《自动控制理论》
Automatic Control Theory

西华大学电气信息学院

Xihua University

School of Electrical and Information Engineering

王军

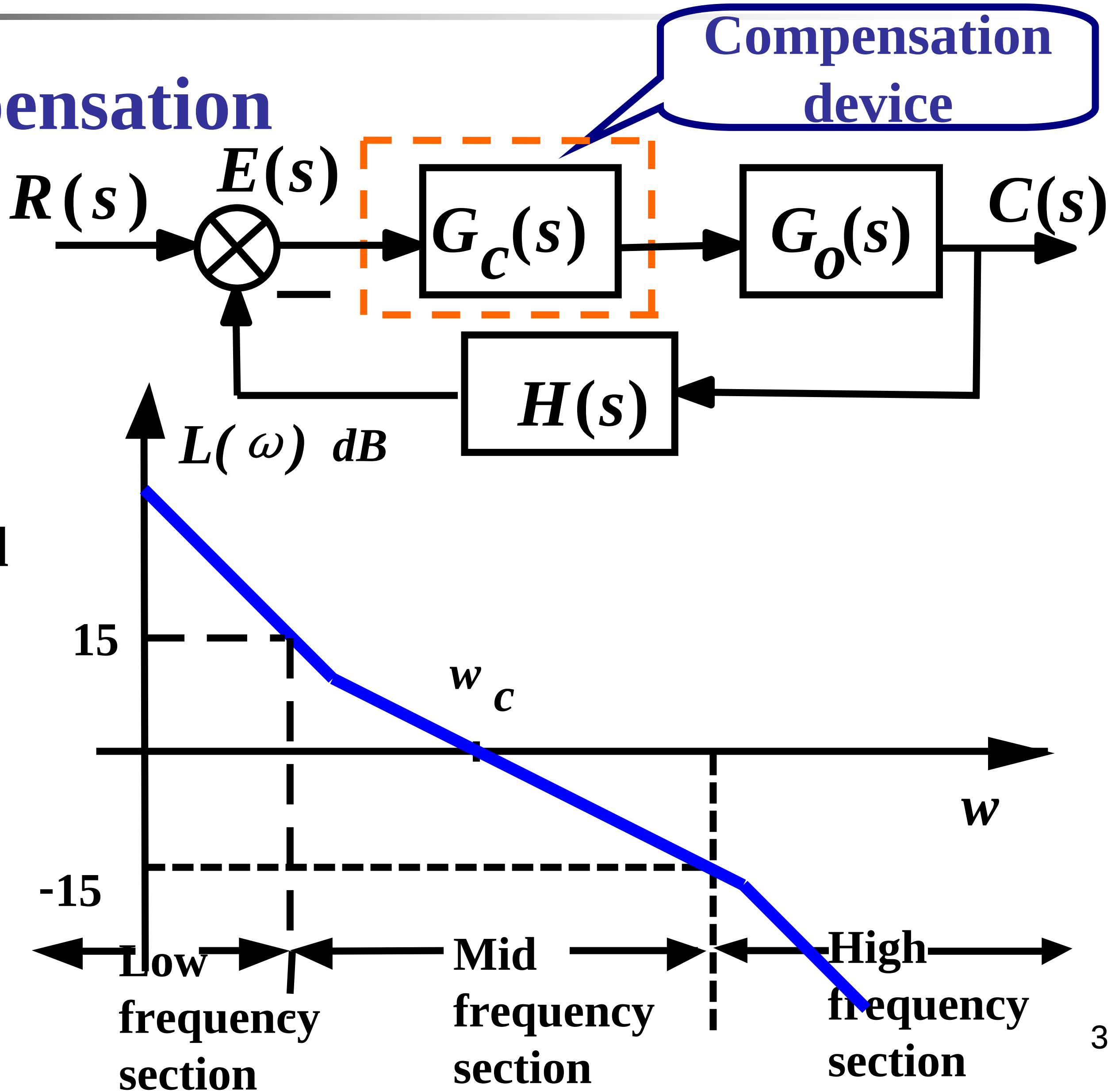
wangjun@mail.xhu.edu.cn

Reviews

➢ **1 Basic concepts**
➢ **2 Cascade phase-lead design**
➢ **3 Cascade phase-lag design**

Reviews

Cascade compensation

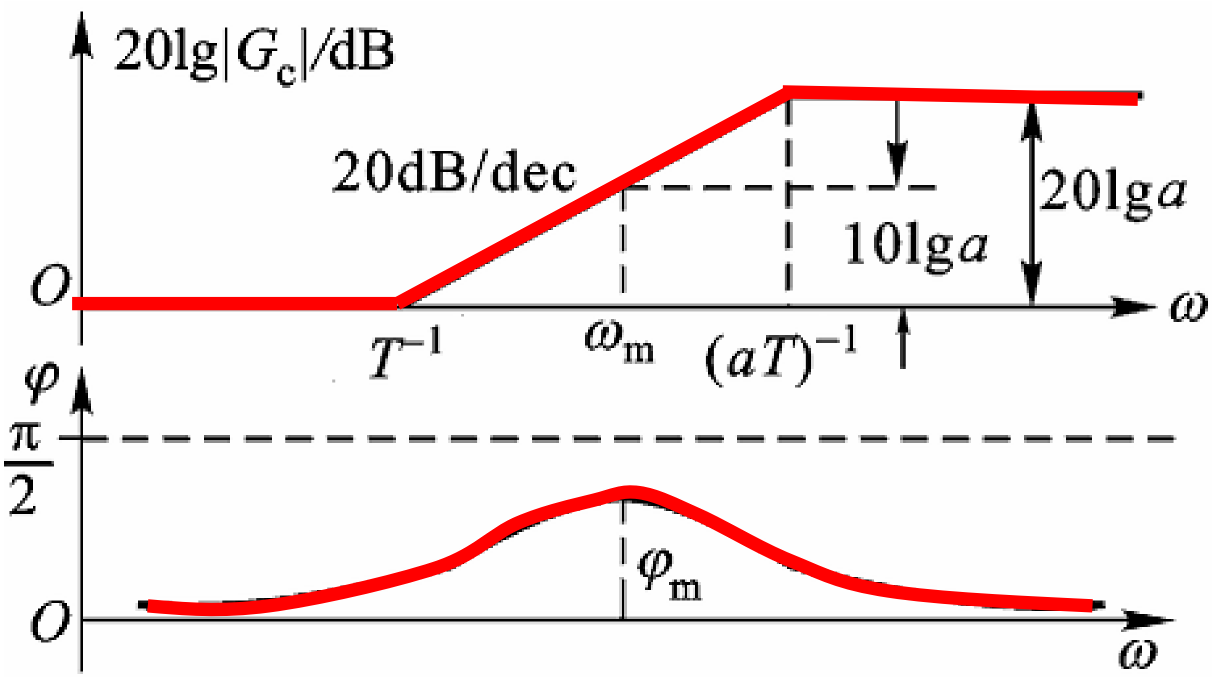


Frequency method

Reviews

Section 2: Cascade phase-lead compensation

$$G_c(s) = \frac{\beta Ts + 1}{Ts + 1} \qquad \beta > 1$$

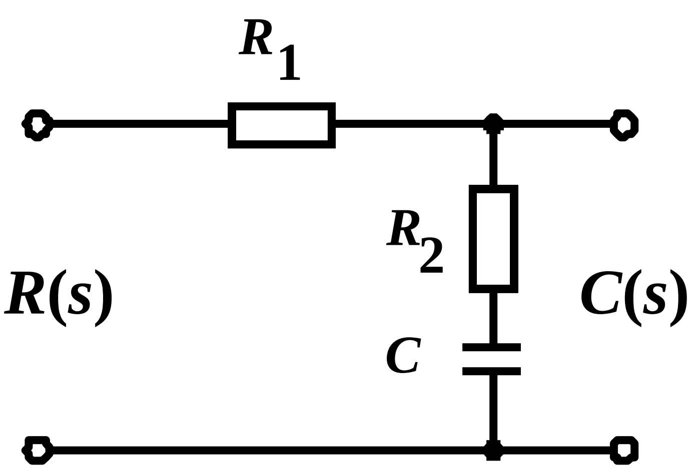


Section 3: Cascade phase-lag compensation

1. Phase-lag compensation device

Phase-lag compensation device is the compensation device with the phase lag character (with a phase frequency character $\varphi(\omega)<0$). It is called integral compensation device too.

Transfer function of passive phase-lag network is



$$G_c(s)=\frac{C(s)}{R(s)}=\frac{\alpha T_1 s+1}{T_1 s+1}$$

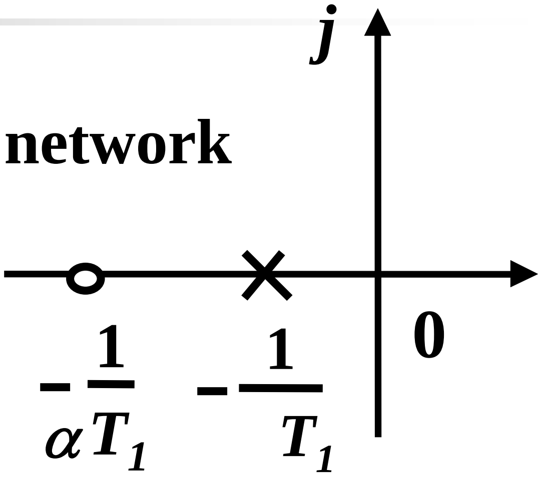
where

$$T_1=(R_1+R_2)C \qquad \alpha=\frac{R_2}{R_1+R_2}<1$$

$G_c(s)$ is the transfer function of phase-lag compensation device.

Section 3: Cascade phase-lag compensation

(1) Distribution of zero and pole in phase-lag network



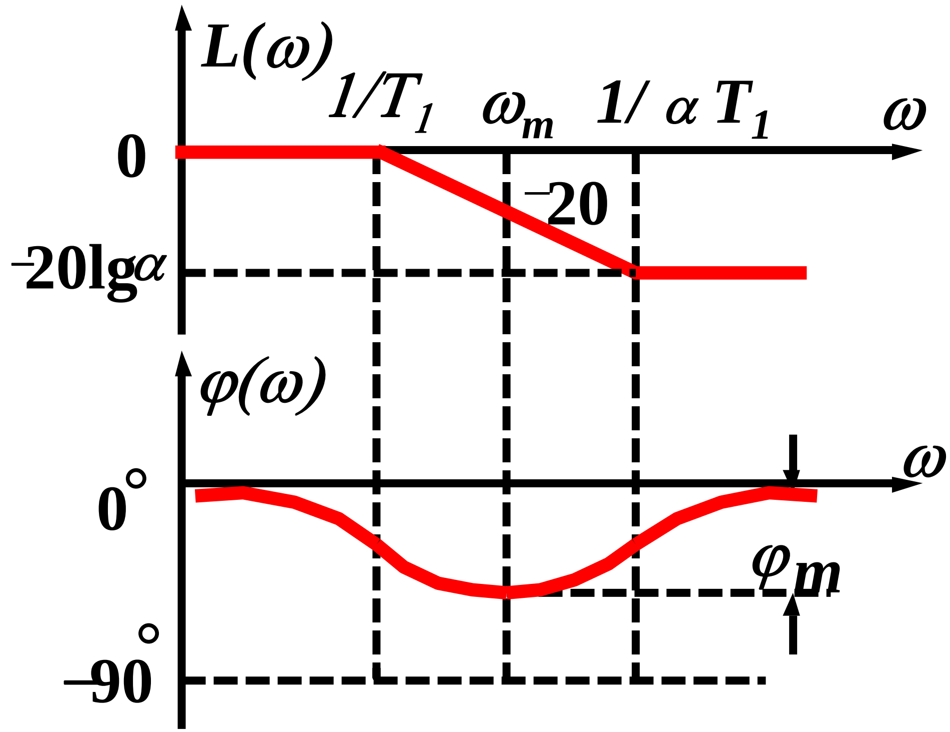
The zeros with negative real part of phase-lag network are on the right side of poles with negative real part. The distance are decided by constant β.

Section 3: Cascade phase-lag compensation

$$\varphi_m = \arcsin\frac{\alpha-1}{\alpha+1}$$

$$\alpha = \frac{1-\sin\varphi_m}{1+\sin\varphi_m}$$

character



1. Magnitude frequency character is small or equals to 0dB.

2. φ(ω) is small or equals to 0. Phase-angle is lag.

3. The max negative phase shift is the geometry midpoint of corner frequency $\frac{1}{\alpha T_1}$ **and** $\frac{1}{T_1}$ **.**

2. Steps of phase-lag compensation design

(1) Confirm open-loop gain *K* though requirement of steady-state error.

(2) Sketch Bode plot of original system though the open-loop gain *K*.

(3) Re-find a new cutoff frequency ω_c' if the γ and ω_c of original system is unsatisfied.

Method: The angle of this frequency is $\phi = -180^0 + \gamma' + \varepsilon$

γ' **is the desired phase margin.**

(4) Calculate α and *T* of phase-lag compensation device.

Calculate log magnitude of original system, then

$$20\lg\alpha = -L(\omega_c')$$ ⟹ **Calculate parameter α**

(5) Ensure the effect of phase-lag compensation device on mid frequency section of original system is small.

2. Steps of phase-lag compensation design

(5) Ensure the effect of phase-lag compensation device on mid frequency section of original system is small.

The max corner frequency of phase-lag compensation device is

$$\frac{1}{\alpha T_1} = (\frac{1}{5} \sim \frac{1}{10})\ \omega_c' \Rightarrow$$ **Calculate** T_1

The transfer function of compensation device is:

$$G_c(s) = \frac{1+\alpha T_1 s}{1+T_1 s}$$

(6) Validate the phase margin γ'' of compensation system.

Sketch Bode plot of compensation system, and validate whether the phase margin satisfy the requirement. If not, augments ε, then re-calculate from step 3.

Example: The open-loop transfer function of a given system is:

$$G_0(s) = \frac{K}{s(s+1)(0.5s+1)}$$

Design the system to make the steady-state speed error $K_V = 5s^{-1}$, $\gamma \geq 40^0$.

(1) Confirm open-loop gain K though desired steady-state error. Sketch Bode plot of original system, then obtain the phase margin and gain margin.

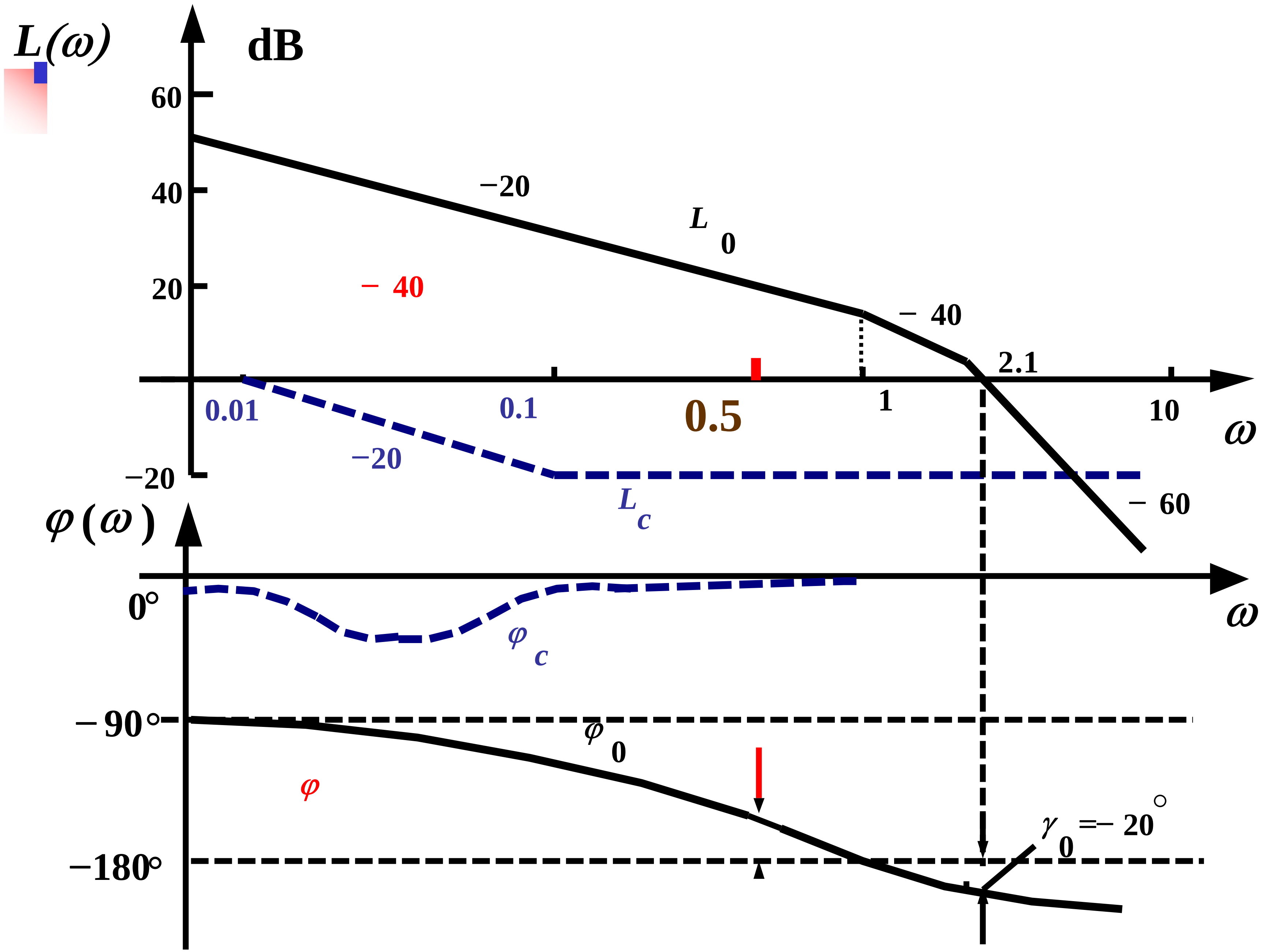
Confirm K. For

$$K_v = \lim_{s\to 0} sG_0(s) = \lim_{s\to 0} \frac{sK}{s(s+1)(0.5s+1)} = K$$

$$K_v = K = 5$$

The solution see blackboard

Sketch Bode plot of original system. Obtain phase margin of original system is $\gamma_0 = -20^0$, the system is unstable.



L(ω)
dB
60
40
20
−20
−20
L 0
− 40
− 40
2.1
0.01
0.1
0.5
1
10
ω
−20
L c
− 60
φ (ω)
ω
0°
φ c
− 90°
φ 0
φ
γ 0 =− 20°
−180°

(2) Confirm cutoff frequency ω_c of compensation system

The requirement phase margin should add (5^0~12^0) on the desired phase margin --- to compensate the phase-lag of compensation network on ω_c.

Confirm ω_c .

The desired phase of compensation system is $\gamma \geq 40^0$. To compensate the phase lag og compensation network, we should add 5^0~12^0

$$\phi = -180^0 + \gamma + \varepsilon = -180^0 + 40^0 + 12^0 = -128^0$$

Obtain from Bode plot, when ω=0.5 (s^{-1}), the phase angle is -128 ^{0}. So, the gain cutoff frequency of compensation system is:

$$\omega_c = 0.5\ (s^{-1})$$

(3) Obtain α

Solution see blackboard

The magnitude frequency gain of original system is 20dB at ω_c=5. To ensure the attenuation equals -20dB at the gain cutoff frequency at ω_c,

$$-20 = 20\lg\alpha \qquad \alpha = 0.1$$

(4) Obtain T. in order to make the effect of phase lag of compensation system small enough, generally

Let

$$\omega_c'' = (5 \sim 10)\frac{1}{T_1}$$

$$\frac{1}{\alpha T_1} = \frac{1}{5}\omega_c'' = 0.1\mathrm{s}^{-1} \qquad \frac{1}{T_1} = 0.01\mathrm{s}^{-1}$$

Section 3: Cascade phase-lag compensation

(5) Confirm the transfer function of phase-lag compensation device.

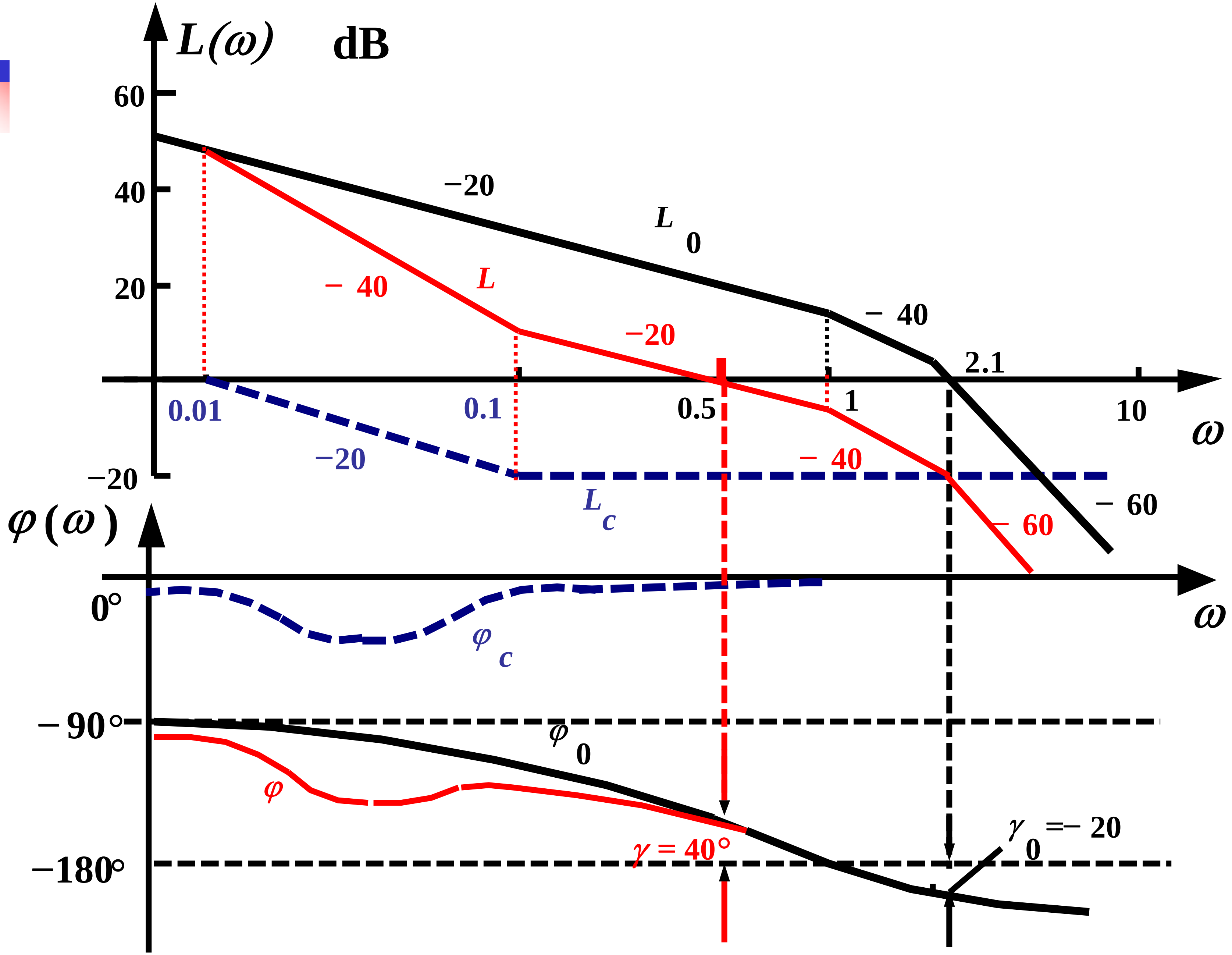
$$G_c(s) = \frac{10s+1}{100s+1} = \frac{1}{10} \times \frac{s+0.1}{s+0.01}$$

The open-loop transfer function of compensation system is

$$G(s) = G_0(s) \cdot G_c(s) = \frac{5(10s+1)}{s(100s+1)(s+1)(0.5s+1)}$$

- **(6) Validation**

Sketch Bode pole of compensation system, obtain $\gamma = 40^0$, K_V=5. Satisfy the requirements.



L(ω) dB
60
40
20
−20
−20
L0
−40
L
−40
−20
−40
2.1
0.01
0.1
0.5
1
10
ω
−20
−40
Lc
−60
−60
φ(ω)
ω
0°
φc
−90°
φ0
φ
−180°
γ = 40°
γ0 = −20

Obtain from analysis of phase-lag compensation:

Under the analysis: in phase-lag compensation, the attenuation character of phase-lag network in high frequency section is adopted but the character of phase lag. After phase-lag compensation:

① Improve the steady-state performance of system

Phase-lag network is a low-pass filter, increases the gain of low frequency signal and decreases steady-state error. It also decrease the gain of high frequency signal, shifts the cutoff frequency to lower frequency and ensure the stability of system.

② Decrease the response speed

The frequency bandwidth become narrow after adopting phase-lag compensation device, and increase the transient response time.

Differences and relations of phase-lead and –lag compensation

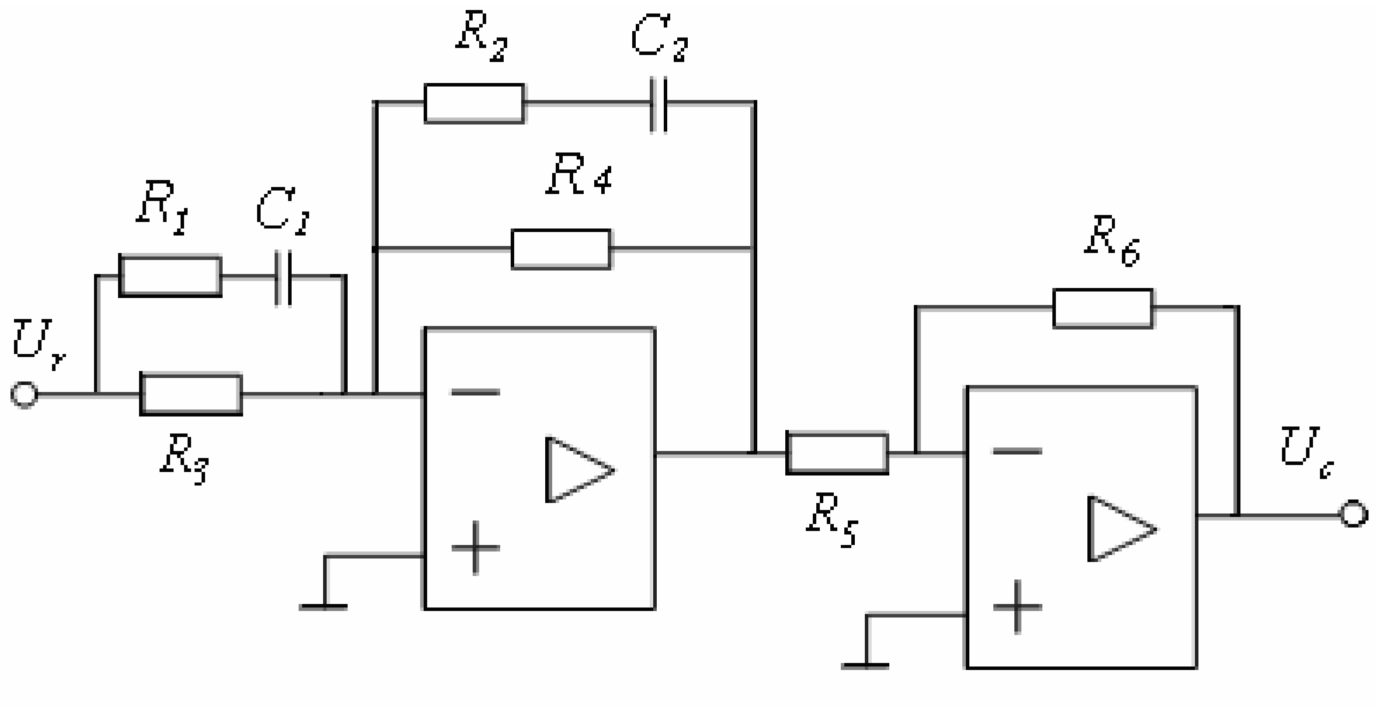
	Phase-lead compensation	Phase-lag compensation
Principle	Adopt phase-lead character to improve the transient performance of system.	Adopt attenuation character of high frequency to improve system steady-state performance.
Effect	(1) Slope of log magnitude character of original system becomes small at ω_c, phase margin γ and magnitude margin K_g become large. (2) Frequency bandwidth become wide. (3) Percent overshoot decreases for increase of γ . (4) Do not affect steady-state character of system, e_{ss} keeps unchanged.	(1) Improve steady-state precision under unchanged relative stability. (2) Gain cutoff frequency ω_c and closed-loop bandwidth decrease. (3) For the given open-loop amplifier coefficient and the magnitude attenuation at ω_c, γ , K_g and resonance peak value M_r are improved.

Differences and relations of phase-lead and -lag compensation

	Phase-lead compensation	Phase-lag compensation
Defect	(1) Widen frequency bandwidth and decrease the ability of high frequency section anti-jamming (2) An amplifier is adopted to compensate the magnitude attenuation of compensation device.	Narrow frequency bandwidth increase the transient response time.
Applications	(1) Near ω_c phase lag of original system changed slowly. The phase-lead would be less than 55^0 for single phase-lead compensation but for multiple cascade phase-lead compensations. (2) Require wide frequency bandwidth and fast transient response. (3) High frequency jam is not the main problem.	(1) Near ω_c phase change of original system is rapidly, and it is difficult to adopt cascade phase-lead compensation. (2) Fit for low requirements of frequency bandwidth and transient response. (3) Requirement on high frequency anti-jamming. (4) Desired phase margin could be found in low frequency section.

Section 3: Cascade phase-lag and -lead compensation

1. Phase-lag and -lead compensation device



$$G_c(s) = K_c \frac{(T_1 s+1)(T_2 s+1)}{(\beta T_1 s+1)(\frac{T_2}{\beta} s+1)}$$

其中： $T_1 = R_1 C_1, T_2 = R_2 C_2,$

$$\beta = \frac{R_2 + R_4}{R_2} > 1$$

Section 3: Cascade phase-lag and -lead compensation

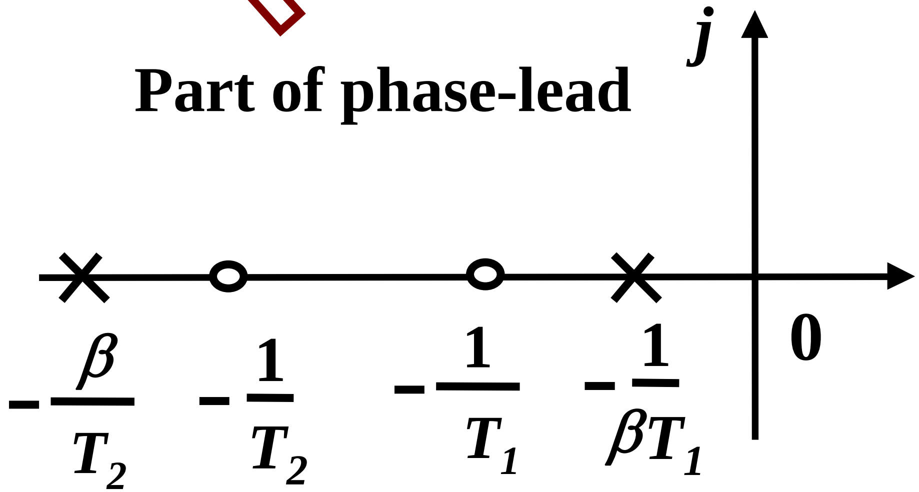
$$G_c(s) = \frac{(T_1 s + 1)(T_2 s + 1)}{(\beta T_1 + 1)(\frac{T_2}{\beta} s + 1)}$$

$T_1 > T_2, \beta > 1$

Part of phase-lag

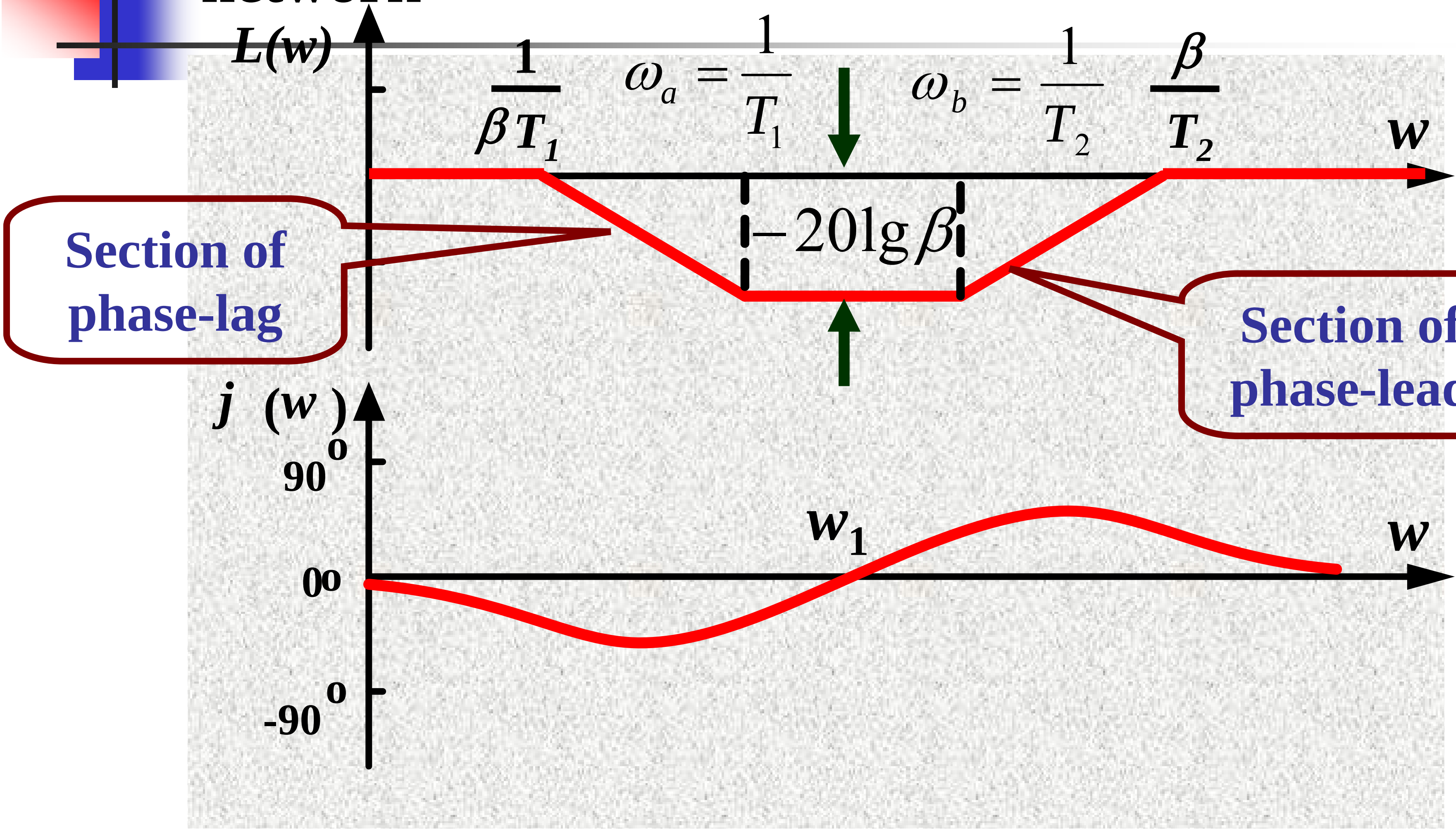
Part of phase-lead

(1) Distribution of zero and pole of phase-lag and –lead network



Cascade phase-lag and –lead compensation combined the character of phase-lag and phase-lead compensations. Adopt part of phase-lead to increase phase margin of system to improve its transient performance; adopt part of phase-lag to improve steady-state performance.

(2) Bode plot of phase-lag and -lead network



$$\omega_1 = \frac{1}{\sqrt{T_1 T_2}}$$

2. Steps of cascade phase-lag and –lead compensation device

(1) According requirement of steady-state performance, confirm open-loop gain K.

(2) Sketch Bode plot of original system, obtain cutoff frequency ω_c, phase margin γ and magnitude margin $h(dB)$.

(3) Select corner frequency on Bode plot of original system, where slope change from -20dB/dec to -40dB/dec as the corner frequency of part of phase-lead network.

It could reduce the order of compensation system, to ensure the slope equals -20dB/dec at mid frequency section and take a wider frequency bandwidth.

(4) According required response speed t_s, select cutoff frequency ω_c'' for system.

and attenuation coefficient of compensation network $\frac{1}{\beta}$

To ensure the selected cutoff frequency ω_c'' of compensation system satisfy the following equation:

$$-20\lg\beta + L'(\omega_c'') + 20\lg T_2\omega_c'' = 0 \Longrightarrow \textbf{Obtain } \beta$$

Max magnitude attenuation of phase-lag and-lead network

Magnitude of original system

Magnitude of part of phase-lead of phase-lag and -lead network

(5) According requirement of phase margin to calculate corner frequency $\omega_b = \frac{1}{T_2}$ of part of phase-lag of compensation network.

(6) Validate performance of compensation open-loop system.

Example: The open-loop transfer of a given unit feedback system

$$G_0(s) = \frac{K}{s(s+1)(0.125s+1)}$$

Requirements: K_v = 20(1 / s), phase margin γ = 50^0, cutoff frequency $\omega_c' \geqslant 2$. Design a phase-lag and -lead compensation device to satisfy the requirements.

(1) According requirement of steady-state performance, confirm open-loop gain K.

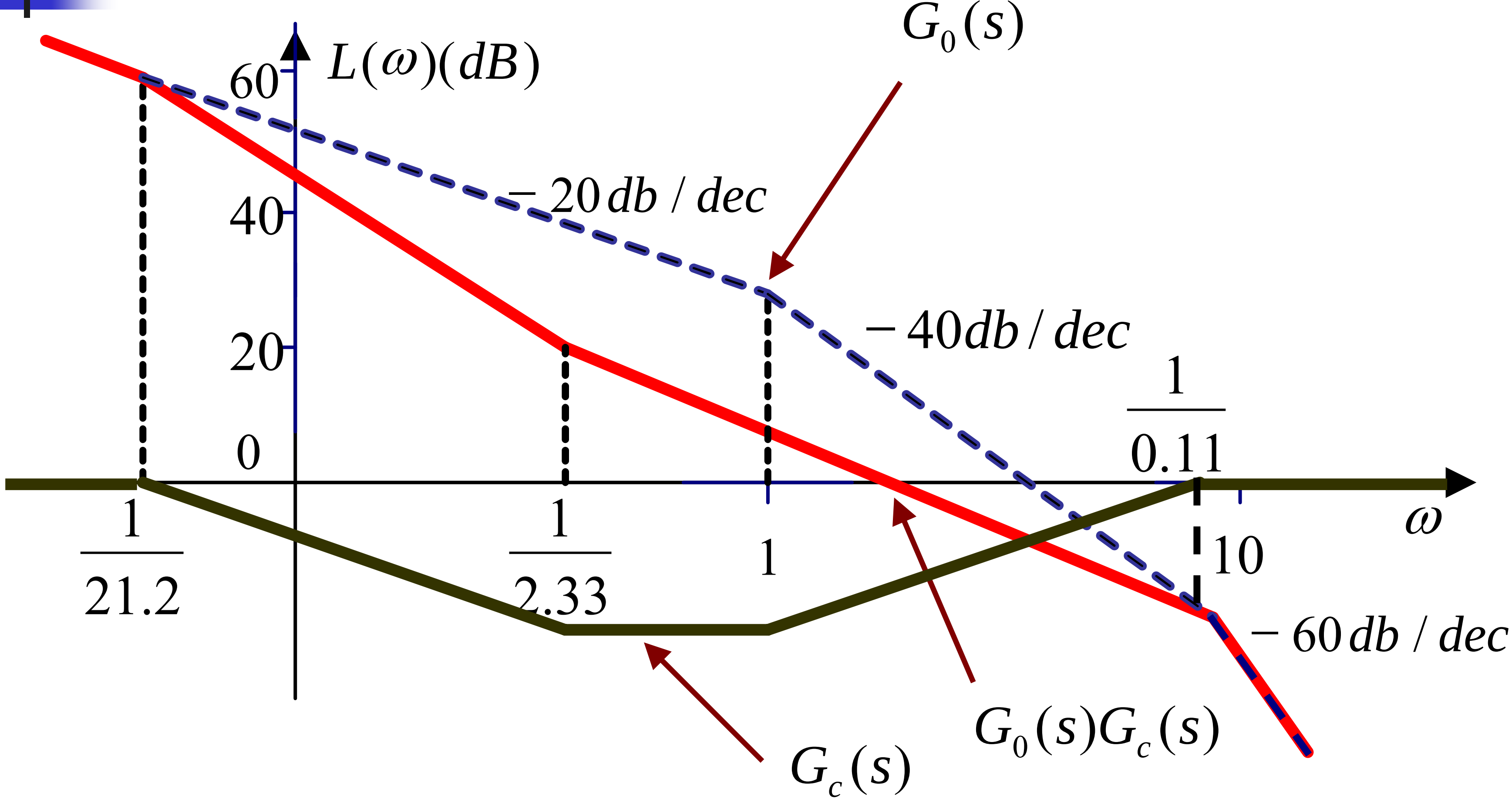
$$K_v = \lim_{s\to 0} sG_0(s) = K = 20$$

(2) Sketch Bode plot of original system. Blue line in figure.

Cutoff frequency of original system $\omega_c = 4.47(rad/s)$

Phase margin is $\gamma = -16.6^0$

The system is unstable.



$G_0(s)$
$L(\omega)(dB)$
60
40
20
0
$-20db/dec$
$-40db/dec$
$-60db/dec$
$\frac{1}{0.11}$
$\frac{1}{21.2}$
$\frac{1}{2.33}$
1
10
ω
$G_0(s)G_c(s)$
$G_c(s)$

(3) Select $\omega_b = \frac{1}{T_2} = 1$ **as the corner frequency of part of phase-lead of compensation device. According the requirements of phase margin and cutoff frequency of compensation system, confirm the cutoff frequency 2.2(*rad/s*). The magnitude of original system at frequency 2.2(*rad/s*) is 12.32*(dB)*, the magnitude of cascade compensation device at frequency 2.2 (rad/s) is 0*dB*. So**

$$-20\lg\beta + 20\lg 2.2 + 12.32 = 0$$

Obtain

$$\beta = 9.1, \frac{T_2}{\beta} = 0.11$$

(4) Obtain T_1

Transfer function of phase-lag and -lead compensation device is:

$$G_c(s)=\frac{(T_1 s+1)(T_2 s+1)}{(\beta T_1 s+1)(\frac{T_2}{\beta}s+1)}=\frac{(\frac{1}{\omega_a}s+1)(s+1)}{(\frac{9.1}{\omega_a}s+1)(0.11s+1)}$$

The open-loop transfer function of compensated system:

$$G_c(s)G_0(s)=\frac{20(\frac{1}{\omega_a}s+1)}{s(0.125s+1)(\frac{9.1}{\omega_a}s+1)(0.11s+1)}$$

(5) Obtain phase margin according requirement of system performances

$$\gamma = 50^0$$

where

$$\gamma = 180^0 + \arctan\frac{\omega_c}{\omega_a} - 90^0 + \arctan 0.125\omega_c - \arctan\frac{\beta\omega_c}{\omega_a} - \arctan 0.11\omega_c$$

$$= 61.01^0 + \arctan\frac{2.2}{\omega_a} - \arctan\frac{19.11}{\omega_a} = 50^0$$

where

$$-\arctan\frac{19.11}{\omega_a} \approx -90^0$$

$$\arctan\frac{2.2}{\omega_a} = 78.99^0 \quad \Rightarrow \quad \omega_a = \frac{1}{T_1} = 0.43(\text{rad/s})$$

(6) Transfer function of compensated device

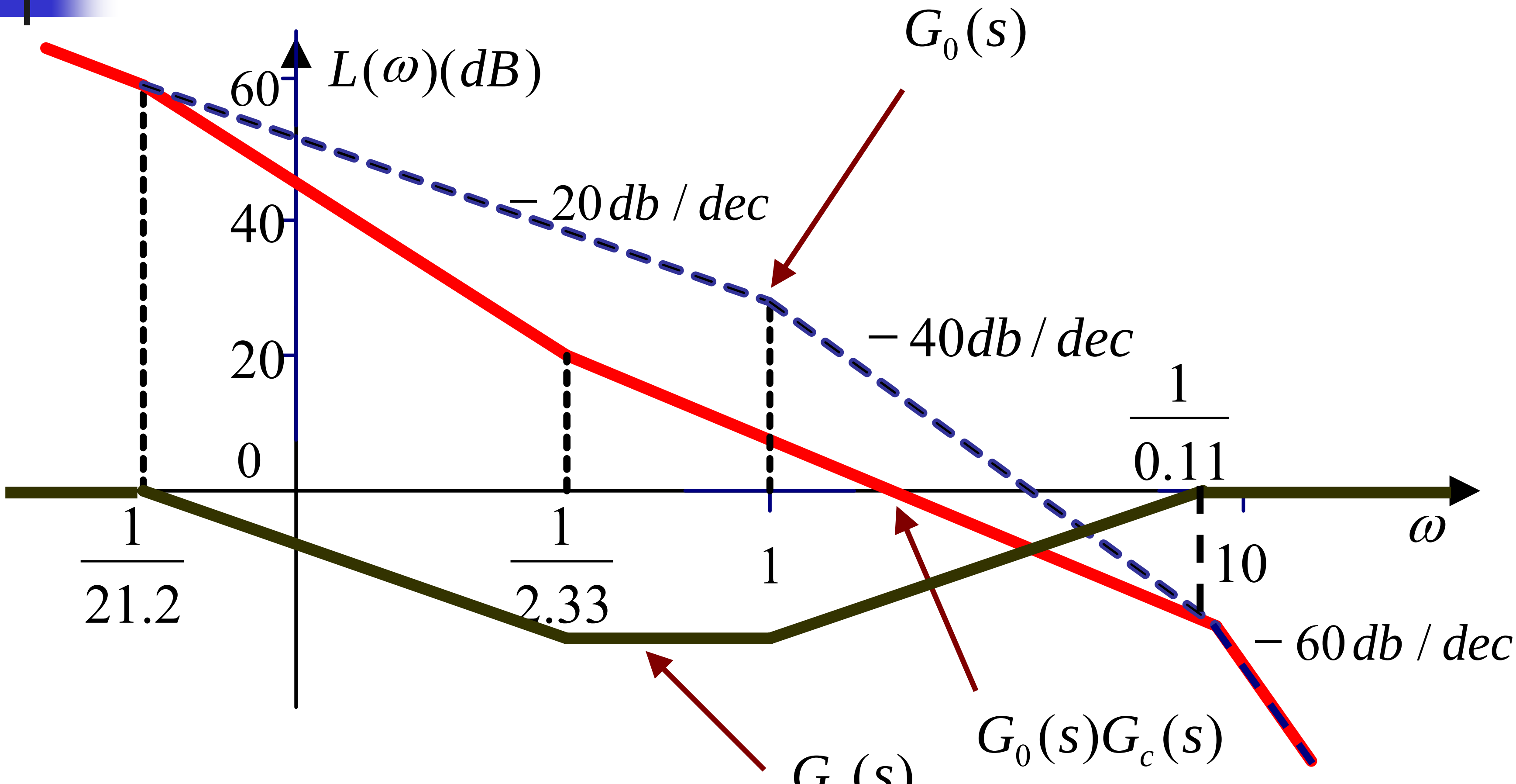
$$G_c(s) = \frac{(2.33s+1)(s+1)}{(21.2s+1)(0.11s+1)}$$

open-loop transfer function of compensated system

$$G_c(s)G_k(s) = \frac{20(2.33s+1)}{s(0.125s+1)(21.2s+1)(0.11s+1)}$$

Bode plot of compensated system is the real line in figure. The phase margin of compensated system is $\gamma = 51.21^0$

Cutoff frequency is 2.2(rad / s), satisfies the requirements of transient and steady-state performance.



$G_0(s)$
$L(\omega)(dB)$
60
40
20
0
$-20db/dec$
$-40db/dec$
$\frac{1}{0.11}$
ω
$\frac{1}{21.2}$
$\frac{1}{2.33}$
1
10
$-60db/dec$
$G_0(s)G_c(s)$
$G_c(s)$

Section 5: Compensation of desired character

Desired character method is based on desired performance indexes, and the confirm a kind of compensation method to obtain a desired open-loop log magnitude frequency character. Then compare the magnitude frequency character with original system, confirm the type of compensation device and log magnitude character, and obtain the parameters.

Section 5: Compensation of desired character

Steps of "desired character":

(1) Sketch Bode plot magnitude character of original system.

(2) Sketch desired character of compensated system.

(3) Obtain compensation device though difference of desired character and original character.

How to obtain desired character?

Sketch desired character curve of system

(1) Sketch low frequency character of desired character though confirmed type and open-loop gain *k*.

(2) According desired system response speed and damp ratio, though desired cutoff frequency ω_c, phase margin γ, mid frequency bandwidth *h* and upper and lower frequency limits ω_a and ω_b. Sketching of character in mid frequency section is to ensure a enough phase margin. Generally, adopts -20dB/dec as the slope in mid frequency section.

(3) Sketch desired character in low and mid frequency section, the slope is general -40dB/dec.

(4) Sketch high frequency section though requirements of system magnitude margin and ability of high frequency anti-jamming. In order to simplify the compensation devices, high frequency character and unchangeable high frequency character of system are superposition or have the same slope.

Example: Open-loop transfer function of given unit feedback system is

$$G_0(S)=\frac{K}{s(0.9s+1)(0.007s+1)}$$

Adopt a cascade compensation device G_c(s) to satisfy the following performance indexes: (1) system is type I system, steady-state error is $K_v \geqslant 1000$, (2) settle time $t_s \leqslant 0.25$(s), percent overshoot $Mp \leqslant 30\%$.

(1) Sketch open-loop log approximate magnitude character.

It is a type I system, let $K=K_v=1000$

The slope of original system is -40dB/dec when cross zero dB line. The phase margin is negative. The system is unstable.

(2) Sketch desired character though transient performance requirement:

$$\sigma_P\%=[0.16+0.4(M_r-1)]100\%$$

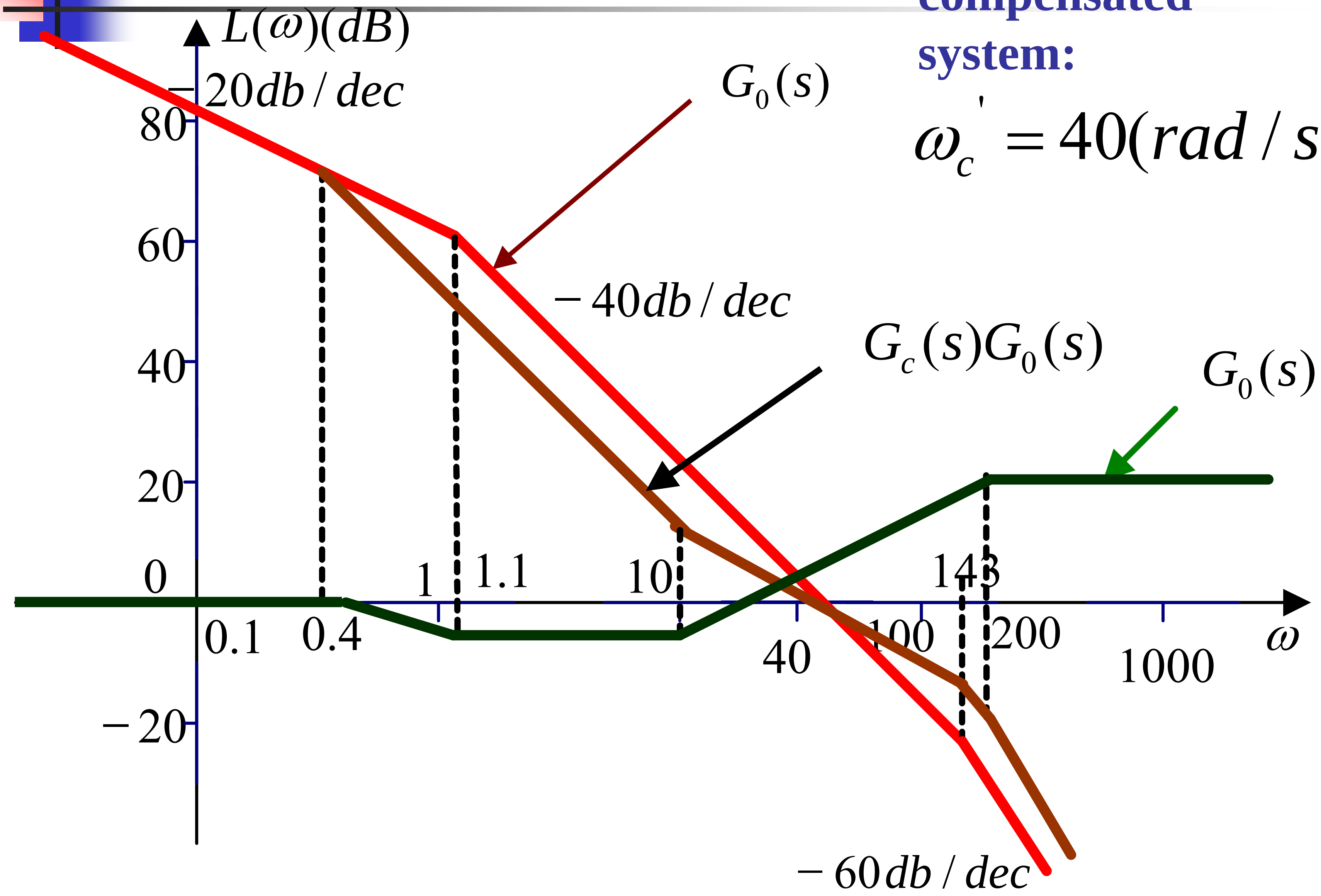
$$t_s=\frac{k\pi}{\omega_c}$$

$$k=2+1.5(M_r-1)+2.5(M_r-1)^2$$

Get $\omega_c = 35.56(rad/s)$

Open-loop cutoff frequency of compensated system:

$\omega_c^{'} = 40(rad/s)$



In order to ensure a enough phase margin of compensated system (to satisfy the requirements of transient performance indexes), near the cutoff frequency ω_c' , the slope should be -20dB/dec and with a proper frequency bandwidth. Then high frequency section have the same slope with original system character.

(3) The slope is -20dB/dec at $\omega_c = 40$(rad/s)

$$\omega_a = \frac{\omega_c'}{2 \sim 5} \quad \textbf{And} \quad \omega_b = (2 \sim 5)\omega_c' \quad \textbf{Select } \omega_c'$$

The corner frequency are ω_a and ω_b, to ensure a proper frequency bandwidth at slope -20dB/dec.

(4) When $\omega_b = \dfrac{1}{0.007}$ =143(*rad/s*), desired character slope changes from -20dB/dec to -40dB/dec; when ω =200(*rad/s*), desired character changes from -40dB/dec to -60dB/dec. The slope desired character of high frequency section is -60dB/dec.

(5) The slope of desired character at ω_a =10(*rad/s*) changes from $-20dB/dec$ to $-40dB/dec$.

The desired character would cross with low frequency character of original system at ω ＝0.4(*rad/s*).

(6) In the frequency section lower than cross point ω =0.4(*rad/s*), make the desired character superposition with original system character.

Validate the performance index:

$K = K_v = 1000$ $\quad \omega_c' = 40$ $\quad \gamma = 49.59^0$

$\boldsymbol{Mp} = 28.5\%$ $\quad t_s = 0.123(s)$

The transfer function of open-loop log approximate magnitude character of compensated system is:

$$G_c(S)G_0(s) = \frac{1000(0.1s+1)}{s(2.5s+1)(0.007s+1)(0.005s+1)}$$

(7) Confirm compensated device.

$$G_c(S) = \frac{(0.9s+1)(0.1s+1)}{(2.5s+1)(0.005s+1)}$$

- **Thanks!**