《自动控制理论》
Automatic Control Theory

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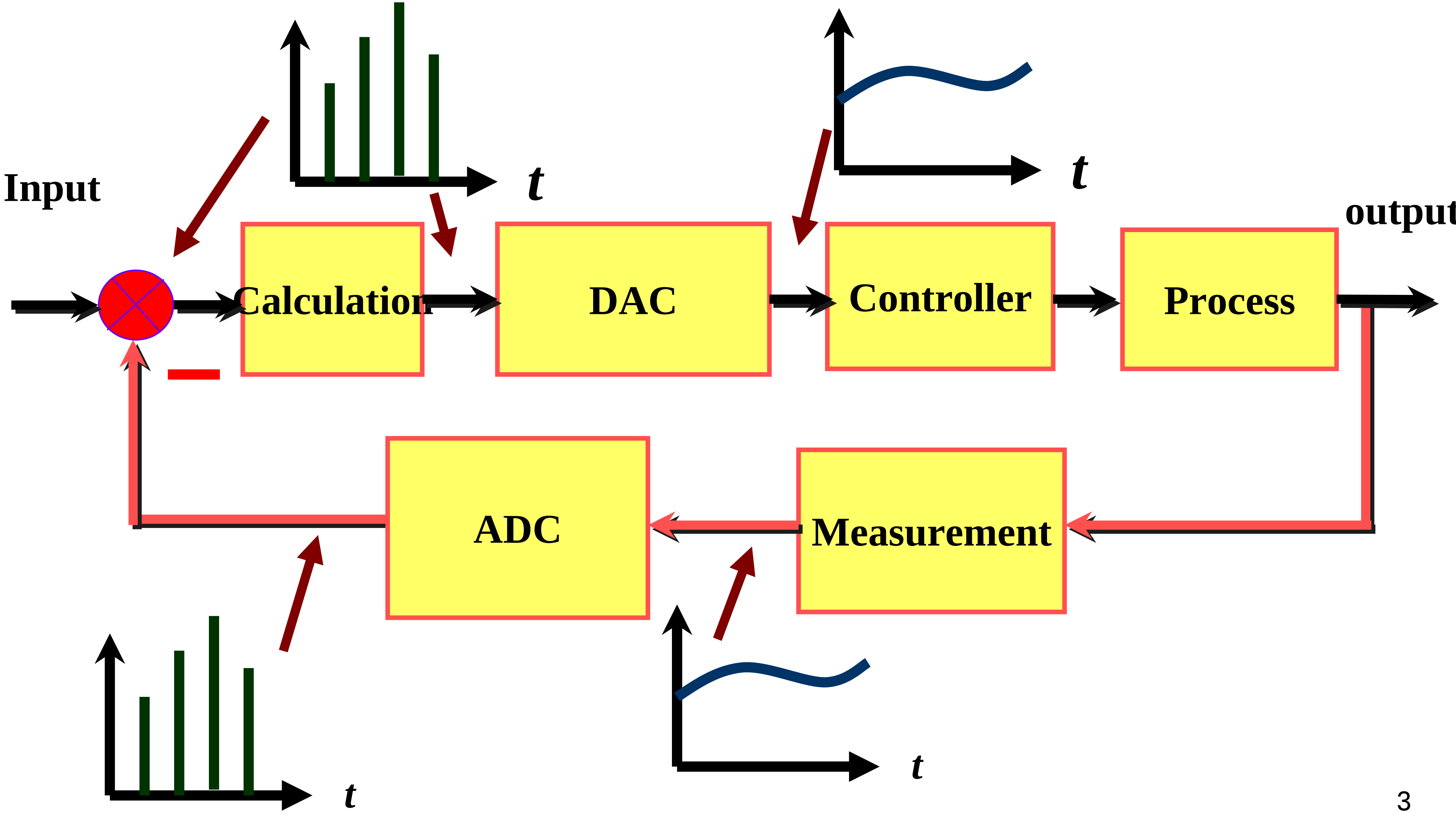
Reviews

- 1 Discrete control system
- 2 Signal Sample and recovery

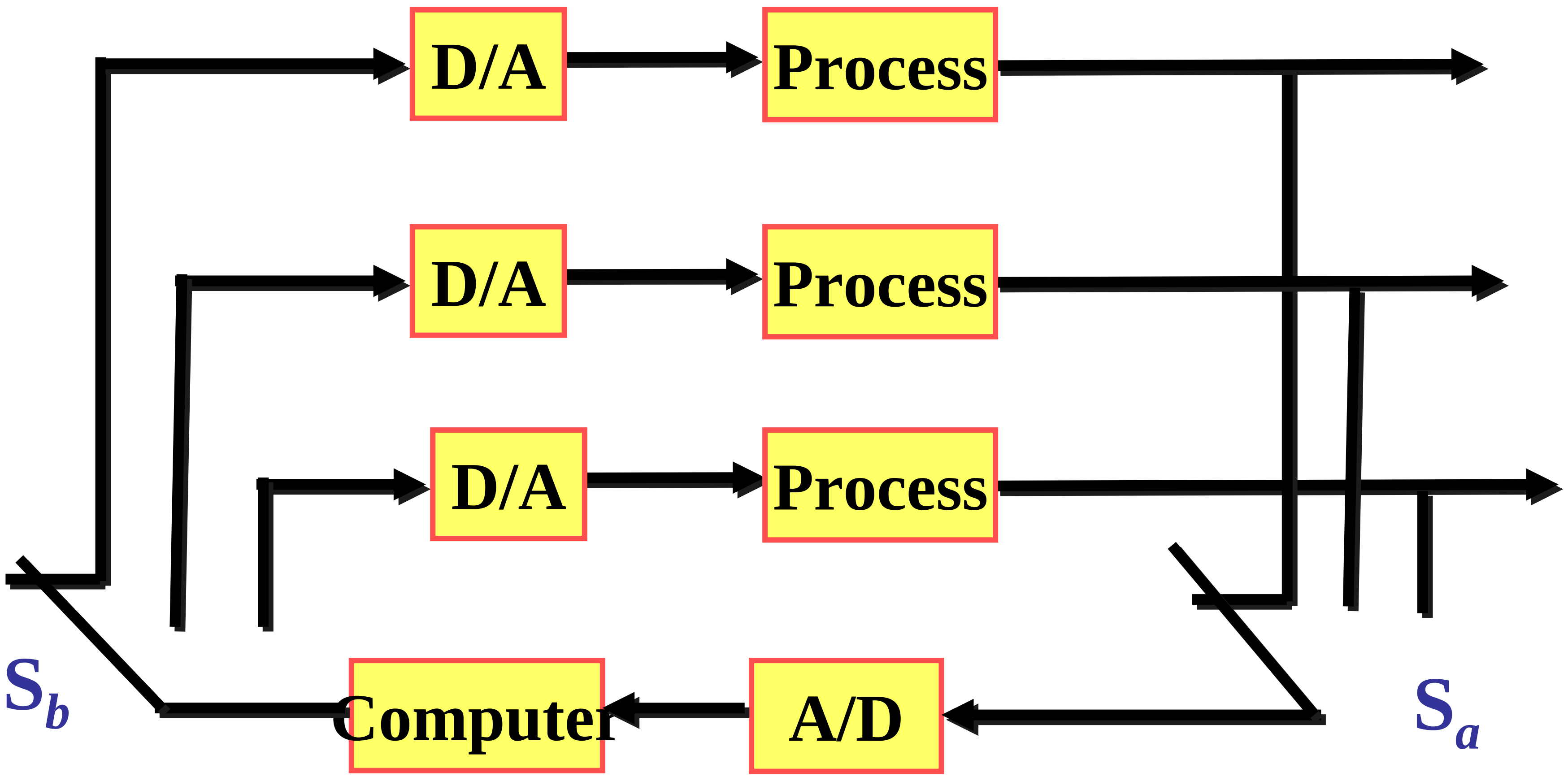
Reviews

1. Structure of computer control system



Reviews

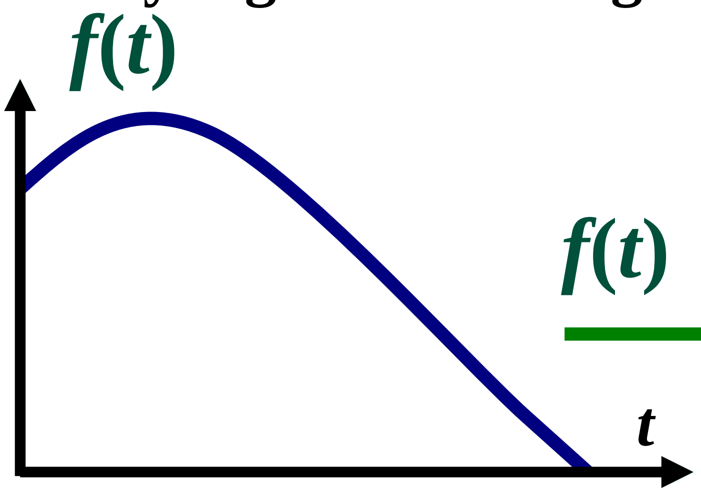
(2) Computer multiple control system

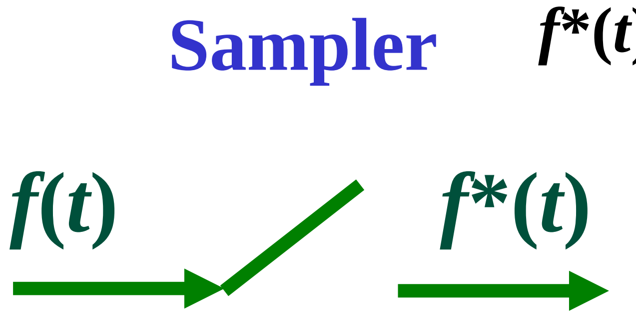


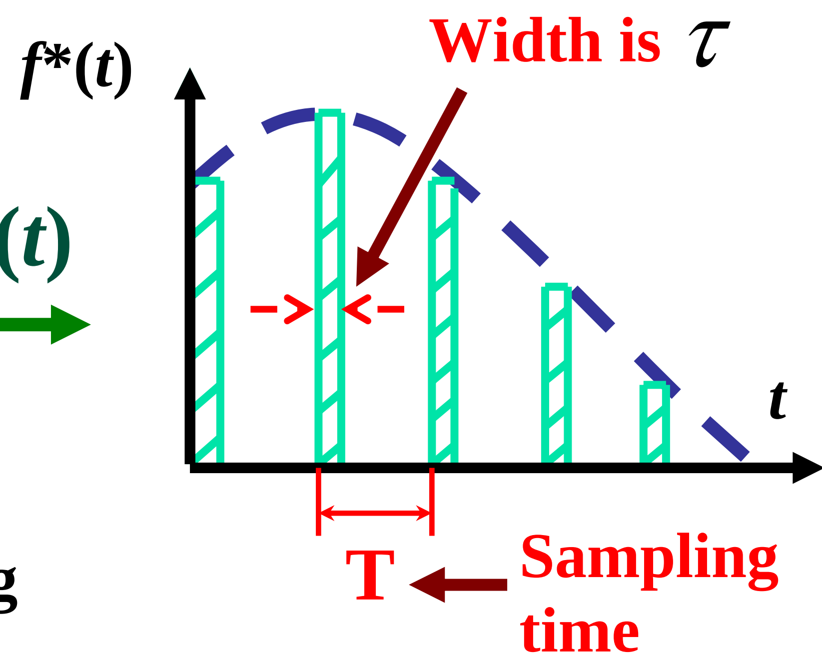
Reviews

1. Sample and its mathematic expression

Sampling process is conversion of continuous signal into sampled signal in sample-data control system. It is realized by digital-to-analog converter.







Describe sampled signal in following mathematic expression

$$f^*(t) = \sum_{k=-\infty}^{\infty} f(t)\delta(t-kT) = \sum_{k=-\infty}^{\infty} f(kT)\delta(t-kT)$$

Reviews

2. Shannon sampling theorem

$$\omega_s \geq 2\omega_{\max}$$

——Shannon sampling theorem

3. Zero-order hold

Transfer function of zero-order hold

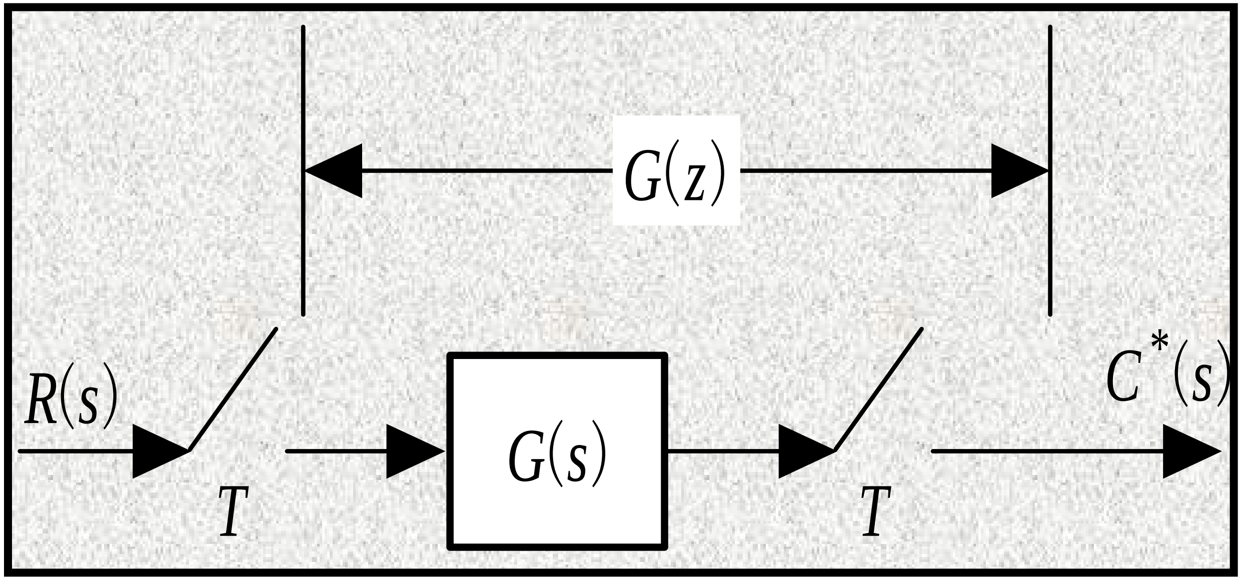
$$G_h(s) = \frac{1}{s} - \frac{e^{-Ts}}{s} = \frac{1-e^{-Ts}}{s}$$

Reviews

1. Impulse transfer function

Under zero initial condition, the ratio of z-transform of output impulse series and z-transform of input impulse series is impulse transfer function.

$$G(z)=\frac{C(z)}{R(z)}$$



2. Steps of obtaining impulse transfer function

2. Impulse transfer function of closed-loop system

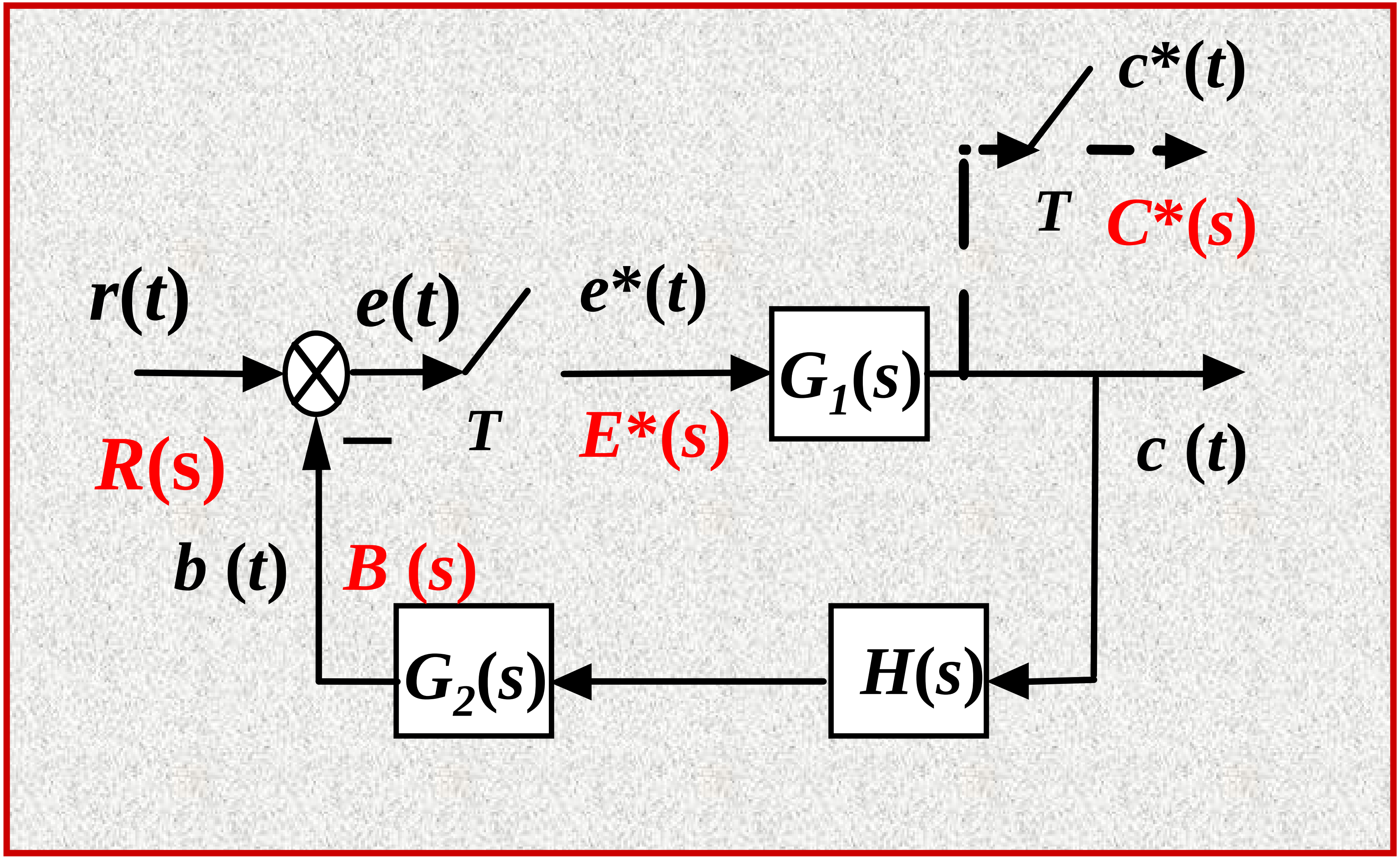
Key points:

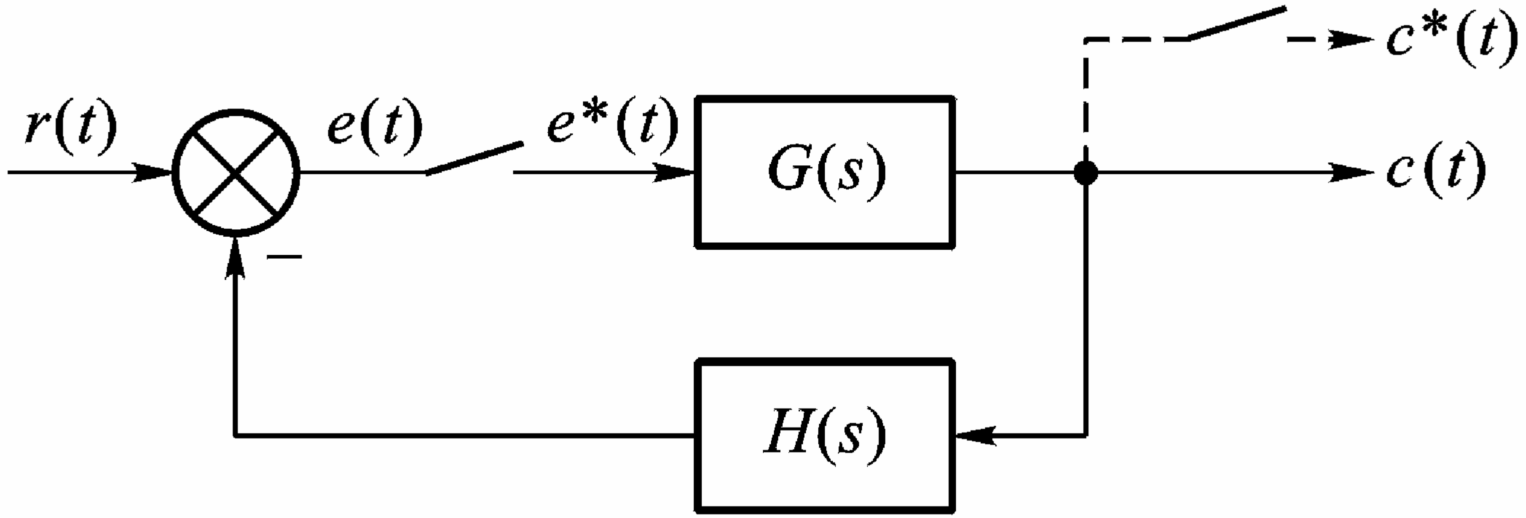
(1) Different position of sampler in system correspond to different closed-loop impulse transfer function .

(2) Obtain relationship of input and output though adopting sampler signal as the middle signal.

(3) Eliminate middle signal and obtain impulse transfer function.

Example 1: Obtain C(z)/R(z)





The input of sampler and output of system are

$$E(s) = R(s) - G(s)H(s)E^*(s)$$

$$C(s) = G(s)E^*(s)$$

$$E^*(s) = R^*(E) - GH^*(s)E^*(s)$$

$$C^*(s) = G^*(s)E^*(s)$$

2. Steps of obtaining impulse transfer function

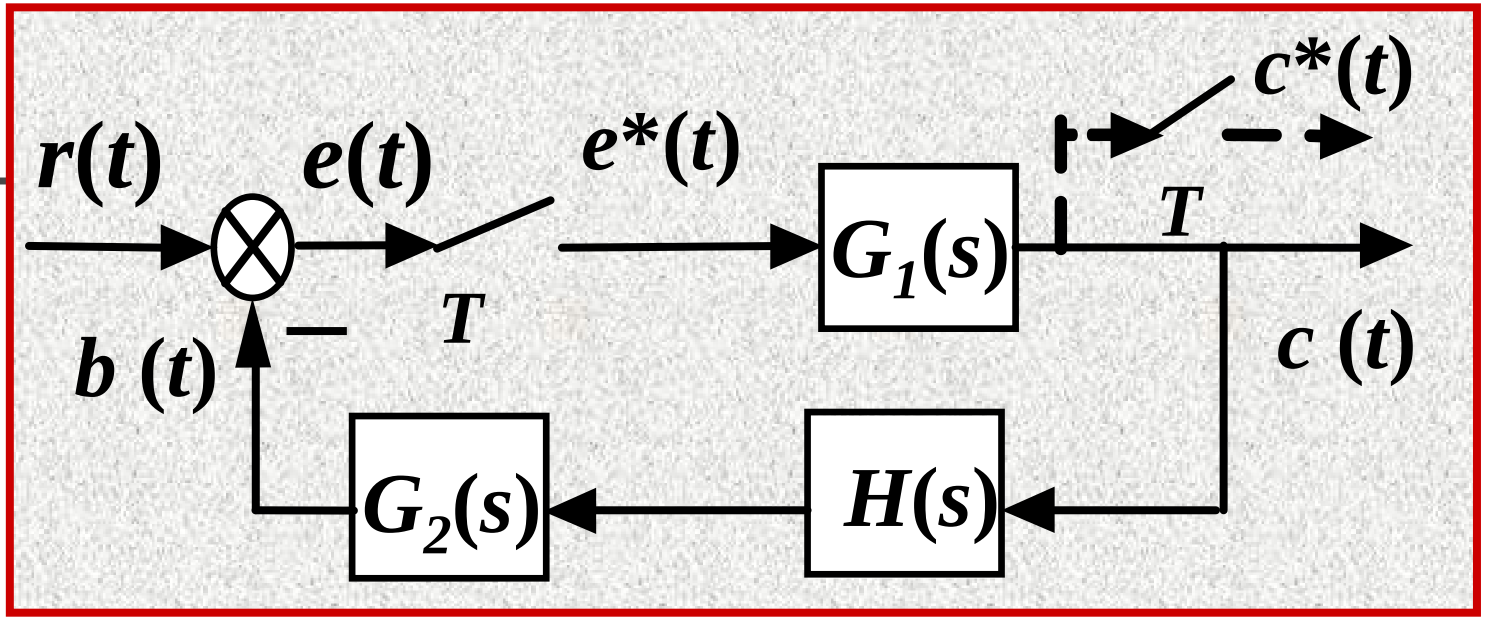
Then

$$C^*(s) = \frac{G^*(s)}{1+GH^*(s)} R^*(s)$$

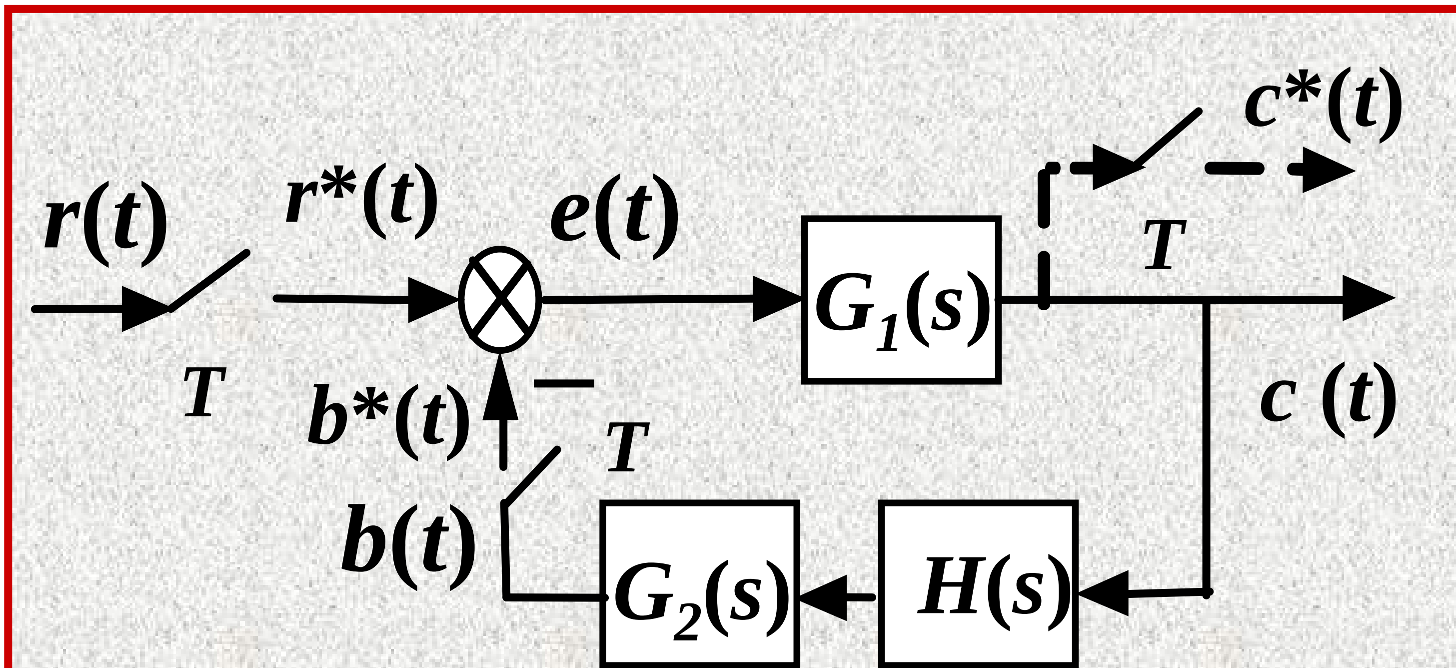
$$C(z) = \frac{G(z)}{1+GH(z)} R(z)$$

The impulse transfer function of closed-loop system is

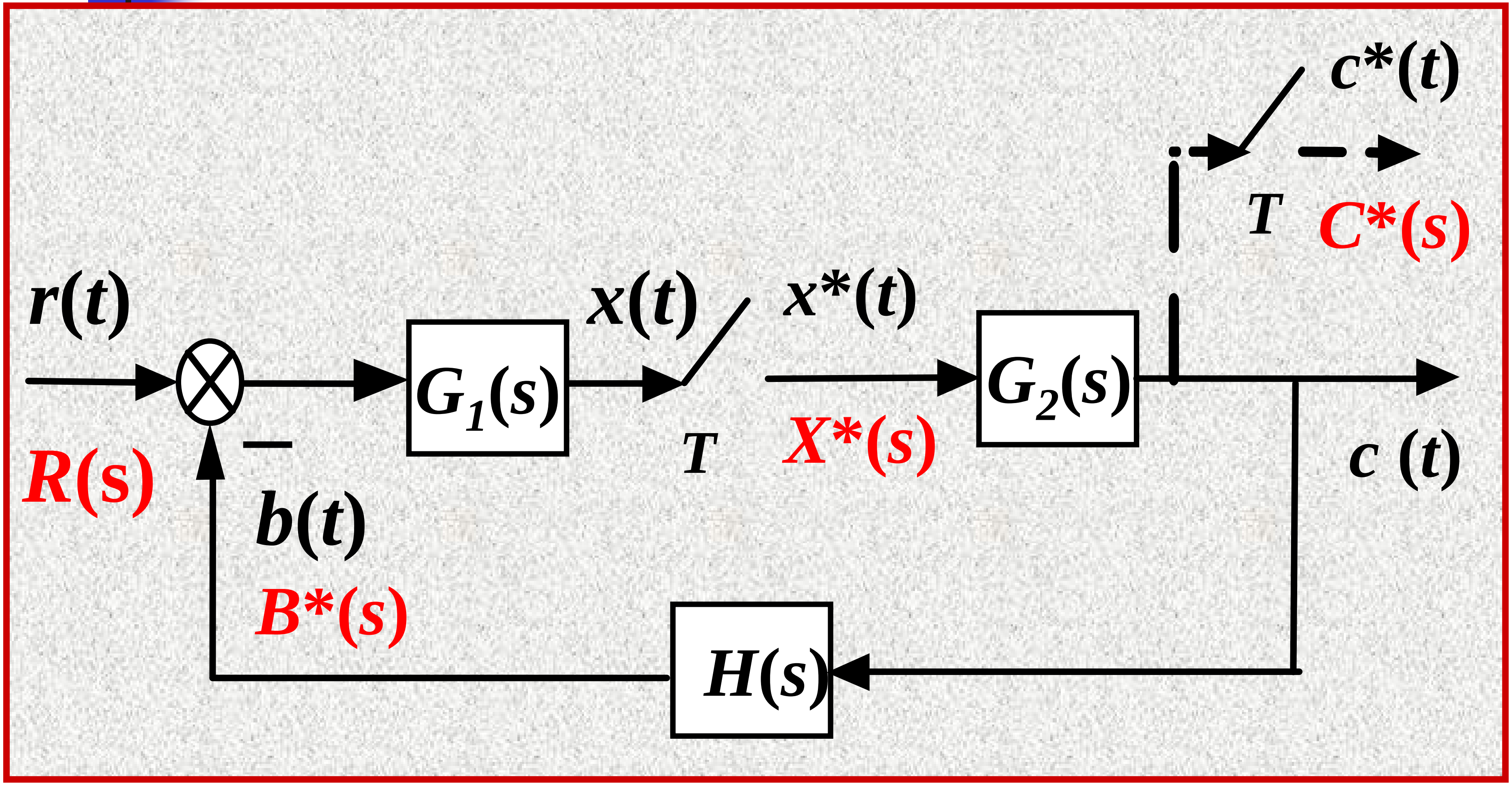
$$C(z) = \frac{G(z)}{1+GH(z)} R(z)$$



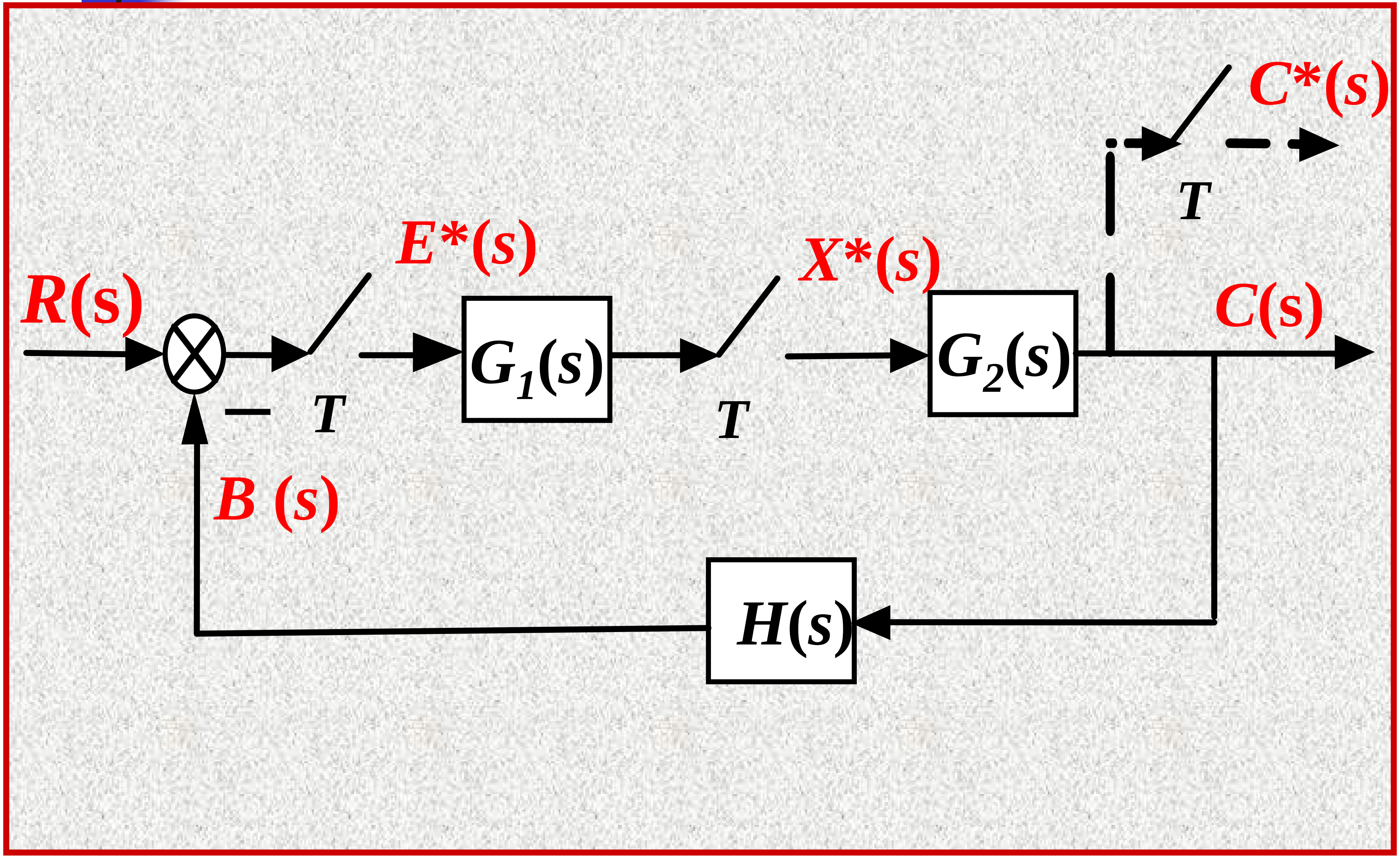
Equivalent



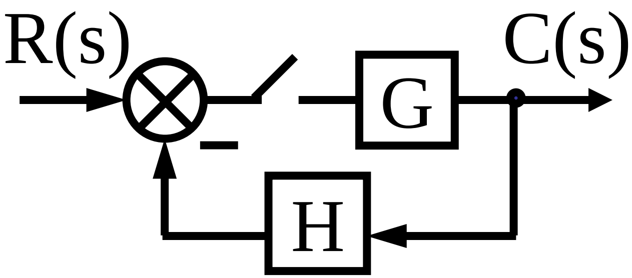
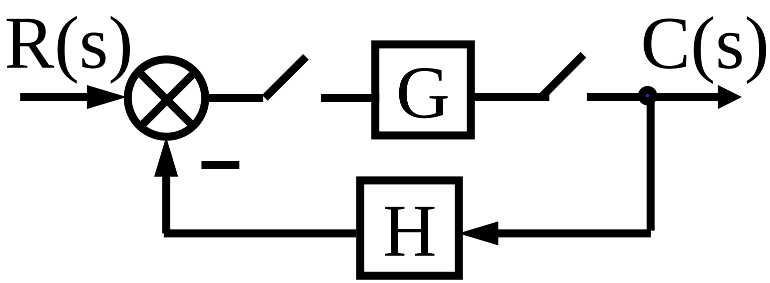
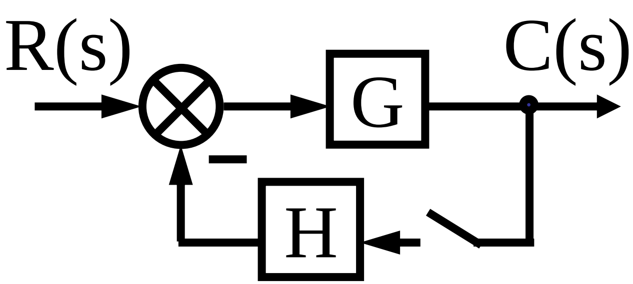
Example 2: Obtain C(z)/R(z)



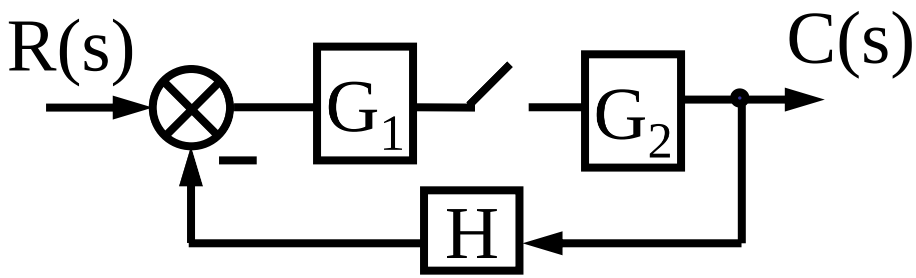
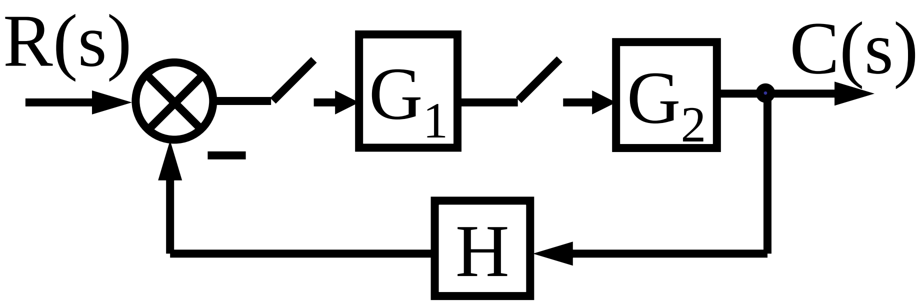
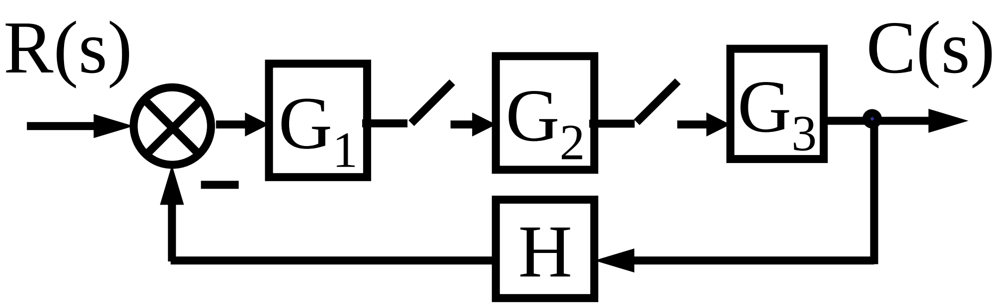
Example 3: Obtain C(z)/R(z)



Impulse transfer functions of common single closed-loop systems

System structures	C(z)
	$C(z)=\dfrac{G(z)}{1+HG(z)}R(z)$
	$C(z)=\dfrac{G(z)}{1+H(z)G(z)}R(z)$
	$C(z)=\dfrac{RG(z)}{1+HG(z)}$

Impulse transfer functions of common single closed-loop systems

System structures	C(z)
	$C(z)=\dfrac{RG_1(z)G_2(z)}{1+G_1G_2H(z)}$
	$C(z)=\dfrac{G_1(z)G_2(z)}{1+G_1(z)HG_2(z)}R(z)$
	$C(z)=\dfrac{G_2(z)C_3(z)RG_1(z)}{1+G_2(z)G_1G_3H(z)}$

Section 5: System performance analysis of discrete control system

2. Stability of linear sampled-data system

(1) Sufficient and necessary condition of stability

The sufficient and necessary condition of stability on z-plane is: all character roots must be inside of unit circle in z-plane.

The closed-loop impulse transfer function of sampled-data system is

$$\phi(z)=\frac{C(z)}{R(z)}=\frac{M(z)}{D(z)}$$

Closed-loop character equation is

$$D(z)=0$$

(2) Mapping relationship between S-plane and z-plane

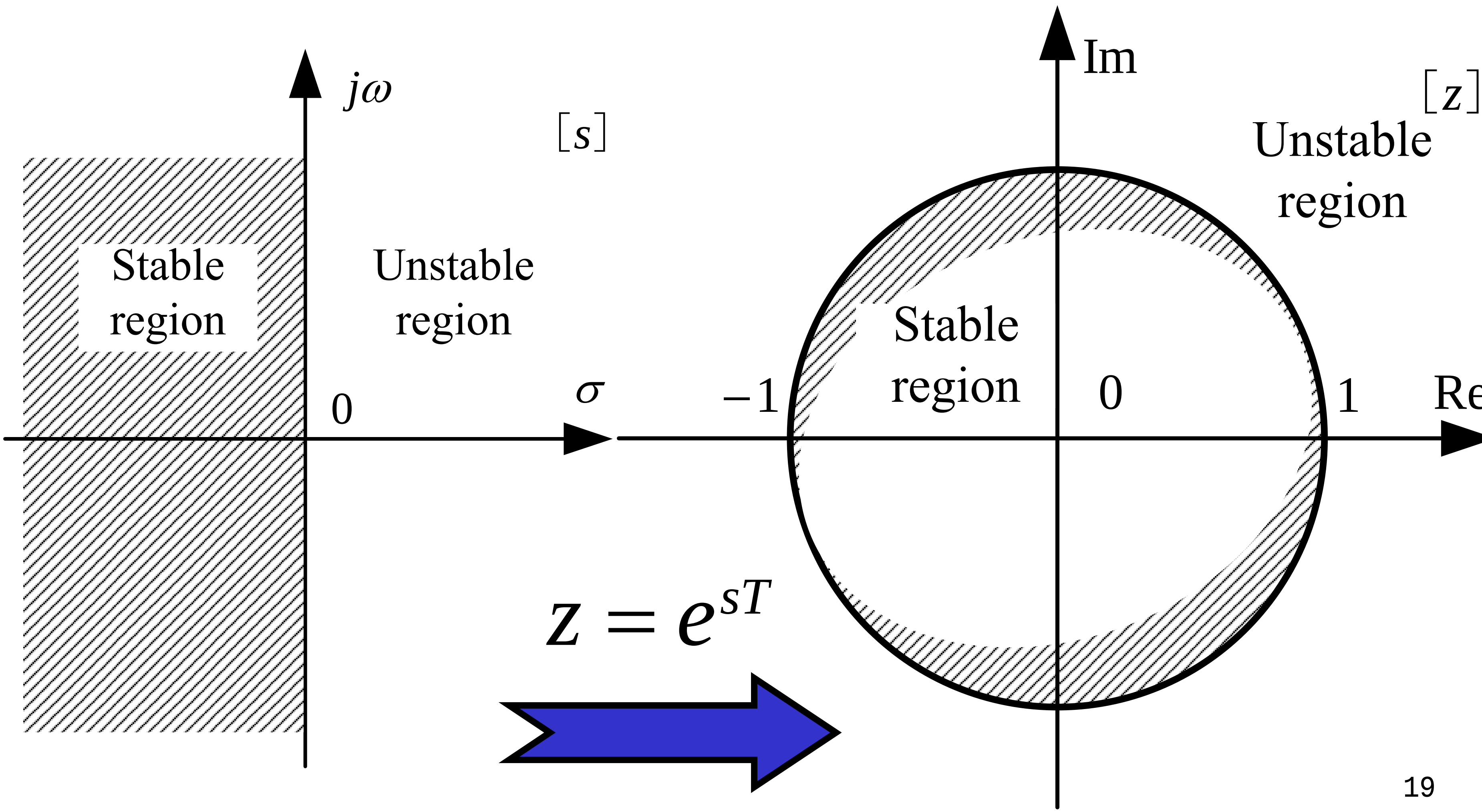
Suppose $z = e^{sT}$ $\qquad s = \sigma + j\omega$

$$z = e^{\sigma T} e^{j\omega T}$$

$$|z| = e^{\sigma T} \qquad \angle z = \omega T$$

$$\begin{cases} \sigma > 0 & |z| > 1 \\ \sigma < 0 & |z| < 1 \\ \sigma = 0 & |z| = 1 \end{cases}$$

Section 5: System performance analysis of discrete control system



2. Routh stability criterion

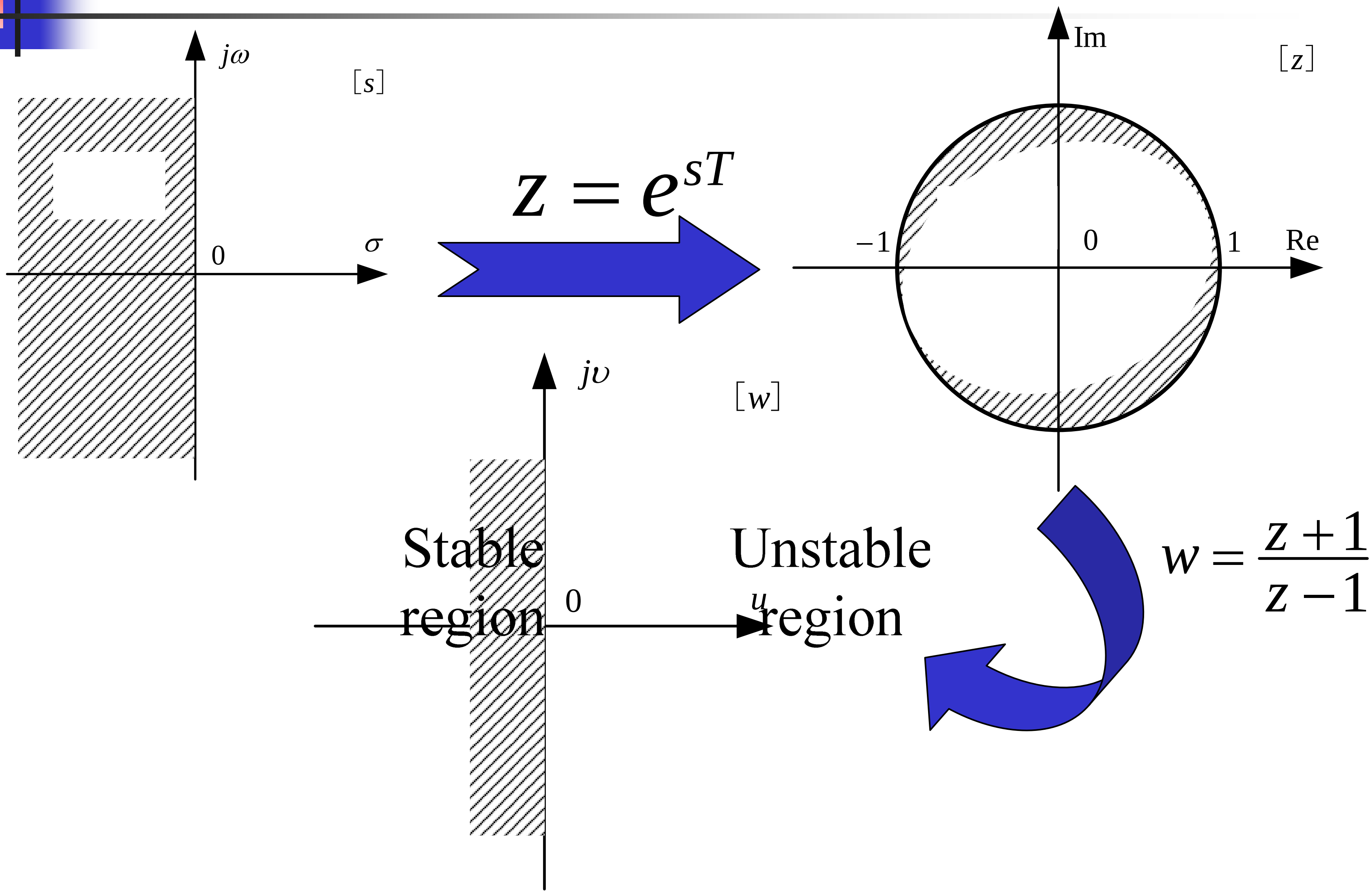
- In analyzing continuous system, Routh stability criterion could estimate system stability though calculating the number of character roots on the right side of s-plane.
- In sampled-data system, Routh criterion could also estimate system stability. For stable region is the inside of unit circle on z-plane but the right side, Routh criterion could not be used directly.

Section 5: System performance analysis of discrete control system

Adopt the following bilinear transform

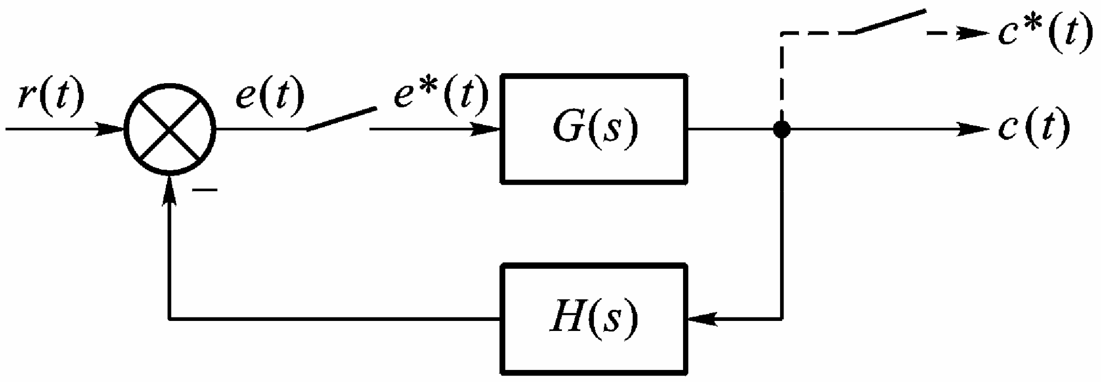
$$z = \frac{w+1}{w-1}$$

The system stability could be estimated though Routh stability criterion.


jω
[s]
0
σ
z = e^{sT}
Im
[z]
−1
0
1
Re
jυ
[w]
Stable region
0
u
Unstable region
w = (z + 1)/(z − 1)

3. Steady-state error of discrete system

Suppose the structure of sampled-data system is



The error impulse transfer function is

$$\phi_{er}(z) = \frac{E(z)}{R(z)} = \frac{1}{1+G(z)}$$

Z-transform of error signal is

$$E(z) = \phi_{er}(z)R(z) = \frac{1}{1+G(z)}R(z)$$

Obtain steady-state error from z-transform final value theorem

$$e_{ss} = \lim_{t\to\infty} e^*(t) = \lim_{z\to 1}(z-1)E(z) = \lim_{z\to 1}(z-1)\frac{R(z)}{1+G(z)}$$

Steady-state error with three typical input

(1) Unit step input (t)=1(t)

Steady-state error is

$$e_{ss} = \lim_{z\to 1}(z-1)\frac{1}{1+G(z)}\cdot\frac{z}{z-1} = \lim_{z\to 1}\frac{1}{1+G(z)} = \frac{1}{1+K_P}$$

Where $K_P = \lim_{z\to 1} G(z)$ —— **The static position error coefficient**

For type 0 system $\nu = 0,\ \ K_p = K,\ \ e_{ss} = \dfrac{1}{1+K_p}$

For systems higher than type 1 $\nu \geq 1,\ \ K_p = \infty,\ \ e_{ss} = 0$

(2) Unit ramp input r(t)=t

$$R(z)=\frac{Tz}{(z-1)^2}$$

Steady-state error is

$$e_{ss}=\lim_{z\to 1}(z-1)\frac{1}{1+G(z)}\cdot\frac{Tz}{(z-1)^2}=\lim_{z\to 1}\frac{Tz}{(z-1)\left[1+G(z)\right]}$$

$$=\lim_{z\to 1}\frac{T}{(z-1)G(z)}=\frac{T}{K_V}$$

Where $$K_V=\lim_{z\to 1}(z-1)G(z)$$

——The speed position error coefficient

Obtain from definition, with ramp input, the system should have at least two $z=1$ **poles in open-loop transfer function to ensure there is no steady-state error.**

(3) Unit parabolic input

$$r(t)=\frac{t^2}{2},\quad R(z)=\frac{T^2 z(z+1)}{2(z-1)^3}$$

Steady-state error is

$$e_{ss}=\lim_{z\to 1}(z-1)\frac{1}{1+G(z)}\frac{T^2 z(z+1)}{2(z-1)^3}$$

$$=\lim_{z\to 1}\frac{T^2}{(z-1)^2 G(z)}=\frac{T^2}{K_a}$$

Where $K_a=\lim_{z\to 1}(z-1)^2 G(z)$

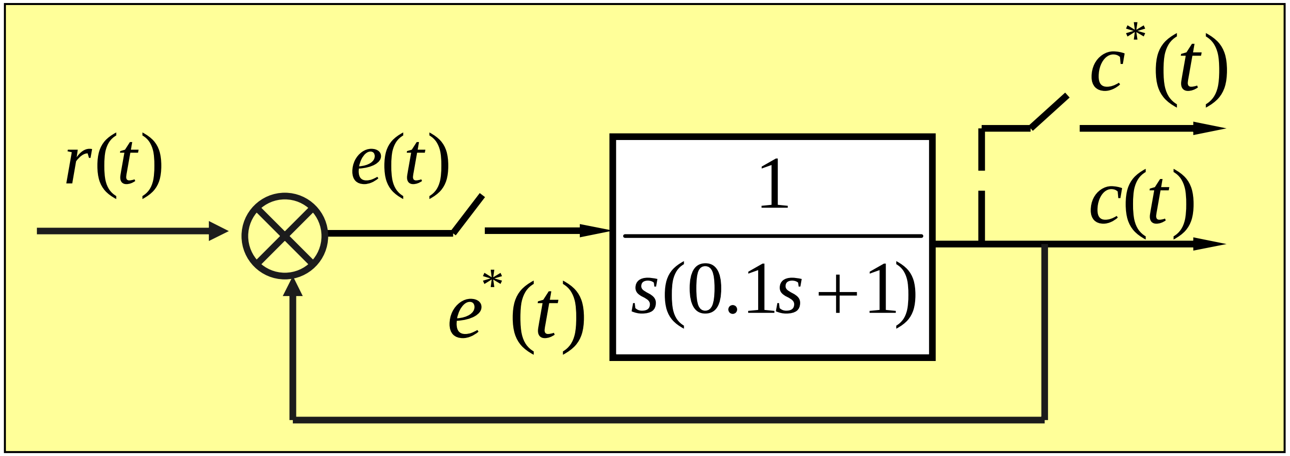
——The speed acceleration error coefficient

Under parabolic input, the system should have at least three $z=1$ **poles in open-loop transfer function to ensure there is no steady-state error.**

Table: Steady-state error of sampled-data system

Type of system	$r(t)=1(t)$	$r(t)=t$	$r(t)=\frac{1}{2}t^2$
0	$\frac{1}{1+K_P}$	∞	∞
1	**0**	$\frac{T}{K_V}$	∞
2	**0**	**0**	$\frac{T^2}{K_a}$

Example: The structure of discrete system is shown in the figure with sampling period T=0.1s. Obtain steady-state error under unit step, unit ramp and unit parabolic input respectively.



Open-loop impulse transfer function is

$$G(z) = Z\left[\frac{1}{s(0.1s+1)}\right] = Z\left[(\frac{1}{s} - \frac{1}{s+10})\right]$$

$$= \frac{z}{z-1} - \frac{z}{z-e^{-10T}}$$

When T=0.1s

$$G(z) = \frac{0.632z}{z^2 - 1.368z + 0.368}$$

Character equation of system is 1+G(z)=0

Then $$z^2 - 0.736z + 0.368 = 0$$

Let $z = \dfrac{1+\omega}{1-\omega}$**, then**

$$D(\omega) = 0.632\omega^2 + 1.264\omega + 2.104 = 0$$

For all coefficients are greater than zero, the system is stable.

Steady-state error coefficient

$$K_P = \lim_{z\to 1} G(z) = \lim_{z\to 1} \frac{0.632z}{(z-1)(z-0.368)} = \infty$$

$$K_V = \lim_{z\to 1}(z-1)G(z) = \lim_{z\to 1} \frac{0.632z}{z-0.368} = 1$$

$$K_a = \lim_{z\to 1}(z-1)^2 G(z) = \lim_{z\to 1}(z-1)\frac{0.632z}{z-0.368} = 0$$

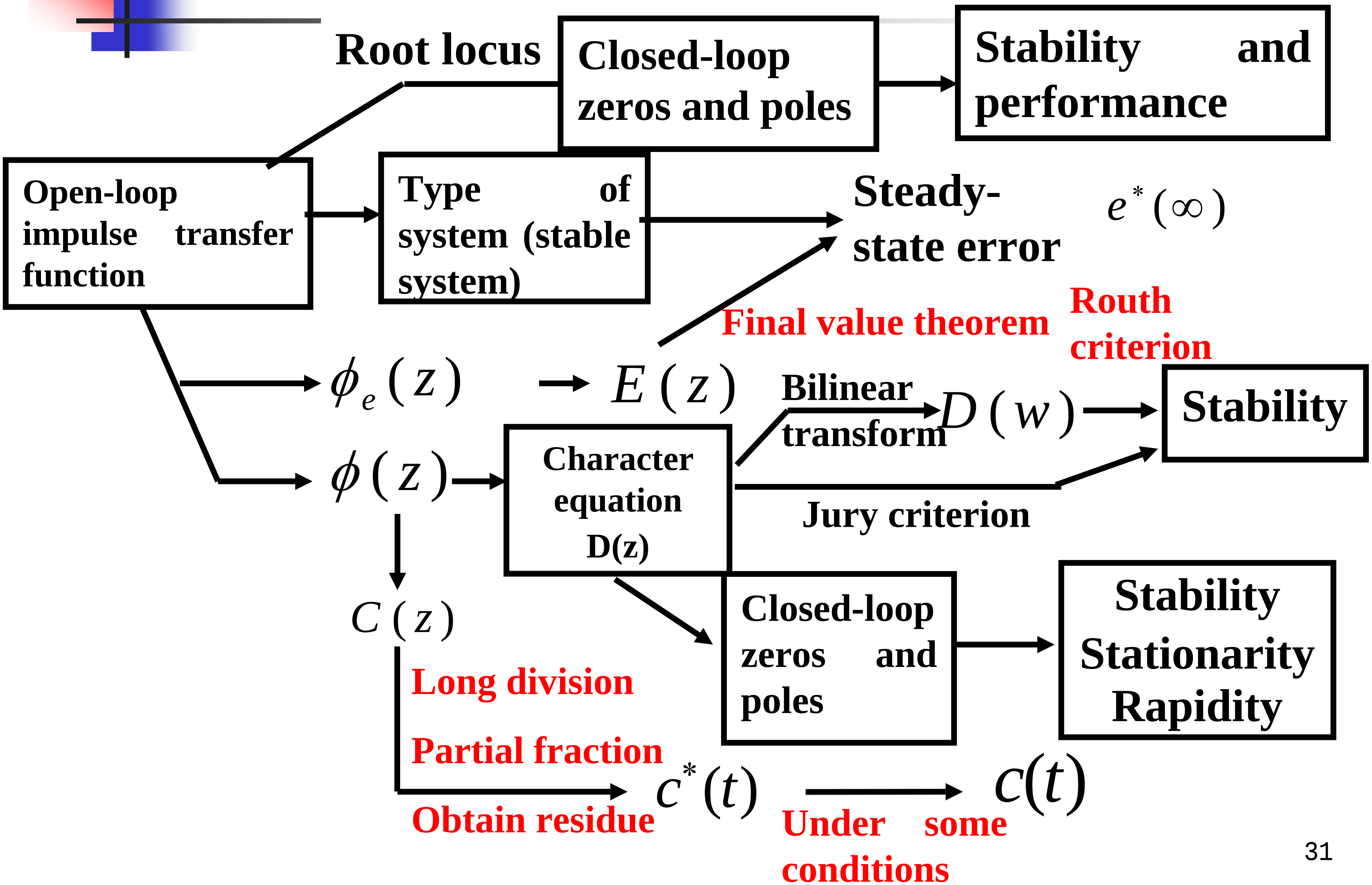
Under different, the steady-state errors are

Unit step input $$e_{ss} = \frac{1}{1+K_P} = 0$$

Unit ramp input $$e_{ss} = \frac{T}{K_V} = \frac{0.1}{1} = 0.1$$

Unit parabolic input $$e_{ss} = \frac{T^2}{K_a} = \infty$$

The key points and thread in this chapter



- **Thanks!**