第4章 线性代数

4.1 线性代数函数包

使用线性代数的命令之前，必须首先加载函数包 LinearAlgebra 或 Student[LinearAlgebra]。

```
> with(LinearAlgebra);
```

(1.1)

```
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix,
   BidiagonalForm, BilinearForm, CharacteristicMatrix,
   CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation,
   ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix,
   ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn,
   DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension,
   Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues,
   Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm,
   GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
   GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt,
   HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm,
   HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis,
   IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix,
   JordanForm, LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map,
   Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse,
   MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply,
   MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,
   NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix,
   Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix,
   RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row,
   RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply,
   ScalarVector, SchurForm, SingularValues, SmithForm,
   StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis,
   SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm,
   UnitVector, VandermondeMatrix, VectorAdd, VectorAngle,
   VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix,
   ZeroVector, Zip]
```

```
> with(Student[LinearAlgebra]);
```

(1.2)

```
[&x, `.`, AddRow, AddRows, Adjoint, ApplyLinearTransformPlot,
```

```
    BackwardSubstitute, BandMatrix, Basis, BilinearForm,
    CharacteristicMatrix, CharacteristicPolynomial, ColumnDimension,
    ColumnSpace, CompanionMatrix, ConstantMatrix, ConstantVector,
    CrossProductPlot, Determinant, Diagonal, DiagonalMatrix, Dimension,
    Dimensions, EigenPlot, EigenPlotTutor, Eigenvalues, EigenvaluesTutor,
    Eigenvectors, EigenvectorsTutor, Equal, GaussJordanEliminationTutor,
    GaussianElimination, GaussianEliminationTutor, GenerateEquations,
    GenerateMatrix, GramSchmidt, HermitianTranspose, Id, IdentityMatrix,
    IntersectionBasis, InverseTutor, IsDefinite, IsOrthogonal, IsSimilar,
    IsUnitary, JordanBlockMatrix, JordanForm, LUDecomposition,
    LeastSquares, LeastSquaresPlot, LinearSolve, LinearSolveTutor,
    LinearSystemPlot, LinearSystemPlotTutor, LinearTransformPlot,
    LinearTransformPlotTutor, MatrixBuilder, MinimalPolynomial, Minor,
    MultiplyRow, Norm, Normalize, NullSpace, Pivot, PlanePlot,
    ProjectionPlot, QRDecomposition, RandomMatrix, RandomVector, Rank,
    ReducedRowEchelonForm, ReflectionMatrix, RotationMatrix, RowDimension,
    RowSpace, SetDefault, SetDefaults, SumBasis, SwapRow, SwapRows, Trace,
    Transpose, UnitVector, VectorAngle, VectorSumPlot, ZeroMatrix,
    ZeroVector]
```

如需向量和向量场运算，还应加载函数包 VectorCalculus 或 Student[VectorCalculus]。

```
> with(VectorCalculus);
[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength,        (1.3)
    BasisFormat, Binormal, Compatibility, ConvertVector, CrossProd,
    CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence,
    DotProd, DotProduct, Flux, GetCoordinateParameters, GetCoordinates,
    GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, Jacobian,
    Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt,
    PlotPositionVector, PlotVector, PositionVector, PrincipalNormal,
    RadiusOfCurvature, RootedVector, ScalarPotential,
    SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt,
    TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion,
    Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff,
    eval, evalVF, int, limit, series]
> with(Student[VectorCalculus]);
[&x, `*`, `+`, `-`, `.`, <,>, <|>, BasisFormat, Binormal, CrossProduct,     (1.4)
    Curl, Curvature, D, Del, Divergence, DotProduct, FlowLine, Flux,
    GetCoordinates, Gradient, Laplacian, LineInt, MapToBasis, Nabla, Norm,
    Normalize, PathInt, PositionVector, PrincipalNormal,
```

RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinates, SpaceCurve, SpaceCurveTutor, SurfaceInt, TNBFrame, TangentVector, Torsion, Vector, VectorField, VectorFieldTutor, VectorPotential, diff, evalVF, int, limit, series]

4.2 向量的定义和运算

输入向量

```
> u := ⟨1, 2⟩; v := [1, 2]; w := [1, 2];
```

$$u := \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$v := \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$w := \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{2.1}$$

```
> whattype(u), whattype(v), whattype(w)
```

$$Vector_{column}, Vector_{column}, Matrix \tag{2.2}$$

```
> u_2 := -u_1; u
```

$$u_2 := -1$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{2.3}$$

注意：当加载函数包VectorCalculus或Student[VectorCalculus]之后，表达式 <1,2> 的含义发生了变化。

定义向量 Vector

```
> u := [1, 2]; v := Vector(u); w := Vector[row](u);
```

$$u := [1, 2]$$

$$v := \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$w := \begin{bmatrix} 1 & 2 \end{bmatrix} \tag{2.4}$$

```
> whattype(u), whattype(v), whattype(w)
```

(2.5)

$$list,\ Vector_{column},\ Vector_{row} \tag{2.5}$$

```
> Vector[row](5, v)
```

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \end{bmatrix} \tag{2.6}$$

```
> Vector[row](5, sin)
```

$$\begin{bmatrix} \sin(1) & \sin(2) & \sin(3) & \sin(4) & \sin(5) \end{bmatrix} \tag{2.7}$$

生成随机向量 RandomVector

```
> RandomVector[row](5, generator = 0..1)
```

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \end{bmatrix} \tag{2.8}$$

```
> RandomVector[row](5, generator = 0..1.0)
```

$$[0.573754663321269453,\ 0.773917130825996002,\ 0.456032823759049522,\ 0.643960953645390854,\ 0.710703873355774007] \tag{2.9}$$

加法、数乘、线性组合 VectorAdd

```
> u := Vector[row](3, symbol = a); v := Vector[row](3, symbol = b);
```

$$u := \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$$

$$v := \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} \tag{2.10}$$

```
> 2 u − 3 v, VectorAdd(u, v, 2, -3);
```

$$\begin{bmatrix} 2\ a_1 - 3\ b_1 & 2\ a_2 - 3\ b_2 & 2\ a_3 - 3\ b_3 \end{bmatrix},\ \begin{bmatrix} 2\ a_1 - 3\ b_1 & 2\ a_2 - 3\ b_2 & 2\ a_3 - 3\ b_3 \end{bmatrix} \tag{2.11}$$

内积 DotProduct、外积 CrossProduct、双线性型 BilinearForm

```
> u.v, DotProduct(u, v);
```

$$\overline{b_1}\ a_1 + \overline{b_2}\ a_2 + \overline{b_3}\ a_3,\ \overline{b_1}\ a_1 + \overline{b_2}\ a_2 + \overline{b_3}\ a_3 \tag{2.12}$$

```
> u &x v, CrossProduct(u, v);
```

$$\begin{bmatrix} a_2\ b_3 - a_3\ b_2 & a_3\ b_1 - a_1\ b_3 & a_1\ b_2 - a_2\ b_1 \end{bmatrix}, \tag{2.13}$$

$$\begin{bmatrix} a_2\ b_3 - a_3\ b_2 & a_3\ b_1 - a_1\ b_3 & a_1\ b_2 - a_2\ b_1 \end{bmatrix}$$

```
> BilinearForm(u, v, Matrix([[1, 2, 3], [4, 5, 6], [7, 8, 9]]));
```

$$\overline{b_1 + 4\ b_2 + 7\ b_3}\ a_1 + \overline{2\ b_1 + 5\ b_2 + 8\ b_3}\ a_2 + \overline{3\ b_1 + 6\ b_2 + 9\ b_3}\ a_3 \tag{2.14}$$

问题：将 u,v 分别换成行向量或列向量，结果有何异同？

向量范数 Norm、夹角 VectorAngle

```
> Norm(u); VectorAngle(u, v);
```

$$\max\left(|a_1|,\ |a_3|,\ |a_2|\right)$$

$$\arccos\left(\frac{\overline{b_1}\ a_1+\overline{b_2}\ a_2+\overline{b_3}\ a_3}{\sqrt{|a_1|^2+|a_2|^2+|a_3|^2}\ \sqrt{|b_1|^2+|b_2|^2+|b_3|^2}}\right) \tag{2.15}$$

```
> Norm(u, 0), Norm(u, 1), Norm(u, 2), Norm(u, infinity)
```

$$3,\ |a_1|+|a_2|+|a_3|,\ \sqrt{|a_1|^2+|a_2|^2+|a_3|^2},\ \max\left(|a_1|,\ |a_3|,\ |a_2|\right) \tag{2.16}$$

问题：将 u,v 分别换成行向量或列向量，结果又有何异同？

向量组生成的子空间的基 Basis、和 SumBasis、交 IntersectionBasis、正交化 GramSchmidt。

```
> u1 := ⟨1|2|1|-2⟩; u2 := ⟨2|3|1|0⟩; u3 := ⟨1|2|2|-3⟩; v1 := ⟨1|1|1|1⟩; v2 := ⟨1|0|1|
      -1⟩; v3 := ⟨1|3|0|-4⟩;
```

$$u1 := \begin{bmatrix} 1 & 2 & 1 & -2 \end{bmatrix}$$

$$u2 := \begin{bmatrix} 2 & 3 & 1 & 0 \end{bmatrix}$$

$$u3 := \begin{bmatrix} 1 & 2 & 2 & -3 \end{bmatrix}$$

$$v1 := \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

$$v2 := \begin{bmatrix} 1 & 0 & 1 & -1 \end{bmatrix}$$

$$v3 := \begin{bmatrix} 1 & 3 & 0 & -4 \end{bmatrix} \tag{2.17}$$

```
> Basis([u1, u2, u3, v1, v2, v3])
```

$$\left[\begin{bmatrix} 1 & 2 & 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 2 & -3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & -1 \end{bmatrix}\right] \tag{2.18}$$

```
> SumBasis([[u1, u2, u3], [v1, v2, v3]])
```

$$\left[\begin{bmatrix} 1 & 2 & 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 2 & -3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & -1 \end{bmatrix}\right] \tag{2.19}$$

```
> IntersectionBasis([[u1, u2, u3], [v1, v2, v3]])
```

$$\left[\begin{bmatrix} 1 & 3 & 0 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}\right] \tag{2.20}$$

```
> GramSchmidt([u1, u2, u3])
```

$$\left[\begin{bmatrix} 1 & 2 & 1 & -2 \end{bmatrix}, \begin{bmatrix} \frac{11}{10} & \frac{6}{5} & \frac{1}{10} & \frac{9}{5} \end{bmatrix}, \begin{bmatrix} \frac{1}{59} & -\frac{15}{59} & \frac{43}{59} & \frac{7}{59} \end{bmatrix}\right] \tag{2.21}$$

4.3 矩阵的定义

输入矩阵

```
> A := <<1|2>, <3|4>>; B := <<1, 2>|<3, 4>>; C := Matrix([[11, 12], [21, 22]]);
```

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B := \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$C := \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} \tag{3.1}$$

```
> whattype(A), whattype(B), whattype(C)
```

$$\mathit{Matrix}, \mathit{Matrix}, \mathit{Matrix} \tag{3.2}$$

定义矩阵 Matrix

```
> Matrix(2);
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.3}$$

```
> Matrix(2, 3)
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

```
> Matrix([[1, 2, 3], [4, 5, 6]]);
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \tag{3.5}$$

```
> Matrix([<1, 2, 3>, <4, 5, 6>]);
```

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \tag{3.6}$$

```
> Matrix(3, (i, j) -> 1/(i+j-1))
```

(3.7)

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \tag{3.7}$$

```
> Matrix(3, 4, symbol = a, shape = band[1, 1])
```

$$\begin{bmatrix} a_{1,1} & a_{1,2} & 0 & 0 \\ a_{2,1} & a_{2,2} & a_{2,3} & 0 \\ 0 & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix} \tag{3.8}$$

随机矩阵 RandomMatrix

```
> randomize( ):
> RandomMatrix(2, 3)
```

$$\begin{bmatrix} 26 & -73 & 40 \\ -6 & 36 & 62 \end{bmatrix} \tag{3.9}$$

```
> RandomMatrix(4, 3, generator = -10..10, outputoptions = [shape = triangular])
```

$$\begin{bmatrix} 9 & 6 & 1 \\ 0 & -2 & -4 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.10}$$

注意：使用RandomMatrix之前，必须先运行randomize；

特殊矩阵（详见课本94页表4-1）

```
> ZeroMatrix(2, 3)
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.11}$$

```
> IdentityMatrix(2, 3)
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{3.12}$$

```
> DiagonalMatrix([1, 2], 2, 3)
```

(3.13)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \tag{3.13}$$

存取矩阵元素 Column、Diagonal、Dimension、Row、SubMatrix

```
> a := 'a' : A := Matrix(3, 4, symbol = a);
```

$$A := \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix} \tag{3.14}$$

```
> A[2, 3]
```

$$a_{2,3} \tag{3.15}$$

```
> A[1..2, 3..4]
```

$$\begin{bmatrix} a_{1,3} & a_{1,4} \\ a_{2,3} & a_{2,4} \end{bmatrix} \tag{3.16}$$

```
> A[1..2, 3..4] := -A[1..2, 3..4]; A
```

$$A_{1..2,\,3..4} := \begin{bmatrix} a_{1,3} & a_{1,4} \\ a_{2,3} & a_{2,4} \end{bmatrix}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix} \tag{3.17}$$

```
> Dimension(A)
```

$$Dimension\left(\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix}\right) \tag{3.18}$$

```
> Diagonal(A)
```

$$\begin{bmatrix} a_{1,1} \\ a_{2,2} \\ a_{3,3} \end{bmatrix} \tag{3.19}$$

```
> A := Matrix(2, symbol = a); B := Matrix(2, symbol = b);
```

$$A := \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$$

$$B := \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \tag{3.20}$$

> $\langle A|B \rangle$;

$$\begin{bmatrix} a_{1,1} & a_{1,2} & b_{1,1} & b_{1,2} \\ a_{2,1} & a_{2,2} & b_{2,1} & b_{2,2} \end{bmatrix} \tag{3.21}$$

> $\langle A, B \rangle$

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \tag{3.22}$$

> $DiagonalMatrix([A, B])$;

$$\begin{bmatrix} a_{1,1} & a_{1,2} & 0 & 0 \\ a_{2,1} & a_{2,2} & 0 & 0 \\ 0 & 0 & b_{1,1} & b_{1,2} \\ 0 & 0 & b_{2,1} & b_{2,2} \end{bmatrix} \tag{3.23}$$

4.4 矩阵的运算

基本运算

> $A := Matrix(2, symbol = a)$; $B := Matrix(2, symbol = b)$;

$$A := \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$$

$$B := \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} \tag{4.1}$$

> $A + B$;

$$\begin{bmatrix} a_{1,1} + b_{1,1} & a_{1,2} + b_{1,2} \\ a_{2,1} + b_{2,1} & a_{2,2} + b_{2,2} \end{bmatrix} \tag{4.2}$$

> $A.B$;

(4.3)

$$\begin{bmatrix} a_{1,1}\,b_{1,1}+a_{1,2}\,b_{2,1} & a_{1,1}\,b_{1,2}+a_{1,2}\,b_{2,2} \\ a_{2,1}\,b_{1,1}+a_{2,2}\,b_{2,1} & a_{2,1}\,b_{1,2}+a_{2,2}\,b_{2,2} \end{bmatrix} \tag{4.3}$$

```
> 2·A;
```

$$\begin{bmatrix} 2\,a_{1,1} & 2\,a_{1,2} \\ 2\,a_{2,1} & 2\,a_{2,2} \end{bmatrix} \tag{4.4}$$

```
> A^2;
```

$$\begin{bmatrix} a_{1,1}^2+a_{1,2}\,a_{2,1} & a_{1,1}\,a_{1,2}+a_{1,2}\,a_{2,2} \\ a_{2,1}\,a_{1,1}+a_{2,2}\,a_{2,1} & a_{1,2}\,a_{2,1}+a_{2,2}^2 \end{bmatrix} \tag{4.5}$$

```
> A^-1;
```

$$\begin{bmatrix} \dfrac{a_{2,2}}{a_{1,1}\,a_{2,2}-a_{1,2}\,a_{2,1}} & -\dfrac{a_{1,2}}{a_{1,1}\,a_{2,2}-a_{1,2}\,a_{2,1}} \\ -\dfrac{a_{2,1}}{a_{1,1}\,a_{2,2}-a_{1,2}\,a_{2,1}} & \dfrac{a_{1,1}}{a_{1,1}\,a_{2,2}-a_{1,2}\,a_{2,1}} \end{bmatrix} \tag{4.6}$$

```
> A^%T;
```

$$\begin{bmatrix} a_{1,1} & a_{2,1} \\ a_{1,2} & a_{2,2} \end{bmatrix} \tag{4.7}$$

```
> A^%H;
```

$$\begin{bmatrix} \overline{a_{1,1}} & \overline{a_{2,1}} \\ \overline{a_{1,2}} & \overline{a_{2,2}} \end{bmatrix} \tag{4.8}$$

```
>
>
```

注意：A^%T 和 A^%H 中的 T 和 H 必须是大写。

矩阵函数（详见课本101页表4-4和106页表4-5）

```
> A := Matrix(2, symbol = a);
```

$$A := \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \tag{4.9}$$

```
> Adjoint(A);
```

$$\begin{bmatrix} a_{2,2} & -a_{1,2} \\ -a_{2,1} & a_{1,1} \end{bmatrix} \tag{4.10}$$

```
> Determinant(A);
```

$$a_{2,\,2}\,a_{1,\,1}-a_{2,\,1}\,a_{1,\,2} \tag{4.11}$$

```
> Permanent(A);
```

$$a_{2,\,2}\,a_{1,\,1}+a_{2,\,1}\,a_{1,\,2} \tag{4.12}$$

```
> Trace(A);
```

$$a_{1,\,1}+a_{2,\,2} \tag{4.13}$$

```
> A := <<-1, 3>|<2, 4>>;
```

$$A := \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} \tag{4.14}$$

```
> Rank(A);
```

$$2 \tag{4.15}$$

```
> CharacteristicPolynomial(A, x);
```

$$-10+x^2-3\,x \tag{4.16}$$

```
> Eigenvalues(A);
```

$$\begin{bmatrix} 5 \\ -2 \end{bmatrix} \tag{4.17}$$

```
> Eigenvectors(A);
```

$$\begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} & -2 \\ 1 & 1 \end{bmatrix} \tag{4.18}$$

```
> JordanForm(A);
```

$$\begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix} \tag{4.19}$$

```
> LUDecomposition(A)
```

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 0 & 10 \end{bmatrix} \tag{4.20}$$

```
> QRDecomposition(A)
```

$$\begin{bmatrix} -\frac{1}{10}\sqrt{10} & \frac{3}{10}\sqrt{10} \\ \frac{3}{10}\sqrt{10} & \frac{1}{10}\sqrt{10} \end{bmatrix}, \begin{bmatrix} \sqrt{10} & \sqrt{10} \\ 0 & \sqrt{10} \end{bmatrix} \tag{4.21}$$

```
> SingularValues(A);
```

(4.22)

$$\begin{bmatrix} \frac{5}{2}\sqrt{2}+\frac{1}{2}\sqrt{10} \\ \frac{5}{2}\sqrt{2}-\frac{1}{2}\sqrt{10} \end{bmatrix} \tag{4.22}$$

注意：Eigenvalues(A)、Eigenvectors(A) 和 JordanForm(A) 返回的特征值次序可能不同。

4.5 线性方程组

课本110页例23

```
> eqns := [x1+3 x2+6 x3+2 x4=-1, 6 x1+3 x2+6 x3+12 x4=24, 3 x1-x2
      -2 x3+6 x4=17];
```

$$eqns := [x1+3\ x2+6\ x3+2\ x4=-1,\ 6\ x1+3\ x2+6\ x3+12\ x4=24,\ 3\ x1-x2-2\ x3+6\ x4=17] \tag{5.1}$$

```
> A, B := GenerateMatrix(eqns, [x1, x2, x3, x4]);
```

$$A,\ B := \begin{bmatrix} 1 & 3 & 6 & 2 \\ 6 & 3 & 6 & 12 \\ 3 & -1 & -2 & 6 \end{bmatrix}, \begin{bmatrix} -1 \\ 24 \\ 17 \end{bmatrix} \tag{5.2}$$

```
> LinearSolve(A, B, free=x);      #` `
```

$$\begin{bmatrix} 5-2\ x_4 \\ -2-2\ x_3 \\ x_3 \\ x_4 \end{bmatrix} \tag{5.3}$$

课本110页例24

```
> eqns := [x1+2 x2=5, 2 x1+x2=6, x1+x2=4];
```

$$eqns := [x1+2\ x2=5,\ 2\ x1+x2=6,\ x1+x2=4] \tag{5.4}$$

```
> A, B := GenerateMatrix(eqns, [x1, x2]);
```

$$A,\ B := \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 4 \end{bmatrix} \tag{5.5}$$

```
> LinearSolve(A, B, free=x);      #` `
Error, (in LinearAlgebra:-LA_Main:-LinearSolve) inconsistent system
> LeastSquares(A, B);
```

$$\begin{bmatrix} \frac{26}{11} \\ \frac{15}{11} \end{bmatrix} \tag{5.6}$$