Chapter 2 Basic Concepts in RF Design

Chapter Outline

Nonlinearity → **Noise** → **Impedance Transformation**

Nonlinearity
- ✓ Harmonic Distortion
- ✓ Compression
- ✓ Intermodulation
- ✓ Dynamic Nonlinear Systems

Noise
- ✓ Noise Spectrum
- ✓ Device Noise
- ✓ Noise in Circuits

Impedance Transformation
- ✓ Series-Parallel Conversion
- ✓ Matching Networks
- ✓ S-Parameter

General Considerations: Units in RF Design

$$A_V|_{\text{dB}} = 20\log\frac{V_{out}}{V_{in}}$$

$$A_P|_{\text{dB}} = 10\log\frac{P_{out}}{P_{in}}.$$

$$A_P|_{\text{dB}} = 10\log\frac{\frac{V_{out}^2}{R_0}}{\frac{V_{in}^2}{R_0}} = 20\log\frac{V_{out}}{V_{in}} = A_V|_{\text{dB}},$$

- This relationship between Power and Voltage only holds when the *input and output impedance are equal*

$$P_{sig}|_{\text{dBm}} = 10\log\left(\frac{P_{sig}}{1\ \text{mW}}\right)$$

An amplifier senses a sinusoidal signal and delivers a power of 0 dBm to a load resistance of 50 Ω. Determine the peak-to-peak voltage swing across the load.

Solution:

$$\frac{V_{pp}^2}{8R_L} = 1\ \text{mW}$$

where R_L= 50 Ω thus,

$$V_{pp} = 632\ \text{mV}$$

Example of Units in RF

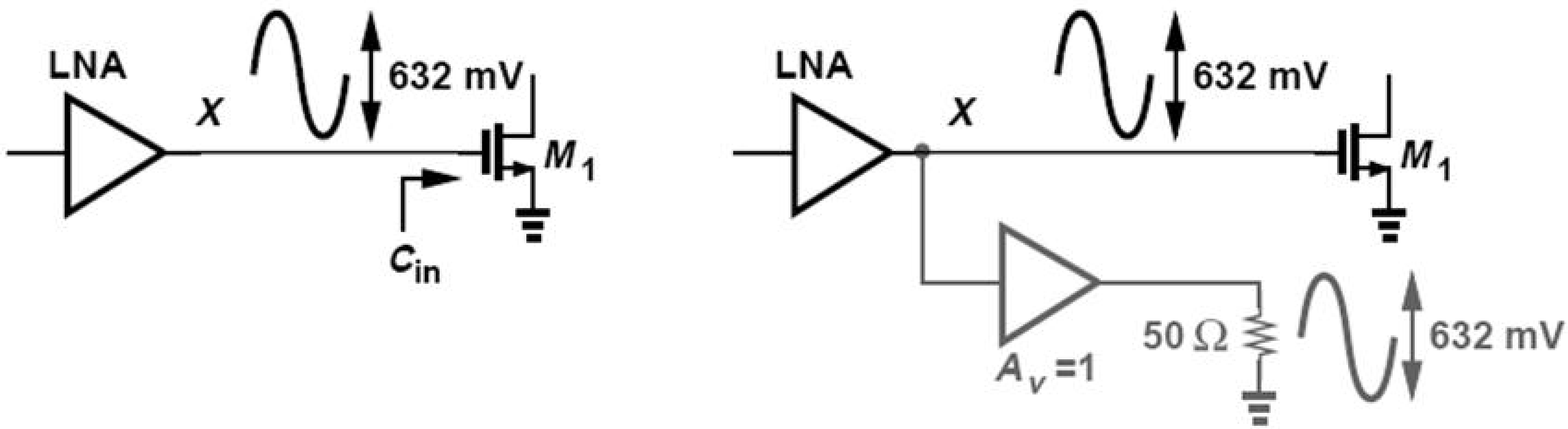
A GSM receiver senses a narrowband (modulated) signal having a level of -100 dBm. If the front-end amplifier provides a voltage gain of 15 dB, calculate the peak-to-peak voltage swing at the output of the amplifier.

Solution:

Since the amplifier output voltage swing is of interest, we first convert the received signal level to voltage. From the previous example, we note that -100 dBm is 100 dB below 632 mV_{pp}. Also, 100 dB for voltage quantities is equivalent to 10^5. Thus, -100 dBm is equivalent to 6.32 μV_{pp}. This input level is amplified by 15 dB (≈ 5.62), resulting in an output swing of 35.5 μV_{pp}.

- **Output *voltage* of the amplifier is of interest in this example**

dBm Used at Interfaces Without Power Transfer



- dBm can be used at interfaces that do not necessarily entail power transfer
- We mentally attach an ideal voltage buffer to node *X* and drive a 50-Ω load. We then say that the signal at node *X* has a level of 0 dBm, tacitly meaning that *if* this signal were applied to a 50-Ω load, *then* it would deliver 1 mW.

General Considerations: Time Variance

- A system is linear if its output can be expressed as a linear combination (superposition) of responses to individual inputs.

$$y_1(t) = f[x_1(t)]$$
$$y_2(t) = f[x_2(t)]$$

$$ay_1(t) + by_2(t) = f[ax_1(t) + bx_2(t)].$$

- A system is time-invariant if a time shift in its input results in the same time shift in its output.

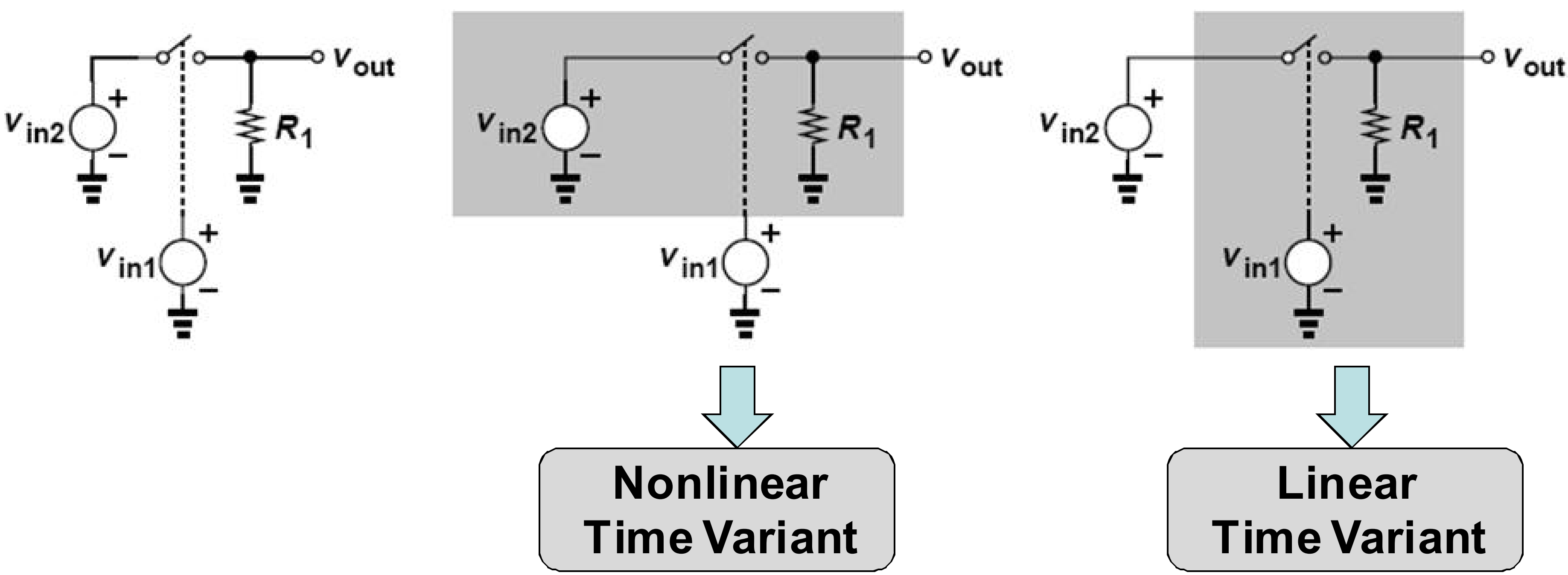
If $\qquad y(t) = f[x(t)]$

then $\qquad y(t-\tau) = f[x(t-\tau)]$

Comparison: Time Variance and Nonlinearity

time variance plays a critical role and must not be confused with nonlinearity:

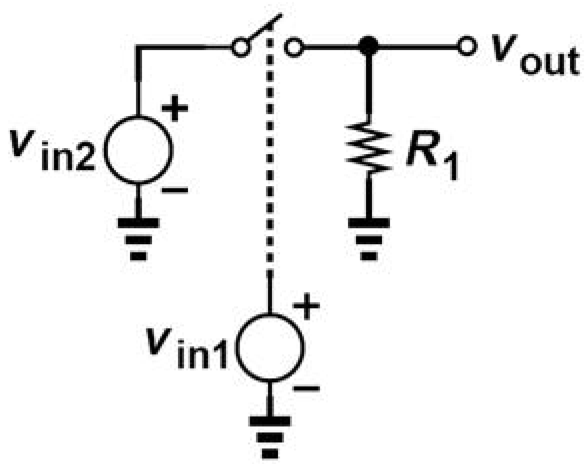
$$v_{in1}(t) = A_1 \cos \omega_1 t \qquad v_{in2}(t) = A_2 \cos \omega_2 t$$

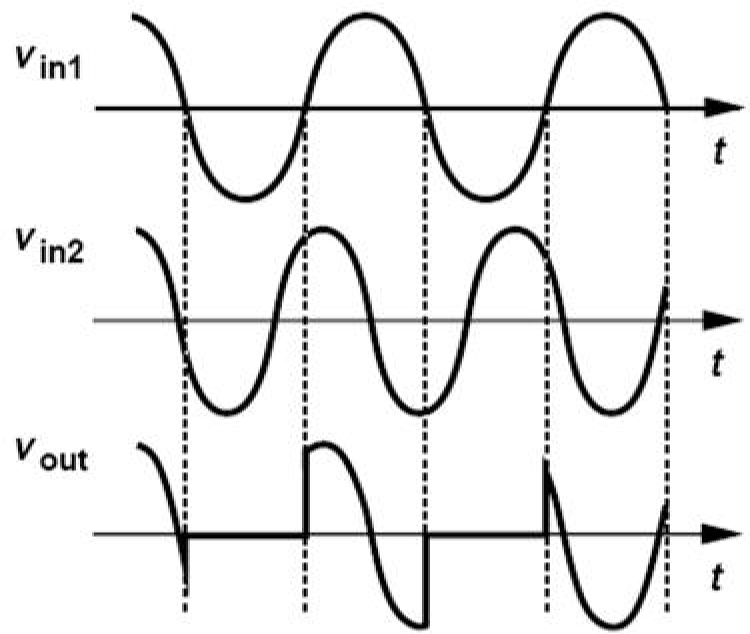


Example of Time Variance

Plot the output waveform of the circuit above if $v_{in1} = A_1 \cos \omega_1 t$ and $v_{in2} = A_2 \cos(1.25\omega_1 t)$.

Solution:



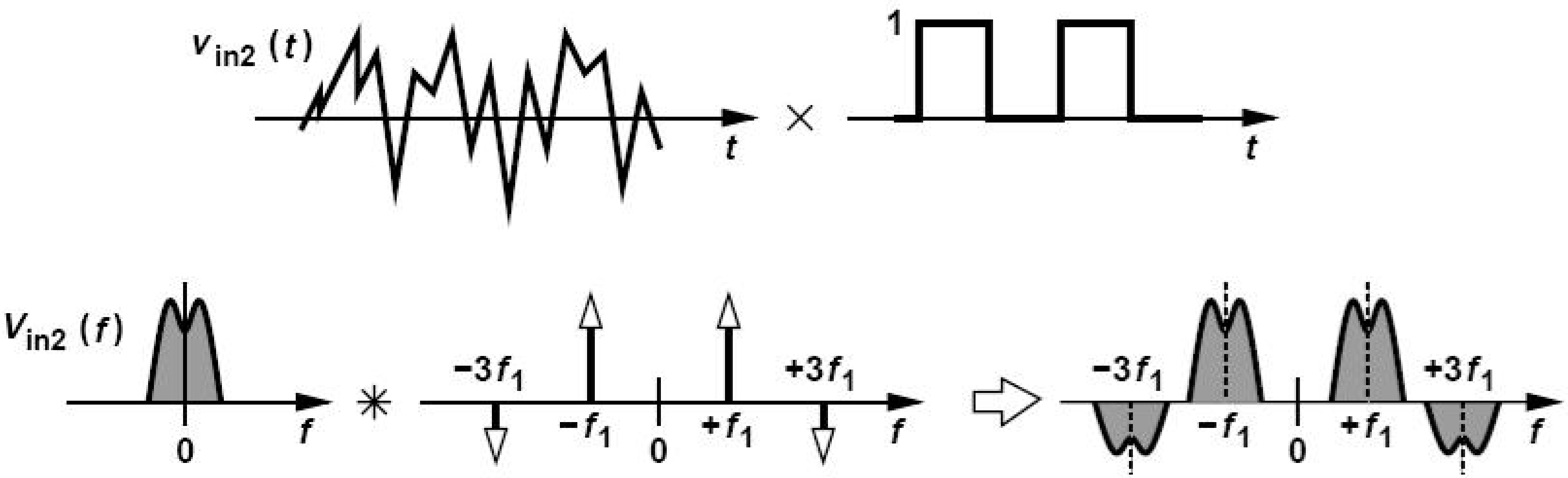


As shown above, v_{out} tracks v_{in2} if $v_{in1} > 0$ and is pulled down to zero by R_1 if $v_{in1} < 0$. That is, v_{out} is equal to the product of v_{in2} and a square wave toggling between 0 and 1.

Time Variance: Generation of Other Frequency Components

$$v_{out}(t) = v_{in2}(t) \cdot S(t).$$

$$\begin{aligned} V_{out}(f) &= V_{in2}(f) * \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} \delta\left(f - \frac{n}{T_1}\right) \\ &= \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} V_{in2}\left(f - \frac{n}{T_1}\right), \end{aligned}$$



- A linear system can generate frequency components that do not exist in the input signal when system is time variant

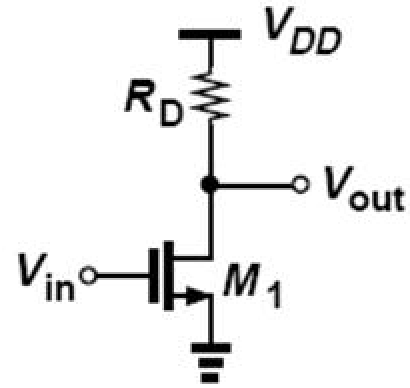
Nonlinearity: Memoryless and Static System

$$y(t) = \alpha x(t).$$ ← **linear**

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$ ← **nonlinear**

- **The input/output characteristic of a memoryless nonlinear system can be approximated with a polynomial**

$$\begin{aligned} V_{out} &= V_{DD} - I_D R_D \\ &= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 R_D \end{aligned}$$



- **In this idealized case, the circuit displays only second-order nonlinearity**

Example of Polynomial Approximation

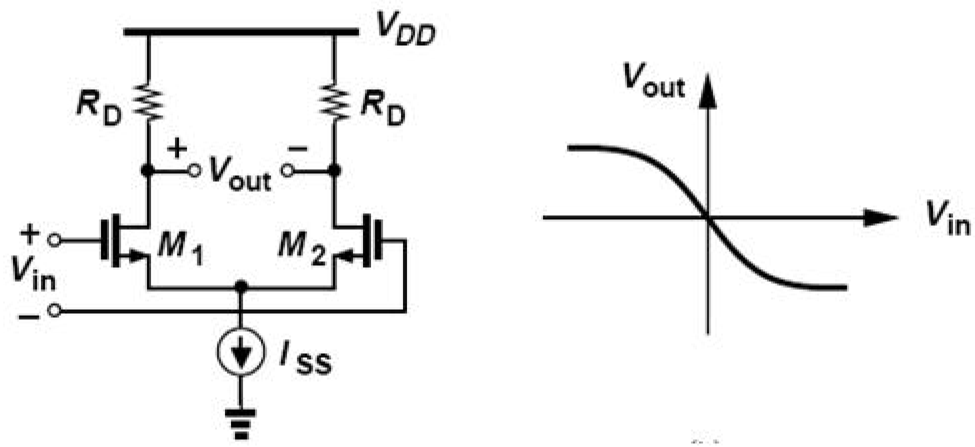
For square-law MOS transistors operating in saturation, the characteristic above can be expressed as

$$V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}R_D$$

If the differential input is small, approximate the characteristic by a polynomial.

Factoring $4I_{ss}$ / $(\mu_n C_{ox} W/L)$ out of the square root and assuming

$$V_{in}^2 \ll \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$



Approximation gives us:

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}V_{in}}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8I_{SS}}V_{in}^2\right)R_D$$

$$\approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}R_D V_{in} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}}R_D V_{in}^3$$

Effects of Nonlinearity: Harmonic Distortion

$$y(t) = h(t) * x(t)$$

Linear Time-invariant

$$y(t) = h(t,\tau) * x(t)$$

Linear Time-variant

- **The output of a "dynamic" system depends on the past values of its input/output**

$$\begin{aligned} y(t) &= \alpha_1 A \cos\omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\ &= \alpha_1 A \cos\omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\ &= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t. \end{aligned}$$

DC | **Fundamental** | **Second Harmonic** | **Third Harmonic**

- **Even-order harmonics result from α_j with even j**
- **nth harmonic grows in proportion to A^n**

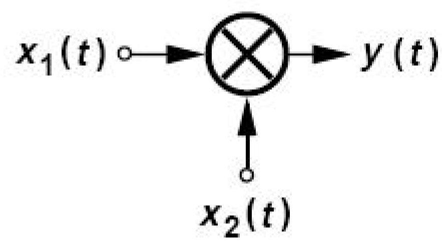
Example of Harmonic Distortion in Mixer

An analog multiplier "mixes" its two inputs below, ideally producing *y(t)* = *k*$x_1(t)x_2(t)$, where *k* is a constant. Assume $x_1(t) = A_1 \cos \omega_1 t$ and $x_2(t) = A_2 \cos \omega_2 t$.

(a) If the mixer is ideal, determine the output frequency components.

(b) If the input port sensing $x_2(t)$ suffers from third-order nonlinearity, determine the output frequency components.

Solution:



(a)

$$y(t) = k(A_1 \cos \omega_1 t)(A_2 \cos \omega_2 t)$$
$$= \frac{kA_1A_2}{2}\cos(\omega_1 + \omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1 - \omega_2)t.$$

(b)

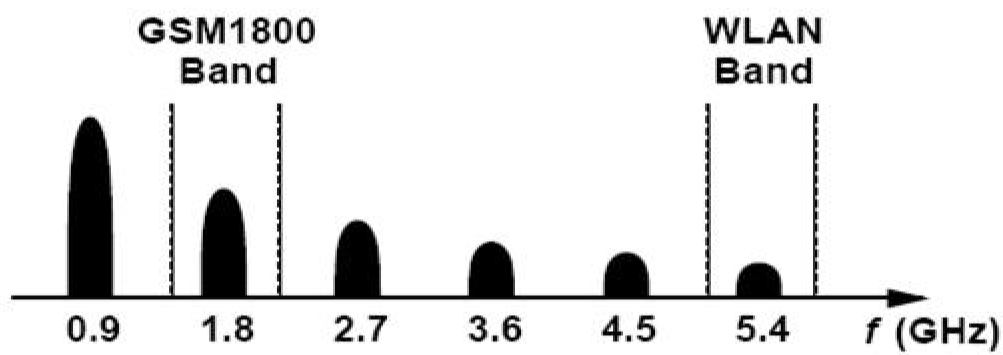
$$y(t) = k(A_1 \cos \omega_1 t)\left(A_2 \cos \omega_2 t + \frac{\alpha_3 A_2^3}{4}\cos 3\omega_2 t\right)$$
$$= \frac{kA_1A_2}{2}\cos(\omega_1 + \omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1 - \omega_2)t + \frac{k\alpha_3 A_1 A_2^3}{8}\cos(\omega_1 + 3\omega_2)t$$
$$+ \frac{k\alpha_3 A_1 A_2^3}{8}\cos(\omega_1 - 3\omega_2)t.$$

Example of Harmonics on GSM Signal

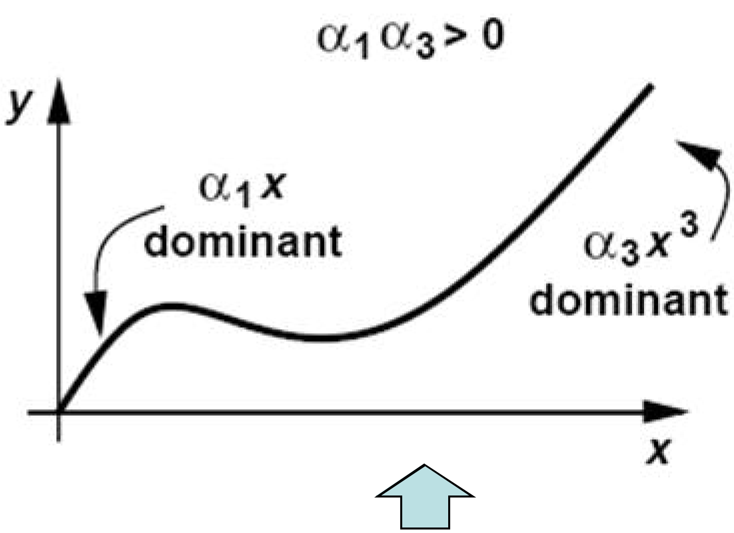
The transmitter in a 900-MHz GSM cellphone delivers 1 W of power to the antenna. Explain the effect of the harmonics of this signal.

Solution:

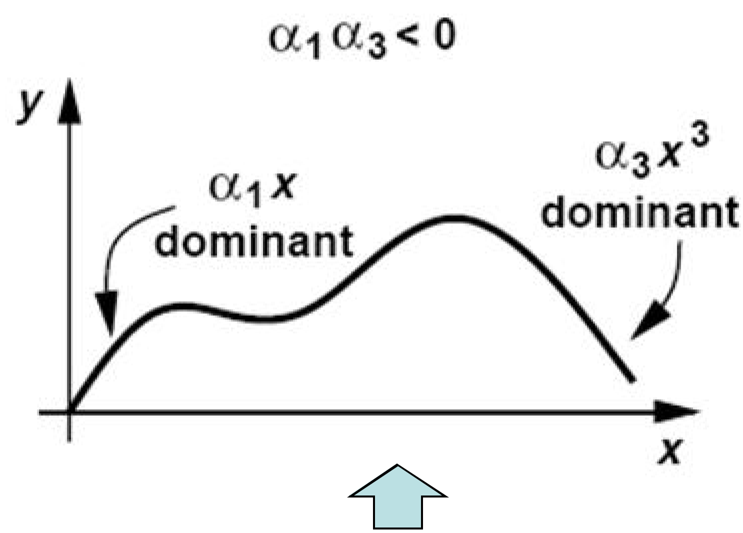
The second harmonic falls within another GSM cellphone band around 1800 MHz and must be sufficiently small to negligibly impact the other users in that band. The third, fourth, and fifth harmonics do not coincide with any popular bands but must still remain below a certain level imposed by regulatory organizations in each country. The sixth harmonic falls in the 5-GHz band used in wireless local area networks (WLANs), e.g., in laptops. Figure below summarizes these results.



Gain Compression– Sign of α_1, α_3



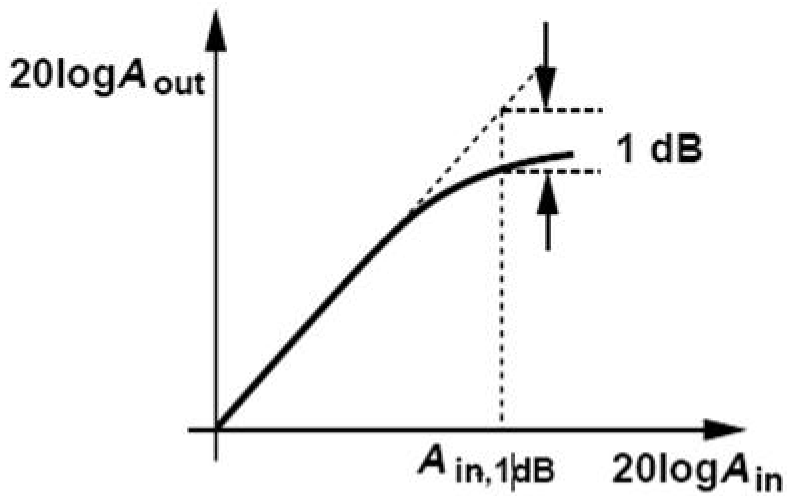
Expansive



Compressive

- **Most RF circuit of interest are compressive, we focus on this type.**

Gain Compression: 1-dB Compression Point

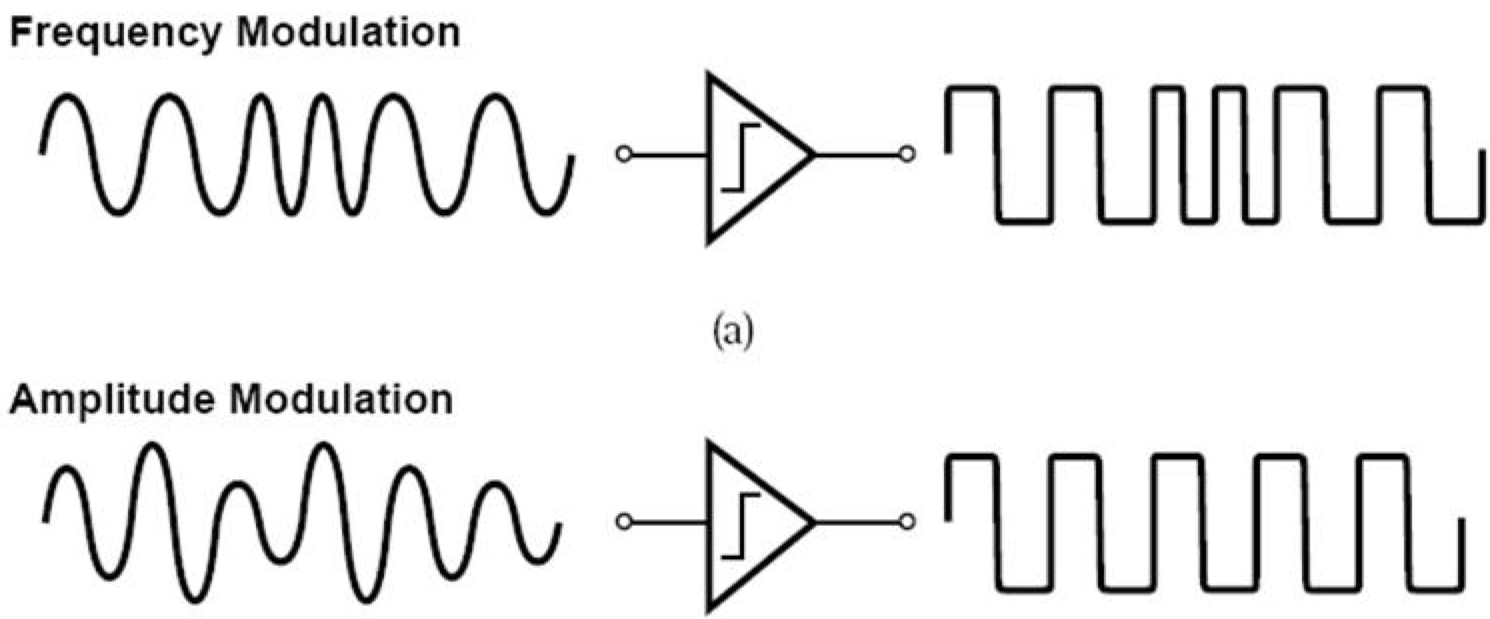


$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in,1dB}^2\right| = 20\log|\alpha_1| - 1\text{ dB}.$$

$$A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

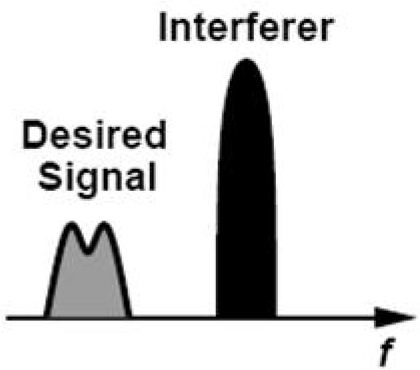
- **Output falls below its ideal value by 1 dB at the 1-dB compression point**
- **Peak value instead of peak-to-peak value**

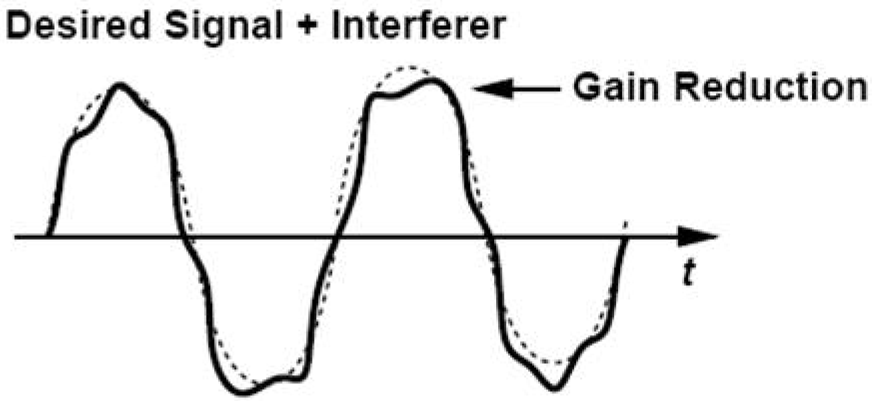
Gain Compression: Effect on FM and AM Waveforms



- **FM signal carries no information in its amplitude and hence tolerates compression.**
- **AM contains information in its amplitude, hence distorted by compression**

Gain Compression: Desensitization





$$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots .$$

For $A_1 << A_2$

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

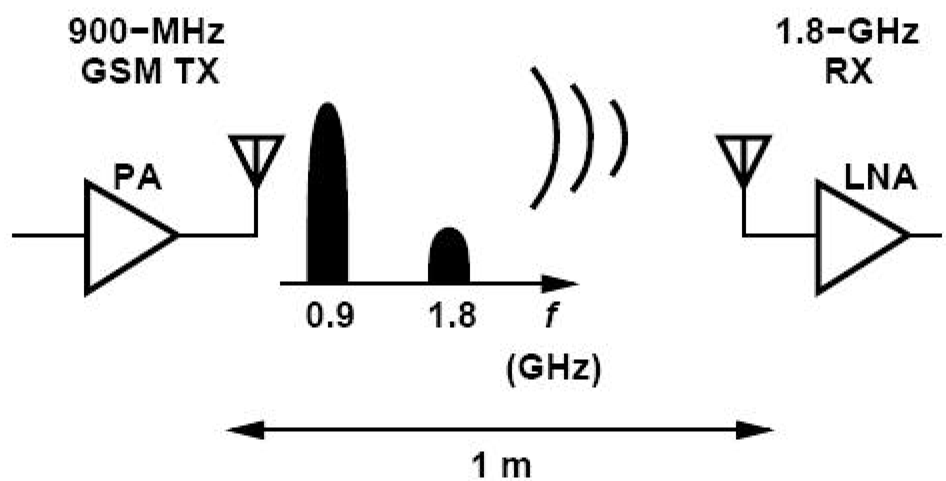
- **Desensitization: the receiver gain is reduced by the large excursions produced by the interferer even though the desired signal itself is small.**

Example of Gain Compression

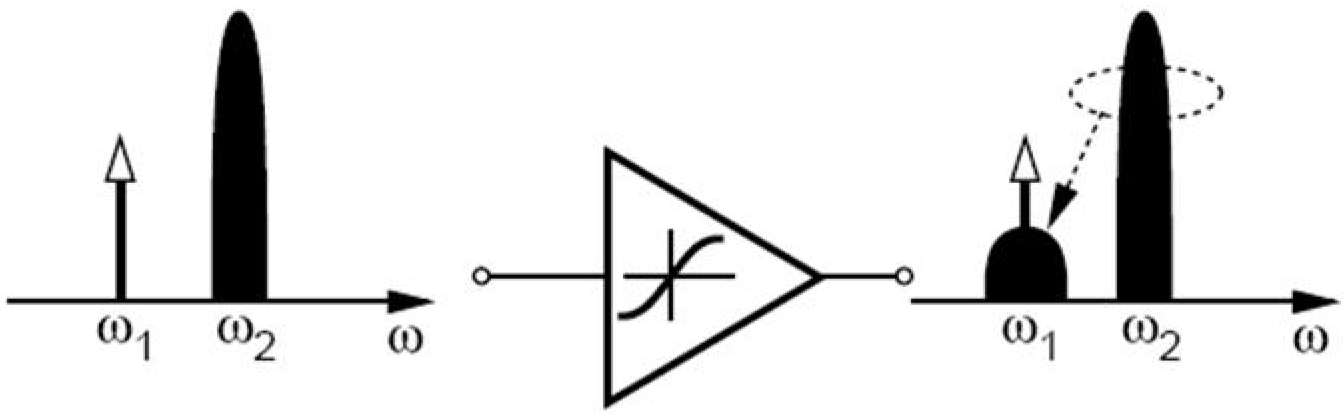
A 900-MHz GSM transmitter delivers a power of 1 W to the antenna. By how much must the second harmonic of the signal be suppressed (filtered) so that it does not desensitize a 1.8-GHz receiver having P_{1dB} = -25 dBm? Assume the receiver is 1 m away and the 1.8-GHz signal is attenuated by 10 dB as it propagates across this distance.

Solution:

The output power at 900 MHz is equal to +30 dBm. With an attenuation of 10 dB, the second harmonic must not exceed -15 dBm at the transmitter antenna so that it is below P_{1dB} of the receiver. Thus, the second harmonic must remain at least 45 dB below the fundamental at the TX output. In practice, this interference must be another several dB lower to ensure the RX does not compress.



Effects of Nonlinearity: Cross Modulation



Suppose that the interferer is an amplitude-modulated signal

$$A_2(1 + m\cos\omega_m t)\cos\omega_2 t$$

Thus

$$y(t) = \left[\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\left(1 + \frac{m^2}{2} + \frac{m^2}{2}\cos 2\omega_m t + 2m\cos\omega_m t\right)\right] A_1\cos\omega_1 t + \cdots.$$

- **Desired signal at output suffers from amplitude modulation**

Example of Cross Modulation

Suppose an interferer contains phase modulation but not amplitude modulation. Does cross modulation occur in this case?

Solution:

Expressing the input as: $x(t) = A_1 \cos\omega_1 t + A_2 \cos(\omega_2 t + \phi)$

where the second term represents the interferer (A_2 is constant but Φ varies with time)

$$\begin{aligned} y(t) &= \alpha_1[A_1 \cos\omega_1 t + A_2 \cos(\omega_2 t + \phi)] + \alpha_2[A_1 \cos\omega_1 t + A_2 \cos(\omega_2 t + \phi)]^2 \\ &+ \alpha_3[A_1 \cos\omega_1 t + A_2 \cos(\omega_2 t + \phi)]^3. \end{aligned}$$

We now note that (1) the second-order term yields components at $\omega_1 \pm \omega_2$ but not at ω_1; (2) the third-order term expansion gives $3\alpha_3 A_1 \cos \omega_1 t \, A_2^2 \cos^2(\omega_2 t+\Phi)$, which results in a component at ω_1. Thus,

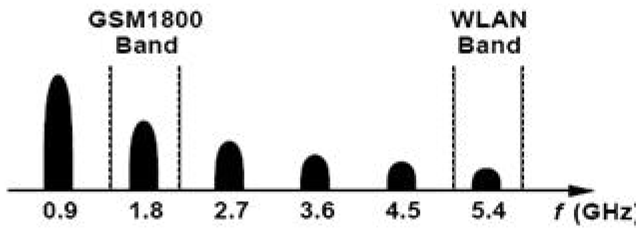
$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots.$$

The desired signal at ω_1 does not experience cross modulation

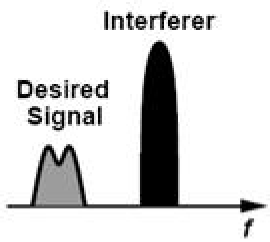
Effects of Nonlinearity: Intermodulation— Recall Previous Discussion

So far we have considered the case of:

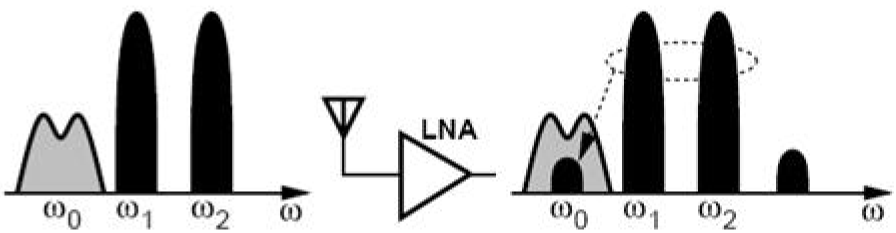
- **Single Signal** → **Harmonic distortion**



- **Signal + one large interferer** → **Desensitization**



- **Signal + two large interferers** → **Intermodulation**



Effects of Nonlinearity: Intermodulation

assume $x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$

Thus

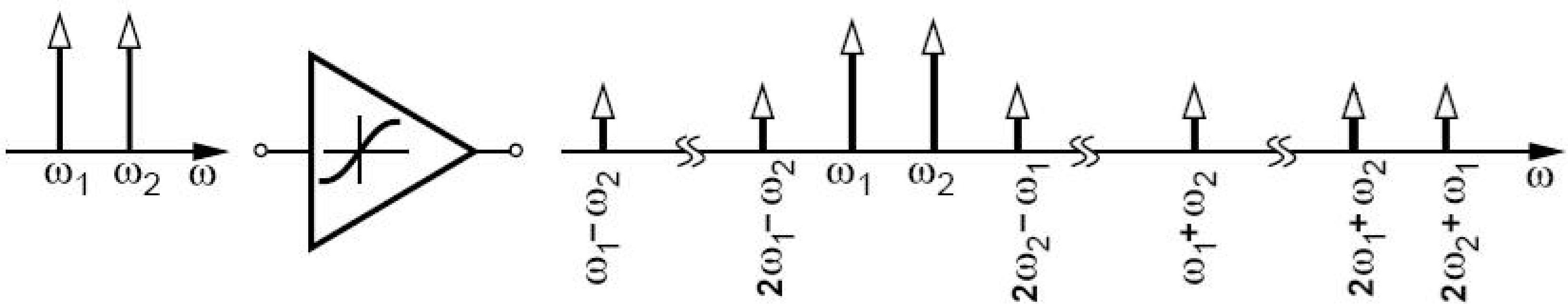
$$y(t) = \alpha_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

Intermodulation products:

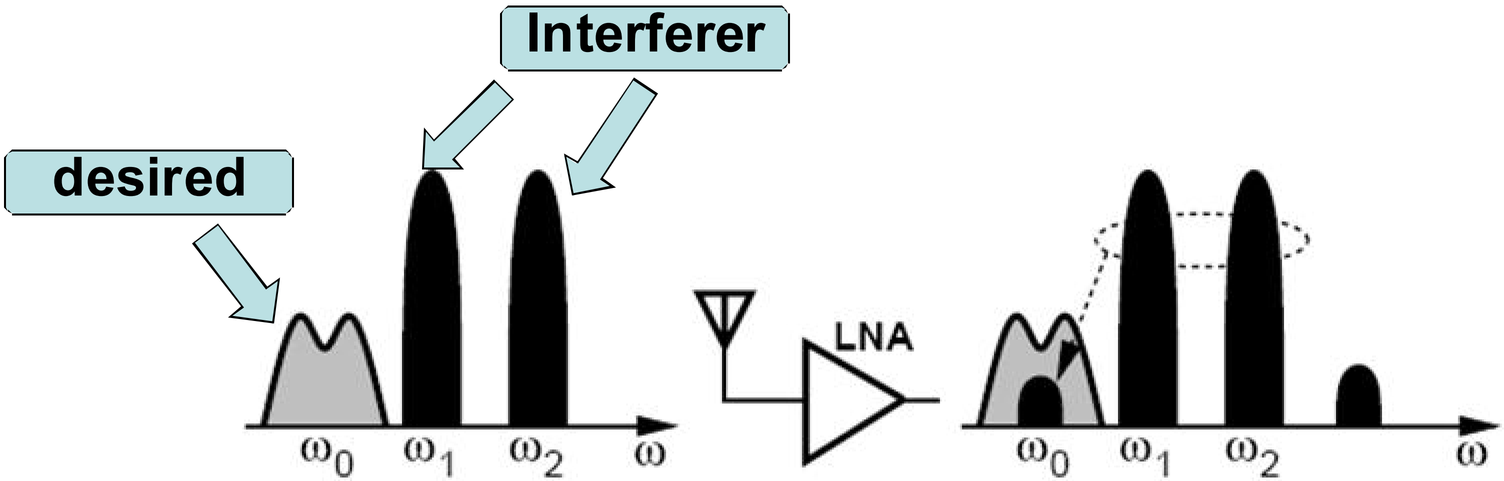
$$\begin{aligned}
\omega &= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 - \omega_2)t \\
\omega &= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 - \omega_1)t
\end{aligned}$$

Fundamental components:

$$\omega = \omega_1, \, \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right) \cos \omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right) \cos \omega_2 t$$



Intermodulation Product Falling on Desired Channel



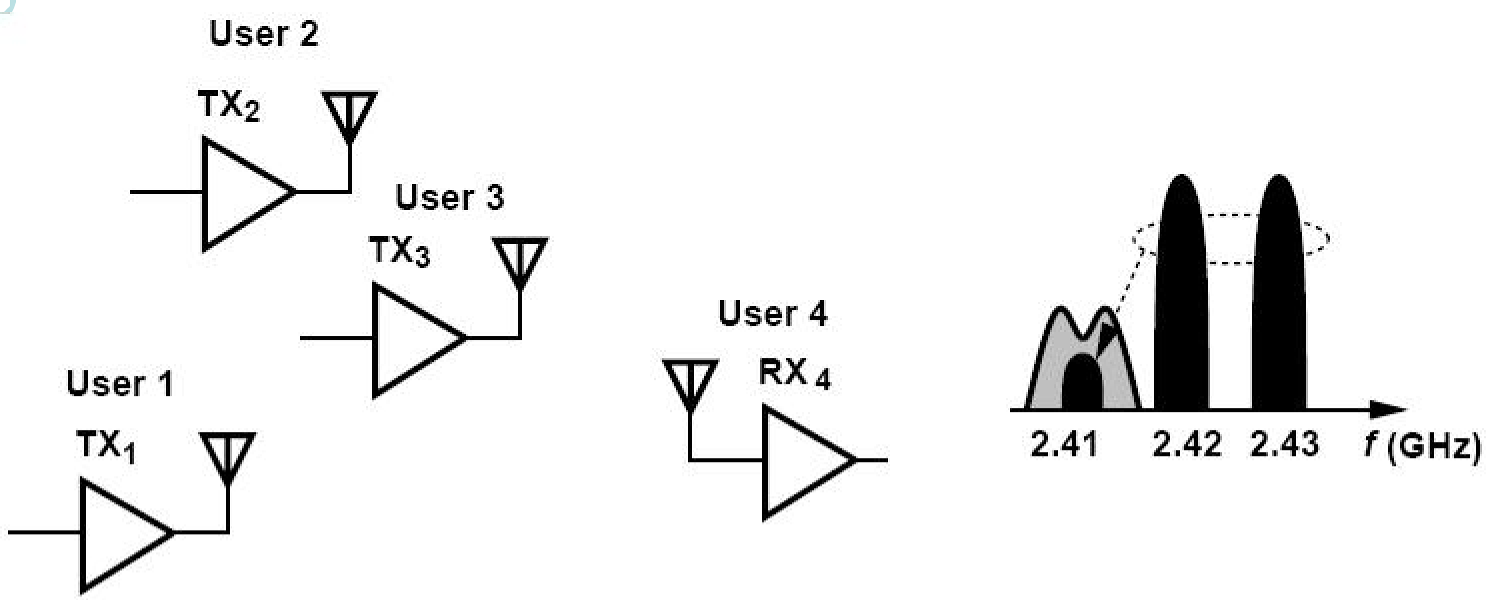
$$2\omega_1 - \omega_2 = \omega_0$$

- **A received small desired signal along with two large interferers**
- **Intermodulation product falls onto the desired channel, corrupts signal.**

Example of Intermodulation

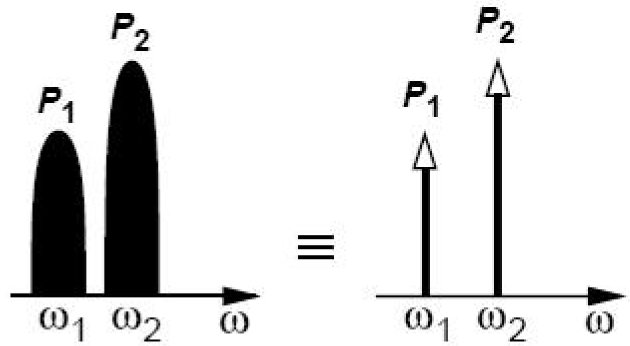
Suppose four Bluetooth users operate in a room as shown in figure below. User 4 is in the receive mode and attempts to sense a weak signal transmitted by User 1 at 2.410 GHz. At the same time, Users 2 and 3 transmit at 2.420 GHz and 2.430 GHz, respectively. Explain what happens.

Solution:

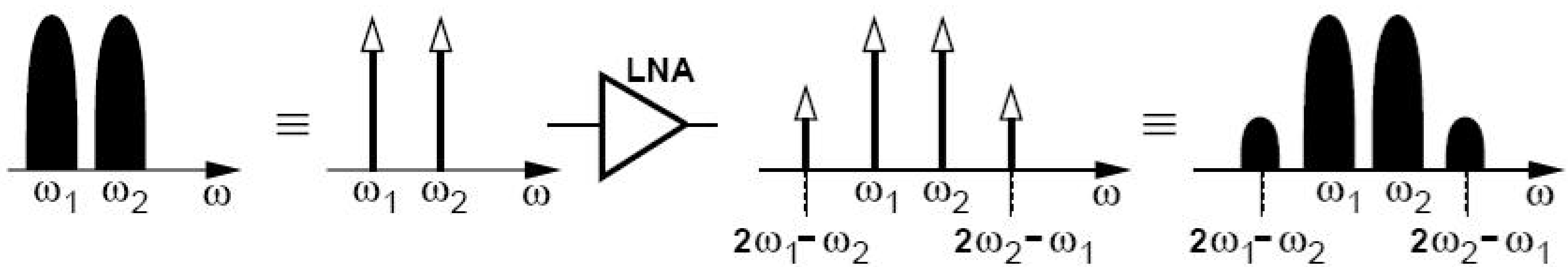


Since the frequencies transmitted by Users 1, 2, and 3 happen to be equally spaced, the intermodulation in the LNA of R_{X4} corrupts the desired signal at 2.410 GHz.

Intermodulation: Tones and Modulated Interferers



(a)



- In intermodulation Analyses:
  - (a) approximate the interferers with tones
  - (b) calculate the level of intermodulation products at the output
  - (c) mentally convert the intermodulation tones back to modulated components so as to see the corruption.

Example of Gain Compression and Intermodulation

A Bluetooth receiver employs a low-noise amplifier having a gain of 10 and an input impedance of 50 Ω. The LNA senses a desired signal level of -80 dBm at 2.410 GHz and two interferers of equal levels at 2.420 GHz and 2.430 GHz. For simplicity, assume the LNA drives a 50-Ω load.
(a) Determine the value of α_3 that yields a P_{1dB} of -30 dBm.
(b) If each interferer is 10 dB below P_{1dB}, determine the corruption experienced by the desired signal at the LNA output.

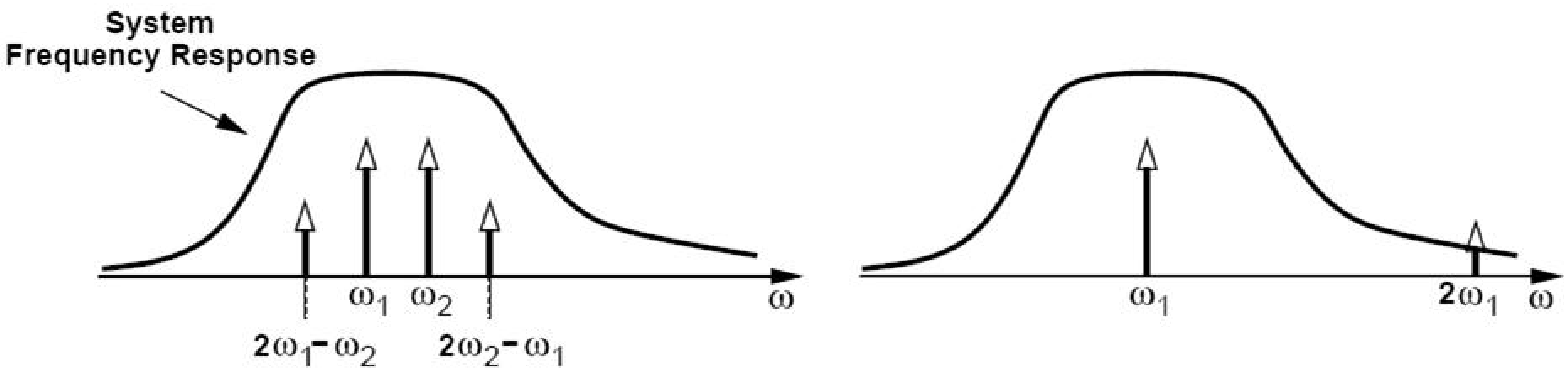
Solution:

(a) From previous equation, α_3 = 14.500 V^{-2}

(b) Each interferer has a level of -40 dBm (= 6.32 m V_{pp}), we determine the amplitude of the IM product at 2.410 GHz as:

$$\frac{3\alpha_3 A_1^2 A_2}{4} = 0.343\,\mathrm{mV_p} = -59.3\ \mathrm{dBm}.$$

Intermodulation: Two-Tone Test and Relative IM



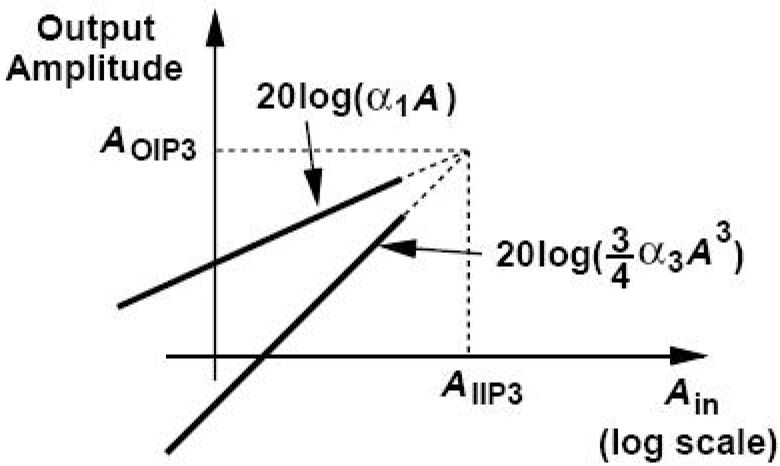
Two-Tone Test **Harmonic Test**

- **Two-Tone Test can be applied to systems with arbitrarily narrow bandwidths**

$$\text{Relative IM} = 20\log\left(\frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2\right)\ \text{dBc}$$

Meaningful only when A is given

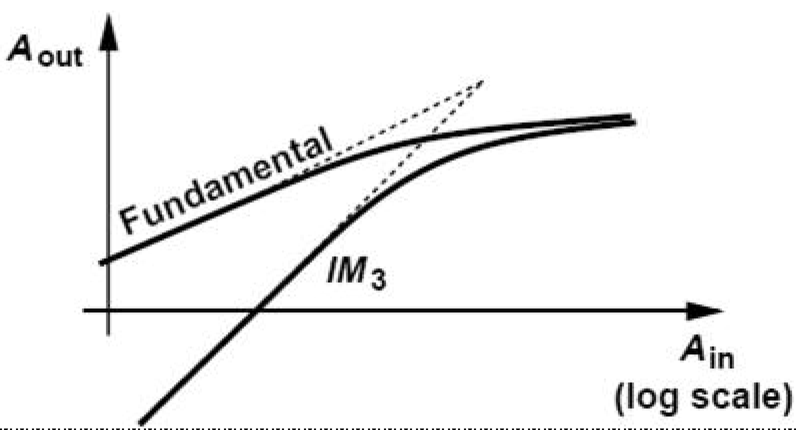
Intermodulation: Third Intercept Point

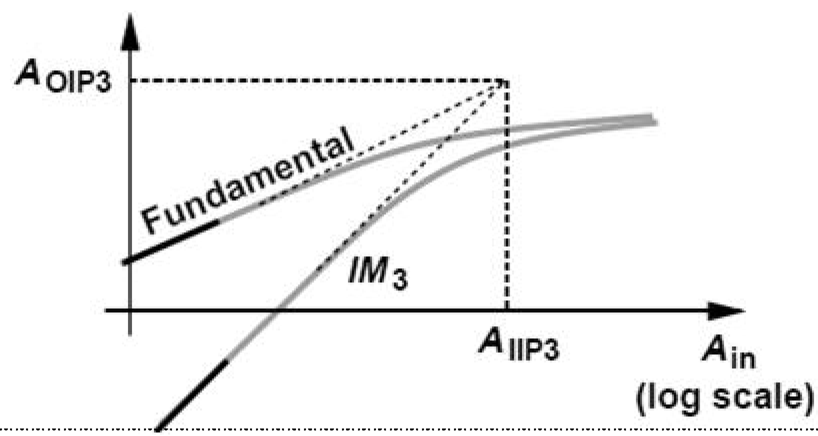


$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A^3_{IIP3}\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}} \approx 9.6 \text{ dB}.$$





- **IP3 is not a directly measureable quantity, but a point obtained by extrapolation**

Example of Third Intercept Point

A low-noise amplifier senses a -80-dBm signal at 2.410 GHz and two -20-dBm interferers at 2.420 GHz and 2.430 GHz. What IIP_3 is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.

Solution:

At the LNA output:

$$20\log|\alpha_1 A_{sig}| - 20\text{ dB} = 20\log\left|\frac{3}{4}\alpha_3 A_{int}^3\right|$$

$$|\alpha_1 A_{sig}| = \left|\frac{30}{4}\alpha_3 A_{int}^3\right|$$

Thus

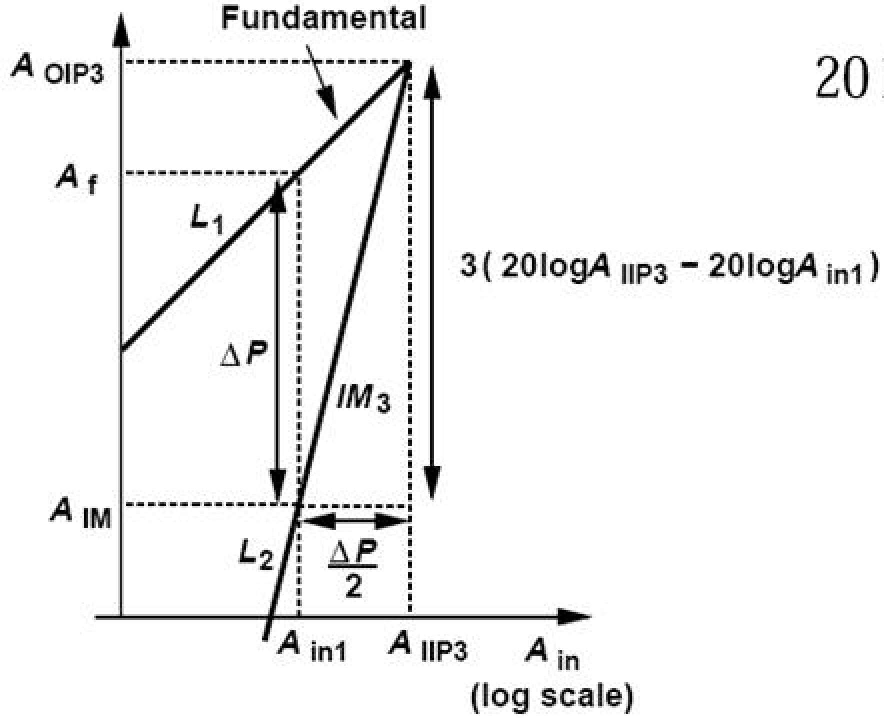
$$\text{IIP}_3 = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$= 3.65\text{ V}_\text{p}$$

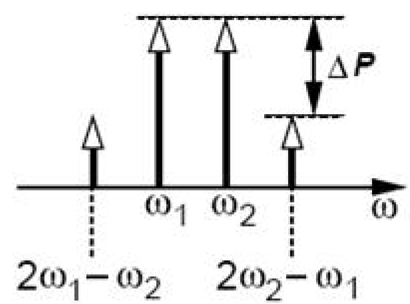
$$= +15.2\text{ dBm}$$

Third Intercept Point: A reasonable estimate

$$\Delta P = 20\log A_f - 20\log A_{IM} = 2(20\log A_{IIP3} - 20\log A_{in1}),$$



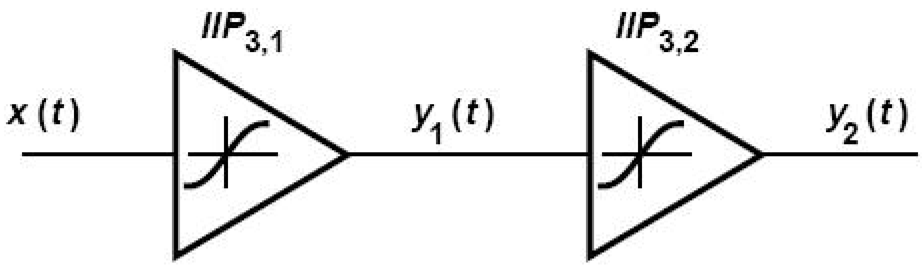
$$20\log A_{IIP3} = \frac{\Delta P}{2} + 20\log A_{in1}$$



$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

- **For a given input level (well below P_{1dB}), the IIP$_3$ can be calculated by halving the difference between the output fundamental and IM levels and adding the result to the input level, where all values are expressed as logarithmic quantities.**

Effects of Nonlinearity: Cascaded Nonlinear Stages



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

$$\begin{aligned} y_2(t) &= \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 \\ &+ \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3. \end{aligned}$$

Considering only the first- and third-order terms, we have:

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \cdots$$

Thus,

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}.$$

Example of Cascaded Nonlinear Stages

Two differential pairs are cascaded. Is it possible to select the denominator of equation above such that IP_3 goes to infinity?

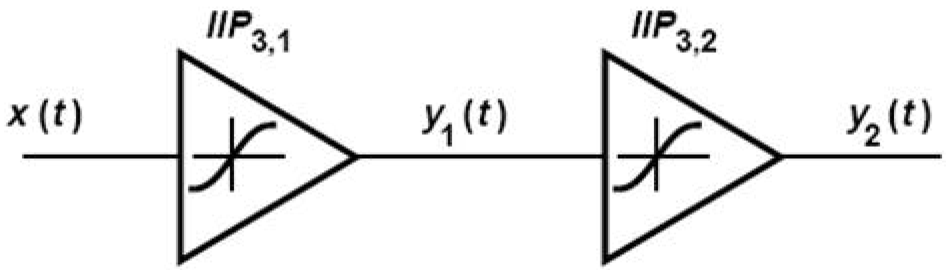
Solution:

With no asymmetries in the cascade, $\alpha_2 = \beta_2 = 0$. Thus, we seek the condition $\alpha_3\beta_1 + \alpha_1^3\beta_3 = 0$, or equivalently,

$$\frac{\alpha_3}{\alpha_1} = -\frac{\beta_3}{\beta_1} \cdot \alpha_1^2$$

Since both stages are compressive, $\alpha_3/\alpha_1 < 0$ and $\beta_3/\beta_1 < 0$. It is therefore impossible to achieve an arbitrarily high IP_3.

Cascaded Nonlinear Stages: Intuitive results

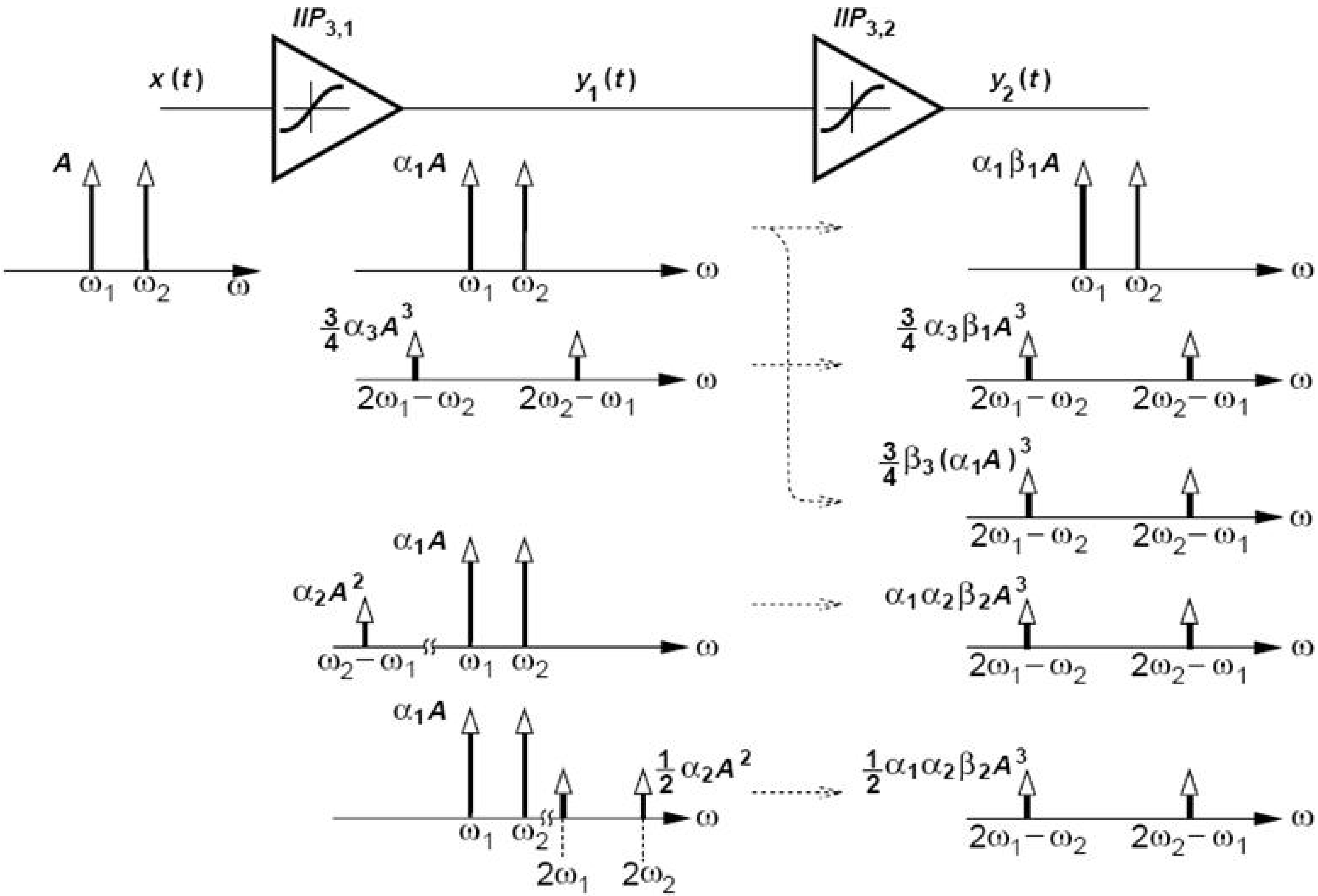


$$\frac{1}{A_{IP3}^2} = \frac{3}{4}\left|\frac{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}{\alpha_1\beta_1}\right|$$
$$= \frac{3}{4}\left|\frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2\beta_2}{\beta_1} + \frac{\alpha_1^2\beta_3}{\beta_1}\right|$$
$$= \left|\frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}\right|$$

- **To "refer" the IP_3 of the second stage to the input of the cascade, we must divide it by α_1. Thus, the higher the gain of the first stage, the more nonlinearity is contributed by the second stage.**

IM Spectra in a Cascade (Ⅰ)

Let us assume $x(t)$ =$A\cos\omega_1 t + A\cos\omega_2 t$ and identify the IM products in a cascade:



IM Spectra in a Cascade (Ⅱ)

Adding the amplitudes of the IM products, we have

$$y_2(t) = \alpha_1\beta_1 A(\cos\omega_1 t + \cos\omega_2 t)$$
$$+ \left(\frac{3\alpha_3\beta_1}{4} + \frac{3\alpha_1^3\beta_3}{4} + \frac{3\alpha_1\alpha_2\beta_2}{2}\right) A^3[\cos(\omega_1 - 2\omega_2)t + \cos(2\omega_2 - \omega_1)t] + \cdots$$

- **Add in phase as worst-case scenario**
- **Heavily attenuated in narrow-band circuits**

For more stages:

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$
$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots$$

- **Thus, if each stage in a cascade has a gain greater than unity, the nonlinearity of the latter stages becomes increasingly more critical because the IP3 of each stage is equivalently scaled down by the total gain preceding that stage.**

Example of Cascaded Nonlinear Stages

A low-noise amplifier having an input IP_3 of -10 dBm and a gain of 20 dB is followed by a mixer with an input IP_3 of +4 dBm. Which stage limits the IP_3 of the cascade more?

Solution:

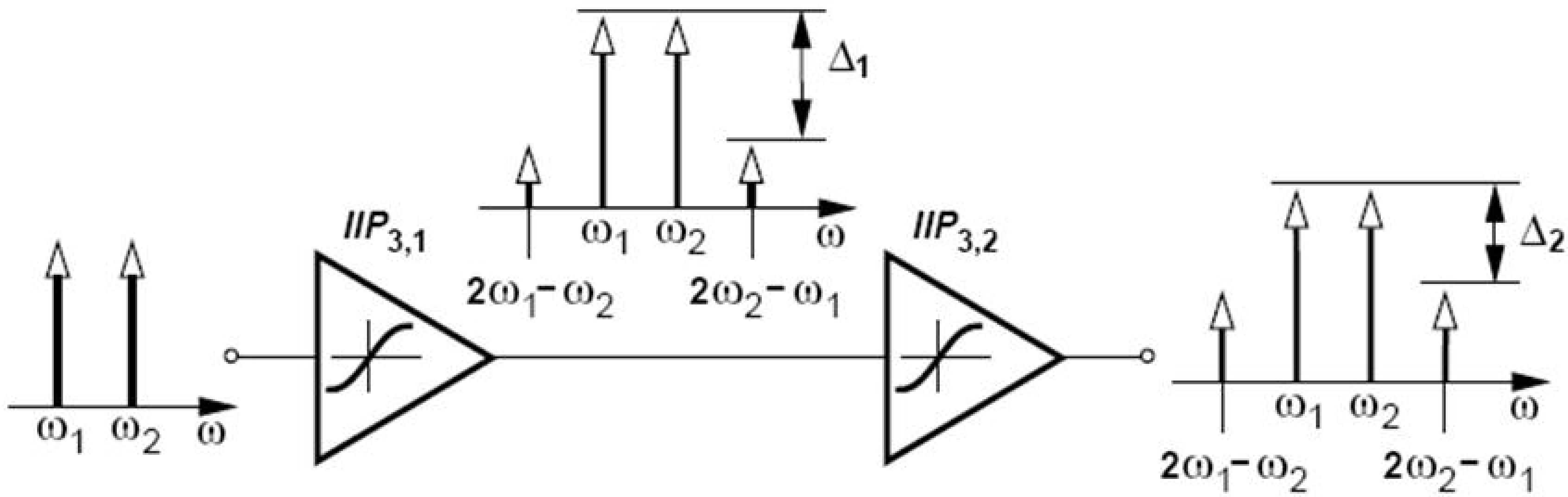
With α_1 = 20 dB, we note that

$$A_{IP3,1} = -10\ \mathrm{dBm}$$

$$\frac{A_{IP3,2}}{\alpha_1} = -16\ \mathrm{dBm}$$

Since the scaled IP_3 of the second stage is lower than the IP_3 of the first stage, we say the second stage limits the overall IP_3 more.

Linearity Limit due to Each Stage

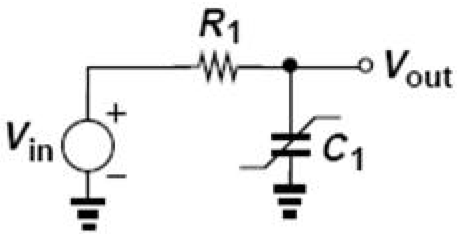


- Examine the relative IM magnitudes at the output of each stage to find out which stage limits the linearity more

Effects of Nonlinearity: AM/PM Conversion

$$V_{out}(t) = V_2 \cos[\omega_1 t + \phi(V_1)]$$

- **AM/PM Conversion arises in systems both dynamic and nonlinear**



$$C_1 = (1 + \alpha V_{out})C_0$$

$$V_{out}(t) \approx \frac{V_1}{\sqrt{1 + R_1^2 C_1^2(t)\omega_1^2}} \cos\{\omega_1 t - \tan^{-1}[R_1 C_1(t)\omega_1]\}$$

If $R_1C_1(t)\omega_1$ << 1 rad

$$V_{out}(t) \approx V_1 \cos[\omega_1 t - R_1(1 + \alpha V_{out})C_0\omega_1]$$

Assume that

$$(1 + \alpha V_{out})C_0 \approx (1 + \alpha V_1 \cos\omega_1 t)C_0.$$

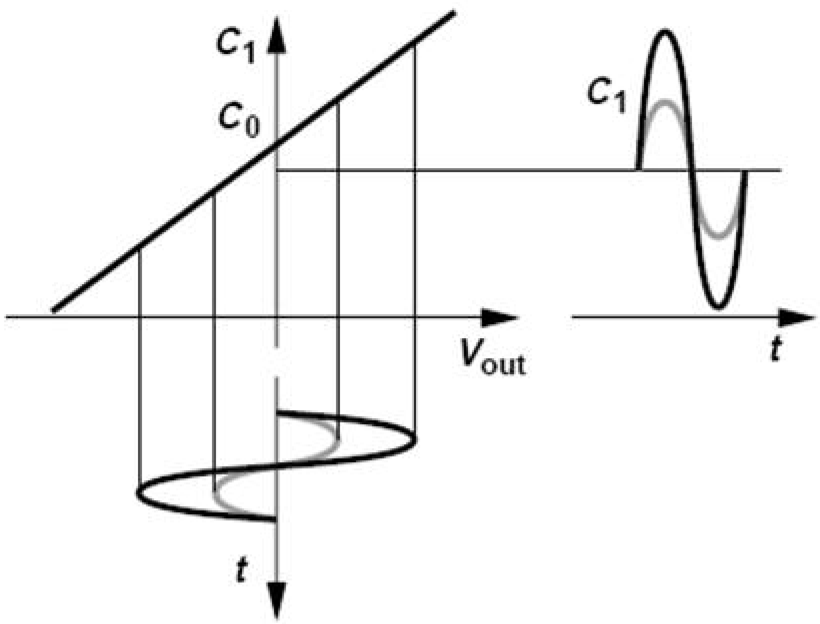
obtaining

$$V_{out}(t) \approx V_1 \cos(\omega_1 t - R_1 C_0 \omega_1 - \alpha R_1 C_0 \omega_1 V_1 \cos\omega_1 t)$$

Phase shift of fundamental, Const.

Higher harmonic

AM/PM Conversion: Time-Variation of Capacitor



First order voltage dependence:

$$C_1 = (1 + \alpha V_{out})C_0$$

$$C_1(t) = C_{avg} + \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_1 t)$$

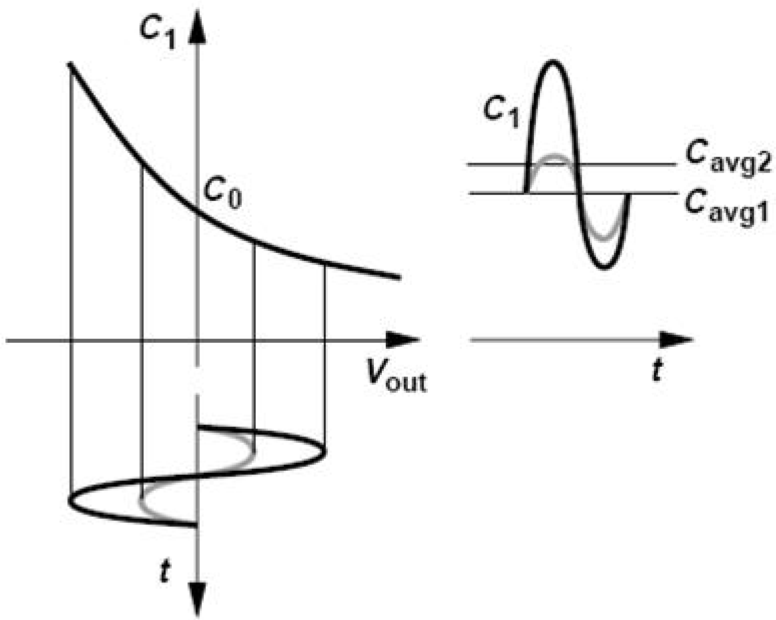
- **No AM/PM conversion because of the first-order dependence of C_1 on V_{out}**

Example of AM/PM Conversion: Second Order Voltage Dependence

Suppose C_1 in above RC section is expressed as $C_1 = C_0(1 + \alpha_1 V_{out} + \alpha_2 V_{out}^2)$. Study the AM/PM conversion in this case if $V_{in}(t) = V_1 \cos \omega_1 t$.

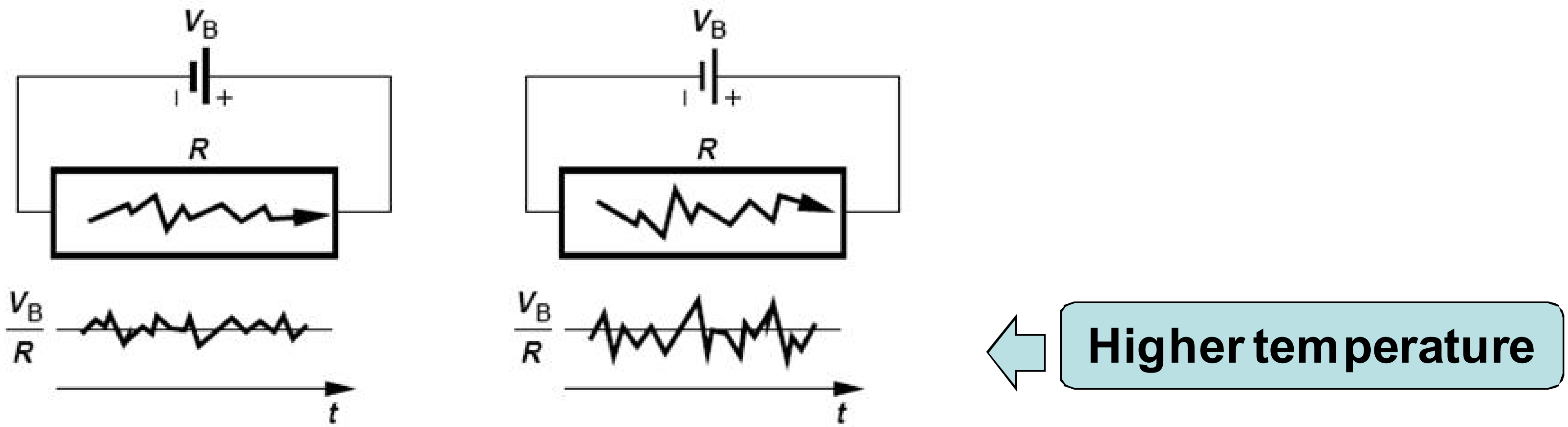
Figure below plots $C_1(t)$ for small and large input swings, revealing that C_{avg} indeed depends on the amplitude.

$$
\begin{aligned}
V_{out}(t) &\approx V_1 \cos[\omega_1 t - R_1 C_0 \omega_1 (1 + \alpha_1 V_1 \cos \omega_1 t + \alpha_2 V_1^2 \cos^2 \omega_1 t)] \\
&\approx V_1 \cos(\omega_1 t - R_1 C_0 \omega_1 - \frac{\alpha_2 R_1 C_0 \omega_1 V_1^2}{2} - \cdots).
\end{aligned}
$$

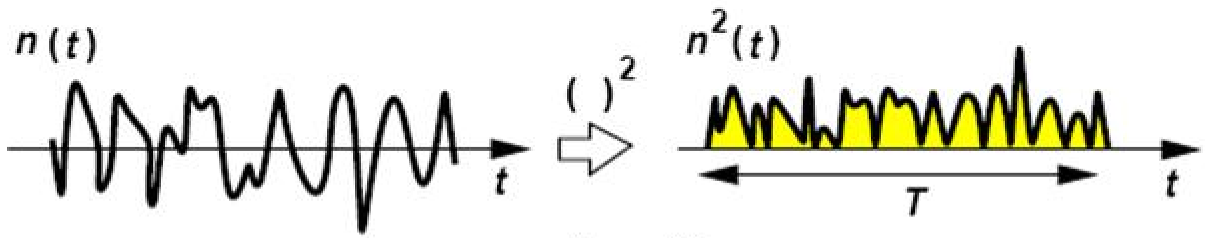


The phase shift of the fundamental now contains an input-dependent term, $-(\alpha_2 R_1 C_0 \omega_1 V_1^2)/2$. This figure also suggests that AM/PM conversion does not occur if the capacitor voltage dependence is odd-symmetric.

Noise: Noise as a Random Process



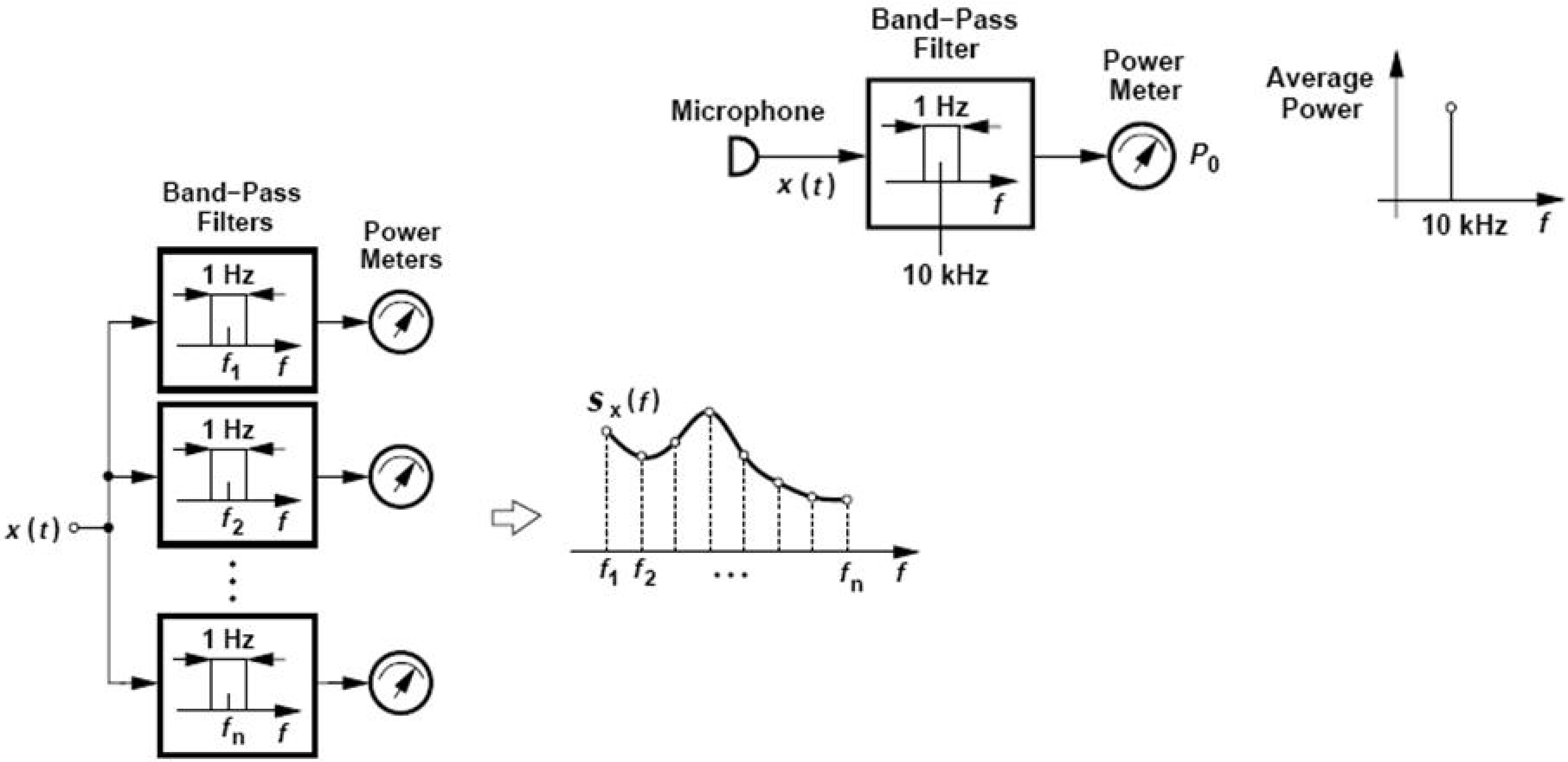
- The average current remains equal to V_B/R but the instantaneous current displays random values



$$P_n = \lim_{T\to\infty} \frac{1}{T} \int_0^T n^2(t)dt$$

- *T* must be long enough to accommodate several cycles of the lowest frequency.

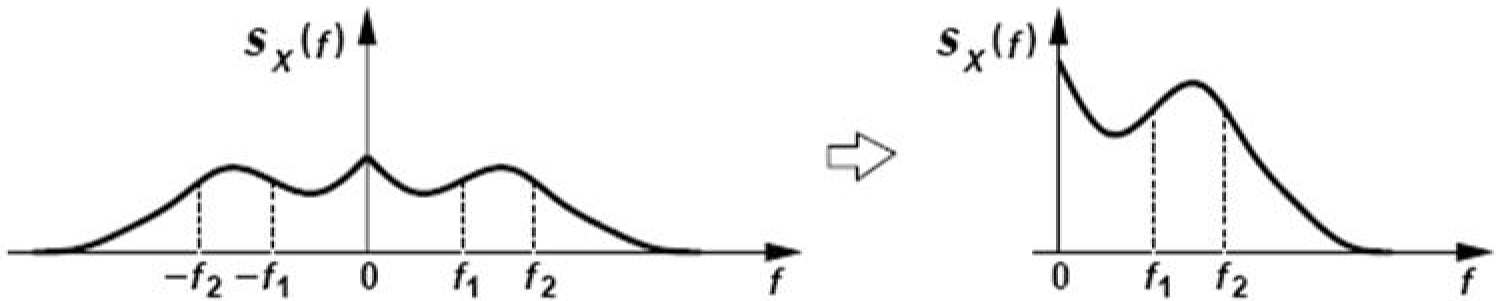
Measurement of Noise Spectrum



- To measure signal’s frequency content at 10 kHz, we need to filter out the remainder of the spectrum and measure the average power of the 10-kHz component.

Noise Spectrum: Power Spectral Density (PSD)

Two-Sided | **One-Sided**



$$\int_0^\infty S_x(f)df = \lim_{T\to\infty}\frac{1}{T}\int_0^T x^2(t)dt$$

- **Total area under $S_x(f)$ represents the average power carried by $x(t)$**

Example of Noise Spectrum

A resistor of value R_1 generates a noise voltage whose one-sided PSD is given by

$$S_v(f) = 4kTR_1$$

where $k = 1.38 \times 10^{-23}$ J/K denotes the Boltzmann constant and T the absolute temperature. Such a flat PSD is called "white" because, like white light, it contains all frequencies with equal power levels.

(a) What is the total average power carried by the noise voltage?

(b) What is the dimension of $S_v(f)$?

(c) Calculate the noise voltage for a 50-Ω resistor in 1 Hz at room temperature.

(a) The area under $S_v(f)$ appears to be infinite, an implausible result because the resistor noise arises from the finite ambient heat. In reality, $S_v(f)$ begins to fall at f > 1 THz, exhibiting a finite total energy, i.e., thermal noise is not quite white.

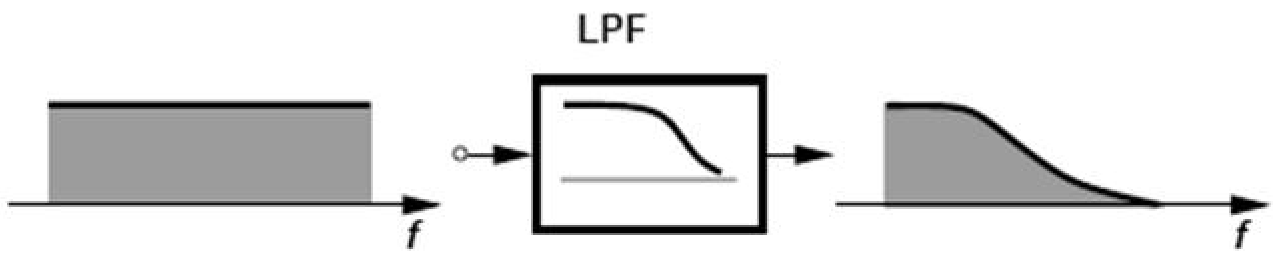
(b) The dimension of $S_v(f)$ is voltage squared per unit bandwidth (V^2/Hz)

(c) For a 50-Ω resistor at T = 300 K

$$\overline{V_n^2} = 8.28 \times 10^{-19}\ \mathrm{V}^2/\mathrm{Hz}$$

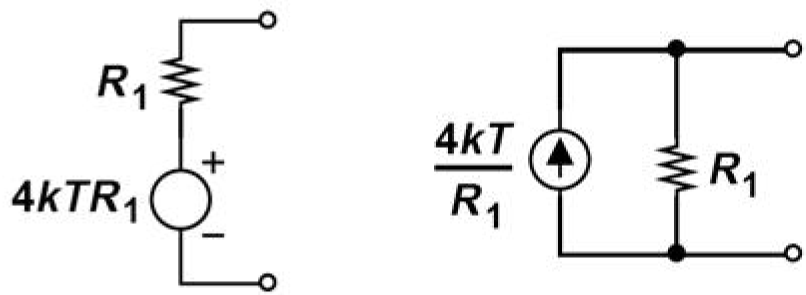
$$\sqrt{\overline{V_n^2}} = 0.91\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$$

Effect of Transfer Function on Noise/ Device Noise



$$S_y(f) = S_x(f)|H(f)|^2$$

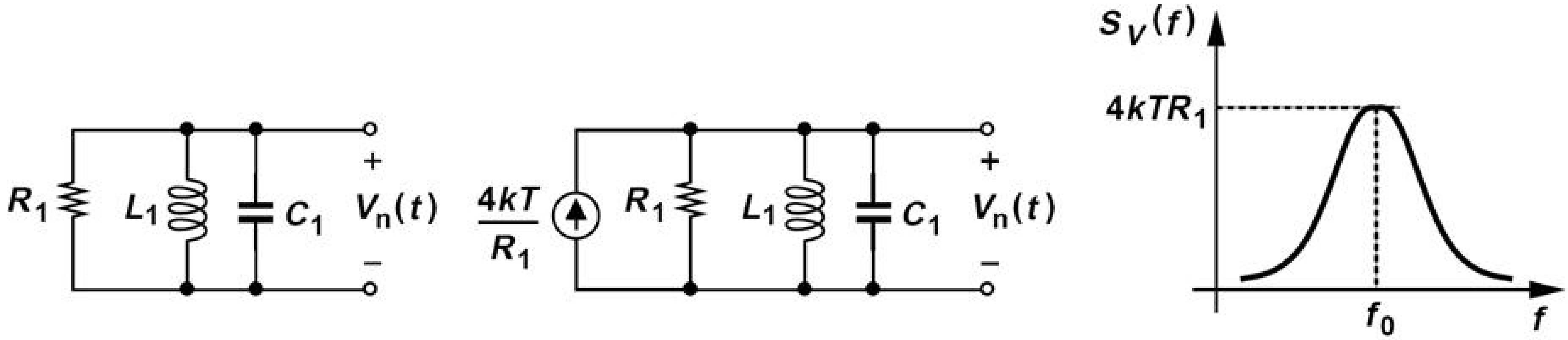
- **Define PSD to allow many of the frequency-domain operations used with deterministic signals to be applied to random signals as well.**



- **Noise can be modeled by a series voltage source or a parallel current source**
- **Polarity of the sources is unimportant but must be kept same throughout the calculations**

Example of Device Noise

Sketch the PSD of the noise voltage measured across the parallel RLC tank depicted in figure below.



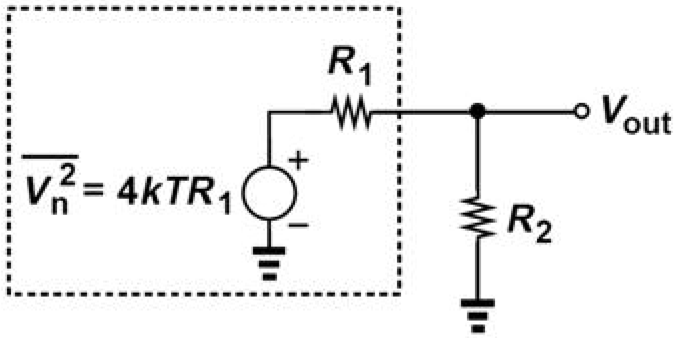
Modeling the noise of R_1 by a current source and noting that the transfer function V_n/I_{n1} is, in fact, equal to the impedance of the tank, Z_T , we write

$$\overline{V_n^2} = \overline{I_{n1}^2}|Z_T|^2$$

At f_0, L_1 and C_1 resonate, reducing the circuit to only R_1. Thus, the output noise at f_0 is simply equal to $4kTR_1$. At lower or higher frequencies, the impedance of the tank falls and so does the output noise.

Can We Extract Energy from Resistor?

Suppose R_2 is held at T = 0 K

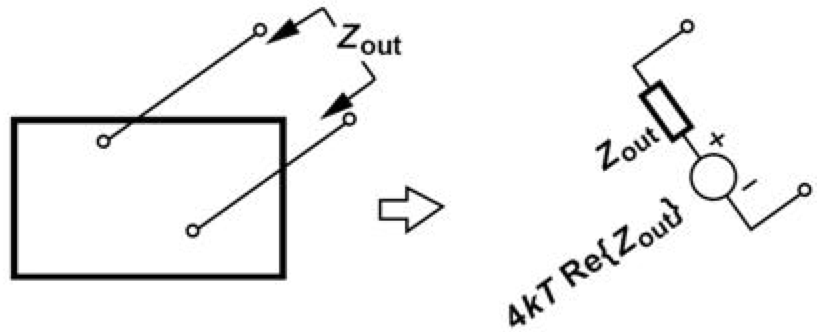


$$\begin{aligned} P_{R2} &= \frac{\overline{V_{out}^2}}{R_2} \\ &= \overline{V_n^2}\left(\frac{R_2}{R_1+R_2}\right)^2 \frac{1}{R_2} \\ &= 4kT\frac{R_1R_2}{(R_1+R_2)^2}. \end{aligned}$$

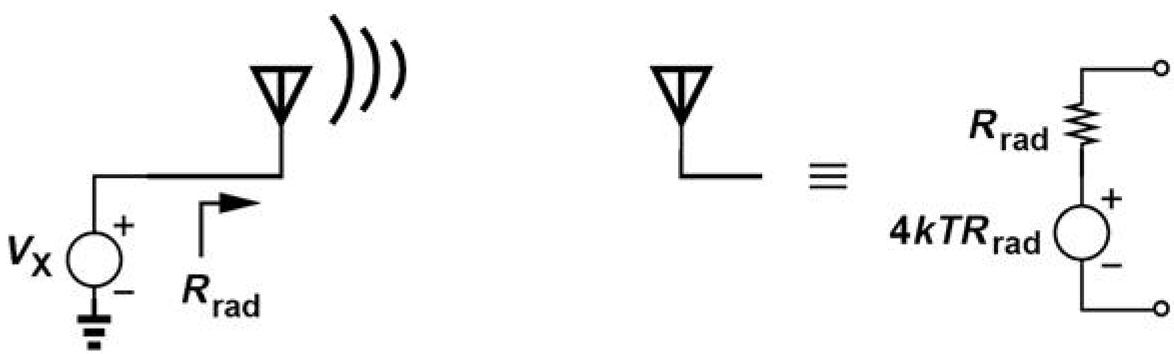
This quantity reaches a maximum if $R_2 = R_1$:
$P_{R2,max} = kT$

Available noise power

A Theorem about Lossy Circuit



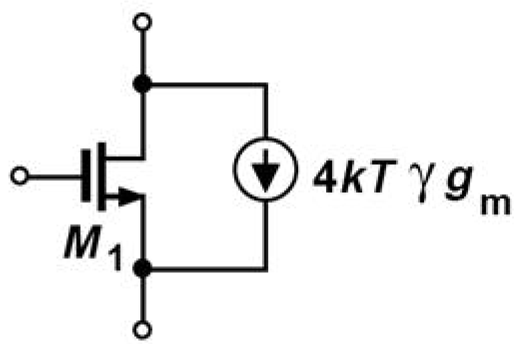
- If the real part of the impedance seen between two terminals of a passive (reciprocal) network is equal to $Re\{Z_{out}\}$, then the PSD of the thermal noise seen between these terminals is given by $4kTRe\{Z_{out}\}$



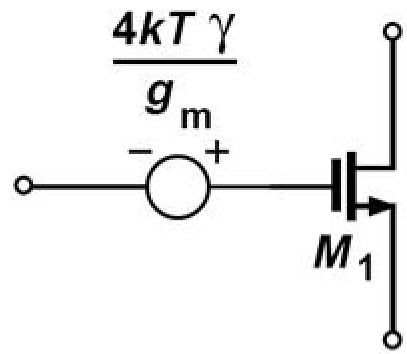
$$\overline{V_{n,ant}^2} = 4kTR_{rad}$$

- An example of transmitting antenna, with radiation resistance R_{rad}

Noise in MOSFETS



$$\overline{I_n^2} = 4kT\gamma g_m$$



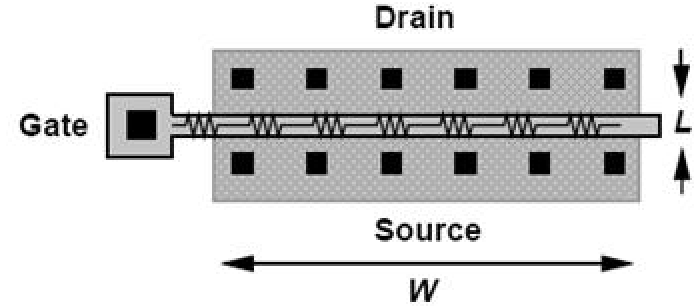
$$\overline{V_n^2} = 4kT\gamma / g_m$$

- **Thermal noise of MOS transistors operating in the saturation region is approximated by a current source tied between the source and drain terminals, or can be modeled by a voltage source in series with gate.**

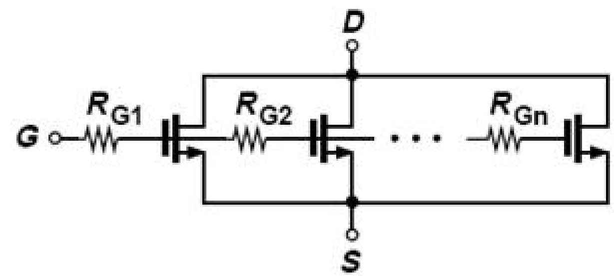
Gate-induced Noise Current

$$R_G = \frac{W}{L} R_{\square}$$

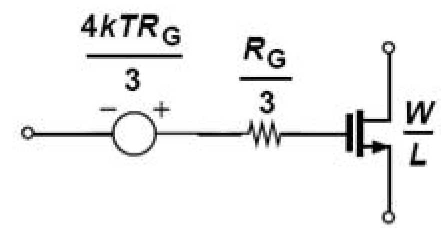
$$4kT\frac{R_G}{3} \ll \frac{4kT\gamma}{g_m}$$

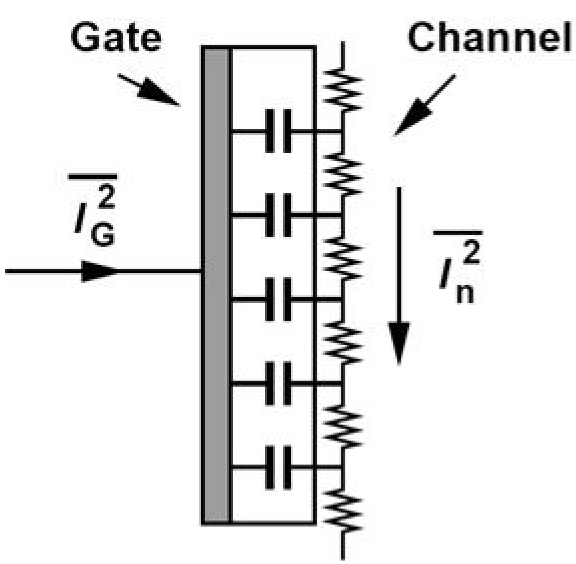


(a)



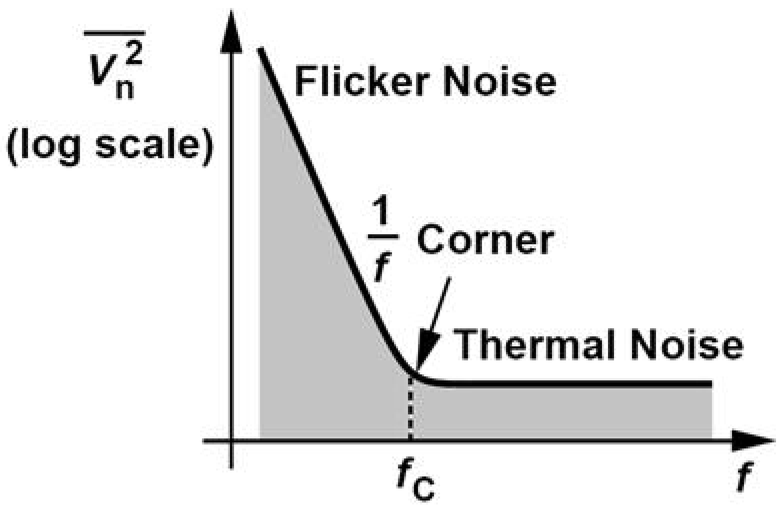
$R_{G1} + R_{G2} + \cdots + R_{Gn} = R_G$





- **At very high frequencies thermal noise current flowing through the channel couples to the gate capacitively**

Flicker Noise and An Example



$$\overline{V_n^2} = \frac{K}{WLC_{ox}}\frac{1}{f}$$

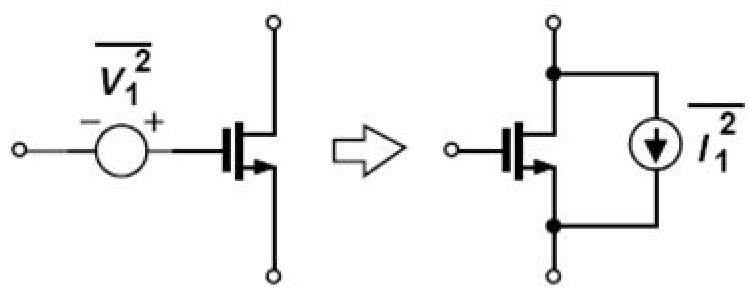
$$\frac{K}{WLC_{ox}}\frac{1}{f_c}g_m^2 = 4kT\gamma g_m$$

$$f_c = \frac{K}{WLC_{ox}}\frac{g_m}{4kT\gamma}$$

Can the flicker noise be modeled by a current source?

Yes, a MOSFET having a small-signal voltage source of magnitude V_1 in series with its gate is equivalent to a device with a current source of value g_mV_1 tied between drain and source. Thus,

$$\overline{I_1^2} = g_m^2\frac{K}{WLC_{ox}}\frac{1}{f}$$



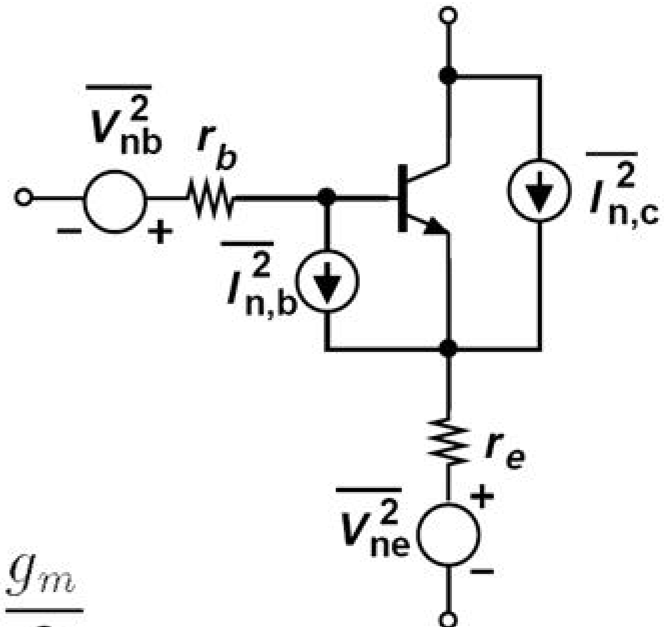
Noise in Bipolar Transistors

Bipolar transistors contain physical resistances in their base, emitter, and collector regions, all of which generate thermal noise. Moreover, they also suffer from "shot noise" associated with the transport of carriers across the base-emitter junction.

$$\overline{I_{n,b}^2} = 2qI_B = 2q\frac{I_C}{\beta}$$

$$\overline{I_{n,c}^2} = 2qI_C,$$

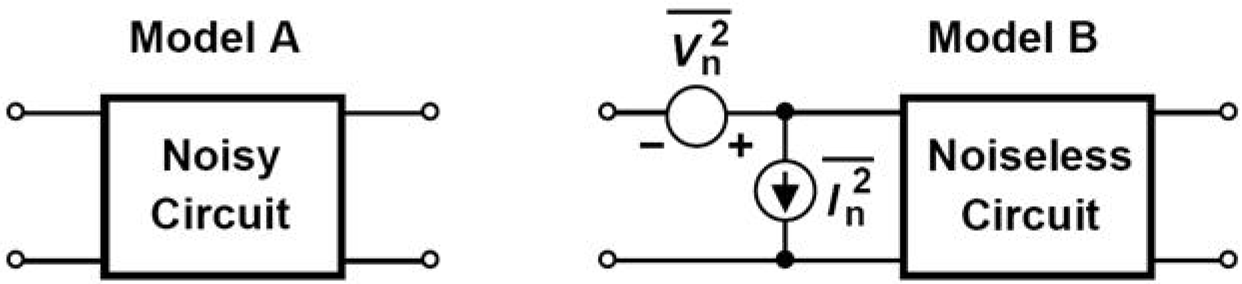
$$g_m = I_C/(kT/q)$$



$$\overline{I_{n,c}^2} = 4kT\frac{g_m}{2}$$

- In low-noise circuits, the base resistance thermal noise and the collector current shot noise become dominant. For this reason, wide transistors biased at high current levels are employed.

Representation of Noise in Circuits: Input-Referred Noise

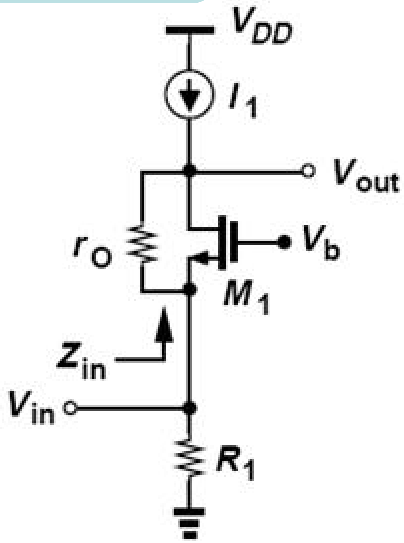


- **Voltage source: short the input port of models A and B and equate their output noise voltage**
- **Current source: leave the input ports open and equate the output noise voltage**

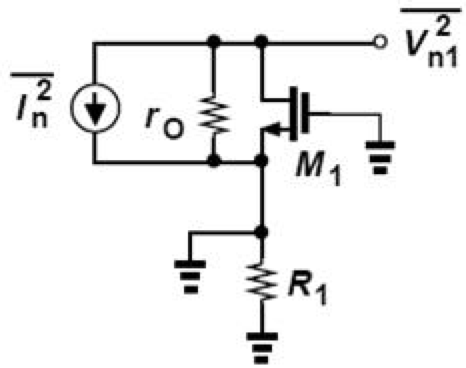
Example of Input-Referred Noise

Calculate the input-referred noise of the common-gate stage depicted in figure below (left). Assume I_1 is ideal and neglect the noise of R_1.

Solution:



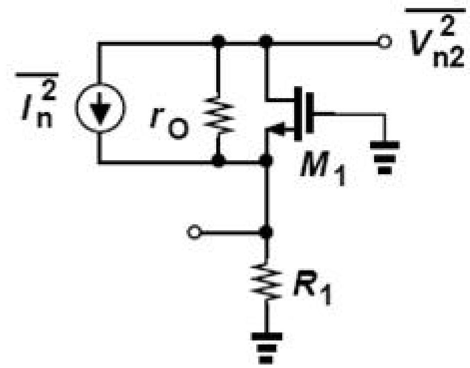
noise voltage



$$\overline{V_{n1}^2} = \overline{I_n^2} \cdot r_O^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{I_n^2} r_O^2}{(1 + g_m r_O)^2} \approx \frac{4kT\gamma}{g_m},$$

noise current



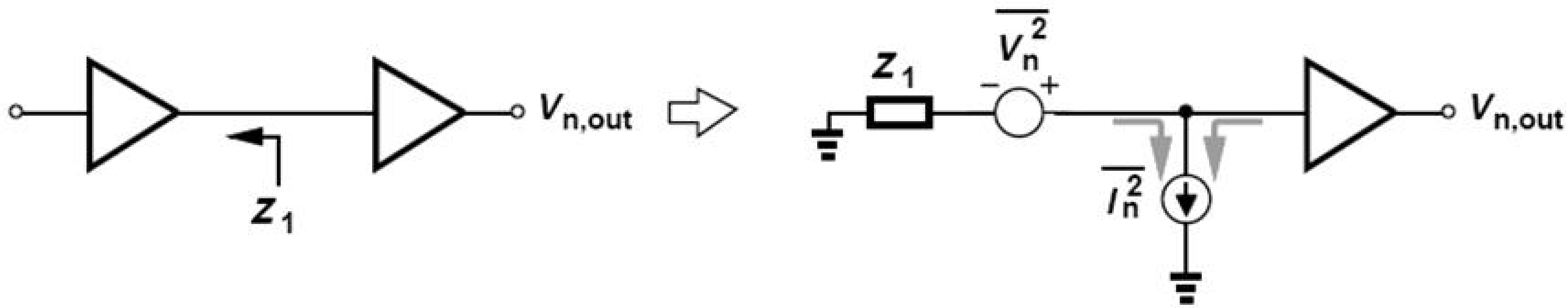
$$\overline{V_{n2}^2} = \overline{I_n^2} r_O^2$$

$$\overline{I_{n,in}^2} = \frac{\overline{I_n^2} r_O^2}{g_m^2 r_O^2 R_1^2} = \frac{4kT\gamma}{g_m R_1^2}.$$

Another Example of Input-Referred Noise

Explain why the output noise of a circuit depends on the output impedance of the preceding stage.

Solution:



Modeling the noise of the circuit by input-referred sources, we observe that some of noise current flows through Z_1, generating a noise voltage at the input that depends on $|Z_1|$. Thus, the output noise, $V_{n,out}$, also depends on $|Z_1|$.

Noise Figure

$$\text{NF} = \frac{SNR_{in}}{SNR_{out}}$$

$$\text{NF}|_{\text{dB}} = 10 \log \frac{SNR_{in}}{SNR_{out}}.$$

- **Depends on not only the noise of the circuit under consideration but the SNR provided by the preceding stage**
- **If the input signal contains no noise, NF=∞**

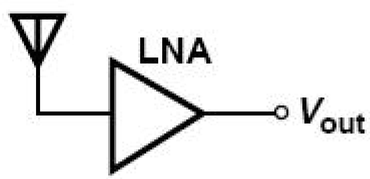
Calculation of Noise Figure

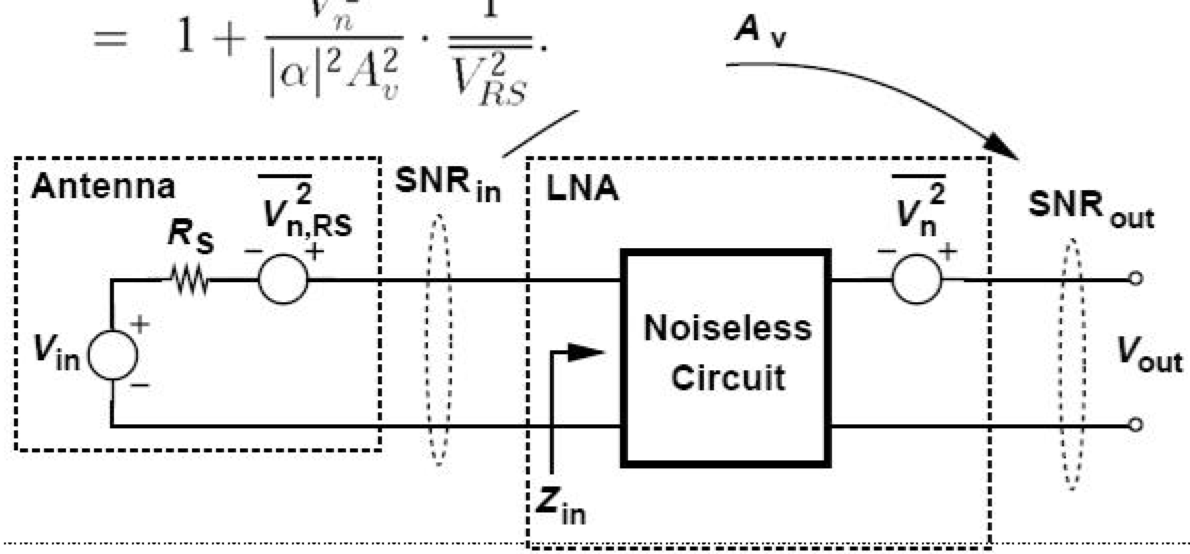
$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}$$

$$\alpha = Z_{in}/(Z_{in} + R_S)$$

$$\begin{aligned} \mathrm{NF} &= \frac{V_{in}^2}{4kTR_S} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{V_{in}^2 |\alpha|^2 A_v^2} \\ &= \frac{1}{\overline{V_{RS}^2}} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{|\alpha|^2 A_v^2} \\ &= 1 + \frac{\overline{V_n^2}}{|\alpha|^2 A_v^2} \cdot \frac{1}{\overline{V_{RS}^2}}. \end{aligned}$$





- **NF must be specified with respect to a source impedance-typically 50 Ω**
- **Reduce the right hand side to a simpler form:**

$$\mathrm{NF} = \frac{1}{4kTR_S} \cdot \frac{\overline{V_{n,out}^2}}{A_0^2}$$

Calculation of NF: Summary

Calculation of NF

- Divide total output noise by the gain from V_{in} to V_{out} and normalize the result to the noise of R_s

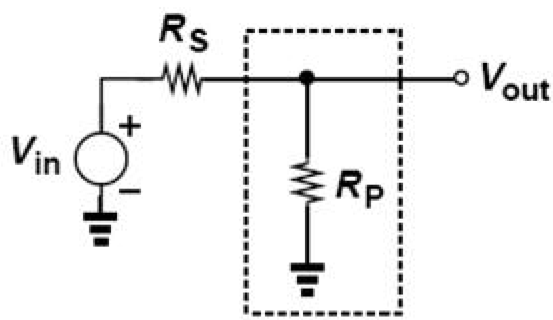
- Calculate the output noise due to the amplifier, divide it by the gain, normalize it to $4kTR_s$ and add 1 to the result

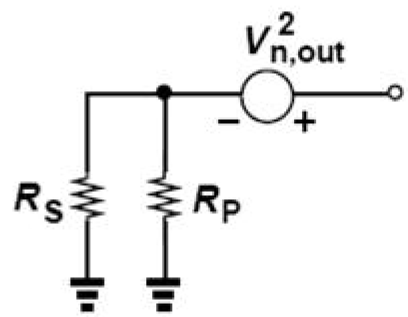
- Valid even if no actual power is transferred. So long as the derivations incorporate noise and signal voltages, no inconsistency arises in the presence of impedance mismatches or even infinite input impedances.

Example of Noise Figure Calculation

Compute the noise figure of a shunt resistor R_P with respect to a source impedance R_S

Solution:





Setting V_{in} to zero:

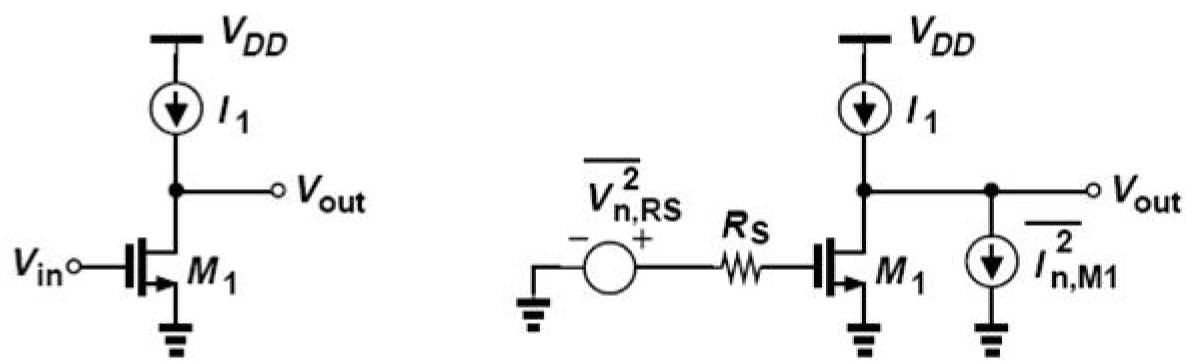
$$\overline{V_{n,out}^2} = 4kT(R_S||R_P) \quad A_0 = \frac{R_P}{R_P + R_S}$$

$$\begin{aligned} \text{NF} &= 4kT(R_S||R_P)\frac{(R_S+R_P)^2}{R_P^2}\frac{1}{4kTR_S} \\ &= 1 + \frac{R_S}{R_P}. \end{aligned}$$

Another Example of Noise Figure Calculation

Determine the noise figure of the common-source stage shown in below (left) with respect to a source impedance R_S. Neglect the capacitances and flicker noise of M_1 and assume I_1 is ideal.

Solution:

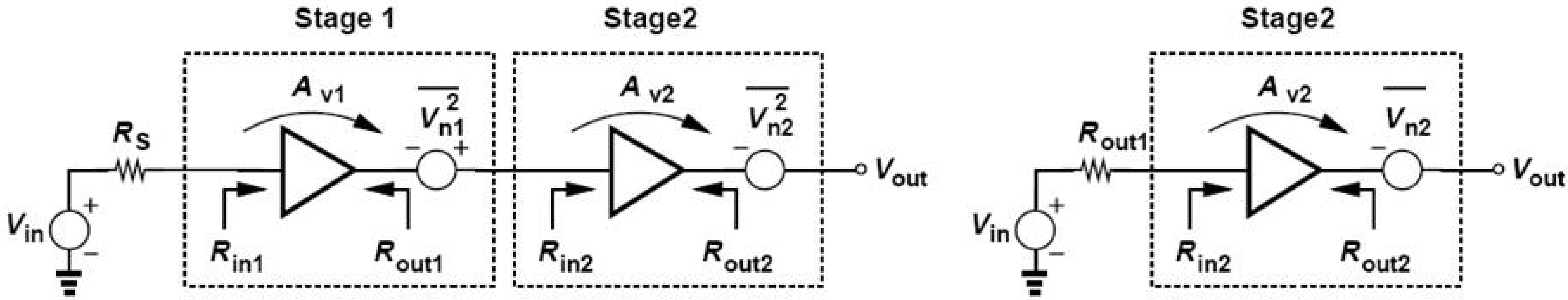


$$\mathrm{NF} = \frac{4kT\gamma g_m r_O^2 + 4kTR_S(g_m r_O)^2}{(g_m r_O)^2} \cdot \frac{1}{4kTR_S}$$

$$= \frac{\gamma}{g_m R_S} + 1.$$

This result implies that the NF falls as R_S rises. Does this mean that, even though the amplifier remains unchanged, the overall system noise performance improves as R_S increases?!

Noise Figure of Cascaded Stages (Ⅰ)



$$A_0 = \frac{V_{out}}{V_{in}} = \frac{R_{in1}}{R_{in1}+R_S}A_{v1}\frac{R_{in2}}{R_{in2}+R_{out1}}A_{v2}$$

$$\overline{V_{n,out}^2} = \overline{V_{n2}^2} + \overline{V_{n1}^2}\frac{R_{in2}^2}{(R_{in2}+R_{out1})^2}A_{v2}^2$$

$$\begin{aligned}
\mathrm{NF_{tot}} &= 1 + \frac{\overline{V_{n,out}^2}}{A_0^2}\cdot\frac{1}{4kTR_S} \\
&= 1 + \frac{\overline{V_{n1}^2}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A_{v1}^2}\cdot\frac{1}{4kTR_S} \\
&+ \frac{\overline{V_{n2}^2}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A_{v1}^2\left(\frac{R_{in2}}{R_{in2}+R_{out1}}\right)^2 A_{v2}^2}\cdot\frac{1}{4kTR_S}
\end{aligned}$$

Noise Figure of Cascaded Stages (Ⅱ)

$$NF_2 = 1 + \frac{\overline{V_{n2}^2}}{\frac{R_{in2}^2}{(R_{in2}+R_{out1})^2}A_{v2}^2}\frac{1}{4kTR_{out1}} \qquad NF_{tot} = NF_1 + \frac{NF_2 - 1}{\frac{R_{in1}^2}{(R_{in1}+R_S)^2}A_{v1}^2\frac{R_S}{R_{out1}}}$$

$$P_{out,av} = V_{in}^2\frac{R_{in1}^2}{(R_S+R_{in1})^2}A_{v1}^2 \cdot \frac{1}{4R_{out1}}$$

$$P_{S,av} = \frac{V_{in}^2}{4R_S}$$

This quantity is in fact the "available power gain" of the first stage, defined as the "available power" at its output, $P_{out,av}$ (the power that it would deliver to a matched load) divided by the available source power, $P_{S,av}$ (the power that the source would deliver to a matched load).

$$NF_{tot} = NF_1 + \frac{NF_2 - 1}{A_{P1}}$$

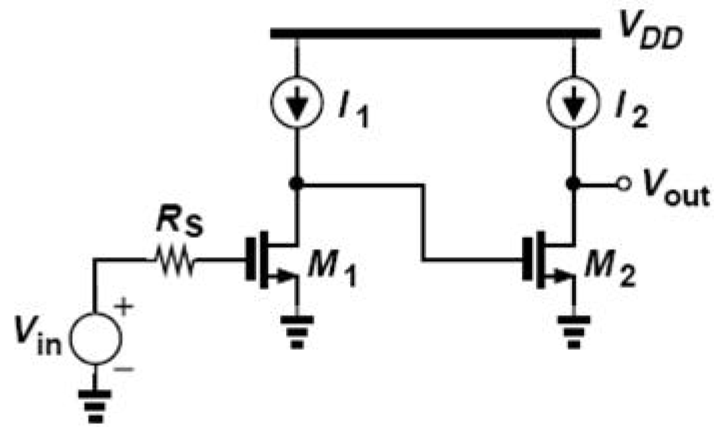
$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1}\cdots A_{P(m-1)}}.$$

Called "Friis' equation", this result suggests that the noise contributed by each stage decreases as the total gain preceding that stage increases, implying that the first few stages in a cascade are the most critical.

Example of Noise Figure of Cascaded Stages

Determine the *NF* of the cascade of common-source stages shown in figure below. Neglect the transistor capacitances and flicker noise.

Solution:



$$R_{in1} = R_{in2} = \infty$$

$$NF = 1 + \frac{\overline{V_{n1}^2}}{A_{v1}^2}\frac{1}{4kTR_S} + \frac{\overline{V_{n2}^2}}{A_{v1}^2 A_{v2}^2}\frac{1}{4kTR_S}$$

where

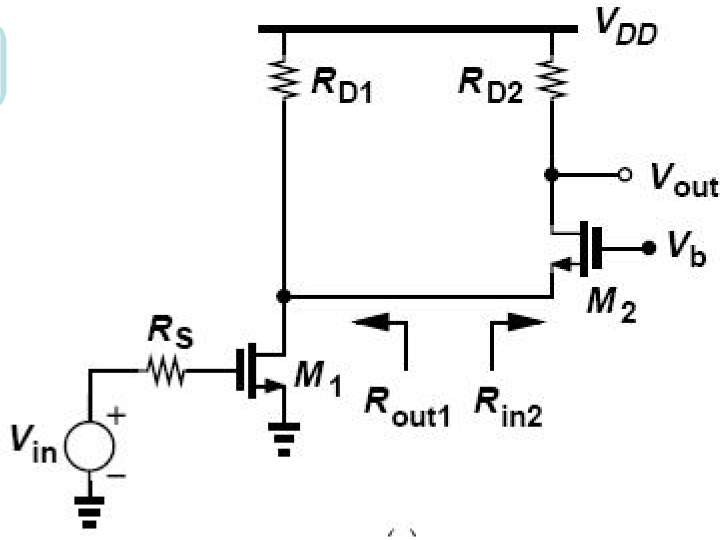
$$\overline{V_{n1}^2} = 4kT\gamma g_{m1} r_{O1}^2,\ \overline{V_{n2}^2} = 4kT\gamma g_{m2} r_{O2}^2,\ A_{v1} = g_{m1} r_{O1},\ \text{and}\ A_{v2} = g_{m2} r_{O2}$$

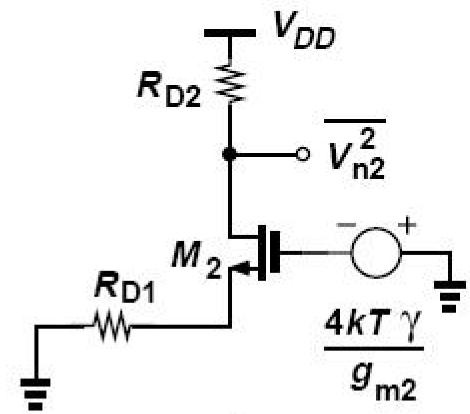
$$NF = 1 + \frac{\gamma}{g_{m1} R_S} + \frac{\gamma}{g_{m1}^2 r_{O1}^2 g_{m2} R_S}$$

Another Example of Noise Figure of Cascaded Stages

Determine the noise figure of the circuit shown below. Neglect transistor capacitances, flicker noise, channel-length modulation , and body effect.

Solution:





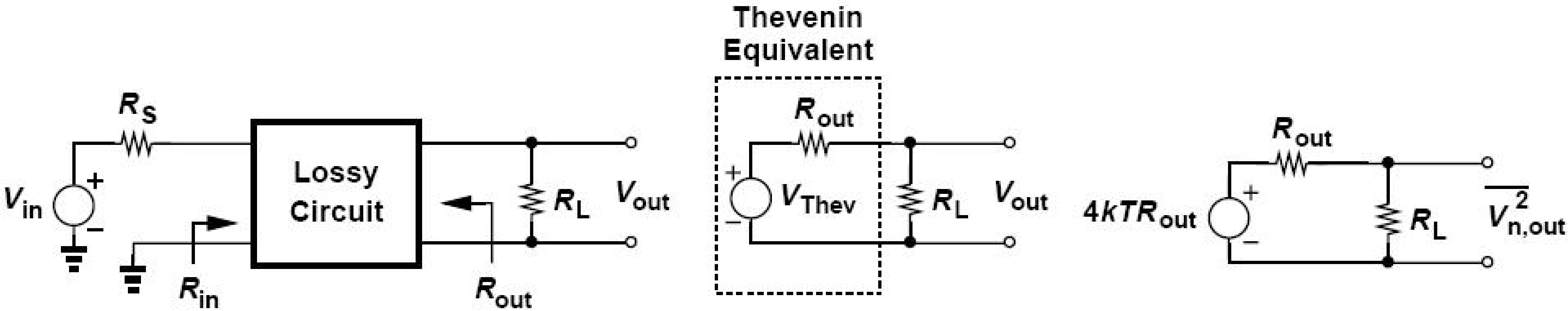
$$\overline{V_{n2}^2} = \frac{4kT\gamma}{g_{m2}}\left(\frac{R_{D2}}{\frac{1}{g_{m2}} + R_{D1}}\right)^2 + 4kTR_{D2}$$

$$\overline{V_{n1}^2} = 4kT\gamma g_{m1}R_{D1}^2 + 4kTR_{D1}$$

$$NF_{tot} = 1 + \frac{4kT\gamma g_{m1}R_D^2 + 4kTR_{D1}}{g_{m1}^2 R_{D1}^2}\cdot\frac{1}{4kTR_S}$$

$$+ \frac{\frac{4kT\gamma}{g_{m2}}\left(\frac{R_{D2}}{g_{m2}^{-1} + R_{D2}}\right)^2 + 4kTR_{D2}}{g_{m1}^2 R_{D1}^2\left(\frac{g_{m2}^{-1}}{g_{m2}^{-1} + R_{D1}}\right)^2 g_{m2}^2 R_{D2}^2}\cdot\frac{1}{4kTR_S}$$

Noise Figure of Lossy Circuits



The power loss is calculated as:

$$L = P_{in}/P_{out}$$

$$L = \frac{V_{in}^2}{V_{Thev}^2}\frac{R_{out}}{R_S}$$

$$\overline{V^2_{n,out}} = 4kTR_{out}\frac{R_L^2}{(R_L+R_{out})^2}$$

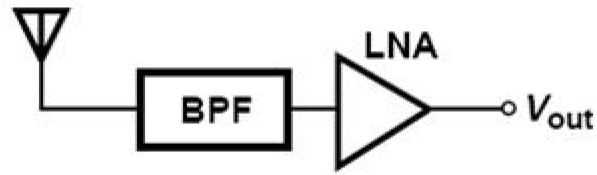
$$A_0 = \frac{V_{Thev}}{V_{in}}\frac{R_L}{R_L+R_{out}}$$

$$\begin{aligned} NF &= 4kTR_{out}\frac{V_{in}^2}{V_{Thev}^2}\frac{1}{4kTR_S} \\ &= L. \end{aligned}$$

Example of Noise Figure of Lossy Circuits

The receiver shown below incorporates a front-end band-pass filter (BPF) to suppress some of the interferers that may desensitize the LNA. If the filter has a loss of *L* and the LNA a noise figure of NF_{LNA}, calculate the overall noise figure.

Solution:



Denoting the noise figure of the filter by NF_{filt}, we write Friis' equation as

$$\begin{aligned} NF_{tot} &= NF_{filt} + \frac{NF_{LNA} - 1}{L^{-1}} \\ &= L + (NF_{LNA} - 1)L \\ &= L \cdot NF_{LNA}, \end{aligned}$$

where NF_{LNA} is calculated with respect to the output resistance of the filter. For example, if *L* = 1.5 dB and NF_{LNA} = 2 dB, then NF_{tot} = 3.5 dB.

Sensitivity and Dynamic Range: Sensitivity

- **The sensitivity is defined as the minimum signal level that a receiver can detect with "acceptable quality."**

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig}/P_{RS}}{SNR_{out}}$$

$$P_{sig} = P_{RS} \cdot NF \cdot SNR_{out}$$

$$P_{sig,tot} = P_{RS} \cdot NF \cdot SNR_{out} \cdot B$$

$$P_{sen}|_{dBm} = P_{RS}|_{dBm/Hz} + NF|_{dB} + SNR_{min}|_{dB} + 10\log B$$

$$P_{sen} = -174\ \text{dBm/Hz} + NF + 10\log B + SNR_{min}$$

Noise Floor

Example of Sensitivity

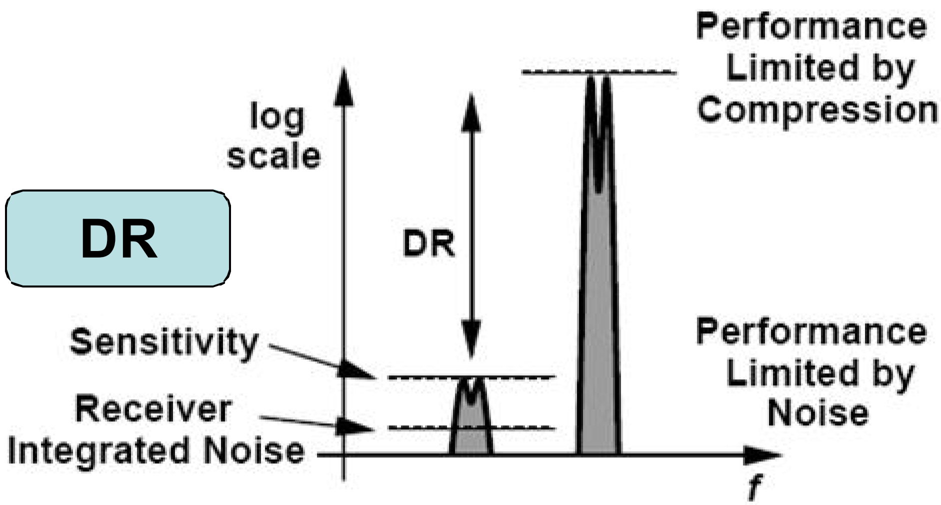
A GSM receiver requires a minimum *SNR* of 12 dB and has a channel bandwidth of 200 kHz. A wireless LAN receiver, on the other hand, specifies a minimum *SNR* of 23 dB and has a channel bandwidth of 20 MHz. Compare the sensitivities of these two systems if both have an *NF* of 7 dB.

Solution:

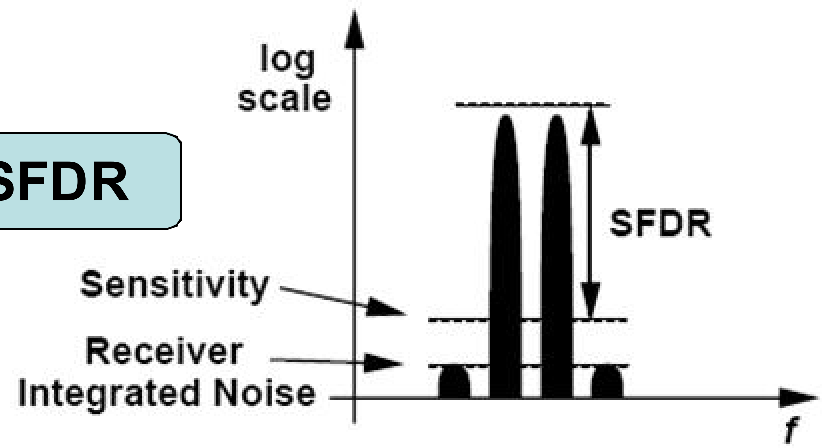
For the GSM receiver, P_{sen} = -102 dBm, whereas for the wireless LAN system, P_{sen} = -71 dBm. Does this mean that the latter is inferior? No, the latter employs a much wider bandwidth and a more efficient modulation to accommodate a data rate of 54 Mb/s. The GSM system handles a data rate of only 270 kb/s. In other words, specifying the sensitivity of a receiver without the data rate is not meaningful.

Dynamic Range Compared with SFDR

DR



SFDR



- **Dynamic Range:**

  Maximum tolerable desired signal power divided by the minimum tolerable desired signal power

- **SFDR:**

  Lower end equal to sensitivity.

  Higher end defined as maximum input level in a *two-tone* test for which the third-order IM products do not exceed the integrated noise of the receiver

SFDR Calculation

Refer output IM magnitudes to input:

$$P_{IIP3} = P_{in} + \frac{P_{out} - P_{IM,out}}{2}$$

$$P_{IM,in} = P_{IM,out} - G. \qquad P_{in} = P_{out} - G.$$

$$P_{IIP3} = P_{in} + \frac{P_{in} - P_{IM,in}}{2}$$

$$= \frac{3P_{in} - P_{IM,in}}{2},$$

$$P_{in} = \frac{2P_{IIP3} + P_{IM,in}}{3}.$$

$$P_{in,max} = \frac{2P_{IIP3} + (-174\ \text{dBm} + NF + 10\log B)}{3}.$$

$$SFDR = P_{in,max} - (-174\ \text{dBm} + NF + 10\log B + SNR_{min})$$

$$= \frac{2(P_{IIP3} + 174\ \text{dBm} - NF - 10\log B)}{3} - SNR_{min}.$$

Example Comparing SFDR and DR

The upper end of the dynamic range is limited by intermodulation in the presence of two interferers or desensitization in the presence of one interferer. Compare these two cases and determine which one is more restrictive.

Solution:

$$P_{1-dB} \overset{?}{\gtrless} P_{in,max}$$

Since $P_{1-dB} = P_{IIP3} - 9.6\ \text{dB}$

$$P_{IIP3} - 9.6\ \text{dB} \overset{?}{\gtrless} \frac{2P_{IIP3} + (-174\ \text{dBm} + NF + 10\log B)}{3}$$

$$P_{IIP3} - 28.8\ \text{dB} \overset{?}{\gtrless} -174\ \text{dBm} + NF + 10\log B$$

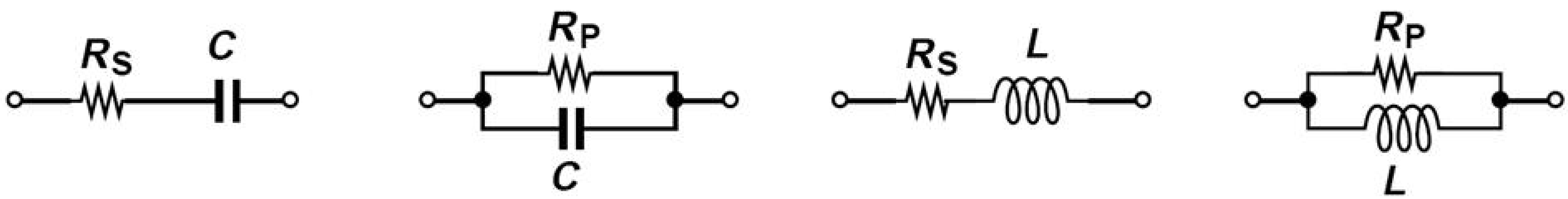
Noise floor

$$P_{1-dB} > P_{in,max}$$

- **SFDR is a more stringent characteristic of system than DR**

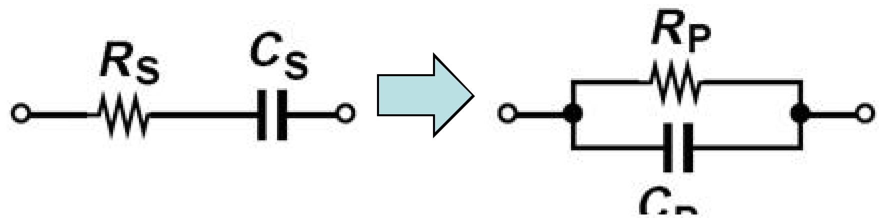
Passive Impedance Transformation: Quality Factor

- **Quality Factor, *Q*, indicates how close to ideal an energy-storing device is.**



$$Q_S = \frac{\frac{1}{C\omega}}{R_S} \qquad Q_P = \frac{R_P}{\frac{1}{C\omega}} \qquad Q_S = \frac{L\omega}{R_S} \qquad Q_P = \frac{R_P}{L\omega}$$

Series-to-Parallel Conversion



$$\frac{R_S C_S s + 1}{C_S s} = \frac{R_P}{R_P C_P s + 1}$$

$$R_P C_S j\omega = 1 - R_P C_P R_S C_S \omega^2 + (R_P C_P + R_S C_S) j\omega.$$

$$R_P C_P R_S C_S \omega^2 = 1$$

$$R_P C_P + R_S C_S - R_P C_S = 0.$$

$Q_s = Q_p$

$$R_P = \frac{1}{R_S C_S^2 \omega^2} + R_S$$

$$R_P = (Q_S^2 + 1) R_S$$

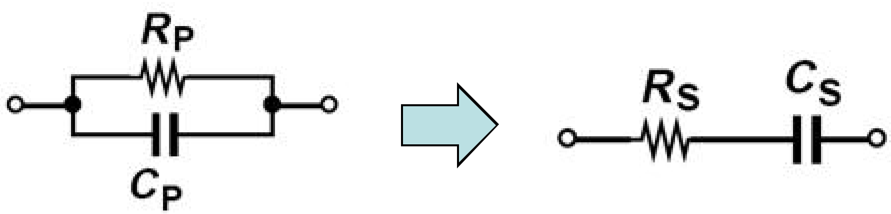
$$C_P = \frac{Q_S^2}{Q_S^2 + 1} C_S$$

$$Q_S^2 \gg 1$$

$$R_P \approx Q_S^2 R_S$$

$$C_P \approx C_S.$$

Parallel-to-Series Conversion



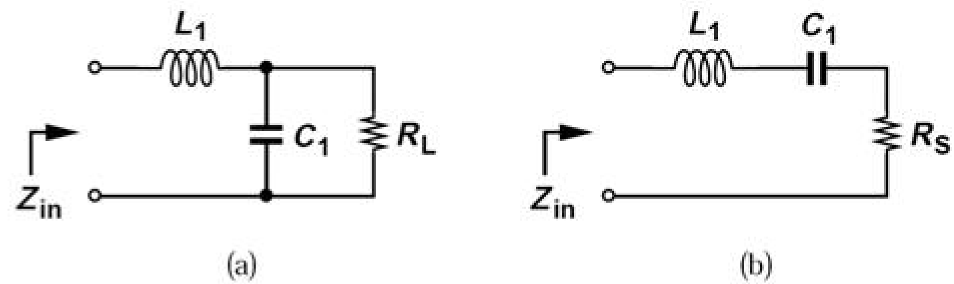
$$R_S = \frac{R_P}{Q_P^2}$$

$$C_s = C_P$$

- **Series-to-Parallel Conversion: will retain the value of the capacitor but raises the resistance by a factor of Q_s^2**
- **Parallel-to-Series Conversion: will reduce the resistance by a factor of Q_P^2**

Basic Matching Networks

$$Z_{in}(j\omega) = \frac{R_L(1 - L_1C_1\omega^2) + jL_1\omega}{1 + jR_LC_1\omega}$$



Thus,

$$Re\{Z_{in}\} = \frac{R_L}{1 + R_L^2C_1^2\omega^2} = \frac{R_L}{1 + Q_P^2},$$

R_L transformed down by a factor

$$L_1 = \frac{R_L^2C_1}{1 + R_L^2C_1^2\omega^2} = \frac{R_L^2C_1}{1 + Q_P^2}.$$

Setting imaginary part to zero

If $Q_P^2 \gg 1$

$$Re\{Z_{in}\} \approx \frac{1}{R_LC_1^2\omega^2}$$

$$L_1 = \frac{1}{C_1\omega^2}.$$

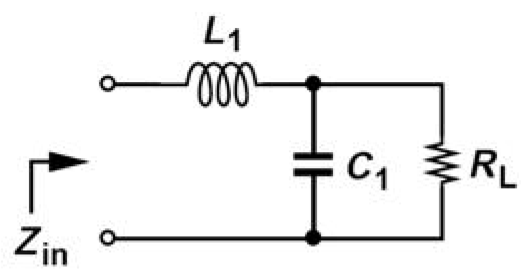
Example of Basic Matching Networks

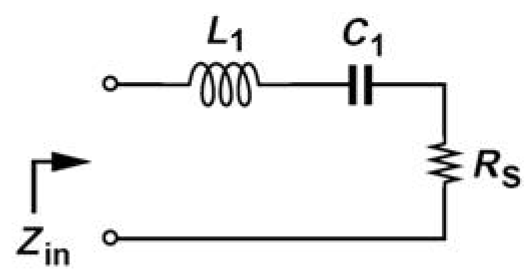
Design the matching network of figure above so as to transform R_L = 50 Ω to 25 Ω at a center frequency of 5 GHz.

Solution:

$$Re\{Z_{in}\} \approx \frac{1}{R_L C_1^2 \omega^2}$$

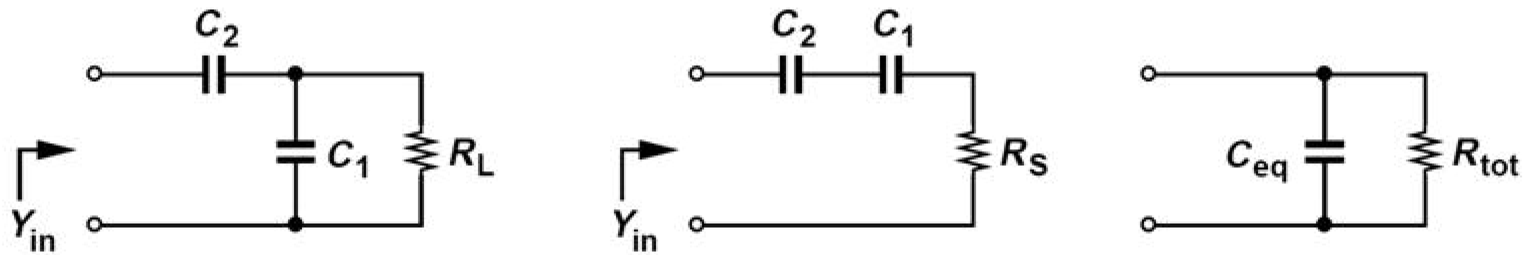
$$L_1 = \frac{1}{C_1 \omega^2}.$$





Assuming Q_P^2 >> 1, we have C_1 = 0:90 pF and L_1 = 1.13 nH, respectively. Unfortunately, however, Q_P = 1.41, indicating the Q_P^2 >> 1 approximation cannot be used. We thus obtain C_1 = 0:637 pF and L_1 = 0:796 nH.

Transfer a Resistance to a Higher Value



If $Q^2 \gg 1$ $R_S \approx [R_L(C_1\omega)^2]^{-1}$ $C_S \approx C_1$

Viewing C_2 and C_1 as one capacitor, C_{eq} $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

$$R_{tot} = \frac{1}{R_S(C_{eq}\omega)^2}$$

$$= \left(1 + \frac{C_1}{C_2}\right)^2 R_L$$

RL boosted

For low Q values

$$Y_{in} = \frac{j\omega C_2(1 + j\omega R_L C_1)}{1 + R_L(C_1 + C_2)j\omega}$$

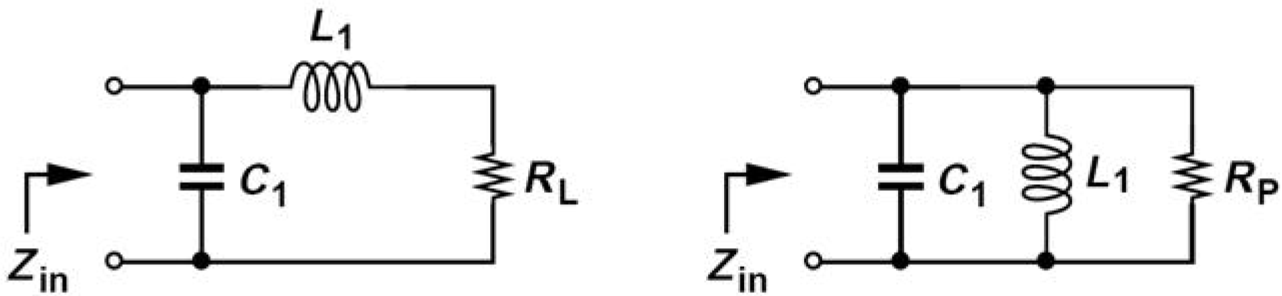
$$R_{tot} = \frac{1}{Re\{Y_{in}\}}$$

$$= \frac{1}{R_L C_2^2 \omega^2} + R_L \left(1 + \frac{C_1}{C_2}\right)^2$$

Another Example of Basic Matching Networks

Determine how the circuit shown below transforms R_L.

Solution:

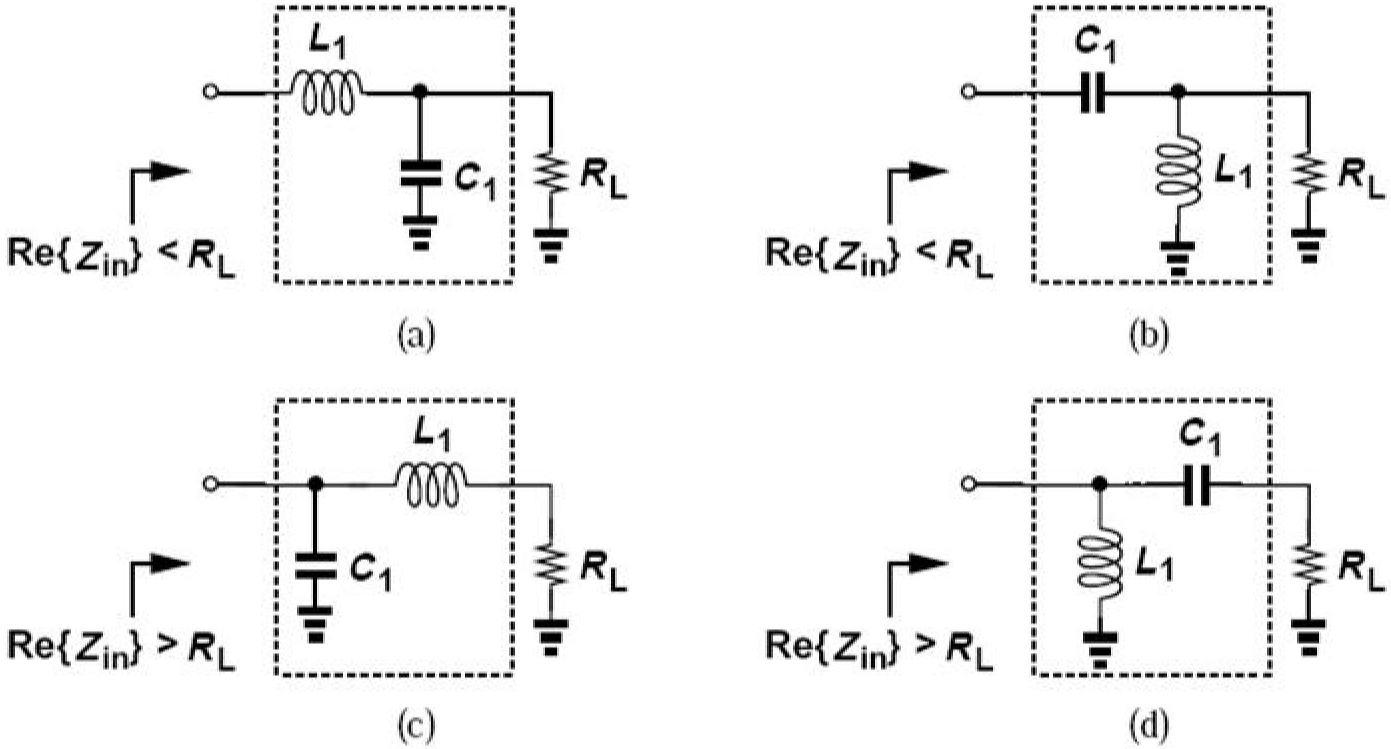


We postulate that conversion of the L_1-R_L branch to a parallel section produces a higher resistance. If $Q_S^2 = (L_1\omega/R_L)^2 >> 1$, then the equivalent parallel resistance is

$$R_P = Q_S^2 R_L = \frac{L_1^2\omega^2}{R_L}.$$

The parallel equivalent inductance is approximately equal to L_1 and is cancelled by C_1

L-Sections



For example, in (a), we have:

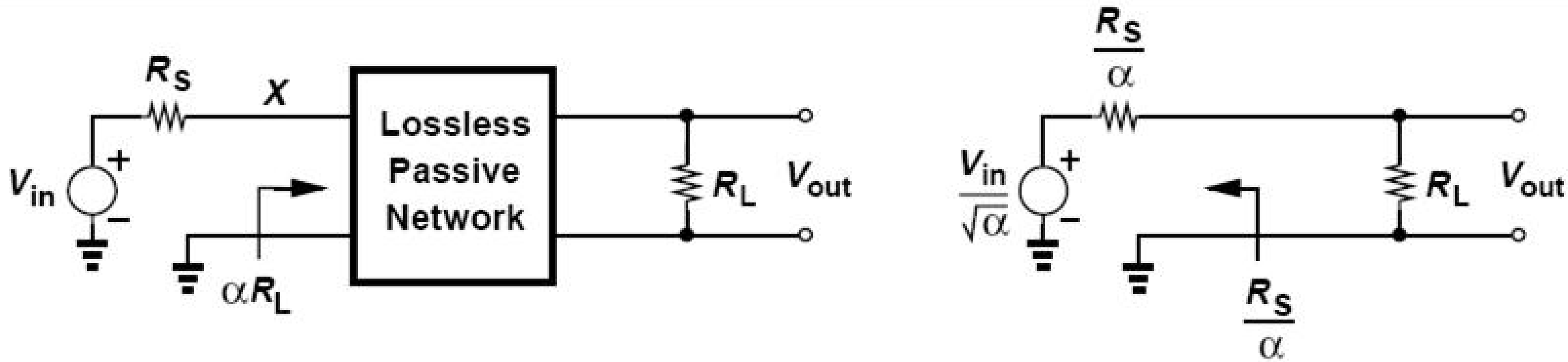
$$\frac{V_{out}}{V_{in}} = \sqrt{\frac{R_L}{Re\{Z_{in}\}}}, \qquad \frac{I_{out}}{I_{in}} = \sqrt{\frac{Re\{Z_{in}\}}{R_L}}$$

a network transforming R_L to a lower value "amplifies" the voltage and attenuates the current by the above factor.

Example of L-Sections

A closer look at the L-sections (a) and (c) suggests that one can be obtained from the other by swapping the input and output ports. Is it possible to generalize this observation?

Solution:

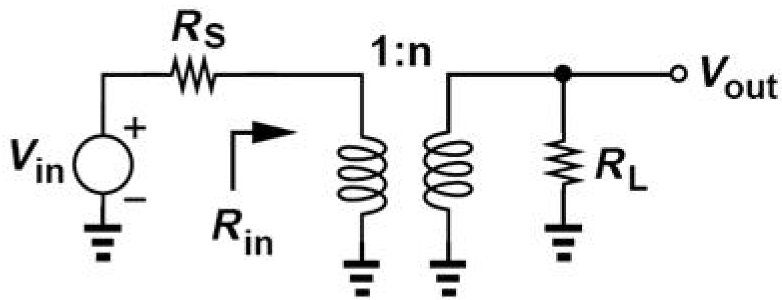


Yes, it is. Consider the arrangement shown above (left), where the passive network transforms R_L by a factor of α. Assuming the input port exhibits no imaginary component, we equate the power delivered to the network to the power delivered to the load:

$$\left(V_{in}\frac{\alpha R_L}{\alpha R_L + R_S}\right)^2 \cdot \frac{1}{\alpha R_L} = \frac{V_{out}^2}{R_L} \Rightarrow V_{out} = \frac{V_{in}}{\sqrt{\alpha}} \cdot \frac{R_L}{R_L + \frac{R_S}{\alpha}}$$

If the input and output ports of such a network are swapped, the resistance transformation ratio is simply inverted.

Impedance Matching by Transformers



$$V_{in}^2/R_{in} = n^2V_{in}^2/R_L$$

$$R_{in} = R_L/n^2$$

- **More on this in Chapter 8**

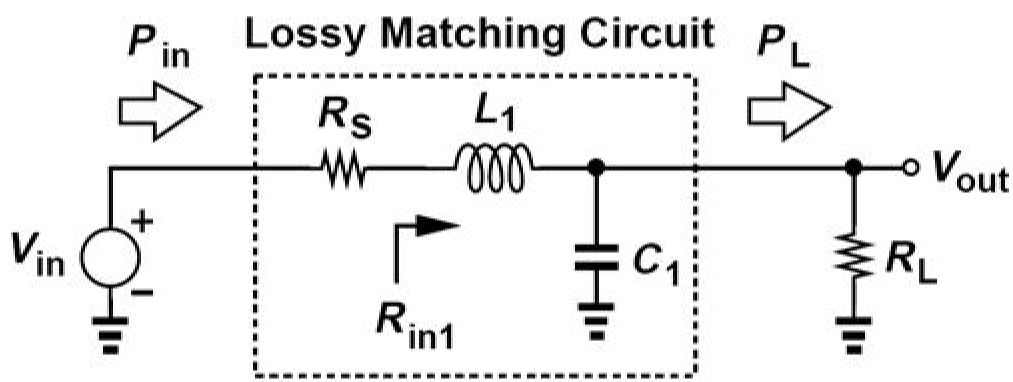
Loss in Matching Networks

We define the loss as the power provided by the input divided by that delivered to R_L

$$P_{in} = \frac{V_{in}^2}{R_S + R_{in1}}$$

$$P_L = \left(V_{in}\frac{R_{in1}}{R_S + R_{in1}}\right)^2 \cdot \frac{1}{R_{in1}}$$

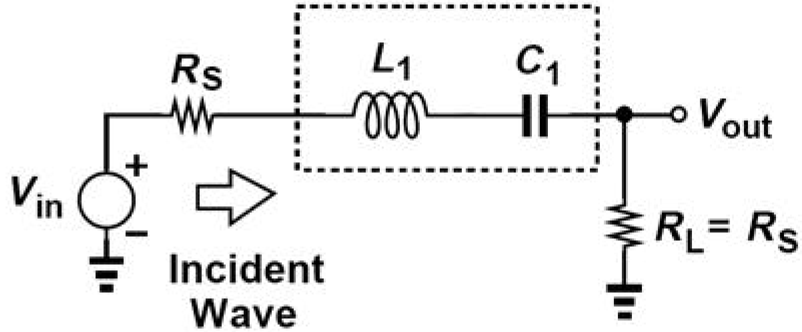
$$\text{Loss} = \frac{P_{in}}{P_L} = 1 + \frac{R_S}{R_{in1}}$$



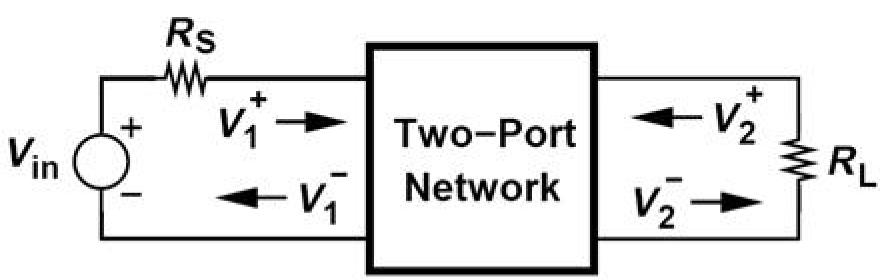
$$P_{in} = \frac{V_{out}^2}{R_P||R_L} = \frac{V_{out}^2}{R_L}\frac{R_P + R_L}{R_P}$$

$$\text{Loss} = 1 + \frac{R_L}{R_P}$$

Scattering Parameters

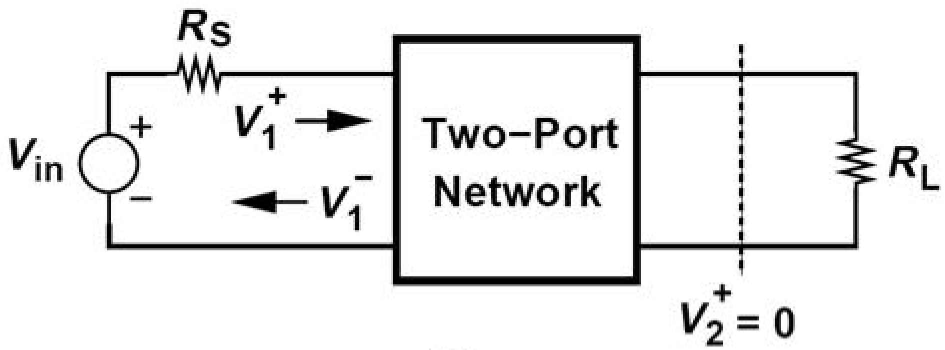


- **S-Parameter: Use power quantities instead of voltage or current**
- **The difference between the incident power (the power that would be delivered to a matched load) and the reflected power represents the power delivered to the circuit.**



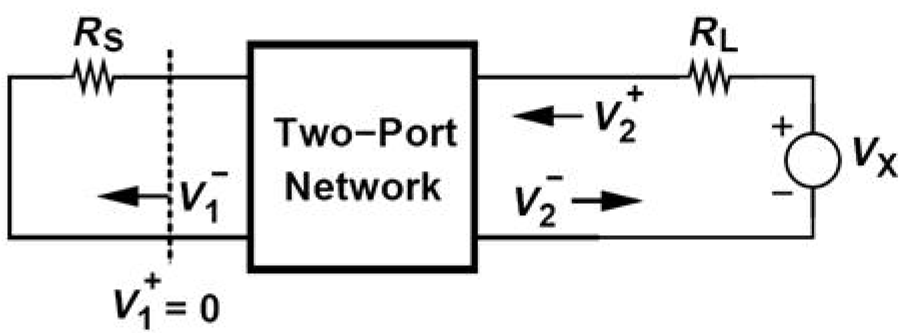
$$
\begin{aligned}
V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ \\
V_2^- &= S_{21}V_1^+ + S_{22}V_2^+.
\end{aligned}
$$

S_{11} and S_{12}



$$S_{11} = \frac{V_1^-}{V_1^+}\Big|_{V_2^+=0}$$

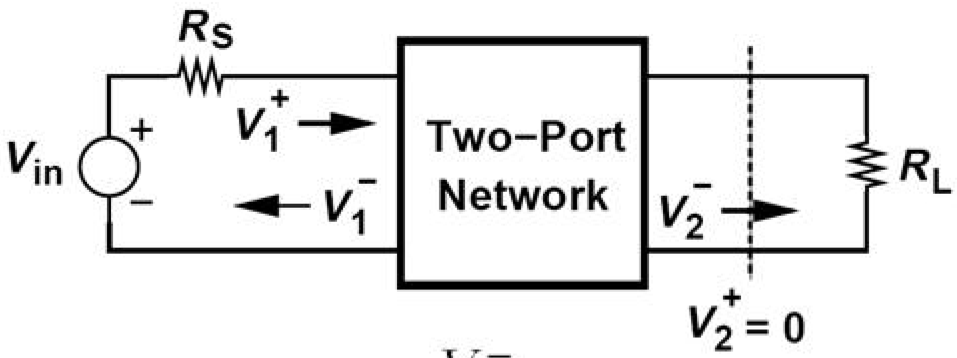
- S_{11} is the ratio of the reflected and incident waves at the input port when the reflection from R_L is zero.
- Represents the accuracy of the input matching



$$S_{12} = \frac{V_1^-}{V_2^+}\Big|_{V_1^+=0}$$

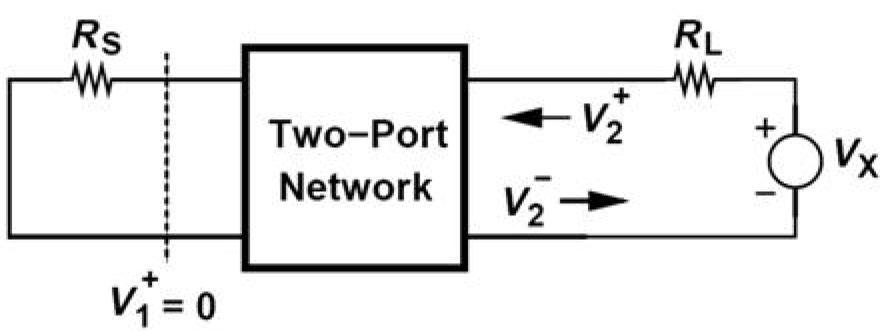
- S_{12} is the ratio of the reflected wave at the input port to the incident wave into the output port when the input is matched
- Characterizes the *reverse isolation*

S_{21} and S_{22}



$$S_{21} = \frac{V_2^-}{V_1^+}|_{V_2^+=0}$$

- S_{21} **is the ratio of the wave incident on the load to that going to the input when the reflection from R_L is zero**
- **Represents the gain of the circuit**



$$S_{22} = \frac{V_2^-}{V_2^+}|_{V_1^+=0}$$

- S_{22} **is the ratio of reflected and incident waves at the output when the reflection from R_s is zero**
- **Represents the accuracy of the output matching**

Scattering Parameters: A few remarks

$$\begin{aligned} V_1^- &= S_{11}V_1^+ + S_{12}V_2^+ \\ V_2^- &= S_{21}V_1^+ + S_{22}V_2^+ \end{aligned}$$

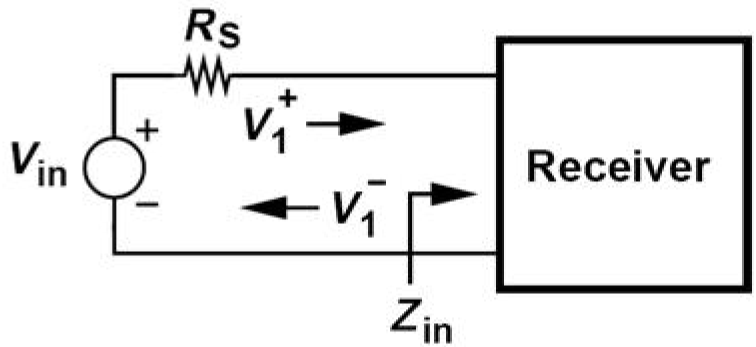
- **S-parameters generally have frequency-dependent complex values**
- **We often express S-parameters in units of dB**

$$S_{mn}|_{dB} = 20\log|S_{mn}|$$

- **The condition $V_2^+=0$ does not mean output port of the circuit must be conjugate-matched to R_L.**

Input Reflection Coefficient

In modern RF design, S_{11} is the most commonly-used S parameter as it quantifies the accuracy of impedance matching at the input of receivers.



$$V_1^- = V_{in}\frac{Z_{in}}{Z_{in}+R_S} - \frac{V_{in}}{2}$$

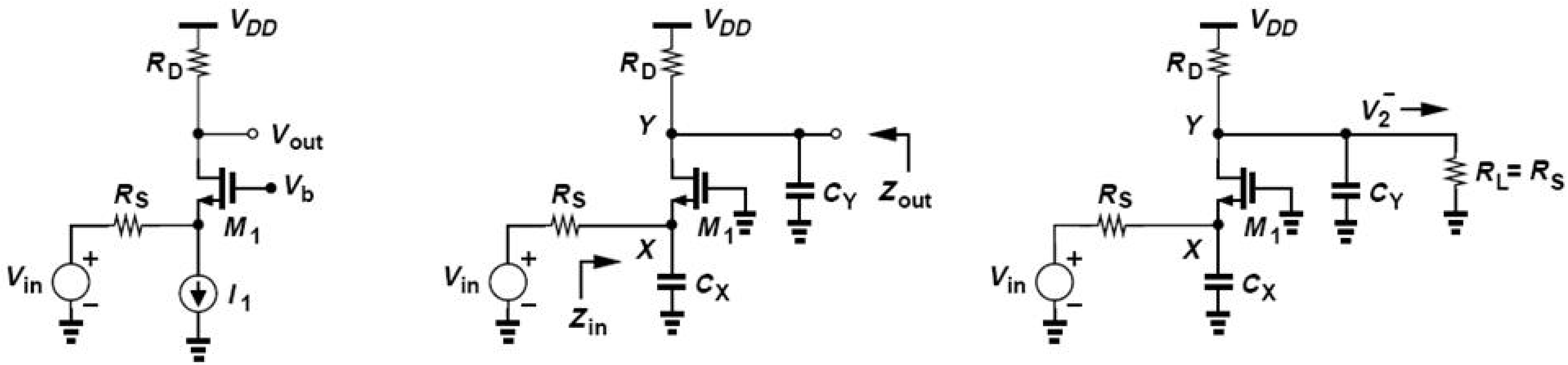
$$= \frac{Z_{in}-R_S}{2(Z_{in}+R_S)}V_{in}.$$

$$\frac{V_1^-}{V_1^+} = \frac{Z_{in}-R_S}{Z_{in}+R_S}.$$

- **Called the "input reflection coefficient" and denoted by G_{in}, this quantity can also be considered to be S_{11} if we remove the condition $V_2^+ = 0$**

Example of Scattering Parameters (Ⅰ)

Determine the S-parameters of the common-gate stage shown in figure below (left). Neglect channel-length modulation and body effect.



Drawing the circuit as shown above (middle), where $C_X = C_{GS} + C_{SB}$ and $C_Y = C_{GD} + C_{DB}$, we write $Z_{in} = (1/g_m)||(C_X s)^{-1}$ and

$$S_{11} = \frac{Z_{in} - R_S}{Z_{in} + R_S} = \frac{1 - g_m R_S - C_X s}{1 + g_m R_S + C_X s}$$

For S_{12}, we recognize that above arrangement yields no coupling from the output to the input if channel-length modulation is neglected. Thus, $S_{12} = 0$.

Example of Scattering Parameters (Ⅱ)

Determine the S-parameters of the common-gate stage shown in figure below (left). Neglect channel-length modulation and body effect.

For S_{22}, we note that $Z_{out} = R_D || (C_Y s)^{-1}$ and hence

$$S_{22} = \frac{Z_{out} - R_S}{Z_{out} + R_S} = -\frac{R_S - R_D + R_S R_D C_Y s}{R_S + R_D + R_S R_D C_Y s}$$

Lastly, S_{21} is obtained according to the configuration of figure above (right). Since $V_2^-/V_{in} = (V_2^-/V_X)(V_X/V_{in})$, $V_2^-/V_X = g_m[R_D||R_S||(C_Y s)^{-1}]$, and $V_X/V_{in} = Z_{in}/(Z_{in} + R_S)$, we obtain

$$\frac{V_2^-}{V_{in}} = g_m \left(R_D || R_S || \frac{1}{C_Y s} \right) \frac{1}{1 + g_m R_S + R_S C_X s}$$

$$S_{21} = 2g_m \left(R_D || R_S || \frac{1}{C_Y s} \right) \frac{1}{1 + g_m R_S + R_S C_X s}.$$

Analysis of Nonlinear Dynamic Systems: Basic Consideration

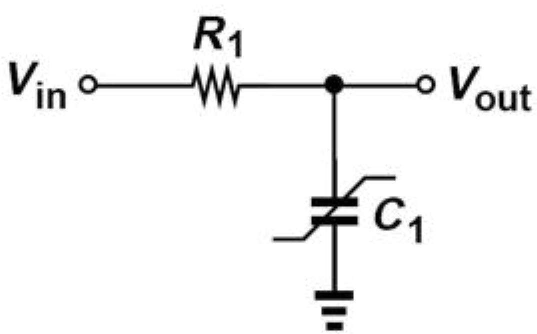
Input: $$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t.$$

Output:

$$y(t) = \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t + \theta_n) + \sum_{n=1}^{\infty} b_n \cos(n\omega_2 t + \phi_n)$$ ⇦ **harmonics**

$$+ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c_{m,n} \cos(n\omega_1 t + m\omega_2 t + \phi_{n,m}).$$ ⇦ **IM products**

- **If the differential equation governing the system is known, we can simply substitute for *y(t)* from this expression, equate the like terms, and compute a_n, b_n, $c_{m,n}$, and the phase shifts.**



$$C_1 = C_0(1 + \alpha V_{out})$$

$$R_1 C_0 (1 + \alpha V_{out}) \frac{dV_{out}}{dt} + V_{out} = V_{in}$$

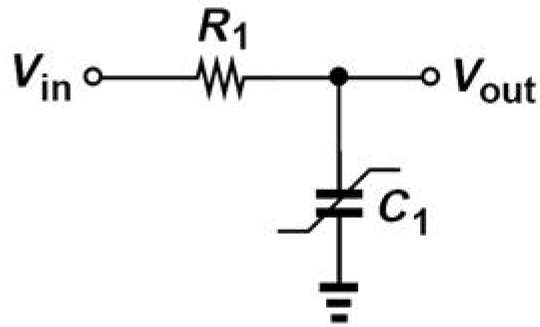
Analysis of Nonlinear Dynamic Systems: Harmonic Balance

$$\begin{aligned} V_{out}(t) &= a_1 \cos(\omega_1 t + \phi_1) + b_1 \cos(\omega_2 t + \phi_2) + c_1 \cos[(\omega_1 + \omega_2)t + \phi_3] + c_2 \cos[(\omega_1 - \omega_2)t + \phi_4] \\ &+ c_3 \cos[(2\omega_1 + \omega_2)t + \phi_5] + c_4 \cos[(\omega_1 + 2\omega_2)t + \phi_6] + c_5 \cos[(2\omega_1 - \omega_2)t + \phi_7] \\ &+ c_6 \cos[(\omega_1 - 2\omega_2)t + \phi_8], \end{aligned}$$

$$R_1 C_0 (1 + \alpha V_{out}) \frac{dV_{out}}{dt} + V_{out} = V_{in}$$

- **We must now substitute for $V_{out}(t)$ and $V_{in}(t)$ in the above equation, convert products of sinusoids to sums, bring all of the terms to one side of the equation, group them according to their frequencies, and equate the coefficient of each sinusoid to zero.**
- **This type of analysis is called "harmonic balance" because it predicts the output frequencies and attempts to balance the two sides of the circuit's differential equation**

Volterra Series(Ⅰ)



$$V_{in}(t) = V_0 \exp(j\omega_1 t)$$

For a linear, time-invariant system, the output is given by

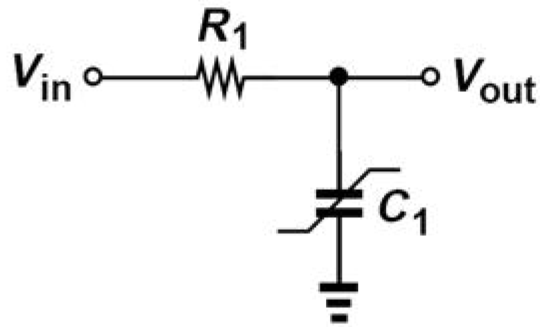
$$V_{out}(t) = H(\omega_1) V_0 \exp(j\omega_1 t)$$

***C*$_1$ = *C*$_0$, then**

$$R_1 C_0 H(\omega_1)(j\omega_1) V_0 \exp(j\omega_1 t) + H(\omega_1) V_0 \exp(j\omega_1 t) = V_0 \exp(j\omega_1 t).$$

$$H(\omega_1) = \frac{1}{R_1 C_0 j\omega_1 + 1}$$

Volterra Series(Ⅱ)



$$V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t)$$

$$V_{out1}(t) = H(\omega_1)V_0 \exp(j\omega_1 t) + H(\omega_2)V_0 \exp(j\omega_2 t)$$

Linear responses

Nonlinear responses

$$V_{out}(t) = H_1(\omega_1)V_0 \exp(j\omega_1 t) + H_1(\omega_2)V_0 \exp(j\omega_2 t) + \underline{H_2(\omega_1, \omega_2)V_0^2 \exp[j(\omega_1 + \omega_2)t] + \cdots}.$$

$$\begin{aligned} V_{out}(t) &= H_1(\omega_1)V_0 \exp(j\omega_1 t) + H_1(\omega_2)V_0 \exp(j\omega_2 t) + H_2(\omega_1, \omega_1)V_0^2 \exp(2j\omega_1 t) \\ &+ H_2(\omega_2, \omega_2)V_0^2 \exp(2j\omega_2 t) + H_2(\omega_1, \omega_2)V_0^2 \exp[j(\omega_1 + \omega_2)t] \\ &+ H_2(\omega_1, -\omega_2)V_0^2 \exp[j(\omega_1 - \omega_2)t] + \cdots. \end{aligned}$$

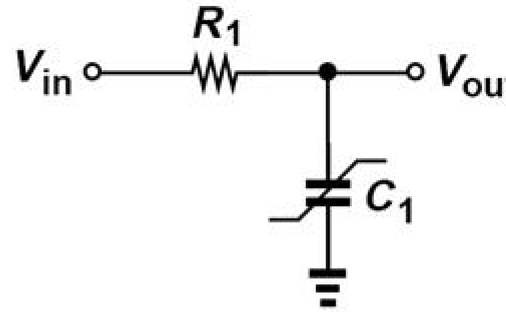
(

Example of Volterra Series(Ⅰ)

Determine *$H_2(\omega_1,\omega_2)$* for the RC circuit with nonlinear capacitor previoius shown

Solution:

We apply the input: $V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t)$



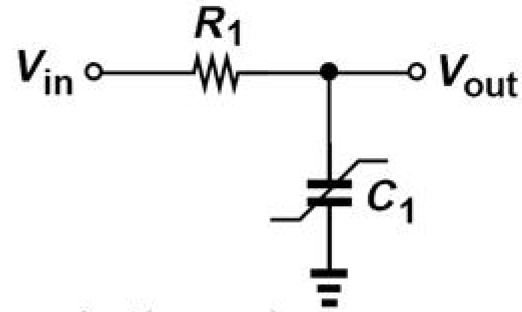
$$V_{out}(t) = H_1(\omega_1)V_0 \exp(j\omega_1 t) + H_1(\omega_2)V_0 \exp(j\omega_2 t) + H_2(\omega_1,\omega_2)V_0^2 \exp[j(\omega_1+\omega_2)t].$$

⇩

$$\begin{aligned}
& R_1C_0[1 + \alpha H_1(\omega_1)V_0 e^{j\omega_1 t} + \alpha H_1(\omega_2)V_0 e^{j\omega_2 t} + \alpha H_2(\omega_1,\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}] \\
\times\ & [H_1(\omega_1)j\omega_1 V_0 e^{j\omega_1 t} + H_1(\omega_2)j\omega_2 V_0 e^{j\omega_2 t} + H_2(\omega_1,\omega_2)j(\omega_1+\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}] \\
+\ & H_1(\omega_1)e^{j\omega_1 t} + H_1(\omega_2)e^{j\omega_2 t} + H_2(\omega_1,\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t} \\
=\ & V_0 e^{j\omega_1 t} + V_0 e^{j\omega_2 t}.
\end{aligned}$$

Example of Volterra Series(Ⅱ)

Determine $H_2(\omega_1,\omega_2)$ for the RC circuit with nonlinear capacitor previoius shown



Consider the terms containing $\omega_1+\omega_2$:

$$R_1C_0[\alpha H_1(\omega_1)H_1(\omega_2)j\omega_1 V_0^2 e^{j(\omega_1+\omega_2)t} + \alpha H_1(\omega_2)H_1(\omega_1)j\omega_2 V_0^2 e^{j(\omega_1+\omega_2)t}$$

$$+ \quad H_2(\omega_1,\omega_2)j(\omega_1+\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}] + H_2(\omega_1,\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t} = 0$$

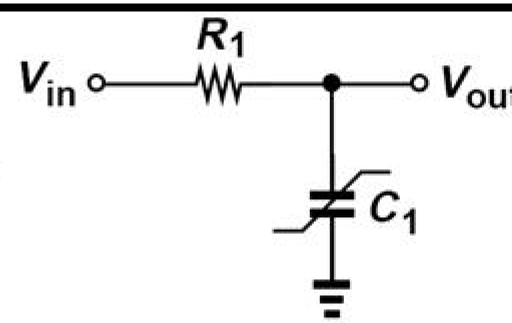
$$H_2(\omega_1,\omega_2) = -\frac{\alpha R_1 C_0 j(\omega_1+\omega_2)H_1(\omega_1)H_1(\omega_2)}{R_1C_0 j(\omega_1+\omega_2)+1}$$

$$H_2(\omega_1,\omega_2) = -\alpha R_1 C_0 j(\omega_1+\omega_2)H_1(\omega_1)H_1(\omega_2)H_1(\omega_1+\omega_2).$$

Another Example of Volterra Series

If an input $V_0\exp(j\omega_1 t)$ is applied to the RC circuit with nonlinear capacitor, determine the amplitude of the second harmonic at the output.

Solution:



$$H_2(\omega_1,\omega_2) = -\alpha R_1 C_0 j(\omega_1+\omega_2)H_1(\omega_1)H_1(\omega_2)H_1(\omega_1+\omega_2).$$

As mentioned earlier, the component at $2\omega_1$ is obtained as $H_2(\omega_1, \omega_1)V_0^2\exp[j(\omega_1 + \omega_1)t]$ Thus, the amplitude is equal to

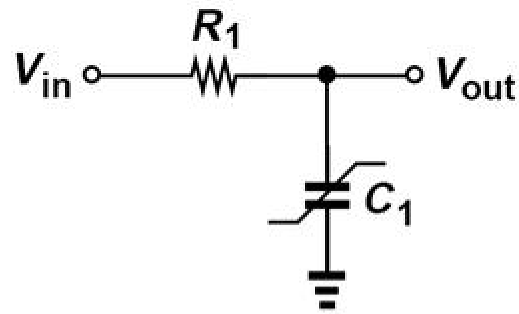
$$\begin{aligned} |A_{2\omega 1}| &= |\alpha R_1 C_0 (2\omega_1) H_1^2(\omega_1) H_1(2\omega_1)| V_0^2 \\ &= \frac{2|\alpha| R_1 C_0 \omega_1 V_0^2}{(R_1^2 C_0^2 \omega_1^2 + 1)\sqrt{4R_1^2 C_0^2 \omega_1^2 + 1}}. \end{aligned}$$

$$\begin{aligned} \left|\frac{A_{\omega1+\omega2}}{A_{\omega1-\omega2}}\right| &= \left|\frac{H_2(\omega_1,\omega_2)}{H_2(\omega_1,-\omega_2)}\right| \\ &= \left|\frac{(\omega_1+\omega_2)H_1(\omega_2)H_1(\omega_1+\omega_2)}{(\omega_1-\omega_2)H_1(-\omega_2)H_1(\omega_1-\omega_2)}\right| \end{aligned}$$

Since $|H_1(\omega_2)| = |H_1(-\omega_2)|$

$$\left|\frac{A_{\omega1+\omega2}}{A_{\omega1-\omega2}}\right| = \frac{(\omega_1+\omega_2)\sqrt{R_1^2C_0^2(\omega_1-\omega_2)^2+1}}{|\omega_1-\omega_2|\sqrt{R_1^2C_0^2(\omega_1+\omega_2)^2+1}}$$

Volterra Series: Nth-Order Terms



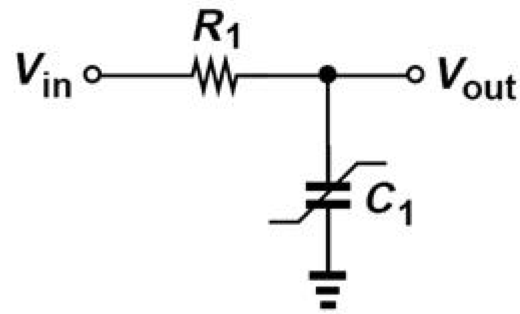
$$V_{in}(t) = V_0 \exp(j\omega_1 t) + \cdots + V_0 \exp(j\omega_N t)$$

$$\begin{aligned} V_{out}(t) &= \sum_{k=1}^{N} H_1(\omega_k) V_0 \exp(j\omega_k t) + \sum_{m=1}^{N}\sum_{k=1}^{N} H_2(\omega_m, \pm\omega_k) V_0^2 \exp[j(\omega_m \pm \omega_k)t] \\ &+ \sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{k=1}^{N} H_3(\omega_n, \pm\omega_m, \pm\omega_k) V_0^3 \exp[j(\omega_n \pm \omega_m \pm \omega_k)t] + \cdots . \end{aligned}$$

- ***H_m* is called the *m*-th *Volterra kernel***

Example of Volterra Kernel Calculation

Determine the third Volterra kernel for the same circuit discussed above.



Solution:

assume $V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t) + V_0 \exp(j\omega_3 t)$

Introduce the short hands

$$H_{3(1,2,3)} = H_3(\omega_1, \omega_2, \omega_3) V_0^3 \exp[j(\omega_1 + \omega_2 + \omega_3)t]$$

$$V_{out}(t) = H_{1(1)} + H_{1(2)} + H_{1(3)} + H_{2(1,2)} + H_{2(1,3)} + H_{2(2,3)} + H_{2(1,1)} + H_{2(2,2)} + H_{2(3,3)} + H_{3(1,2,3)} + \cdots$$

Substitute for V_{out} and V_{in}, grouping all of the terms

$$\begin{aligned} & H_3(\omega_1, \omega_2, \omega_3) \\ = \quad & -j\alpha R_1 C_0 \frac{H_2(\omega_1, \omega_2)\omega_3 H_1(\omega_3) + H_2(\omega_2, \omega_3)\omega_1 H_1(\omega_1) + H_2(\omega_1, \omega_3)\omega_2 H_1(\omega_2)}{R_1 C_0 j(\omega_1 + \omega_2 + \omega_3) + 1} \\ - \quad & j\alpha R_1 C_0 \frac{H_1(\omega_1)(\omega_2 + \omega_3) H_2(\omega_2, \omega_3) + H_1(\omega_2)(\omega_1 + \omega_3) H_2(\omega_1, \omega_3)}{R_1 C_0 j(\omega_1 + \omega_2 + \omega_3) + 1} \\ - \quad & j\alpha R_1 C_0 \frac{H_1(\omega_3)(\omega_1 + \omega_2) H_2(\omega_1, \omega_2)}{R_1 C_0 j(\omega_1 + \omega_2 + \omega_3) + 1}. \end{aligned}$$

Volterra Series: Harmonic Method

1. Assume $V_{in}(t) = V_0 \exp(j\omega_1 t)$ and $V_{out}(t) = H_1(\omega_1)V_0 \exp(j\omega_1 t)$. Substitute for V_{out} and V_{in} in the system's differential equation, group the terms that contain $\exp(j\omega_1 t)$, and compute the first (linear) kernel, $H_1(\omega_1)$.

2. Assume $V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t)$ and $V_{out}(t) = H_1(\omega_1)V_0 \exp(j\omega_1 t) + H_1(\omega_2)V_0 \exp(j\omega_2 t) + H_2(\omega_1; \omega_2)V_0^2 \exp[j(\omega_1 + \omega_2)t]$. Make substitutions in the differential equation, group the terms that contain $\exp[j(\omega_1 + \omega_2)t]$, and determine the second kernel, $H_2(\omega_1; \omega_2)$.

3. Assume $V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t) + V_0 \exp(j\omega_3 t)$ and $V_{out}(t)$ is given. Make substitutions, group the terms that contain $\exp[j(\omega 1 + \omega 2 + \omega 3)t]$, and calculate the third kernel, $H_3(\omega 1; \omega 2; \omega 3)$.

4. To compute the amplitude of harmonics and IM components, choose $\omega_1, \omega_2, \cdots$ properly. For example, $H_2(\omega_1; \omega_1)$ yields the transfer function for $2\omega_1$ and $H_3(\omega_1; -\omega_2; \omega_1)$ the transfer function for $2\omega_1 - \omega_2$.

Volterra Series: Method of Nonlinear Currents

1. Assume $V_{in}(t) = V_0 \exp(j\omega_1 t)$ and determine the linear response of the circuit by ignoring the nonlinearity. The "response" includes both the output of interest and the voltage across the nonlinear device.

2. Assume $V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t)$ and calculate the voltage across the nonlinear device, assuming it is linear. Now, compute the nonlinear component of the current flowing through the device, assuming the device is nonlinear.

3. Set the main input to zero and place a current source equal to the nonlinear component found in Step 2 in parallel with the nonlinear device.

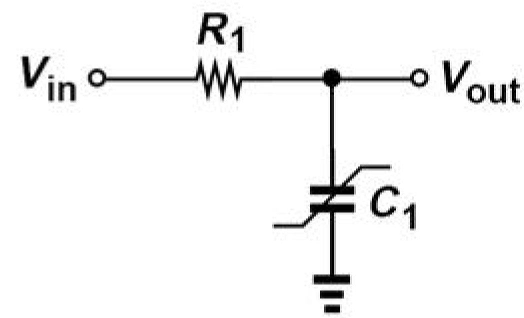
4. Ignoring the nonlinearity of the device again, determine the circuit's response to the current source applied in Step 3. Again, the response includes the output of interest and the voltage across the nonlinear device.

5. Repeat Steps 2, 3, and 4 for higher-order responses. The overall response is equal to the output components found in Steps 1, 4, etc.

Example Using Method of Nonlinear Currents (Ⅰ)

Determine $H_3(\omega_1,\omega_2,\omega_3)$ for the circuit below.

Solution:



Step 1

The voltage across the capacitor is equal to:

$$V_{C1}(t) = \frac{V_0}{R_1C_0j\omega_1 + 1}e^{j\omega_1 t}.$$

Step 2

$$V_{in}(t) = V_0\exp(j\omega_1 t) + V_0\exp(j\omega_2 t)$$

$$V_{C1}(t) = \frac{V_0 e^{j\omega_1 t}}{R_1C_0j\omega_1 + 1} + \frac{V_0 e^{j\omega_2 t}}{R_1C_0j\omega_2 + 1}$$

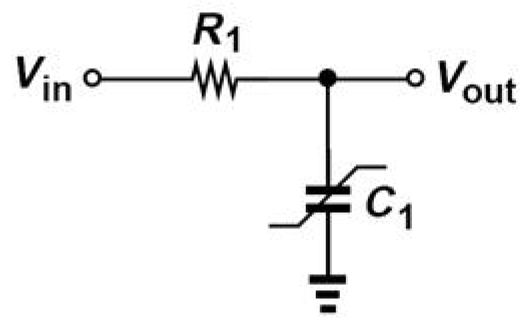
Compute the nonlinear current flowing through C_1

$$\begin{aligned} I_{C1,non}(t) &= \alpha C_0 V_{C1}\frac{dV_{C1}}{dt} \\ &= \alpha C_0\left(\frac{V_0 e^{j\omega_1 t}}{R_1C_0j\omega_1 + 1} + \frac{V_0 e^{j\omega_2 t}}{R_1C_0j\omega_2 + 1}\right) \\ &\times \left(\frac{j\omega_1 V_0 e^{j\omega_1 t}}{R_1C_0j\omega_1 + 1} + \frac{j\omega_2 V_0 e^{j\omega_2 t}}{R_1C_0j\omega_2 + 1}\right). \end{aligned}$$

Example Using Method of Nonlinear Currents (Ⅱ)

Determine $H_3(\omega_1,\omega_2,\omega_3)$ for the circuit below.

Solution:

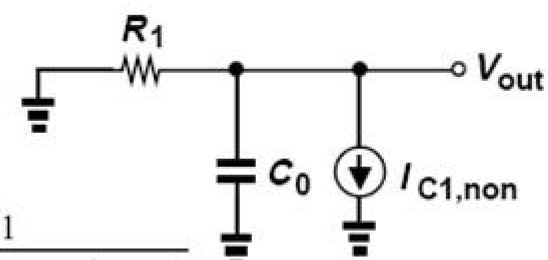


$$I_{C1,non}(t) = \alpha C_0\left[\frac{j(\omega_1+\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}}{(R_1C_0j\omega_1+1)(R_1C_0j\omega_2+1)}+\cdots\right]$$
$$= \alpha C_0[j(\omega_1+\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}H_1(\omega_1)H_1(\omega_2)+\cdots]$$

Step 3

Set the input to zero, assume a linear capacitor, and apply $I_{C1,non(t)}$ in parallel with C_1

Step 4

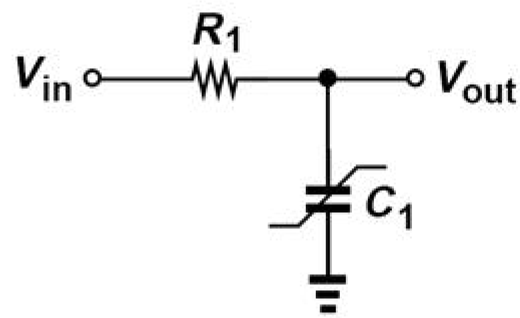


$$V_{C1,non}(t) = -\alpha C_0 j(\omega_1+\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}H_1(\omega_1)H_1(\omega_2)\frac{R_1}{R_1C_0j(\omega_1+\omega_2)+1}$$
$$= -\alpha R_1C_0 j(\omega_1+\omega_2)H_1(\omega_1)H_1(\omega_2)H_1(\omega_1+\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}.$$

Example Using Method of Nonlinear Currents (Ⅲ)

Determine $H_3(\omega_1,\omega_2,\omega_3)$ for the circuit below.

Step 5



$$V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t) + V_0 \exp(j\omega_3 t)$$

$$\begin{aligned} V_{C1}(t) &= H_1(\omega_1)V_0 e^{j\omega_1 t} + H_1(\omega_2)V_0 e^{j\omega_2 t} + H_1(\omega_3)V_0 e^{j\omega_3 t} + H_2(\omega_1,\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t} \\ &+ H_2(\omega_1,\omega_3)V_0^2 e^{j(\omega_1+\omega_3)t} + H_2(\omega_2,\omega_3)V_0^2 e^{j(\omega_2+\omega_3)t}. \end{aligned}$$

The nonlinear current through C_1 is thus equal to $I_{C1,non}(t) = \alpha C_0 V_{C1} \dfrac{dV_{C1}}{dt}$

$$\begin{aligned} I_{C1,non}(t) &= \alpha C_0[H_1(\omega_1)H_2(\omega_2,\omega_3)j(\omega_2+\omega_3) + H_2(\omega_2,\omega_3)j\omega_1 H_1(\omega_1) \\ &+ H_1(\omega_2)H_2(\omega_1,\omega_3)j(\omega_1+\omega_3) + H_2(\omega_1,\omega_3)j\omega_2 H_1(\omega_2) \\ &+ H_1(\omega_3)H_2(\omega_1,\omega_2)j(\omega_1+\omega_2) + H_2(\omega_1,\omega_2)j\omega_3 H_1(\omega_3)]V_0^3 e^{j(\omega_1+\omega_2+\omega_3)t} \\ &+ \cdots. \end{aligned}$$

Another Example Using Method of Nonlinear Currents (Ⅰ)

Figure below shows the input network of a commonly-used LNA (Chapter 5). Assuming that $g_mL_1/C_{GS} = R_S$ (Chapter 5) and $I_D = \alpha(V_{GS}-V_{TH})^2$, determine the nonlinear terms in I_{out}. Neglect other capacitances, channel-length modulation, and body effect.

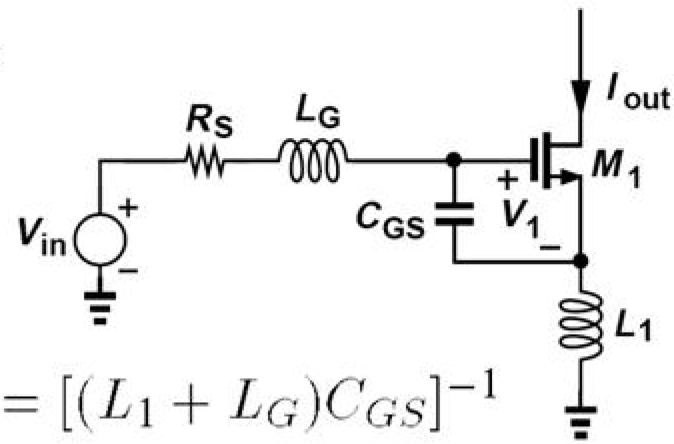
Solution:

Step 1

$$V_{in} = (R_S + L_G s)V_1 C_{GS} s + V_1 + (V_1 C_{GS} s + g_m V_1)L_1 s$$

$$\frac{V_1}{V_{in}} = \frac{1}{(L_1 + L_G)C_{GS}s^2 + (R_S C_{GS} + g_m L_1)s + 1}$$

$$\frac{V_1}{V_{in}}(j\omega) = \frac{1}{2g_m L_1 j\omega + 1 - \frac{\omega^2}{\omega_0^2}} = H_1(\omega)$$



where $\omega_0^2 = [(L_1 + L_G)C_{GS}]^{-1}$

Step 2

assume $V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t)$ $\quad V_1(t) = H_1(\omega_1)V_0 e^{j\omega_1 t} + H_1(\omega_2)V_0 e^{j\omega_2 t}$

This voltage results in a nonlinear current given by

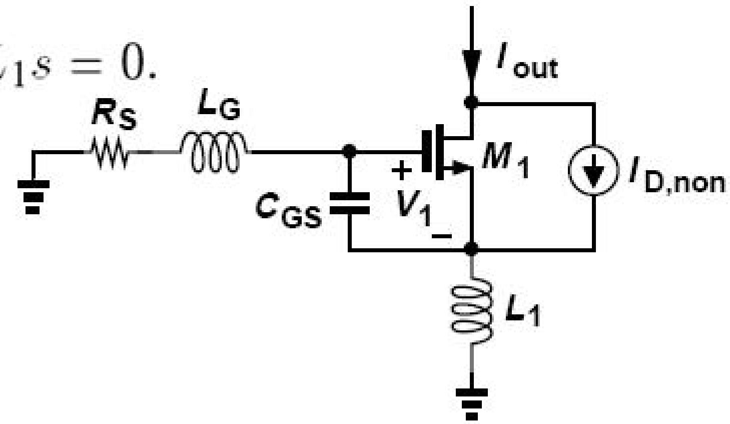
$$I_{D,non} = 2\alpha H_1(\omega_1)H_1(\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t}$$

Another Example Using Method of Nonlinear Currents (Ⅱ)

Figure below shows the input network of a commonly-used LNA (Chapter 5). Assuming that $g_m L_1/C_{GS} = R_S$ (Chapter 5) and $I_D = \alpha(V_{GS}-V_{TH})^2$, determine the nonlinear terms in I_{out}. Neglect other capacitances, channel-length modulation, and body effect.

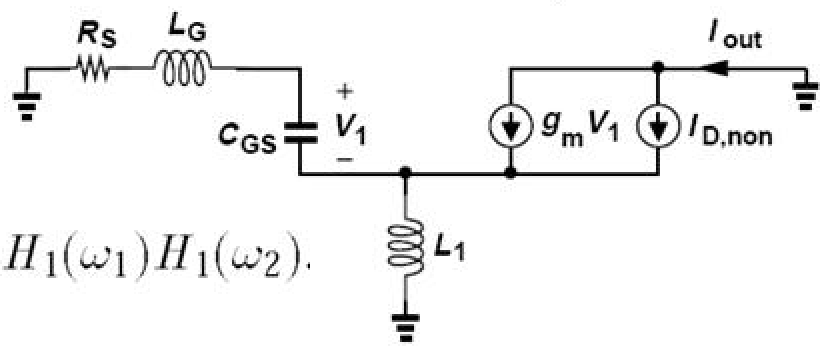
Solution: **Step 3&4**

$$(R_S + L_G s)V_1 C_{GS} s + V_1 + (g_m V_1 + I_{D,non} + V_1 C_{GS} s)L_1 s = 0.$$



Thus, for $s = j\omega$

$$\frac{V_1}{I_{D,non}}(j\omega) = \frac{-jL_1\omega}{2g_m L_1 j\omega + 1 - \frac{\omega^2}{\omega_0^2}}$$

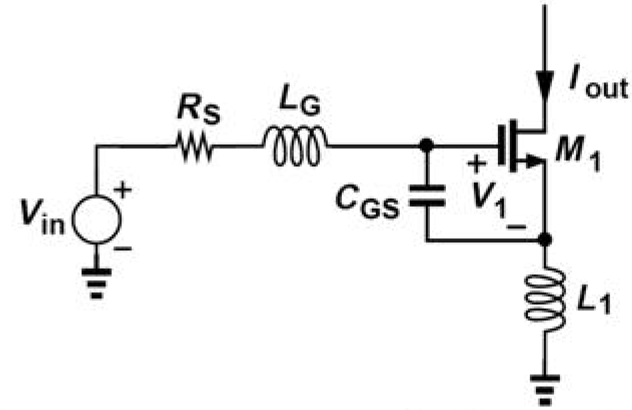


$$H_2(\omega_1,\omega_2) = \frac{-jL_1(\omega_1+\omega_2)}{2g_m L_1 j(\omega_1+\omega_2) + 1 - \frac{(\omega_1+\omega_2)^2}{\omega_0^2}} 2\alpha H_1(\omega_1)H_1(\omega_2).$$

Another Example Using Method of Nonlinear Currents (Ⅳ)

Figure below shows the input network of a commonly-used LNA (Chapter 5). Assuming that $g_m L_1/C_{GS} = R_S$ (Chapter 5) and $I_D = \alpha(V_{GS}-V_{TH})^2$, determine the nonlinear terms in I_{out}. Neglect other capacitances, channel-length modulation, and body effect.

Solution: **Step 5**



assume

$$V_{in}(t) = V_0 \exp(j\omega_1 t) + V_0 \exp(j\omega_2 t) + V_0 \exp(j\omega_3 t)$$

$$\begin{aligned} V_1(t) &= H_1(\omega_1)V_0 e^{j\omega_1 t} + H_1(\omega_2)V_0 e^{j\omega_2 t} + H_1(\omega_3)V_0 e^{j\omega_3 t} + H_2(\omega_1,\omega_2)V_0^2 e^{j(\omega_1+\omega_2)t} \\ &+ H_2(\omega_1,\omega_3)V_0^2 e^{j(\omega_1+\omega_3)t} + H_2(\omega_2,\omega_3)V_0^2 e^{j(\omega_2+\omega_3)t}. \end{aligned}$$

Since $I_D = \alpha V_1^2$, the nonlinear current at $\omega_1 + \omega_2 + \omega_3$ is expressed as

$$I_{D,non} = 2\alpha[H_1(\omega_1)H_2(\omega_2,\omega_3) + H_1(\omega_2)H_2(\omega_1,\omega_3) + H_1(\omega_3)H_2(\omega_1,\omega_2)]V_0^3 e^{j(\omega_1+\omega_2+\omega_3)t}.$$

References (I)

[1] B. Razavi, *Design of Analog CMOS Integrated Circuits,* Boston: MgGraw-Hill, 2001.

[2] L. W. Couch, *Digital and Analog Communication Systems,* Fourth Edition, New York: Macmillan Co., 1993.

[3] A. van der Ziel, "Thermal Noise in Field Effect Transistors," *Proc. IRE*, vol. 50, pp. 1808-1812, Aug. 1962.

[4] A. A. Abidi, "High-Frequency Noise Measurements on FETs with Small Dimensions," *IEEE Trans. Electron Devices,* vol. 33, pp. 1801-1805, Nov. 1986.

[5] A. J. Sholten et al, "Accurate Thermal Noise Model of Deep-Submicron CMOS," *IEDM Dig. Tech. Papers,* pp. 155-158, Dec. 1999.

[6] B. Razavi, "Impact of Distributed Gate Resistance on the Performance of MOS Devices," *IEEE Trans. Circuits and Systems- Part I*, vol. 41, pp. 750-754, Nov. 1994.

References (Ⅱ)

[7] H. T. Friis, "Noise Figure of Radio Receivers," *Proc. IRE*, vol. 32, pp. 419-422, July 1944.

[8] A. Papoulis, *Probability, Random Variables, and Stochastic Processes,* Third Edition, New York: McGraw-Hill, 1991.

[9] R. W. Bennet, "Methods of Solving Noise Problems," *Proc. IRE*, vol. 44, pp. 609-638, May 1956.

[10] S. Narayanan, "Application of Volterra Series to Intermodulation Distortion Analysis of Transistor Feedback Amplifiers," *IEEE Tran. Circuit Theory,* vol. 17, pp. 518-527, Nov. 1970.

[11] P. Wambacq et al, "High-Frequency Distortion Analysis of Analog Integrated Circuits," *IEEE Tran. Circuits and Systems, II*, vol. 46, pp. 335-334, March 1999.

[12] P. Wambaq and W. Sansen, *Distortion Analysis of Analog Integrated Circuits,* Norwell, MA, Kluwer, 1998.

References (Ⅲ)

[13] J. Bussganag, L. Ehrman, and J. W. Graham, "Analysis of Nonlinear Systems with Multiple Inputs," *Proc. IEEE*, vol. 62, pp. 1088-1119, Aug. 1974.

[14] E. Bedrosian and S. O. Rice, "The Output Properties of Volterra Systems (Nonlinear Systems with Memory) Driven by Harmonic and Gaussian Inputs," *Proc. IEEE*, vol. 59, pp. 1688-1707, Dec. 1971.