§11-3 位移法的基本体系

一、超静定结构计算的总原则:

欲求超静定结构先取一个基本体系,然后让基本体系在受力方面和变形方面与原结构完全一样。

力法的特点:

基本未知量——多余未知力;

基本体系——静定结构;

基本方程——位移条件

（变形协调条件）

位移法的特点:

基本未知量—— 独立结点位移

基本体系——一组单跨超静定梁

基本方程—— 平衡条件

二、基本未知量的选取

1、结点角位移数:

结构上可动刚结点数即为位移法计算的结点角位移数。

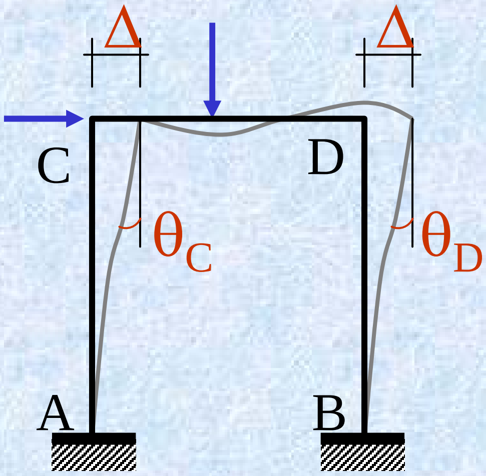
2、结构独立线位移:

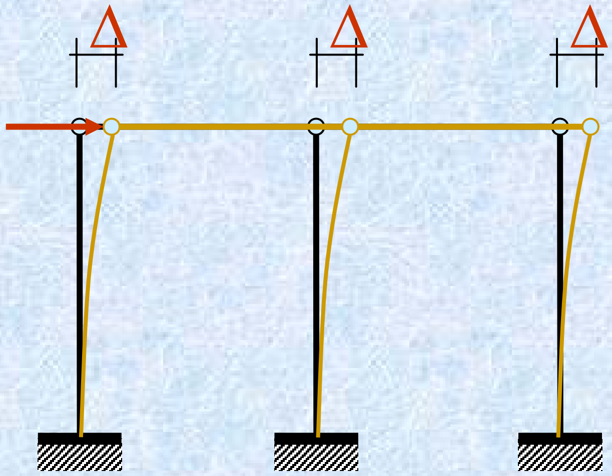
每个结点有两个线位移，为了减少未知量，引入与实际相符的两个假设:

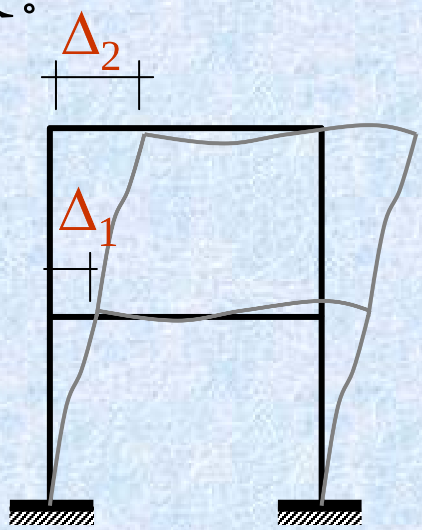
（1）忽略轴向力产生的轴向变形---变形后的曲杆与原直杆等长;

（2）变形后的曲杆长度与其弦等长。

上面两个假设导致杆件变形后两个端点距离保持不变。

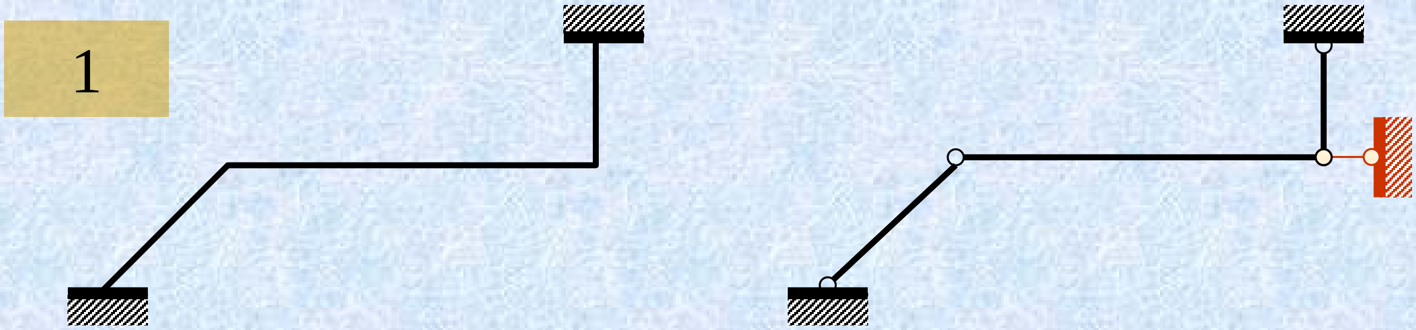


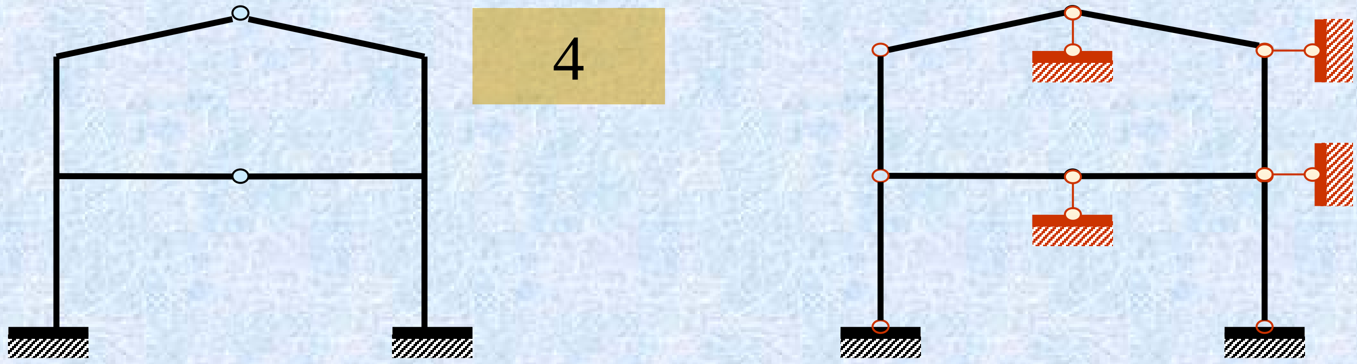


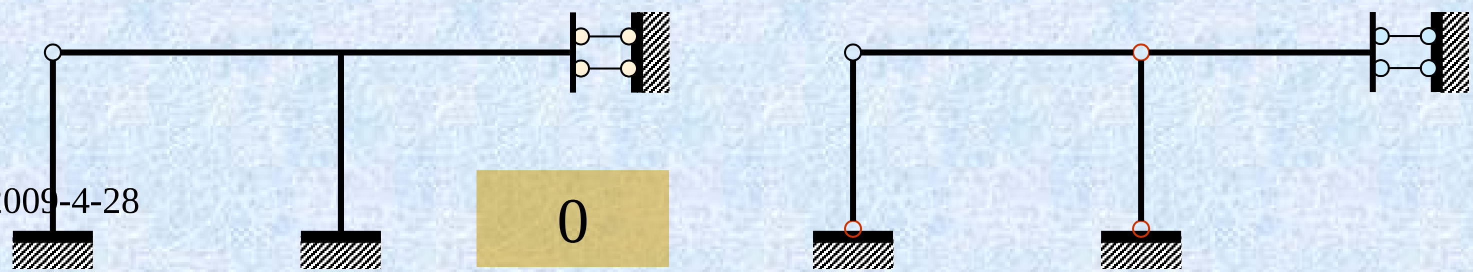


线位移数也可以用几何方法确定。

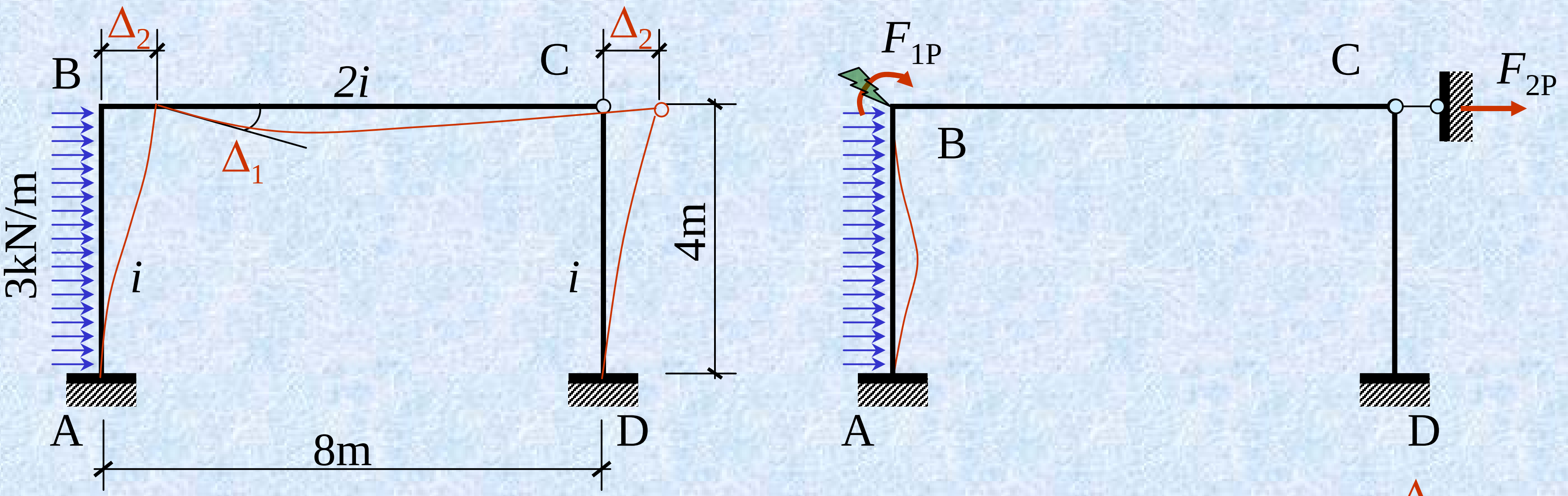
将结构中所有刚结点和固定支座，代之以铰结点和铰支座，分析新体系的几何构造性质，若为几何可变体系，则通过增加支座链杆使其变为无多余联系的几何不变体系，所需增加的链杆数，即为原结构位移法计算时的线位移数。

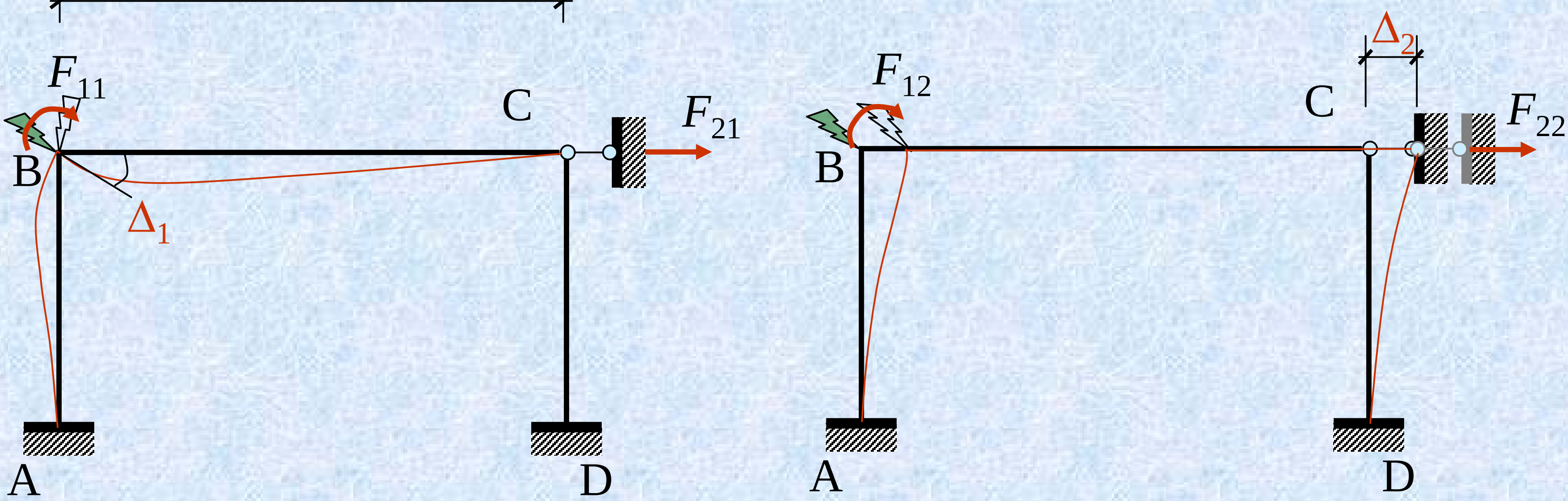






三、选择基本体系

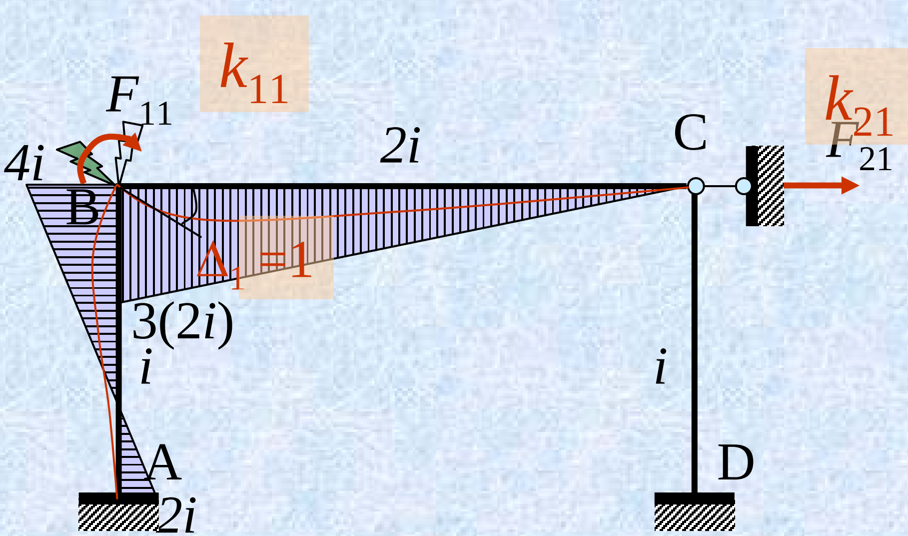


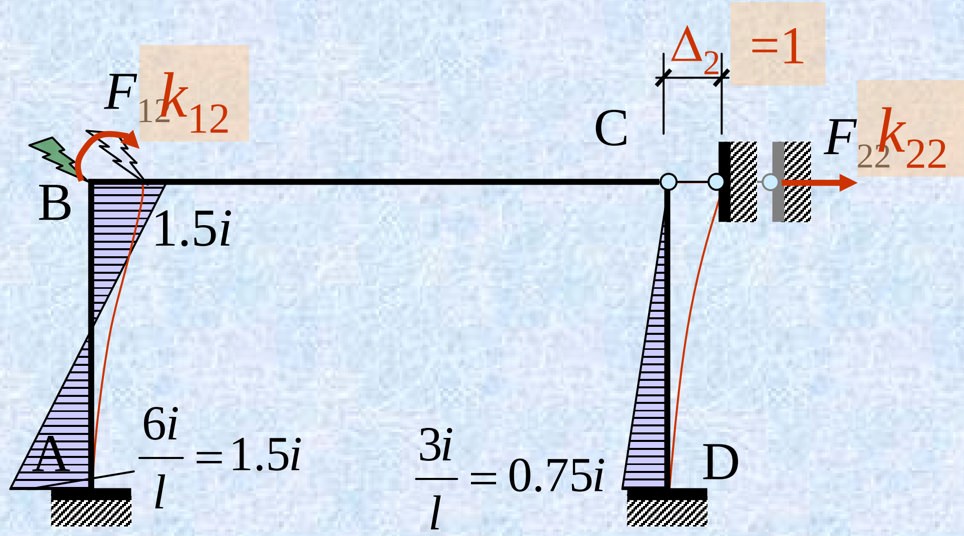


四、建立基本方程

$$F_{11}+F_{12}+F_{1P}=0 \ldots\ldots\ldots\ldots\ldots\ldots(1a)$$

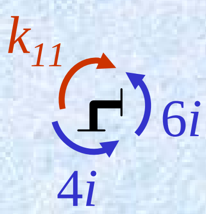
$$F_{21}+F_{22}+F_{2P}=0 \ldots\ldots\ldots\ldots\ldots\ldots(2a)$$

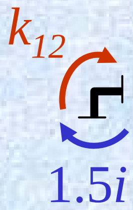




$$F_{11}+F_{12}+F_{1P}=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1a)$$

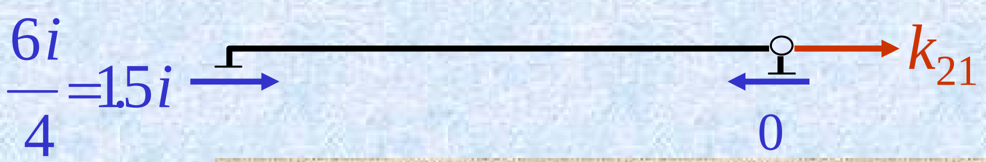
$$F_{21}+F_{22}+F_{2P}=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2a)$$



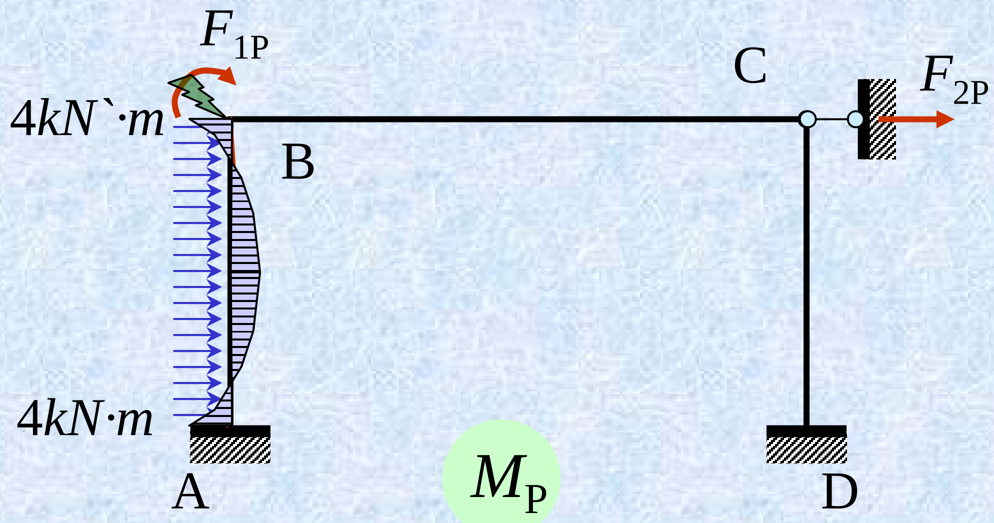


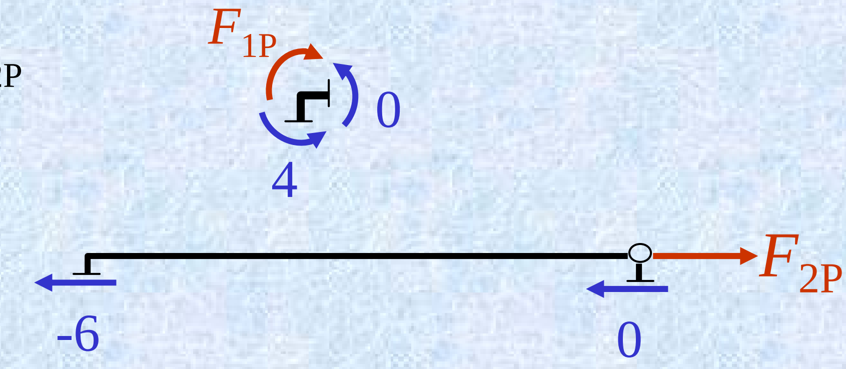
$$k_{11}\Delta_1 + k_{12}\Delta_2 + F_{1P} = 0 \ldots\ldots\ldots(1)$$

$$k_{21}\Delta_1 + k_{22}\Delta_2 + F_{2P} = 0 \ldots\ldots\ldots(2)$$



$k_{11}=10i$ $k_{21}= -1.5i$ $k_{12}= -1.5i$ $k_{22} = \dfrac{15}{16} i$





F_{1P}=4kN·m　F_{2P}=-6kN

位移法方程:

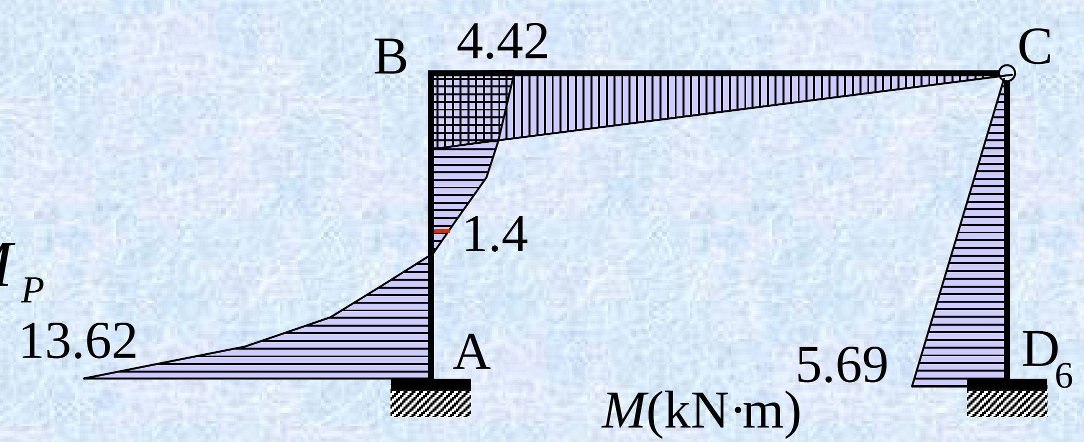
$$\left.\begin{aligned} 10\ i\Delta_1 - 1.5\ i\Delta_2 + 4 = 0 \\ -1.5 i\Delta_1 + \frac{15}{16} i\Delta_2 - 6 = 0 \end{aligned}\right\}$$

$$\Delta_1 = 0.737\frac{1}{i} \qquad \Delta_2 = 7.580\frac{1}{i}$$

五、计算结点位移

六、绘制弯矩图

$$M = \overline{M}_1\Delta_1 + \overline{M}_2\Delta_2 + M_P$$

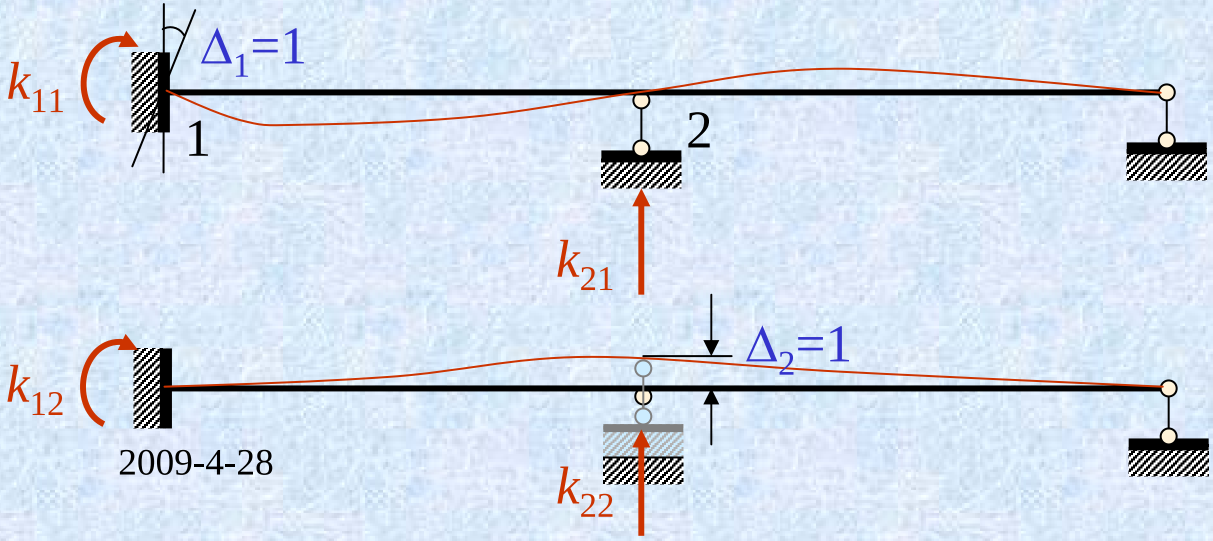


具有n个独立结点位移的超静定结构:

$$\left.\begin{aligned} k_{11}\Delta_1+k_{12}\Delta_2+\cdots\cdots+k_{1n}\Delta_n+F_{1P}=0 \\ k_{21}\Delta_1+k_{22}\Delta_2+\cdots\cdots+k_{2n}\Delta_n+F_{2P}=0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ k_{n1}\Delta_1+k_{n2}\Delta_2+\cdots\cdots+k_{nn}\Delta_n+F_{nP}=0 \end{aligned}\right\}$$

$$\begin{bmatrix} k_{11} & k_{12} & \cdots\cdots & k_{1n} \\ k_{21} & k_{22} & \cdots\cdots & k_{2n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ k_{n1} & k_{n2} & \cdots\cdots & k_{nn} \end{bmatrix}$$

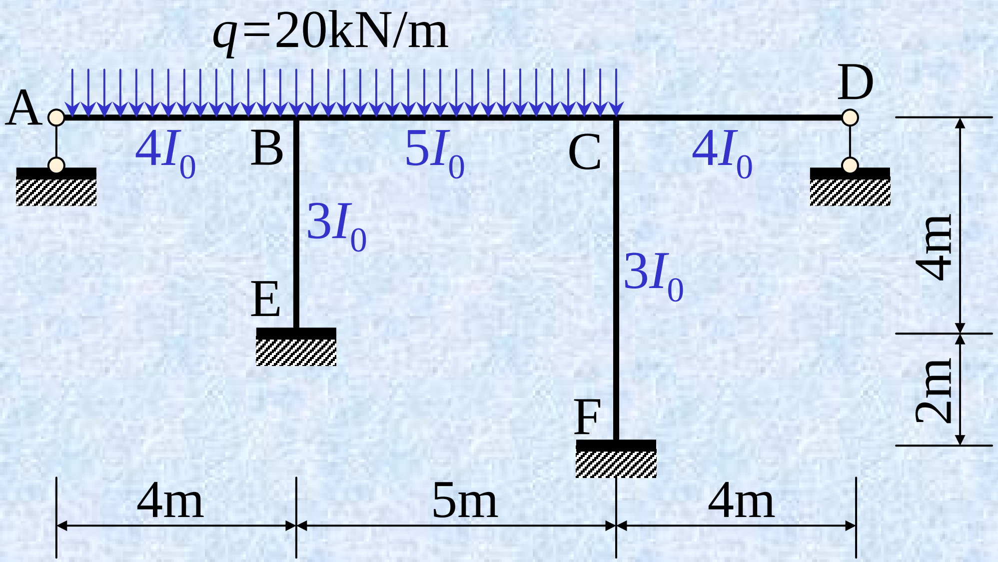
$k_{ij}=k_{ji}$



$$k_{11}\times 0+k_{21}\times 1$$
$$=k_{12}\times 1+k_{22}\times 0$$

$k_{21}=k_{12}$

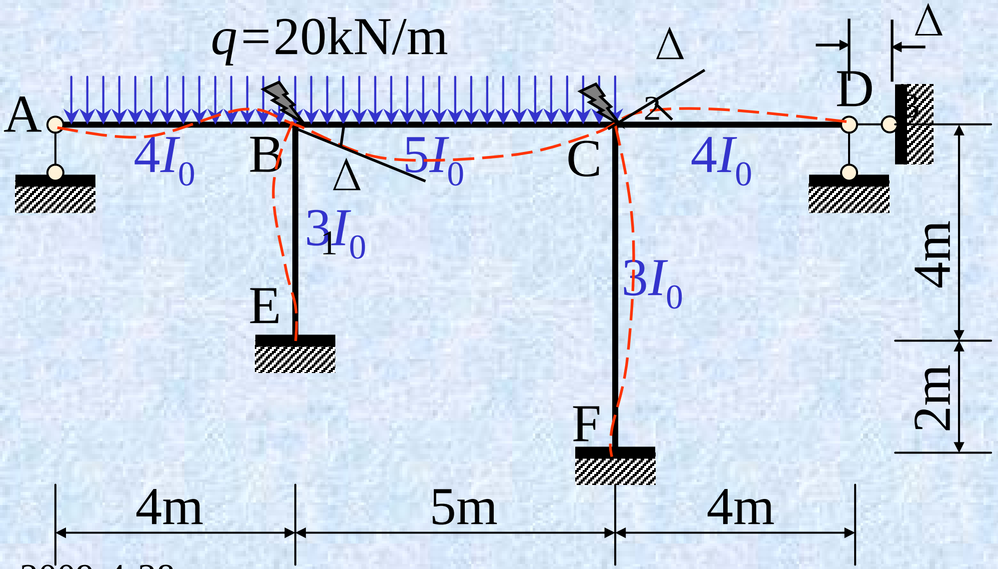
例1、试用位移法分析图示刚架。



（1）基本未知量

Δ1、 Δ2、 Δ3

（2）基本体系



计算杆件线性刚度i，设$EI_0=1$，则

$$i_{AB}=\frac{EI_{AB}}{l_{AB}}=\frac{E\cdot 4I_0}{4}=1$$

$$i_{BC}=1,\ i_{CD}=1,$$

$$i_{BE}=\frac{3}{4},\ i_{CF}=\frac{1}{2}$$

（3）位移法方程

$$k_{11}\Delta_1 + k_{12}\Delta_2 + k_{13}\Delta_3 + F_{1P} = 0$$

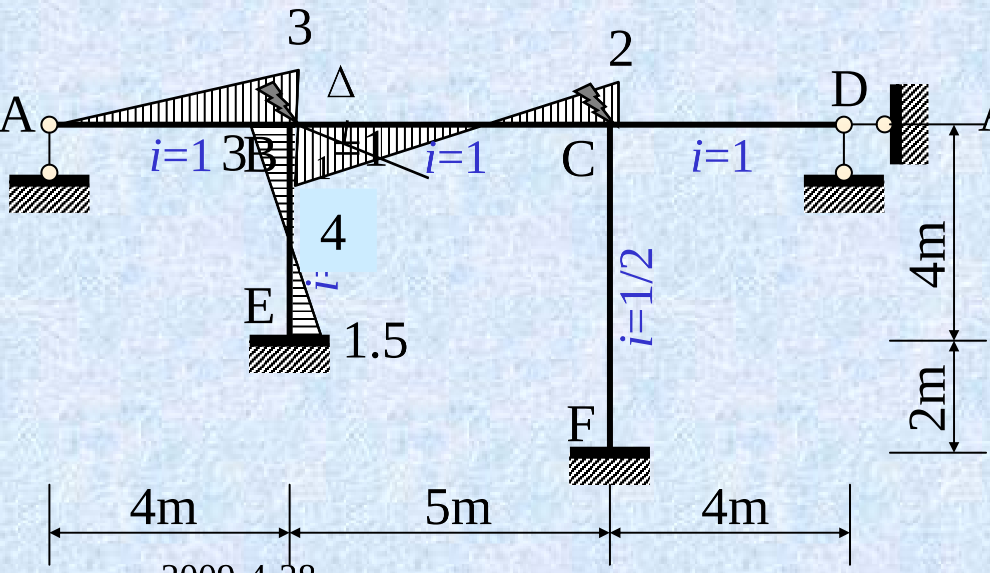
$$k_{21}\Delta_1 + k_{22}\Delta_2 + k_{23}\Delta_3 + F_{2P} = 0$$

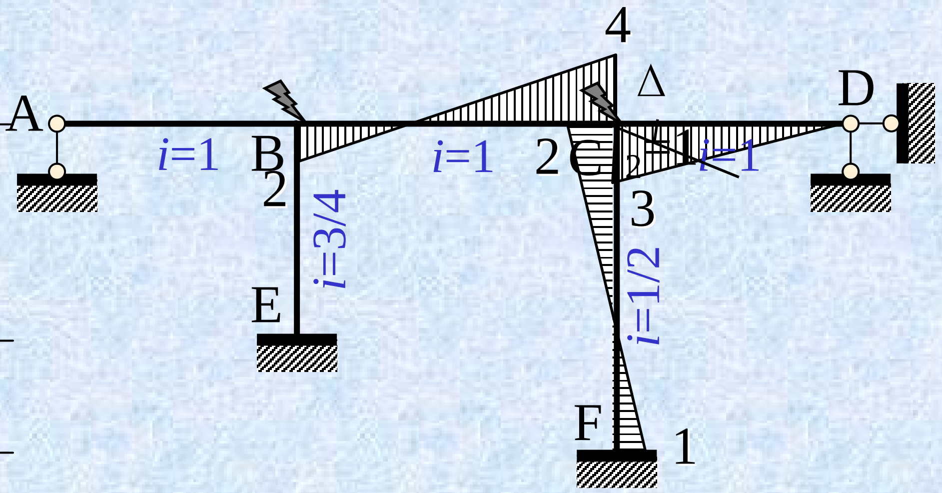
$$k_{31}\Delta_1 + k_{32}\Delta_2 + k_{33}\Delta_3 + F_{3P} = 0$$

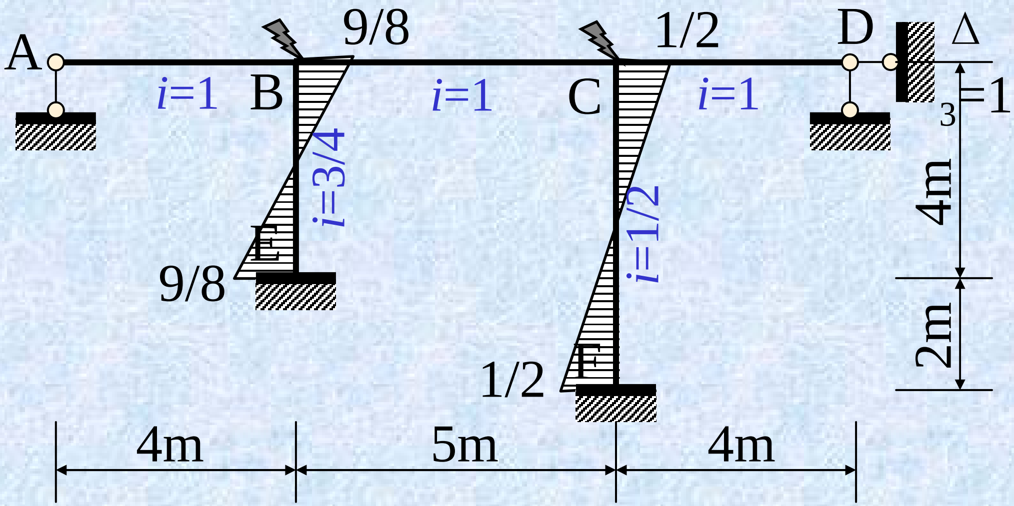
（4）计算系数：$\boldsymbol{k_{11}}$、$\boldsymbol{k_{12}}$、$\boldsymbol{k_{13}}$、$\boldsymbol{k_{21}}$、$\boldsymbol{k_{22}}$、$\boldsymbol{k_{23}}$、$\boldsymbol{k_{31}}$、$\boldsymbol{k_{32}}$、$\boldsymbol{k_{33}}$

$k_{11}=3+4+3=10$　　$k_{22}=4+3+2=9$

$k_{12}=k_{21}=2$　　$k_{13}=k_{31}=?$　　$k_{23}=k_{32}=?$





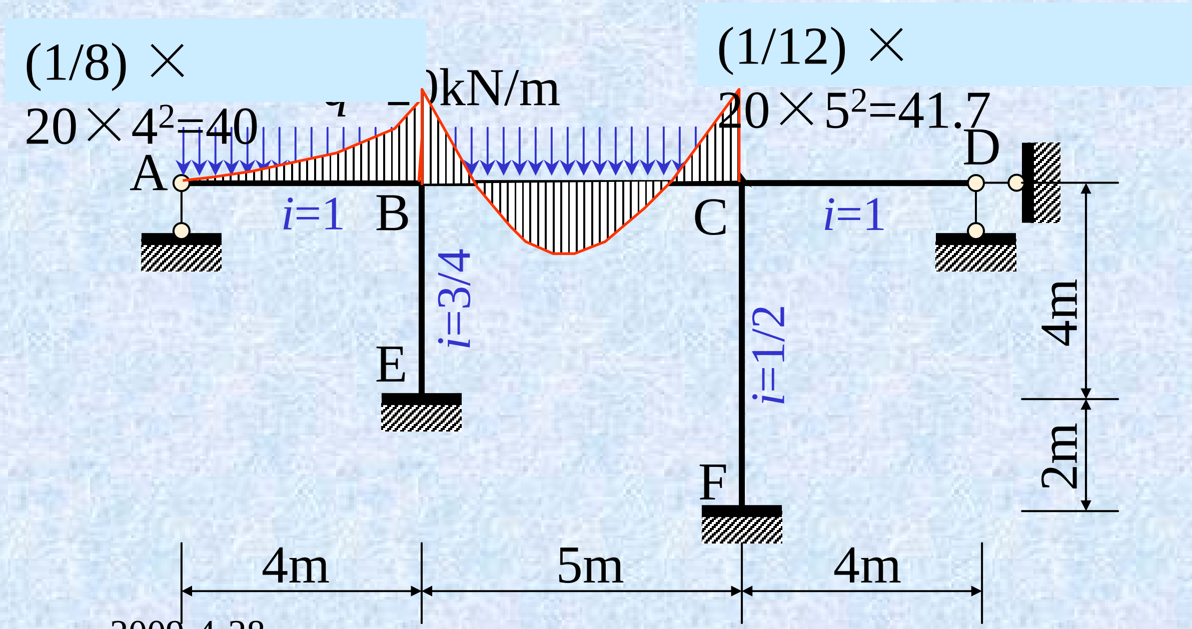


$k_{33}=(1/6)+(9/16)=35/48$

$k_{31}=k_{13}=-9/8$

$k_{32}=k_{23}=-1/2$

（5）计算自由项：F_{1P}、F_{2P}、F_{3P}



$F_{1P}=40-41.7=-1.7$

$F_{2P}=41.7$

$F_{3P}=0$

（6）建立位移法基本方程：

$$10\Delta_1 + 2\Delta_2 - \frac{9}{8}\Delta_3 - 1.7 = 0$$

$$2\Delta_1 + 9\Delta_2 - \frac{1}{2}\Delta_3 + 41.7 = 0$$

$$-\frac{9}{8}\Delta_1 - \frac{1}{2}\Delta_2 + \frac{35}{48}\Delta_3 = 0$$

（7）解方程求结点位移：

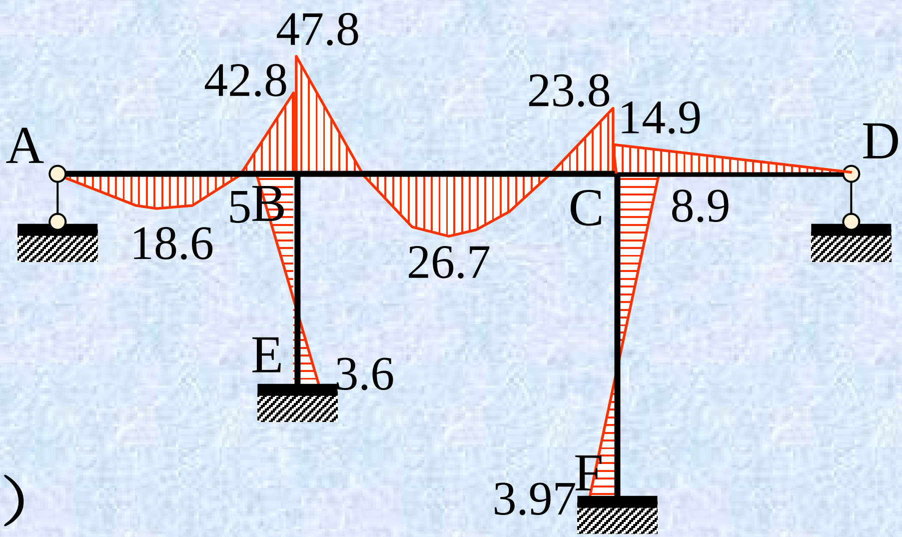
$$\Delta_1 = 0.94$$

$$\Delta_2 = -4.94$$

$$\Delta_3 = -1.94$$

（8）绘制弯矩图

$$M = \overline{M_1}\Delta_1 + \overline{M_2}\Delta_2 + M_P$$

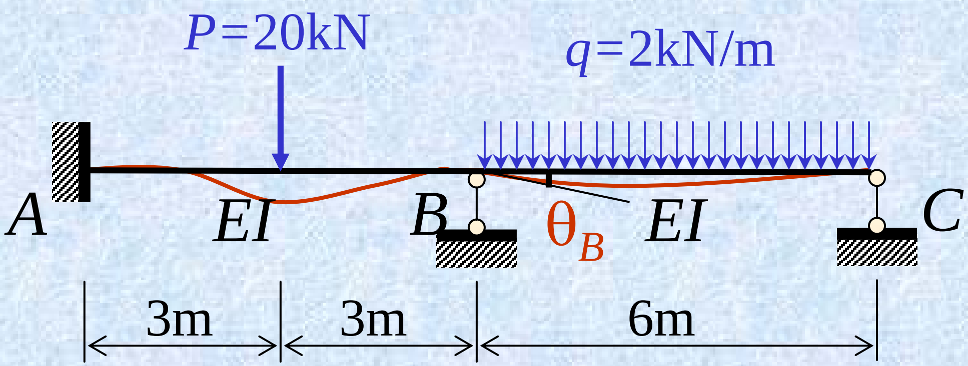


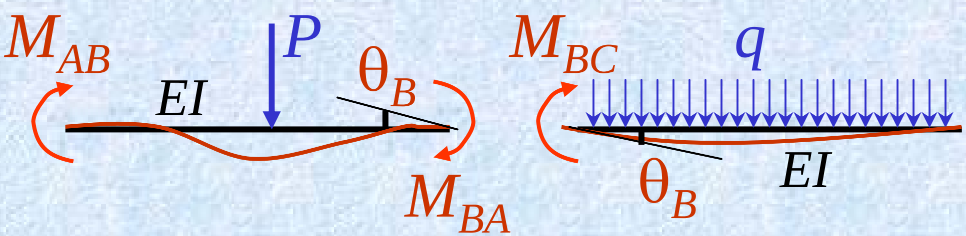
***M*图（kN•m）**

（9）校核 结点及局部杆件的静力平衡条件的校核。

§ 11-4　无侧移刚架的计算

如果除支座以外，刚架的各结点只有角位移而没有线位移，这种刚架称为无侧移刚架。





1、基本未知量 θ_B

2、固端弯矩

$$m_{BA}=\frac{Pl}{8}=\frac{20\times 6}{8}=15kN\cdot m$$

$$m_{AB}=-15kN\cdot m$$

$$m_{BC}=-\frac{ql^2}{8}=-9kN\cdot m$$

3、列杆端转角位移方程

设 $i=\frac{EI}{6}$

$$M_{AB}=2i\theta_B-15$$

$$M_{BA}=4i\theta_B+15$$

$$M_{BC}=3i\theta_B-9$$

$$M_{BC}=3i\theta_B-\frac{3i}{l}\Delta+m_{BC}$$

4、位移法基本方程（平衡条件）

超静定结构必须满足的三个条件:

(1) 变形连续条件: 在确定基本未知量时得到满足;

(2) 物理条件: 即刚度方程;

(3) 平衡条件: 即位移法基本方程。

4、位移法基本方程（平衡条件）

$$\sum M_B = 0 \qquad M_{BA} + M_{BC} = 0$$

$$4i\theta_B + 15 + 3i\theta_B - 9 = 0$$

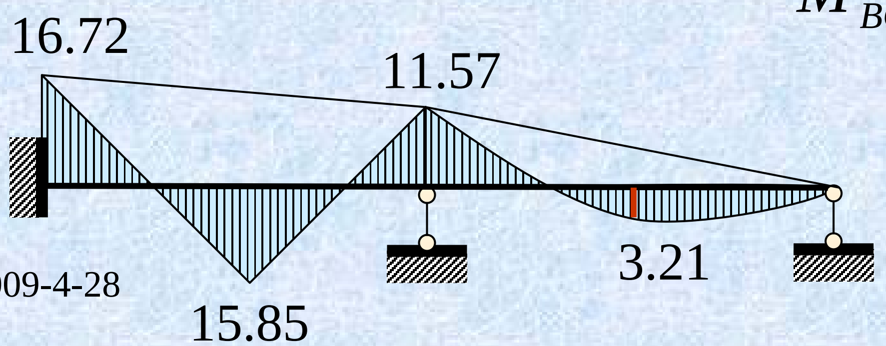
$$\therefore \theta_B = -\frac{6}{7i}$$

5、各杆端弯矩及弯矩图

$$M_{AB} = 2i\left(-\frac{6}{7i}\right) - 15 = -16.72kN\cdot m$$

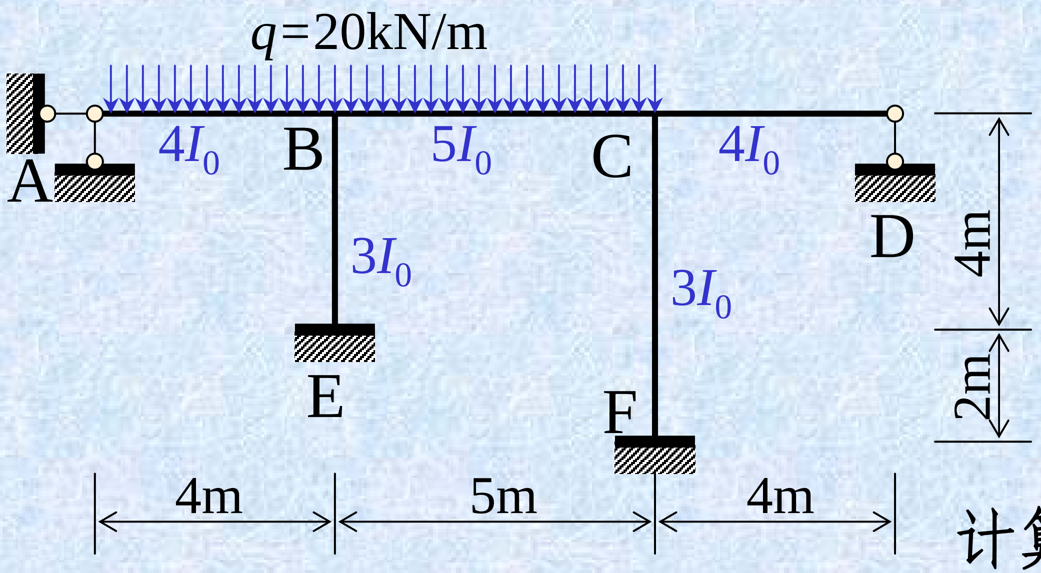
$$M_{BA} = 4i\left(-\frac{6}{7i}\right) + 15 = 11.57kN\cdot m$$

$$M_{BC} = 3i\left(-\frac{6}{7i}\right) - 9 = -11.57kN\cdot m$$



M图 $(kN\cdot m)$

例1、试用位移法分析图示刚架。



（1）基本未知量 θ_B、θ_C

（2）杆端弯矩 M_{ij}

$$m_{BA}=\frac{ql^2}{8}=\frac{20\times4^2}{8}=40$$

$$m_{BC}=-\frac{ql^2}{12}=-41.7$$

$$m_{CB}=41.7$$

计算线性刚度 i，设 $EI_0=1$，则

$$i_{AB}=\frac{EI_{AB}}{l_{AB}}=\frac{E\cdot4I_0}{4}=1$$

$$i_{BC}=1,\ i_{CD}=1,\ i_{BE}=\frac{3}{4},\ i_{CF}=\frac{1}{2}$$

梁

$$M_{BA}=3i_{AB}\cdot\theta_B+m_{BA}=3\theta_B+40$$

$$M_{CB}=4\theta_C+2\theta_B+41.7$$

$$M_{BC}=4\theta_B+2\theta_C-41.7$$

$$M_{CD}=3\theta_C$$

梁

$$M_{BA}=3i_{AB}\cdot\theta_B+m_{BA}=3\theta_B+40 \qquad M_{CD}=3\theta_C$$

$$M_{BC}=4\theta_B+2\theta_C-41.7 \qquad M_{CB}=4\theta_C+2\theta_B+41.7$$

柱

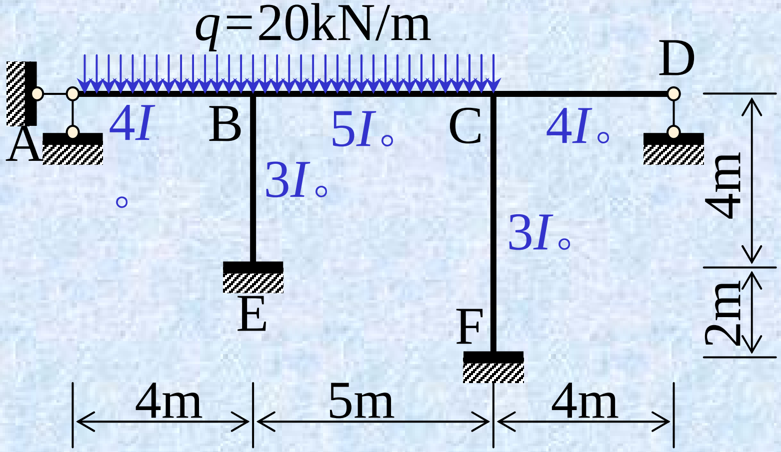
$$M_{BE}=4\cdot\frac{3}{4}\theta_B=3\theta_B \qquad M_{CF}=4\cdot\frac{1}{2}\theta_C=2\theta_C$$

$$M_{EB}=2\cdot\frac{3}{4}\theta_B=1.5\theta_B \qquad M_{FC}=2\cdot\frac{1}{2}\theta_C=\theta_C$$

(3) 位移法方程

$$\begin{aligned}\sum M_B=0 \qquad & M_{BA}+M_{BC}+M_{BE}=0\\ \sum M_C=0 \qquad & M_{CB}+M_{CD}+M_{CF}=0\end{aligned}$$

$$\left.\begin{aligned}10\theta_B+2\theta_C-1.7=0\\ 2\theta_B+9\theta_C+41.7=0\end{aligned}\right\}$$



(4) 解方程

$$\theta_B = 1.15 \qquad \theta_C = -4.89 \qquad \text{(相对值)}$$

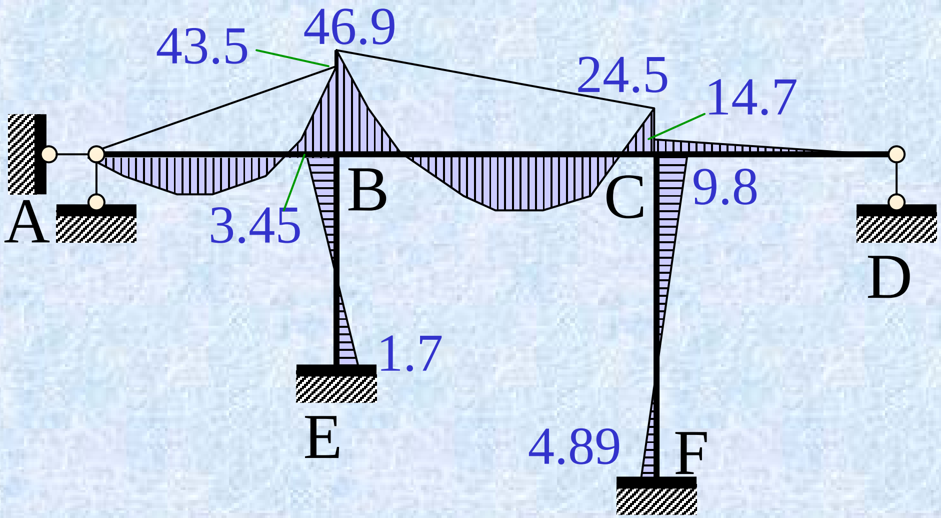
(5)杆端弯矩及弯矩图

梁

$$\begin{cases} M_{BA} = 3i_{AB} \cdot \theta_B + m_{BA} = 3\theta_B + 40 = 3\times1.15 + 40 = 43.5kN\cdot m \\ M_{BC} = 4\theta_B + 2\theta_C - 41.7 = 4\times1.15 + 2\times\left(-4.89\right) - 41.7 = -46.9kN\cdot m \\ \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{cases}$$

柱

$$\begin{cases} M_{BE} = 4\cdot\frac{3}{4}\theta_B = 3\theta_B = 3\times1.15 = 3.45kN\cdot m \\ M_{CF} = 4\cdot\frac{1}{2}\theta_C = 2\theta_C = 2\times(-4.89) = -9.8kN\cdot m \end{cases}$$



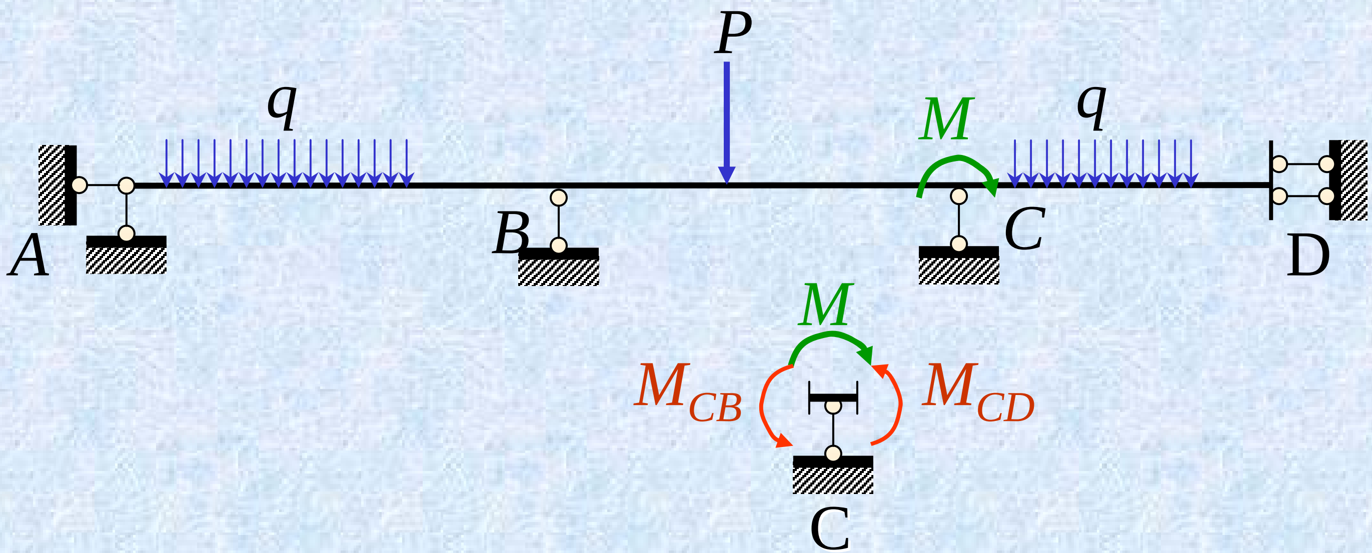
M图 $(kN\cdot m)$

小 结

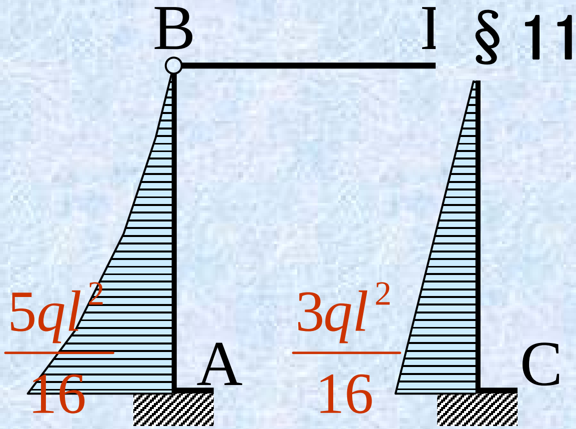
1、有几个未知结点位移就应建立几个平衡方程;

2、单元分析、建立单元刚度方程是基础;

3、当结点作用有集中外力矩时，结点平衡方程式中应包括外力矩。

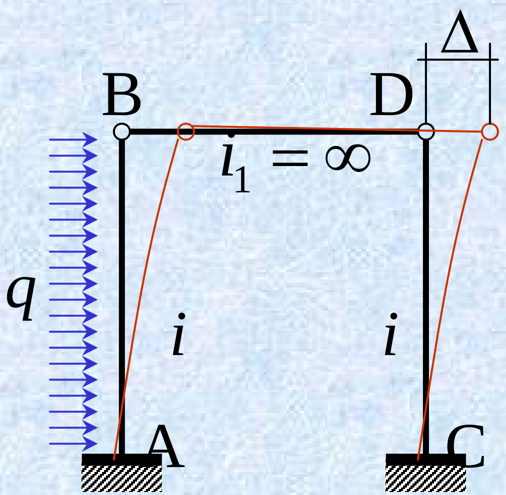


§11-5 有侧移刚架的计算



$$M_{AB}=-\frac{3i}{l}\Delta+m_{AB}=-\frac{3i}{l}\times\frac{ql^3}{16i}-\frac{ql^2}{8}=-\frac{5ql^2}{16}$$

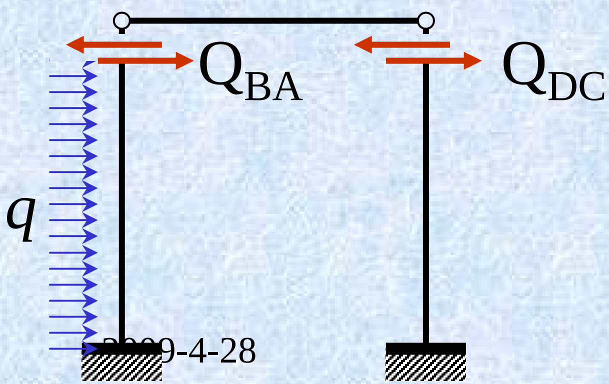
$$M_{CD}=-\frac{3i}{l}\Delta=-\frac{3ql^2}{16}$$



$$\sum x=0 \qquad Q_{BA}+Q_{DC}=0$$

其中 $Q_{BA}=\frac{3i}{l^2}\Delta-\frac{3}{8}ql \qquad Q_{DC}=\frac{3i}{l^2}\Delta$

$$\frac{6i}{l^2}\Delta-\frac{3}{8}ql=0 \qquad \Delta=\frac{ql^3}{16i}$$

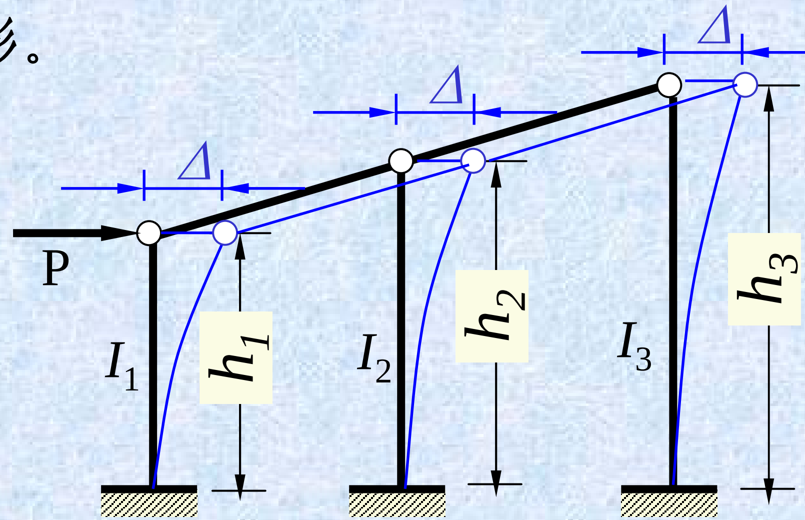


绘制弯矩图的方法:

（1）直接由外荷载及剪力计算;

（2）由角变位移方程计算。

例：作图示刚架的弯矩图。忽略梁的轴向变形。



解：1）基本未知量：Δ

2）各柱的杆端剪力

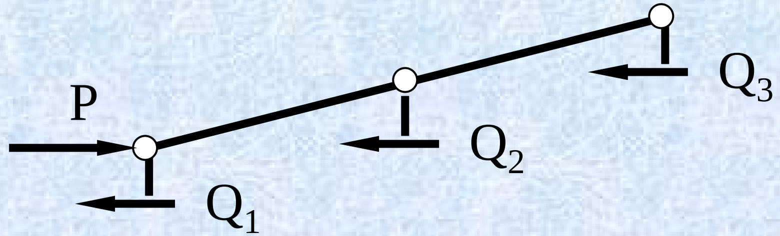
侧移刚度**$J=3i/h^2$**，则：

$Q_1=J_1\Delta$，$Q_2=J_2\Delta$，$Q_3=J_3\Delta$

3）位移法方程

$\Sigma X=0$ $\quad Q_1+Q_2+Q_3=P$

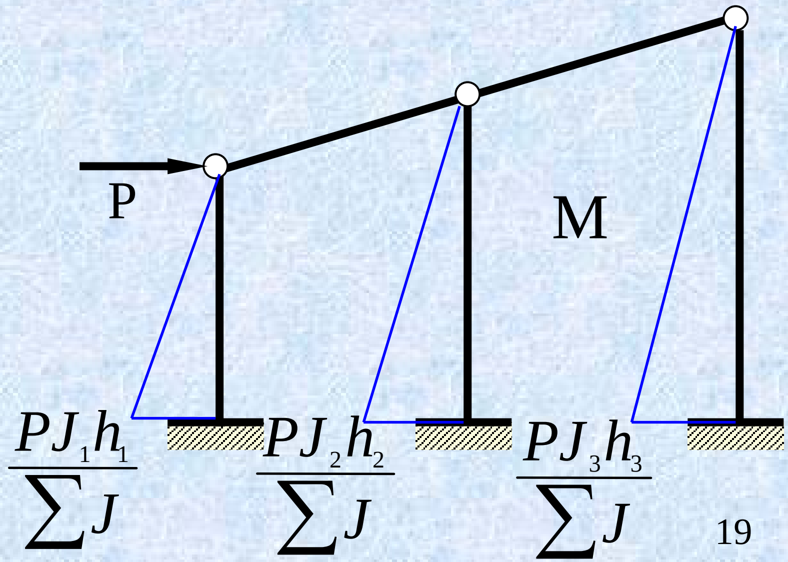
$$J_1\Delta+J_2\Delta+J_3\Delta=P \qquad \Delta=\frac{P}{\sum J_i}$$

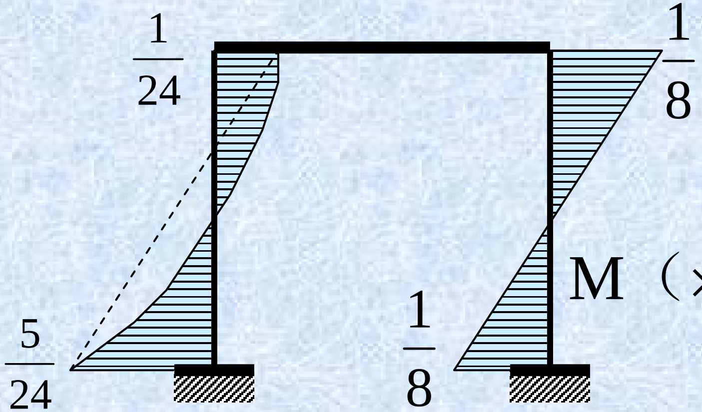


柱顶剪力：$Q_i=\dfrac{PJ_i}{\sum J}$

柱底弯矩：$M=Q_ih_i=\dfrac{PJ_i}{\sum J}h_i$

结点集中力作为各柱总剪力，按各柱的侧移刚度分配给各柱。再由反弯点开始即可作出弯矩图。



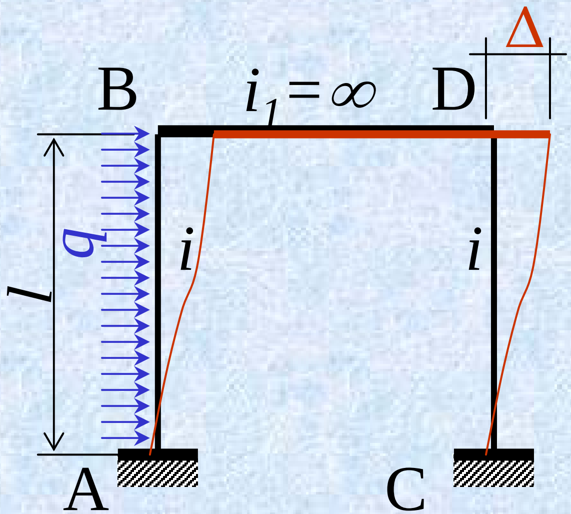


绘制弯矩图

$$M_{AB}=-\frac{6i}{l}\Delta+m_{AB}=-\frac{5}{24}ql^2$$

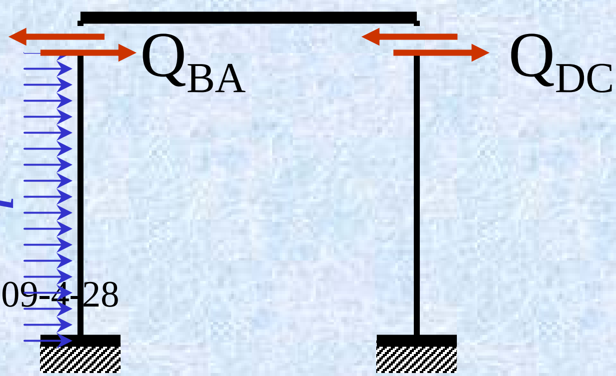
$$M_{BA}=-\frac{6i}{l}\Delta+m_{BA}=-\frac{1}{24}ql^2$$

……………………………………



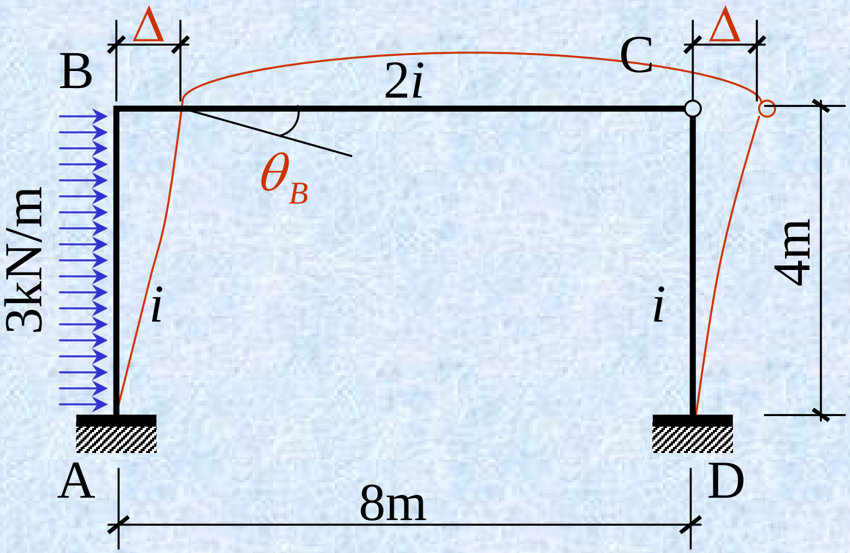
$$\sum x=0 \qquad Q_{BA}+Q_{DC}=0$$

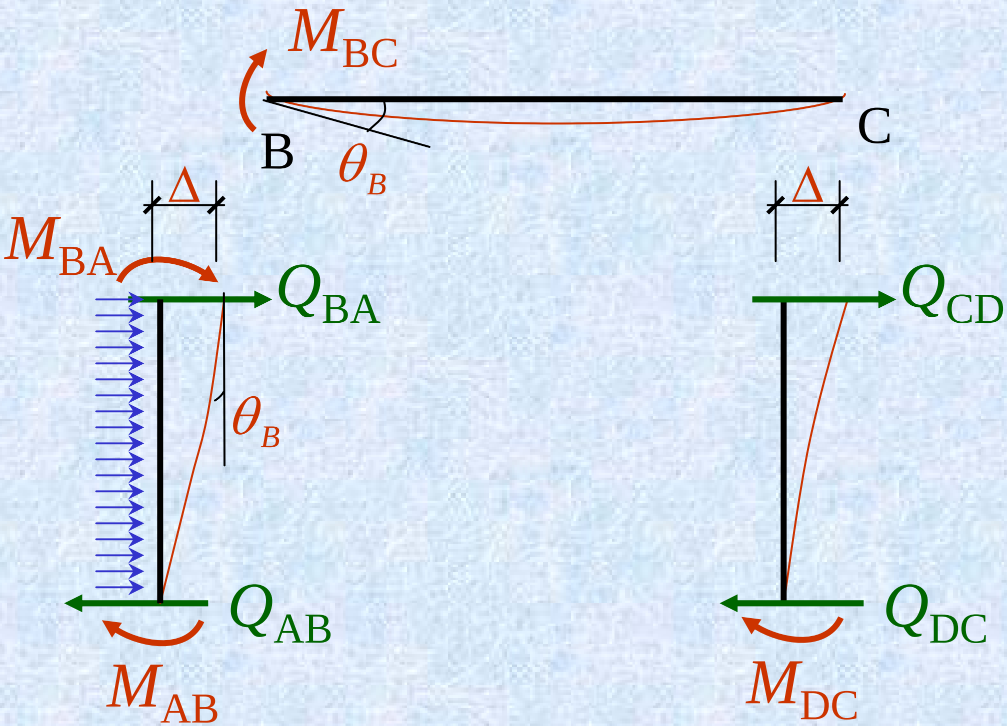
其中 $Q_{BA}=\frac{12i}{l^2}\Delta-\frac{ql}{2}$ $\qquad Q_{DC}=\frac{12i}{l^2}\Delta$



$$\frac{24i}{l^2}\Delta-\frac{ql}{2}=0 \qquad \Delta=\frac{ql^3}{48i}$$

例1. 用位移法分析图示刚架。





[解]（1）基本未知量θ_B、Δ

（2）单元分析

$$M_{AB}=2i\theta_B-\frac{6i}{4}\Delta-\frac{3\times4^2}{12}\qquad M_{BC}=3(2i)\theta_B$$

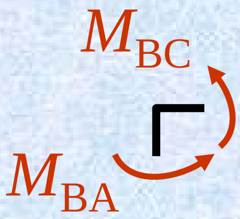
$$M_{BA}=4i\theta_B-\frac{6i}{4}\Delta+\frac{3\times4^2}{12}\qquad M_{DC}=-\frac{3i}{4}\Delta\qquad Q_{CD}=\frac{3i}{4^2}\Delta$$

$$Q_{BA}=-\frac{6i}{4}\theta_B+\frac{12i}{4^2}\Delta-\frac{ql}{2}=1.5i\theta_B+0.75i\Delta-6$$

(3）位移法方程

$$M_{AB}=2i\theta_B-1.5i\Delta-4 \qquad M_{BC}=6i\theta_B$$

$$M_{BA}=4i\theta_B-1.5i\Delta+4 \qquad M_{DC}=-0.75i\Delta$$



$$\sum M_B=0$$

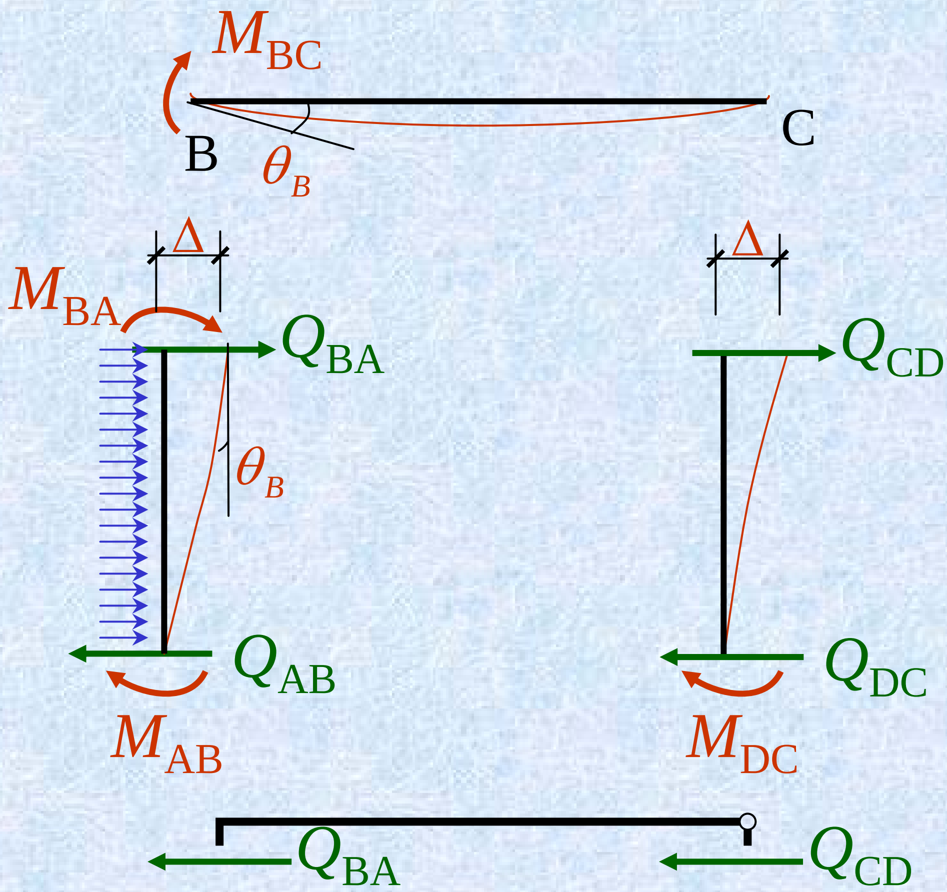
$$M_{BA}+M_{BC}=0 \ldots\ldots\ldots\ldots\ldots(1a)$$

$$10i\theta_B-15i\Delta+4=0 \ldots\ldots\ldots\ldots(1)$$

$$\sum x=0$$

$$Q_{BA}+Q_{CD}=0 \ldots\ldots\ldots\ldots\ldots(2a)$$

$$6i\theta_B-3.75i\Delta+24=0 \ldots\ldots(2)$$



（4）解位移法方程

$$Q_{BA}=1.5i\theta_B+0.75i\Delta-6 \qquad Q_{CD}=\frac{3i}{4^2}\Delta$$

（4）解位移法方程

$$10i\theta_B - 1.5i\Delta + 4 = 0 \text{..........}(1)$$

$$6i\theta_B - 3.75i\Delta + 24 = 0 \text{........}(2)$$

$$\theta_B = \frac{0.737}{i} \qquad \Delta = \frac{7.58}{i}$$

（5）弯矩图

M_{AB}= -13.896 kN·m

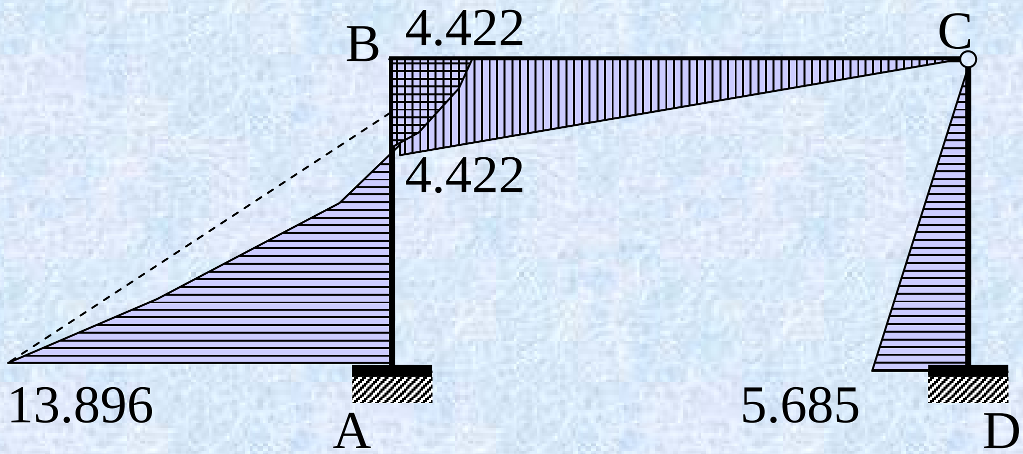
M_{BA}= -4.422kN·m

M_{BC}= 4.422kN·m

M_{DC}= -5.685kN·m

Q_{BA}= -1.42kN

Q_{CD}= -1.42kN

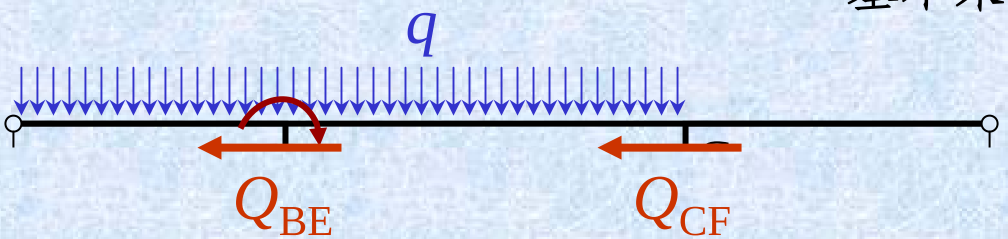


M图（kN·m）

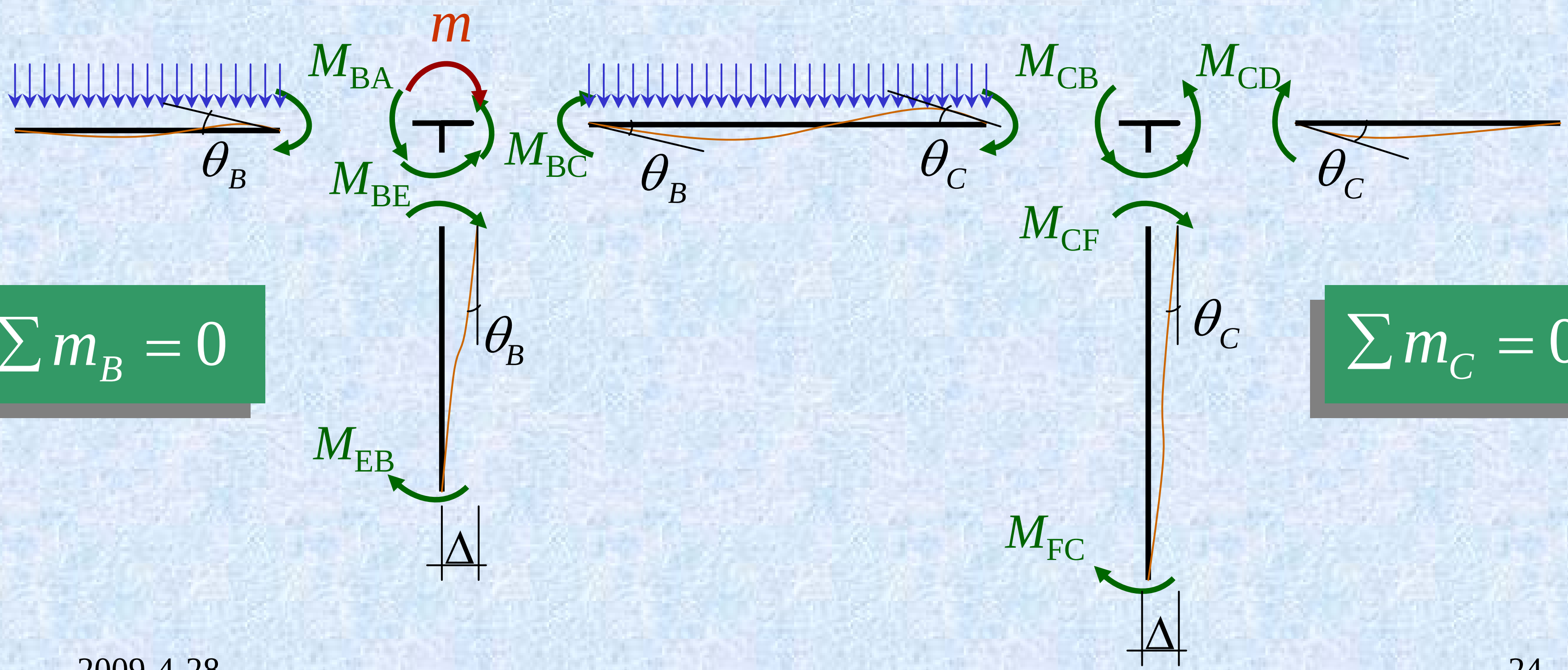
例2. 用位移法分析图示刚架。——思路

基本未知量为:

$\theta_B \quad \theta_C \quad \Delta$



$$\sum x = 0$$



$$\sum m_B = 0$$

$$\sum m_C = 0$$

基本未知量为：　θ_C　Δ