers makes the $1 / f$ noise contribution of the subsequent stage critical.
$>$ Passive MOS mixers require large (rail-to-rail) LO swings, a disadvantage with respect to active mixers.

## Current Driven Passive Mixers: Input Impedance

Voltage-driven and current-driven passive mixers entail a number of interesting differences.
> First, the input impedance of the currentdriven mixer shown here is quite different from that of the voltage-driven counterpart.

In a passive mixer, we cannot calculate the input
 impedance of an LNA by applying a voltage or a current source to the input port because it is a time-variant circuit.

In The input current is routed to the upper arm for $50 \%$ of the time and flows through $\mathbf{Z}_{B B}$.

$$
V_{1}(t)=\left[i_{i n}(t) \times S(t)\right] * h(t)
$$

In the frequency domain:

$$
V_{1}(f)=\left[I_{i n}(f) * S(f)\right] \cdot Z_{B B}(f)
$$



## Current Driven Passive Mixers: Spectra at Input and Output

$>$ The switches in figure above also mix the baseband waveforms with the LO, delivering the upconverted voltages to node $A$. Thus, $V_{1}(t)$ is multiplied by $S(t)$ as it returns to the input, and its spectrum is translated to RF. The spectrum of $V_{2}(t)$ is also upconverted and added to this result.


## Current Driven Passive Mixers: Noise and Nonlinearity Contribution, Duty Cycle

$>$ The second property of current-driven passive mixers is that their noise and nonlinearity contribution is reduced.
> Passive mixers need not employ a $50 \%$ LO duty cycle. In fact, both voltagedriven and current driven mixers utilizing a $25 \%$ duty cycle provide a higher gain.


Voltage-driven the RF current entering each switch generates an IF current given by

$$
I_{I F}(t)=\frac{2}{\pi} \frac{\sin \pi d}{2 d} I_{R F 0} \cos \omega_{I F} t
$$

As expected, $d=0.5$ yields a gain of $2 / \pi$. More importantly, for $d=0.25$, the gain reaches $2 \sqrt{2} / \pi, 3 \mathrm{~dB}$ higher.

## Active Downconversion Mixers: Function and Typical Realization

> Mixers can be realized so as to achieve conversion gain in one stage.
> Called active mixers, such topologies perform three functions: they convert the RF voltage to a current, "commutate" (steer) the RF current by the LO, and convert the IF current to voltage.

$>$ We call $M_{2}$ and $M_{3}$ the "switching pair."
$>$ The switching pair does not need rail-to-rail LO swings.

## Active Downconversion Mixers: Double-Balanced Topology


$>$ One advantage of double-balanced mixers over their single-balanced counterparts stems from their rejection of amplitude noise in the LO waveform.

## Conversion Gain

With abrupt LO switching, the circuit reduces to that shown in figure below (left).


$$
I_{2}=I_{R F} \cdot S\left(t-\frac{T_{L O}}{2}\right)
$$

We have for $\boldsymbol{R}_{\mathbf{1}}=\mathbf{R}_{\mathbf{2}}=\boldsymbol{R}_{\boldsymbol{D}} \quad V_{\text {out }}(t)=I_{R F} R_{D}\left[S\left(t-\frac{T_{L O}}{2}\right)-S(t)\right]$
The waveform exhibits a fundamental amplitude equal to $4 / \pi$, yielding an output given by

$$
\begin{gathered}
V_{\text {out }}(t)=I_{R F}(t) R_{D} \cdot \frac{4}{\pi} \cos \omega_{L O} t+\cdots \\
V_{I F}(t)=\frac{2}{\pi} g_{m 1} R_{D} V_{R F} \cos \left(\omega_{R F}-\omega_{L O}\right) t \Leftrightarrow \frac{V_{I F, p}}{V_{R F, p}}=\frac{2}{\pi} g_{m 1} R_{D}
\end{gathered}
$$

## Active Mixer with LO at CM Level ( I )



If $M_{1}$ is at the edge of saturation, then

$$
V_{C M, L O}-V_{G S 2,3} \geq V_{G S 1}-V_{T H 1}
$$

For $\boldsymbol{M}_{\mathbf{2}}$ to remain in saturation up to this point, its drain voltage

$$
V_{X, \min }=V_{C M, L O}+\frac{\sqrt{2}}{2}\left(V_{G S 2,3}-V_{T H 2}\right)-V_{T H 2}
$$

which reduces to

$$
V_{X, \min }=V_{G S 1}-V_{T H 1}+\left(1+\frac{\sqrt{2}}{2}\right)\left(V_{G S 2,3}-V_{T H 2}\right)
$$

## Active Mixer with LO at CM Level (II)

The maximum allowable dc voltage across each load resistor is equal to

$$
V_{R, \text { max }}=V_{D D}-\left[V_{G S 1}-V_{T H 1}+\left(1+\frac{\sqrt{2}}{2}\right)\left(V_{G S 2,3}-V_{T H 2}\right)\right]
$$

Since each resistor carries half of $I_{D 1}$

$$
R_{D, \max }=\frac{2 V_{R, \max }}{I_{D 1}}
$$

we obtain the maximum voltage conversion gain as

$$
\begin{aligned}
A_{V, \max } & =\frac{2}{\pi} g_{m 1} R_{D, \max } \\
& =\frac{8}{\pi} \frac{V_{R, \max }}{V_{G S 1}-V_{T H 1}}
\end{aligned}
$$

> Low supply voltages severely limit the gain of active mixers.

## RF Current as a CM Component

The conversion gain may also fall if the LO swing is lowered.

$>$ While $M_{2}$ and $M_{3}$ are near equilibrium, the RF current produced by $M_{1}$ is split approximately equally between them, thus appearing as a common-mode current and yielding little conversion gain for that period of time.
$>$ Reduction of the LO swing tends to increase this time and lower the gain

## Dual-Gate Mixer

Figure below shows a "dual-gate mixer," where $M_{1}$ and $M_{2}$ can be viewed as one transistor with two gates. Identify the drawbacks of this circuit.

For $M_{2}$ to operate as a switch, its gate voltage must fall to $V_{T H 2}$ above zero regardless of the overdrive voltages of the two transistors. For this reason, the dual-gate mixer typically calls for larger LO swings than the single-balanced active topology does. Furthermore, since the RF current of $M_{1}$ is now multiplied by a square wave toggling between 0 and 1 , the conversion gain is half:

$$
A_{V}=\frac{1}{\pi} g_{m 1} R_{D}
$$

Additionally, all of the frequency components produced by $M_{1}$ appear at the output without translation because they are multiplied by the average value of the square wave, $1 / 2$. Thus, half of the flicker noise of $M_{1}$-a high-frequency device and hence small-emerges at IF. Also, low-frequency beat components resulting from even-order distortion in $M_{1}$ directly corrupt the output, leading to a low $\mathrm{IP}_{2}$. The dual-gate mixer does not require differential LO waveforms, a minor advantage. For these reasons, this topology is rarely used in modern RF design.


## Effect of Gradual LO Transitions

> With a sinusoidal LO, the drain currents of the switching devices depart from square waves, remaining approximately equal for a fraction of each half cycle, $\Delta T$. The circuit exhibits little conversion gain during these periods.


We surmise that the overall gain of the mixer is reduced to

$$
\begin{aligned}
A_{V} & =\frac{2}{\pi} g_{m 1} R_{D}\left(1-\frac{2 \Delta T}{T_{L O}}\right) \\
& =\frac{2}{\pi} g_{m 1} R_{D}\left[1-\frac{\left(V_{G S}-V_{T H}\right)_{e q}}{5 \pi V_{p, L O}}\right]
\end{aligned}
$$

## Examples of Conversion Gain Calculation

A single-balanced active mixer requires an overdrive voltage of 300 mV for the input V/I converter transistor. If each switching transistor has an equilibrium overdrive of 150 mV and the peak LO swing is 300 mV , how much conversion gain can be obtained with a 1-V supply?

$V_{R, \max }=444 \mathrm{mV}$ and hence $\quad$| $A_{V, \max }$ | $=3.77$ |
| ---: | :--- |
|  | $\approx 11.5 \mathrm{~dB}$ |,$~$

Owing to the relatively low conversion gain, the noise contributed by the load resistors and following stages may become significant.

Repeat the above example but take the gradual LO edges into account.
The gain expressed above must be multiplied by $1-0.0318 \approx 0.97$ :


Thus, the gradual LO transitions lower the gain by about 0.2 dB .

## Gain Degradation Due to Capacitance at Drain of Input Transistor



With abrupt LO edges, $M_{2}$ is on and $M_{3}$ is off, yielding a total capacitance at node $P$ equal to:

$$
C_{P}=C_{D B 1}+C_{G S 2}+C_{G S 3}+C_{S B 2}+C_{S B 3} .
$$

The RF current produced by $M_{1}$ is split between $C_{P}$ and the resistance seen at the source of $M_{2}, \mathbf{1} / \boldsymbol{g}_{m 2}$. The voltage conversion gain is modified as:

$$
A_{V, \text { max }}=\frac{2}{\pi} g_{m 1} R_{D}\left[1-\frac{2\left(V_{G S}-V_{T H}\right)_{e q}}{5 \pi V_{P, L O}}\right] \frac{g_{m 2}}{\sqrt{C_{P}^{2} \omega^{2}+g_{m 2}^{2}}}
$$

## Calculation with Output Resistance of $\boldsymbol{M}_{\mathbf{2}}$

If the output resistance of $\boldsymbol{M}_{2}$ in figure above is not neglected, how should it be included in the calculations?

## Solution:

Since the output frequency of the mixer is much lower than the input and LO frequencies, a capacitor is usually tied from each output node to ground to filter the unwanted components. As a result, the resistance seen at the source of $\boldsymbol{M}_{2}$ in figure below is simply equal to $\left(1 / g_{m}\right) \| r_{o 2}$ because the output capacitor establishes an ac ground at the drain of $M_{2}$ at the input frequency.

## Comparison: Voltage Conversion Gains of SingleBalanced and Double-Balanced Active Mixers

Compare the voltage conversion gains of single-balanced and double-balanced active mixers.

From previous discussion, we know that $\left(V_{X 1}-V_{Y 1}\right) / V_{R F}{ }^{+}$is equal to the voltage conversion gain of a single-balanced mixer. Also, $V_{X 1}=V_{Y 2}$ and $V_{Y 1}=V_{X 2}$ if $V_{R F}=-V_{R F^{+}}$. Thus, if $Y_{2}$ is shorted to $X_{1}$, and $X_{2}$ to $Y_{1}$, these node voltages remain unchanged. The differential voltage conversion gain of the double-balanced topology is therefore given by

$$
\frac{V_{X}-V_{Y}}{V_{R F}^{+}-V_{R F}^{-}}=\frac{V_{X 1}-V_{Y 1}}{2 V_{R F}^{+}}
$$

which is half of that of the single-balanced counterpart. This reduction arises because the limited voltage headroom disallows a load resistance of $R_{D}$


## Noise in Active Mixers: Starting Analysis

$>$ The noise components of interest lie in the RF range before downconversion and in the IF range after downconversion.

> The frequency translation of RF noise by the switching devices prohibits the direct use of small-signal ac and noise analysis in circuit simulators, necessitating simulations in the time domain.
$>$ Moreover, the noise contributed by the switching devices exhibits time-varying statistics,

## Noise in Active Mixers: Qualitative Analysis (I)



First assume abrupt LO transitions and consider the representation in figure above for half of the LO cycle.

$$
C_{P}=C_{G D 1}+C_{D B 1}+C_{S B 2}+C_{S B 3}+C_{G S 3} .
$$

$>$ In this phase, the circuit reduces to a cascode structure, with $\boldsymbol{M}_{\mathbf{2}}$ contributing some noise because of the capacitance at node $P$. At frequencies well below $f_{T}$, the output noise current generated by $M_{2}$ is equal to $V_{n, M 2} C_{P} s$.
$>$ This noise and the noise current of $M_{1}$ (which is dominant) are multiplied by a square wave toggling between 0 and 1.

## Noise in Active Mixers: Qualitative Analysis (II)

Consider a more realistic case where the LO transitions are not abrupt.

> The circuit now resembles a differential pair near equilibrium, amplifying the noise of $M_{2}$ and $M_{3}$-while the noise of $M_{1}$ has little effect on the output because it behaves as a common-mode disturbance.

## Comparison: Noise Behavior of Single-Balanced and Double-Balanced Active Mixers ( I )

Compare single-balanced and double-balanced active mixers in terms of their noise behavior. Assume the latter's total bias current is twice the former's.


Let us first study the output noise currents of the mixers. If the total differential output noise current of the single-balanced topology is $\overline{I_{n, s i n g}^{2}}$
then that of the double-balanced circuit is equal to

$$
\overline{I_{n, \text { doub }}^{2}}=2 \overline{I_{n, \text { sing }}^{2}}
$$

## Comparison: Noise Behavior of Single-Balanced and Double-Balanced Active Mixers (II)



Next, we determine the output noise voltages, bearing in mind that the load resistors differ by a factor of two We have

$$
\begin{aligned}
& \overline{V_{n, \text { out }, \text { sing }}^{2}}=\overline{I_{n, \text { sing }}^{2}}\left(R_{D}\right)^{2} \\
& \overline{V_{n, \text { out }, \text { doub }}^{2}}=\overline{I_{n, \text { ooub }}^{2}}\left(\frac{R_{D}}{2}\right)^{2}
\end{aligned}
$$

Recall that the voltage conversion gain of the double-balanced mixer is half of that of the single-balanced topology. Thus, the input-referred noise voltages of the two circuits are related by

$$
\overline{V_{n, i n, \operatorname{sing}}^{2}}=\frac{1}{2} \overline{V_{n, i n, d o u b}^{2}}
$$

## Effect of LO Buffer Noise on Single-Balanced Mixer

$>$ The noise generated by the local oscillator and its buffer becomes indistinguishable from the noise of $M_{2}$ and $M_{3}$ when these two transistors are around equilibrium.

$>$ A differential pair serving as the LO buffer may produce an output noise much higher than that of $M_{2}$ and $M_{3}$. It is therefore necessary to simulate the noise behavior of mixers with the LO circuitry present.

## Effect of LO Noise on Double-Balanced Mixer

Study the effect of LO noise on the performance of double-balanced active mixers.

## Solution:

Drawing the circuit as shown below, we note that the LO noise voltage is converted to current by each switching pair and summed with opposite polarities. Thus, the doublebalanced topology is much more immune to LO noise-a useful property obtained at the cost of the $3-\mathrm{dB}$ noise and the higher power dissipation.


## Noise in Active Mixers: Quantitative Analysis ( I )

$>$ To estimate the input-referred noise voltage, we apply the following procedure:
(1) for each source of noise, determine a "conversion gain" to the IF output;
(2) multiply the magnitude of each noise by the corresponding gain and add up all of the resulting powers, thus obtaining the total noise at the IF output;
(3) divide the output noise by the overall conversion gain of the mixer to refer it to the input.
half of the noise powers (squared current quantities) of $M_{1}$ and $M_{2}$ is injected into node $X$, the total noise at node $X$ is equal to

$$
\overline{V_{n, X}^{2}}=\frac{1}{2}\left(\overline{I_{n, M 1}^{2}}+\overline{V_{n, M 2}^{2}} C_{P}^{2} \omega^{2}\right) R_{D}^{2}+4 k T R_{D}
$$



## Noise in Active Mixers: Quantitative Analysis (II)

The noise power must be doubled to account for that at node $Y$ and then divided by the square of the conversion gain. The input-referred noise voltage is

$$
\begin{aligned}
\overline{V_{n, i n}^{2}} & =\frac{\left(4 k T \gamma g_{m 1}+\frac{4 k T \gamma}{g_{m 2}} C_{P}^{2} \omega^{2}\right) R_{D}^{2}+8 k T R_{D}}{\frac{4}{\pi^{2}} g_{m 1}^{2} R_{D}^{2} \frac{g_{m 2}^{2}}{C_{P}^{2} \omega^{2}+g_{m 2}^{2}}} \\
& =\pi^{2}\left(\frac{C_{P}^{2} \omega^{2}}{g_{m 2}^{2}}+1\right) k T\left(\frac{\gamma}{g_{m 1}}+\frac{\gamma C_{P}^{2} \omega^{2}}{g_{m 2} g_{m 1}^{2}}+\frac{2}{g_{m 1}^{2} R_{D}}\right)
\end{aligned}
$$

If the effect of $C_{P}$ is negligible:

$$
\overline{V_{n, i n}^{2}}=\pi^{2} k T\left(\frac{\gamma}{g_{m 1}}+\frac{2}{g_{m 1}^{2} R_{D}}\right)
$$

Compare equation above with the input-referred noise voltage of a commonsource stage having the same transconductance and load resistance.
For the CS stage,

$$
\overline{V_{n, i n, C S}^{2}}=4 k T\left(\frac{\gamma}{g_{m 1}}+\frac{1}{g_{m 1}^{2} R_{D}}\right)
$$

Even if the second term in the parentheses is negligible, the mixer exhibits 3.92 dB higher noise power. Chapter 6 Mixers

## Noise in Active Mixers: Effect of Gradual LO Transitions on the Noise Behavior

$M_{2}$ and $M_{3}$ operate as a differential pair


The input-referred noise is given by:

$$
\overline{V_{n, i n}^{2}}=\frac{8 k T\left(\gamma g_{m 2} R_{D}^{2}+R_{D}\right) \frac{2 \Delta T}{T_{L O}}+\left[4 k T \gamma\left(g_{m 1}+\frac{C_{P}^{2} \omega^{2}}{g_{m 2}}\right) R_{D}^{2}+8 k T R_{D}\right]\left(1-\frac{2 \Delta T}{T_{L O}}\right)}{\frac{4}{\pi^{2}} g_{m 1}^{2} R_{D}^{2} \frac{g_{m 2}^{2}}{C_{P}^{2} \omega^{2}+g_{m 2}^{2}}\left(1-\frac{2 \Delta T}{T_{L O}}\right)^{2}}
$$

## Effect of Scaling on Noise

A single-balanced mixer is designed for a certain $\mathrm{IP}_{3}$, bias current, LO swing, and supply voltage. Upon calculation of the noise, we find it unacceptably high. What can be done to lower the noise?

## Solution:

The overdrive voltages and the dc drop across the load resistors offer little flexibility. We must therefore sacrifice power for noise by a direct scaling of the design. the idea is to scale the transistor widths and currents by a factor of $\alpha$ and the load resistors by a factor of $1 / \alpha$.



Unfortunately, this scaling also scales the capacitances seen at the RF and LO ports, making the design of the LNA and the LO buffer more difficult and/or more power-hungry.

## Flicker Noise ( I )



Only the flicker noise of $M_{2}$ and $M_{3}$ must be considered. computing the time at which the gate voltages of $\boldsymbol{M}_{1}$ and $\boldsymbol{M}_{2}$ are equal

$$
\begin{aligned}
V_{C M}+V_{p, L O} \sin \omega_{L O} t+V_{n 2}(t) & =V_{C M}-V_{p, L O} \sin \omega_{L O} t \\
\triangleleft 2 V_{p, L O} \sin \omega_{L O} t & =-V_{n 2}(t)
\end{aligned}
$$

In the vicinity of $\boldsymbol{t}=\mathbf{0}$ :

$$
2 V_{p, L O} \omega_{L O} t \approx-V_{n 2}(t)
$$

Crossing is displaced by $\Delta T$

$$
2 V_{p, L O} \omega_{L O} \Delta T \approx-V_{n 2}(t)
$$

$$
\Rightarrow|\Delta T|=\frac{\left|V_{n 2}(t)\right|}{2 V_{p, L O} \omega_{L O}}
$$

## Flicker Noise ( II )



If each narrow pulse is approximated by an impulse, the noise waveform in $I_{D 2}-I_{D 3}$ can be expressed as

$$
\begin{aligned}
I_{n, \text { out }}(t) & =\sum_{k=-\infty}^{+\infty} \frac{2 I_{S S} V_{n 2}(t)}{S_{L O}} \delta\left(t-k \frac{T_{L O}}{2}\right) \\
\qquad I_{n, \text { out }}(f) & =\frac{4 I_{S S}}{T_{L O} S_{L O}} \sum_{k=-\infty}^{+\infty} V_{n 2}(f) \delta\left(t-2 k f_{L O}\right)
\end{aligned}
$$

The baseband component is obtained for $k=0$ because $V_{n 2}(f)$ has a low-pass spectrum.

$$
\begin{aligned}
\left.I_{n, \text { out }}(f)\right|_{k=0} & =\frac{I_{S S}}{\pi V_{p, L O}} V_{n 2}(f) \\
\left.V_{n, \text { out }}(f)\right|_{k=0} & =\frac{I_{S S} R_{D}}{\pi V_{p, L O}} V_{n 2}(f)
\end{aligned}
$$

## Flicker Noise Referred to the Input

## Refer the noise found above to the input of the mixer.

## Solution:

Considering Noise of $M_{3}$ and dividing the conversion gain, we have

$$
\begin{aligned}
\left.V_{n, i n}(f)\right|_{k=0} & =\frac{\sqrt{2} I_{S S}}{2 g_{m 1} V_{p, L O}} V_{n 2}(f) \\
& =\frac{\sqrt{2}\left(V_{G S}-V_{T H}\right)_{1}}{4 V_{p, L O}} V_{n 2}(f)
\end{aligned}
$$

(1) $V_{n 2}(f)$ is typically very large because $M_{2}$ and $M_{3}$ are relatively small, and (2) the noise voltage found above must be multiplied by $\sqrt{2}$ to account for the noise of $M_{3}$.
> Another flicker noise mechanism in active mixers arises from the finite capacitance at node $P$.

The differential output current in this case includes a flicker noise component

$$
I_{n, \text { out }}(f)=2 f_{L O} C_{P} V_{n 2}(f)
$$

## Linearity

$>$ The input transistor imposes a direct trade-off between nonlinearity and noise.

$$
\begin{aligned}
\mathrm{IP}_{3} & \propto V_{G S}-V_{T H} \\
\frac{4 k T \gamma}{V_{n, i n}^{2}} & =\frac{4 k T \gamma}{g_{m}}=\frac{4 I_{D}}{2 I_{D}}\left(V_{G S}-V_{T H}\right)
\end{aligned}
$$

$>$ The linearity of active mixers degrades if the switching transistors enter the triode region. Thus, the LO swings cannot be arbitrarily large.


## Compression

$>$ If the output swings become excessively large, the circuit begins to compress at the output rather than at the input.

> An active mixer exhibits a voltage conversion gain of 10 dB and an input 1-dB compression point of $355 \mathrm{mV}_{p p}$ ( $=-5 \mathrm{dBm}$ ). Is it possible that the switching devices contribute compression?

## Solution:

At an input level of -5 dBm , the mixer gain drops to 9 dB , leading to an output differential swing of $355 \mathrm{mV}_{p p} \times 2.82 \approx 1 \mathrm{~V}_{p p}$. Thus, each output node experiences a peak swing of 250 mV . If the LO drive is large enough, the switching devices enter the triode region and compress the gain.
> The input transistor may introduce compression even if it satisfies the quadratic characteristics of long-channel MOSFETs.

$$
V_{P} \approx-g_{m 1} R_{P} V_{R F}
$$

With a large input level, the gate voltage of the device rises while the drain voltage falls, possibly driving it into the triode region.

## Active Mixer Design Example ( I ): Design Parameter Assignment

Design a 6-GHz active mixer in $65-\mathrm{nm}$ technology with a bias current of 2 mA from a 1.2-V supply. Assume direct downconversion with a peak single-ended sinusoidal LO swing of 400 mV .

## Solution:

The design of the mixer is constrained by the limited voltage headroom. We begin by assigning an overdrive voltage of 300 mV to the input transistor, $M_{1}$, and 150 mV to the switching devices, $M_{2}$ and $M_{3}$ (in equilibrium).

From previous equation, we obtain a maximum allowable dc drop of about 600 mV for each load resistor, $R_{D}$. With a total bias current of 2 mA , we conservatively choose $R_{D}=500 \Omega$.

The overdrives chosen above lead to $W_{1}=15 \mu \mathrm{~m}$ and $W_{2,3}$ $=20 \mu \mathrm{~m}$. Capacitors $C_{1}$ and $C_{2}$ have a value of 2 pF to suppress the LO component at the output (which would otherwise help compress the mixer at the output).


## Active Mixer Design Example (II): Conversion Gain and NF

We can now estimate the voltage conversion gain and the noise figure of the mixer.

$$
\begin{aligned}
A_{v} & =\frac{2}{\pi} g_{m 1} R_{D} \\
& =4.1(=12.3 \mathrm{~dB})
\end{aligned}
$$

To compute the noise figure due to thermal noise, we first estimate the input-referred noise voltage as

$$
\begin{aligned}
\overline{V_{n, i n}^{2}} & =\pi^{2} k T\left(\frac{\gamma}{g_{m 1}}+\frac{2}{g_{m 1}^{2} R_{D}}\right) \\
& =4.21 \times 10^{-18} \mathrm{~V}^{2} / \mathrm{Hz}
\end{aligned}
$$

We now write the single-sideband NF with respect to $R_{S}=50 \Omega$ as:

$$
\begin{aligned}
\mathrm{NF}_{S S B} & =1+\frac{\overline{V_{n, i n}^{2}}}{4 k T R_{S}} \\
& =6.1(=7.84 \mathrm{~dB})
\end{aligned}
$$

The double-sideband NF is $\mathbf{3 d B}$ less.

## Active Mixer Design Example (III): Simulation

> From the compression characteristic of the mixer, the uncompressed gain is $\mathbf{1 0 . 3} \mathbf{d B}$, about $\mathbf{2 d B}$ less than our estimate, falling by 1 dB at $V_{i n, p}=170$ mV .
$>$ We reduce the load resistors by 5 and scale the output voltage swings and simulation shows the output port reach compression first.
> Use the two-tone test to measure the input $\mathrm{IP}_{3}$, here shows the downconverted spectrum.
$>$ We obtain IIP $_{3}=711 \mathrm{mV}_{\mathrm{p}}$. The $\mathrm{IIP}_{3}$ is 12.3 dB higher than the input $P_{1 d B}$ in this design.


## Active Mixer Design Example (IV): Simulation


> The flicker noise heavily corrupts the baseband up to several megahertz.
$>$ The NF at 100 MHz is equal to 5.5 dB , about 0.7 dB higher than our prediction.

## Improved Mixer Topologies: Active Mixers with Current-Source Helpers

$>$ The principal difficulty in the design of active mixers stems from the conflicting requirements between the input transistor current and the load resistor current.
$>$ We therefore surmise that adding current sources ("helpers") in parallel with the load resistors alleviates this conflict
how about the noise contributed by $M_{4}$ and $M_{5}$ ?

$$
\overline{V_{n, X}^{2}}=4 k T \gamma \frac{2 \alpha I_{0}}{V_{0}} R_{D}^{2}+4 k T R_{D}
$$



The above noise power must be normalized to $R_{D}{ }^{2}$.

$$
\begin{aligned}
\frac{\overline{V_{n, X}^{2}}}{R_{D}^{2}} & =4 k T \gamma \frac{2 \alpha I_{0}}{V_{0}}+\frac{4 k T}{R_{D}} \\
& =4 k T \frac{I_{0}}{V_{0}}(2 \alpha \gamma+1-\alpha) \\
& =4 k T \frac{I_{0}}{V_{0}}[(2 \gamma-1) \alpha+1]
\end{aligned}
$$

## Active Mixers with Current-Source Helpers: Flicker Noise Contribution of $M_{4}$ and $M_{5}$

Study the flicker noise contribution of $\boldsymbol{M}_{4}$ and $\boldsymbol{M}_{5}$ in figure above.

## Solution:

Modeled by a gate-referred voltage, the flicker noise of each device is multiplied by $g^{2}{ }_{m 4,5} R^{2}{ }_{D}$ as it appears at the output. As with the above derivation, we normalize this result to $R^{2}{ }_{D}$ :

$$
\frac{\overline{V_{n, X}^{2}}}{R_{D}^{2}}=\overline{V_{n, 1 / f}^{2}}\left(\frac{2 \alpha I_{0}}{V_{0}}\right)^{2}
$$

Since the voltage headroom, $V_{0}$, is typically limited to a few hundred millivolts, the helper transistors tend to contribute substantial $1 / f$ noise to the output, a serious issue in directconversion receivers.
$>$ The addition of the helpers also degrades the linearity. The circuit is likely to compress at the output rather than at the input.

## Active Mixers with Enhanced Transconductance

$>$ We can insert the current-source helper in the RF path rather than in the IF path.

The above approach nonetheless faces two issues.
$>$ First, transistor $M_{4}$ contributes additional capacitance to node $P$, exacerbating the difficulties mentioned earlier.


As a smaller bias current is allocated to $M_{2}$ and $M_{3}$, raising the impedance seen at their source, $C_{P}$ "steals" a greater fraction of the RF current generated by $M_{1}$, reducing the gain.
$>$ Second, the noise current of $\boldsymbol{M}_{4}$ directly adds to the RF signal.

$$
\begin{aligned}
\overline{I_{n, M 1}^{2}}+\overline{I_{n, M 4}^{2}} & =4 k T \gamma g_{m 1}+4 k T \gamma g_{m 4} \\
& =4 k T \gamma\left[\frac{2 I_{D 1}}{\left(V_{G S}-V_{T H}\right)_{1}}+\frac{2 \alpha I_{D 1}}{\left|V_{G S}-V_{T H}\right|_{2}}\right]
\end{aligned}
$$

## Current Mirror Voltage Limitations

A student eager to minimize the noise of $M_{4}$ in the above equation selects $\mid V_{G S}$ $V_{T H} l_{2}=0.75 \mathrm{~V}$ with $V_{D D}=1 \mathrm{~V}$. Explain the difficulty here.

## Solution:

The bias current of $M_{4}$ must be carefully defined so as to track that of $M_{1}$. Poor matching may "starve" $M_{2}$ and $M_{3}$, i.e., reduce their bias currents considerably, creating a high impedance at node $P$ and forcing the RF current to ground through $C_{P}$. Now, consider the simple current mirror shown below. If $\left|V_{G S}-V_{T H}\right|_{4}=0.75 \mathrm{~V}$, then $\left|V_{G S 4}\right|$ may exceed $V_{D D}$, leaving no headroom for $I_{R E F}$. In other words, $\mid V_{G S}-V_{T H} \|_{4}$ must be chosen less than $V_{D D}-$ $\left|V_{G S 4}\right|-V_{\text {IREF }}$, where $V_{\text {IREF }}$ denotes the minimum acceptable voltage across $I_{\text {REF }}$.


## Use of Inductive Resonance at Tail with Helper Current Source

$>$ In order to suppress the capacitance and noise contribution of $M_{4}$ in figure above, an inductor can be placed in series with its drain, allowing the inductor to resonate with $C_{p}$.


The choice of the inductor is governed by the following conditions:

$$
\begin{aligned}
L_{1} C_{P, \text { tot }} & =\frac{1}{\omega_{R F}^{2}} \\
R_{1} & =Q L_{1} \omega_{R F} \gg \frac{1}{g_{m 2,3}}
\end{aligned}
$$

$$
\frac{4 k T}{R_{1}}=\frac{4 k T}{Q L_{1} \omega_{R F}} \ll 4 k T \gamma g_{m 1}
$$

## Active Mixer Using Capacitive Coupling with

 Resonance> Shown below is a topology wherein capacitive coupling permits independent bias currents for the input transistor and the switching pair.

$>$ Here $C_{1}$ acts as a short circuit at RF and $L_{1}$ resonates with the parasitics at nodes $P$ and $N$.
$>$ Furthermore, the voltage headroom available to $M_{1}$ is no longer constrained by ( $\left.V_{G S}-V_{T H}\right)_{2,3}$ and the drop across the load resistors.

## Active Mixers with High $\mathrm{IP}_{2}$ : IP2 Calculation ( I )

It is instructive to compute the IP2 of a single-balanced mixer in the presence of asymmetries.


Shown above (right), the vertical shift of $V_{L O}$ displaces the consecutive crossings of $L O$ and LO by $\pm \Delta T$. This forces $M_{2}$ to remain on for $T_{L O} / 2+2 \Delta T$ seconds and $M_{3}$ for $T_{L O} / 2-2 \Delta T$ seconds.

The differential output current, $I_{D 2}-I_{D 3}$ contains a dc component equal to $\left(4 \Delta T / T_{L O}\right) I_{S S}=V_{O S} I_{s S} /\left(\pi V_{p, L O}\right)$, and the differential output voltage a dc component equal to $V_{O S} I_{s S} R_{D} /\left(\pi V_{p, L O}\right)$.


## Active Mixers with High $\mathbf{I P}_{2}$ : IP2 Calculation (II)

We now replace $I_{s s}$ with a transconductor device as depicted here and assume

$$
V_{R F}=V_{m} \cos \omega_{1} t+V_{m} \cos \omega_{2} t+V_{G S 0}
$$

The $\mathbf{I M}_{2}$ product emerges in the current of $M_{1}$ as

$$
I_{I M 2}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L} V_{m}^{2} \cos \left(\omega_{1}-\omega_{2}\right) t
$$

The direct feedthrough to the output:

$V_{I M 2, \text { out }}=\left[\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L} V_{m}^{2} \cos \left(\omega_{1}-\omega_{2}\right) t\right] \frac{V_{O S} R_{D}}{\pi V_{p, L O}}$
$\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L} V_{I I P 2}^{2} \frac{V_{O S} R_{D}}{\pi V_{p, L O}}=\frac{2}{\pi} g_{m 1} R_{D} V_{I I P 2} \quad \triangleleft \quad V_{I I P 2}=4\left(V_{G S}-V_{T H}\right)_{1} \frac{V_{p, L O}}{V_{O S}}$
The foregoing analysis also applies to asymmetries in the LO waveforms that would arise from mismatches within the LO circuitry and its buffer. Replace $I_{s s}$ with the $\mathbf{I M}_{\mathbf{2}}$ component
$V_{I M 2, \text { out }}=\left[\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L} V_{m}^{2} \cos \left(\omega_{1}-\omega_{2}\right) t\right] \frac{2 \Delta T}{T_{L O}} R_{D} \quad \triangleleft \quad V_{I I P 2}=\frac{2 T_{L O}}{\pi \Delta T}\left(V_{G S}-V_{T H}\right)_{1}$

## Active Mixers with High $\mathrm{IP}_{2}$ : Double-Balanced Topology

$>$ In order to raise the $\mathrm{IP}_{2}$, the input transconductor of an active mixer can be realized in differential form, leading to a double-balanced topology.

Assuming square-law devices, determine the $\mathrm{IM}_{2}$ product generated by $\boldsymbol{M}_{1}$ and $\boldsymbol{M}_{2}$ in figure below if the two transistors suffer from an offset voltage of $V_{\text {os1 }}$.

$$
\begin{aligned}
& \text { The differential output: } \\
& I_{D 1}-I_{D 2}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(\Delta V_{\text {in }}-V_{O S 1}\right) \sqrt{\frac{4 I_{S S}}{\mu_{n} C_{o x}(W / L)}-\left(\Delta V_{i n}-V_{O S 1}\right)^{2}} \\
& I_{D 1}-I_{D 2} \approx \sqrt{\mu_{n} C_{o x} \frac{W}{L} I_{S S}}\left[\Delta V_{\text {in }}-V_{O S 1}-\frac{\mu_{n} C_{o x}(W / L)}{8 I_{S C}}\left(\Delta V_{i n}-V_{O S 1}\right)^{3}\right. \\
& V_{I M 2}=\frac{3\left[\mu_{n} C_{o x}(W / L)\right]^{3 / 2}}{8 \sqrt{I_{S S}}} V_{m}^{2} V_{O S 1} \cos \left(\omega_{1}-\omega_{2}\right) t \\
& =\frac{3 I_{S S}}{8\left(V_{G S}-V_{T H}\right)_{e q}^{3}} V_{m}^{2} V_{O S 1} \cos \left(\omega_{1}-\omega_{2}\right) t, \\
& \sqrt{V_{I I P 2}}=\frac{16\left(V_{G S}-V_{T H}\right)_{e q}^{2} V_{p, L O}}{3 V_{O S 1} V_{O S 2}}
\end{aligned}
$$

## Double-Balanced Mixer Using Quasi-Differential Input Pair

$>$ While improving the $\mathrm{IP}_{\mathbf{2}}$ significantly, the use of a differential pair degrades the $\mathrm{IP}_{3}$.

$$
\begin{aligned}
& I_{D 1}=\frac{1}{2} \mu_{n} C_{o x}\left(\frac{W}{L}\right)_{1}\left(V_{m} \cos \omega_{1} t+V_{m} \cos \omega_{2} t+V_{O S 1}+V_{G S 0}-V_{T H}\right)^{2} \\
& I_{D 2}=\frac{1}{2} \mu_{n} C_{o x}\left(\frac{W}{L}\right)_{2}\left(V_{m} \cos \omega_{1} t+V_{m} \cos \omega_{2} t+V_{G S 0}-V_{T H}\right)^{2} \\
& V_{I M 2, o u t}=\left[\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L} V_{m}^{2} \cos \left(\omega_{1}-\omega_{2}\right) t\right] \frac{R_{D}}{\pi V_{p, L O}}\left(V_{O S 2}+V_{O S 3}\right) \\
& \text { Chapter } 6 M i x e r s
\end{aligned}
$$

## Effect of Low-Frequency Beat in a Mixer Using Capacitive Coupling and Resonance

$>$ The high-pass filter consisting of $L_{1}, C_{1}$, and the resistance seen at node $P$ suppresses low-frequency beats generated by the even-order distortion in $M_{1}$.

$$
\begin{aligned}
\frac{I_{m}}{I_{\text {beat }}} & =\frac{L_{1} s}{L_{1} s+\frac{1}{C_{1} s}+\frac{1}{2 g_{m}}} \\
& =\frac{L_{1} C_{1} s^{2}}{L_{1} C_{1} s^{2}+\frac{C_{1} s}{2 g_{m}}+1}
\end{aligned}
$$

At low frequencies, this result can be approximated as


$$
\frac{I_{m}}{I_{\text {beat }}} \approx L_{1} C_{1} s^{2}
$$

revealing a high attenuation.

## Capacitive Degeneration

$>$ Another approach to raising the $\mathrm{IP}_{\mathbf{2}}$ is to degenerate the transconductor capacitively.

The degeneration capacitor, $\boldsymbol{C}_{d}$, acts as a short circuit at RF but nearly an open circuit at the low-frequency beat components.

$$
\begin{aligned}
G_{m} & =\frac{g_{m 1}}{1+\frac{g_{m 1}}{C_{d} s}} \\
& =\frac{g_{m 1} C_{d} s}{C_{d} s+g_{m 1}}
\end{aligned}
$$



The gain at low frequencies falls in proportion to $C_{d s}$, making $\boldsymbol{M}_{1}$ incapable of generating second-order intermodulation components.

## Example of Worst-Case $\mathrm{IM}_{2}$ Product

The mixer above is designed for a $900-\mathrm{MHz}$ GSM system. What is the worst-case attenuation that capacitive degeneration provides for $\mathrm{IM}_{2}$ products that would otherwise be generated by $M_{1}$ ? Assume a low-IF receiver.

We may surmise that the highest beat frequency experiences the least attenuation, thereby creating the largest $\mathrm{IM}_{2}$ product.


This situation arises if the two interferers remain within the GSM band but as far from each other as possible. Assume the pole frequency is around 900 MHz . The $\mathrm{IM}_{2}$ product therefore falls at 25 MHz and, experiences an attenuation of roughly 36 by capacitive degeneration. However, in a low-IF receiver, the downconverted $200-\mathrm{kHz}$ GSM channel is located near zero frequency. Thus, this case proves irrelevant.

We seek two interferers that bear a frequency difference of 200 kHz . We place the adjacent interferers near the edge of the GSM band. Located at a center frequency of 200 kHz , the beat experiences an attenuation of roughly $4675 \approx 73 \mathrm{~dB}$.

## Use of Inductor at Sources of Switching Quad

> Even with capacitive coupling between the transconductor stage and the switching devices, the capacitance at the common source node of the switching pair ultimately limits the $\mathrm{IP}_{2}$.

> Figure above shows a double-balanced mixer employing both capacitive degeneration and resonance to achieve an $\mathrm{IP}_{2}$ of +78 dBm

## Active Mixers with Low Flicker Noise: Use of a Diode-Connected Device

> Can we turn on the PMOS current source only at the zero crossings of the LO so that it lowers the bias current of the switching devices and hence the effect of their flicker noise?

$>M_{H}$ can provide most of the bias current of $M_{1}$ near the crossing points of LO and LO while injecting minimal noise for the rest of the period.
> Unfortunately, the diode-connected transistor in figure above does not turn off abruptly as LO and LO depart from their crossing point.

## Active Mixers with Low Flicker Noise: Use of CrossCoupled Pair

$>M_{H 1}$ and $M_{H 2}$ turn on and off simultaneously because $V_{P}$ and $V_{Q}$ vary identically. These two transistors provide most of the bias currents of $M_{1}$ and $M_{4}$ at the crossing points of LO and LO.
$>$ The cross-coupled pair acts as a negative resistance, partially canceling the positive resistance presented by the switching pairs at $P$ and $Q$.

$>$ The circuit above nonetheless requires large LO swings to ensure that $V_{P}$ and $V_{Q}$ rise rapidly and sufficiently so as to turn off $M_{H_{1}}$ and $M_{H_{2}}$.

## Latchup Calculation

The positive feedback around $M_{H 1}$ and $M_{H 2}$ in figure above may cause latchup, i.e., a slight imbalance between the two sides may pull $P$ (or $Q$ ) toward $V_{D D}$, turning $\boldsymbol{M}_{H 2}$ (or $\boldsymbol{M}_{H 1}$ ) off. Derive the condition necessary to avoid latchup.

The impedance presented by the switching pairs at $P$ and $Q$ is at its highest value when either transistor in each differential pair is off. Shown below is the resulting worst case. For a symmetric circuit, the loop gain is equal to $\left(g_{m H}\left(g_{m 2,5}\right)^{2}\right.$, where $g_{m H}$ represents the transconductance of $\boldsymbol{M}_{\boldsymbol{H 1}}$ and $\boldsymbol{M}_{H 2}$. To avoid latchup, we must ensure that

$$
\left(\frac{g_{m H}}{g_{m 2}}\right)^{2}<1
$$



## Active Mixers with Low Flicker Noise: Use of a Switch to Turn Off the Switching Pair

$>$ The notion of reducing the current through the switching devices at the crossing points of LO and LO can alternatively be realized by turning off the transconductor momentarily.
$>$ The flicker noise of $M_{2}$ and $M_{3}$ is heavily attenuated.
$>M_{2}$ and $M_{3}$ inject no thermal noise to the output near the equilibrium.


## Upconversion Mixers: Performance Requirements

$>$ The upconversion mixers must :
(1) translate the baseband spectrum to a high output frequency (unlike downconversion mixers) while providing sufficient gain,
(2) drive the input capacitance of the PA,
(3) deliver the necessary swing to
 the PA input,
(4) not limit the linearity of the TX.
$>$ The interface between the mixers and the PA: the designer must decide at what point and how the differential output of the mixers must be converted to a single-ended signal.
$>$ The noise requirement of upconversion mixers is generally much more relaxed than that of downconversion mixers.
$>$ The interface between the baseband DACs and the upconversion mixers: the DACs must be directly coupled to the mixers to avoid a notch in the signal spectrum.

## Upconversion Mixer Topologies: Passive Mixers

Consider a low-frequency baseband sinusoid applied to a sampling mixer. The output appears to contain mostly the input waveform and little high- frequency energy.


The component of interest in $Y_{1}(f)$ still occurs at $\boldsymbol{k}= \pm \mathbf{1}$ and is given by

$$
\left.Y_{1}(f)\right|_{k= \pm 1}=\frac{X\left(f-f_{L O}\right)}{j \pi}-\frac{X\left(f+f_{L O}\right)}{j \pi}
$$

$$
\begin{aligned}
& \text { For } \boldsymbol{Y}_{\mathbf{2}}(\mathbf{f}) \text {, we must also set } \mathbf{k} \text { to } \pm \mathbf{1} \text { : } \\
& \qquad\left.Y_{2}(f)\right|_{k= \pm 1}=\frac{1}{T_{L O}}\left[-X\left(f-f_{L O}\right)+X\left(f+f_{L O}\right)\right]\left[\frac{1}{j \omega}\left(1-e^{-j \omega T_{L O} / 2}\right)\right] \\
& {\left[Y_{1}(f)+Y_{2}(f)\right]_{k= \pm 1} \approx \frac{\omega_{B B}}{\omega_{L O}+\omega_{B B}}\left[\left(\frac{1}{j \pi}+\frac{1}{2}\right) X\left(f-f_{L O}\right)+\left(-\frac{1}{j \pi}+\frac{1}{2}\right) X\left(f+f_{L O}\right)\right]}
\end{aligned}
$$

Thus, such a mixer is not suited to upconversion.

## Double-Balanced Passive Mixer: Issues (I)

While simple and quite linear, the circuit below must deal with a number of issues.

$>$ First, the bandwidth at nodes $X$ and $Y$ must accommodate the upconverted signal frequency so as to avoid additional loss.

It is possible to null the capacitance at nodes $X$ and $Y$ by means of resonance. As illustrated in figure above (right), inductor $L_{1}$ is chosen to yield

$$
\omega_{I F}=\frac{1}{\sqrt{\frac{L_{1}}{2}\left(C_{X, Y}+C_{i n}\right)}}
$$

## Double-Balanced Passive Mixer: Issues ( II )

$>$ The second issue relates to the use of passive mixers in a quadrature upconverter: passive mixers sense and produce voltages, making direct summation difficult.

$>$ A drawback of the above current-summing topology is that its bias point is sensitive to the input common-mode level.

## Double-Balanced Passive Mixer: Issues (III)

$>$ Alternatively, we can resort to true differential pairs, with their common-source nodes at ac ground.

$>$ Defined by the tail currents, the bias conditions now remain relatively independent of the input CM level, but each tail current source consumes voltage headroom.

## Double-Balanced Passive Mixer: Issues (IV)

> The third issue concerns the available overdrive voltage of the mixer switches: if the peak LO level is equal to $V_{D D}$, the switch experiences an overdrive of only $V_{D D}-\left(V_{T H 5}+V_{B B}\right)$, thereby suffering from a tight trade-off between its on-resistance and capacitance.

> A small overdrive also degrades the linearity of the switch.

## Double-Balanced Passive Mixer: Issues (V)

$>$ The foregoing difficulty can be alleviated if the peak LO level can exceed $V_{D D}$. This is accomplished if the LO buffer contains a load inductor tied to $V_{D D}$.

$>$ The above- $V_{D D}$ swings in figure above do raise concern with respect to device voltage stress and reliability.

## Carrier Feedthrough

An ideal double-balanced passive mixer upconverts both the signal and the offset, producing at its output the RF (or IF) signal and a carrier (LO) component. If modeled as a multiplier, the mixer generates an output given by

$V_{\text {out }}(t)=\alpha\left(V_{a} \cos \omega_{B B} t+V_{O S, D A C}\right) \cos \omega_{L O} t$

$V_{\text {out }}(t)=\frac{\alpha V_{a}}{2} \cos \left(\omega_{L O}+\omega_{B B}\right) t+\frac{\alpha V_{a}}{2} \cos \left(\omega_{L O}-\omega_{B B}\right) t+\alpha V_{O S, D A C} \cos \omega_{L O} t$.
Since $\alpha / 2=2 / \pi$ for a double-balanced mixer, we note that the carrier feedthrough has a peak amplitude of $\alpha V_{O S, D A C}=(4 / \pi) V_{O S, D A C}$.

## Effect of Threshold Mismatches


$>$ The threshold mismatch in one pair shifts the LO waveform vertically, distorting the duty cycle.
> Carrier feedthrough can occur only if a dc component in the baseband is mixed with the fundamental LO frequency. We therefore conclude that threshold mismatches within passive mixers introduce no carrier feedthrough.

## LO Feedthrough Paths in a Passive Mixer

$>$ The carrier feedthrough in passive upconversion mixers arises primarily from mismatches between the gate-drain capacitances of the switches.

The LO feedthrough observed at $\boldsymbol{X}$ is equal to:

$$
V_{X}=V_{L O} \frac{C_{G D 1}-C_{G D 3}}{C_{G D 1}+C_{G D 3}+C_{X}}
$$



Calculate the relative carrier feedthrough for a $C_{G D}$ mismatch of $5 \%, C_{X} \approx 10 C_{G D}$, peak LO swing of 0.5 V , and peak baseband swing of 0.1 V .
At the output, the LO feedthrough is given by equation above and approximately equal to $(5 \% / 12) V_{L O}=2.1 \mathrm{mV}$. The upconverted signal has a peak amplitude of $0.1 \mathrm{~V} \times(2 / \pi)=63.7 \mathrm{mV}$. Thus, the carrier feedthrough is equal to $\mathbf{- 2 9 . 6} \mathbf{d B}$.

## Upconversion with Active Mixers: Double-Balanced Topology with Quasi-Differential Pair

The inductive loads serve two purposes: they relax voltage headroom issues and raise the conversion gain (hence the output swings) by nulling the capacitance at the output node.

$>$ The circuit is quite tolerant of capacitance at nodes $P$ and $Q$. However, stacking of the transistors limits the voltage headroom.

## Voltage Excursions in an Active Upconversion Mixer

Equation above allocates a drain-source voltage to the input transistors equal to their overdrive voltage. Explain why this is inadequate.

The voltage gain from each input to the drain of the corresponding transistor is about -1. Thus, as depicted in figure below, when one gate voltage rises by $V_{a}$, the corresponding drain falls by approximately $V_{a}$, driving the transistor into the triode region by $2 V_{a}$. In other words, the $V_{D S}$ of the input devices in the absence of signals must be at least equal to their overdrive voltage plus $2 V_{a}$, further limiting equation above as:
$V_{X, \text { min }}=V_{G S 1}-V_{T H 1}+2 V_{a}+\left(1+\frac{\sqrt{2}}{2}\right)\left(V_{G S 3}-V_{T H 3}\right)$

The output swing is therefore small. If $V_{a}=100 \mathrm{mV}$, then the above numerical example yields a peak output swing of 160 mV .


## Gilbert Cell as Upconversion Mixer


$>$ This circuit faces two difficulties. First, the current source consumes additional voltage headroom.
$>$ Second, since node A cannot be held at ac ground by a capacitor at low baseband frequencies, the nonlinearity is more pronounced.
$>$ We therefore fold the input path and degenerate the differential pair to alleviate these issues.

## Max Allowable Input and Output Swings in Circuit Above

## Determine the maximum allowable input and output swings in the circuit above

In the absence of signals, the maximum gate voltage of $M_{1}$ with respect to ground is equal to $V_{D D}-\left|V_{G S 1}\right|-\left|V_{11}\right|$, where $\left|V_{11}\right|$ denotes the minimum allowable voltage across $I_{1}$. Also, $V_{P}=V_{13}$. Note that, due to source degeneration, the voltage gain from the baseband input to $P$ is quite smaller than unity. We therefore neglect the baseband swing at node $P$. For $M_{1}$ to remain in saturation as its gate falls by $V_{a}$ volts,

$$
\begin{aligned}
& V_{D D}-\left|V_{G S 1}\right|-\left|V_{I 1}\right|-V_{a}+\left|V_{T H 1}\right| \geq V_{P} \\
& V_{a} \leq V_{D D}-\left|V_{G S 1}-V_{T H 1}\right|-\left|V_{I 1}\right|-\left|V_{I 3}\right|
\end{aligned}
$$

For the output swing,

$$
V_{X, \min }=\left(1+\frac{\sqrt{2}}{2}\right)\left(V_{G S 3}-V_{T H 3}\right)+V_{I 3}
$$

## Active Upconversion Mixers: Mixer Carrier Feedthrough ( I )

Figure below (left) shows a more detailed implementation of the folded mixer. Determine the input-referred offset in terms of the threshold mismatches of the transistor pairs. Neglect channel-length modulation and body effect.

## Solution:



As depicted above (right), we insert the threshold mismatches and seek the total mismatch between $I_{P}$ and $I_{Q}$. To obtain the effect of $V_{o s 10}$, we first recognize that it generates an additional current of $g_{m 10} V_{o s 10}$ in $M_{10}$.

## Active Upconversion Mixers: Mixer Carrier Feedthrough (II)

This current is split between $M_{2}$ and $M_{1}$ according to the small-signal impedance seen at node $E$,

$$
\begin{aligned}
\left|I_{D 2}\right|_{V O S 10} & =g_{m 10} V_{O S 10} \frac{R_{S}+\frac{1}{g_{m 1}}}{R_{S}+\frac{1}{g_{m 1}}+\frac{1}{g_{m 2}}} \\
\left|I_{D 1}\right|_{V O S 10} & =g_{m 10} V_{O S 10} \frac{\frac{1}{g_{m 2}}}{R_{S}+\frac{1}{g_{m 1}}+\frac{1}{g_{m 2}}}
\end{aligned}
$$

The resulting mismatch between $I_{P}$ and $I_{Q}$ is given by the difference between these two:

$$
\left|I_{P}-I_{Q}\right|_{V O S 10}=g_{m 10} V_{O S 10} \frac{R_{S}}{R_{S}+\frac{2}{g_{m 1,2}}}
$$

The mismatch between $M_{3}$ and $M_{4}$ simply translates to a current mismatch of $g_{m 4} V_{\text {OS4 }}$. we arrive at the input-referred offset:

$$
V_{O S, \text { in }}=g_{m 10} R_{S} V_{O S 10}+g_{m 4} V_{O S 4}\left(\frac{R_{S}}{2}+\frac{1}{g_{m 1,2}}\right)+V_{O S 1}
$$

## Summation of Quadrature Outputs


$>$ Active mixers readily lend themselves to quadrature upconversion because their outputs can be summed in the current domain.

## Design Procedure ( I )

> The design of upconversion mixers typically follows that of the power amplifier.

With the input capacitance of the PA (or PA driver) known, the mixer output inductors are designed to resonate at the frequency of interest. At this point, the capacitance contributed by the switching quads, $\boldsymbol{C}_{q}$, is unknown and must be guessed.

$$
L_{1}=L_{2}=\frac{1}{\omega_{0}^{2}\left(C_{q}+C_{L}\right)}
$$

The finite $Q$ of the inductors introduces a parallel equivalent resistance given by

$$
R_{p}=\frac{Q}{\omega_{0}\left(C_{q}+C_{L}\right)}
$$

If sensing quadrature baseband inputs with a peak single-ended swing of $V_{a}$, the output swing is given by

$$
V_{p, \text { out }}=\sqrt{2} \frac{2}{\pi} \frac{R_{p}}{\frac{R_{S}}{2}+\frac{1}{g_{m p}}}\left(2 V_{a}\right)
$$

## Design Procedure ( II ): Bias Current

The tail current of figure below varies with time as $I_{S S}=I_{0}+I_{0} \cos \omega_{B B} t$. Calculate the voltage swing of the upconverted signal.

Solution:


We know that $I_{S S}$ is multiplied by $(2 / \pi) R_{p}$ as it is upconverted. Thus, the output voltage swing at $\omega_{L O}-\omega_{B B}$ or $\omega_{L O}+\omega_{B B}$ is equal to $(2 / \pi) I_{0} R_{p}$. We have assumed that $I_{S S}$ swings between zero and $2 I_{0}$, but an input transistor experiencing such a large current variation may become quite nonlinear.

## Design Procedure (III): Transistor Dimensions

$>$ First consider the switching devices, noting that each switching pair carries a current of nearly $I_{3}\left(=I_{4}\right)$ at the extremes of the baseband swings. These transistors must therefore be chosen wide enough.
$>$ Next, the transistors implementing $I_{3}$ and $I_{4}$ are sized according to their allowable voltage headroom.
$>$ Lastly, the dimensions of the input differential pair and the transistors realizing $I_{1}$ and $I_{2}$ are chosen.

An engineer designs a quadrature upconversion mixer for a given output frequency, a given output swing, and a given load capacitance, $C_{L}$. Much to her dismay, the engineer's manager raises $C_{L}$ to $2 C_{L}$ because the following power amplifier must be redesigned for a higher output power. If the upconverter output swing must remain the same, how can the engineer modify her design to drive $2 C_{L}$ ?

The load inductance and hence $R_{p}$ must be halved. Thus, all bias currents and transistor widths must be doubled so as to maintain the output voltage swing. This in turn translates to a higher load capacitance seen by the LO. In other words, the larger $P$ input capacitance "propagates" to the LO port. Now, the engineer designing the LO is in trouble.

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