# 《自动控制理论》
# Automatic Control Theory

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# Reviews

- 1 Drawing open-loop Bode plot
- 2 Nyquist stability criterion
- 3 Relative stability of control system

# Section 4: Drawing open-loop Bode plot

**Example A given open-loop transfer function is**

$$G(s)H(s)=\frac{K}{s(T_1s+1)(T_2s+1)} \qquad (T_1>T_2)$$

**It is series-wound with a amplifier element, a integral element and two inertia element. The corresponding frequency character is**

$$G(j\omega)H(j\omega)=\frac{K}{j\omega(T_1\omega+1)(jT_2\omega+1)}$$

**Magnitude and phase character are**

$$\left|G(j\omega)H(j\omega)\right|=\frac{K}{(j\omega)\sqrt{(jT_1\omega)^2+1}\sqrt{(jT_2\omega)^2+1}}$$

$$\angle G(j\omega)H(j\omega)=-90^0-arctgT_1\omega-arctgT_2\omega$$

# Section 4: Drawing open-loop Bode plot

**1) Open-loop log magnitude frequency character**

$$L(\omega)=L_1(\omega)+L_1(\omega)+L_1(\omega)+L_1(\omega)$$

$$=20\lg K-20\lg\omega-20\lg\sqrt{(T_1\omega)^2+1}-20\lg\sqrt{(T_2\omega)^2+1}$$

$$=L_1(\omega)+L_2(\omega)+L_3(\omega)+L_4(\omega)$$

**Open-loop log phase frequency character**

$$\varphi(\omega)=\varphi_1(\omega)+\varphi_2(\omega)+\varphi_3(\omega)+\varphi_4(\omega)$$

$$=0^0-90^0-\tan^{-1}1/\sqrt{(T_1\omega)^2+1}-\tan^{-1}1/\sqrt{(T_2\omega)^2+1}$$

**2) Draw open-loop log magnitude and phase frequency character. (See Blackboard)**

# 顺序斜率迭加法

## Steps:

**① Analyze the components of system, transfer the transfer function of typical element into standard forms.**
**The constants in transfer function of typical element are 1.**

**② Calculate $20\lg K$ though proportion element.**

**③ Find the point A with abscissa equals $\omega = 1$ ,and ordinate equals $L(\omega)\big|_{\omega=1} = 20\lg K$. Sketch a beeline though point A with the slope equals -20vdB/dec. v is the number of integral element.**

## Section 4: Drawing open-loop Bode plot

**④ Calculate the angle frequency of typical element, and arrange them from low to high corroding angle frequency. Change the slope corroding following principle:**

**Shift -20dB/dec when pass a first-order inertia element;**

**Shift 20dB/dec when pass proportion derivativeelement;**

**Shift -40dB/dec when pass a second-order vibrate element.**

**⑤ Revise the approximate if needed.**

# 4. Minimum and non-minimum phase system

**(1) When these is not poles and zeros on the right side of s-plane in open-loop transfer function, it is a minimum phase system.**

$$G_1(s) = \frac{K(T_1 s + 1)}{s(T_2 s + 1)(T_3 s + 1)}$$

**(2) When these are some poles or zeros on the right side of s-plane in open-loop transfer function, or contains $e^{-Ts}$, it is a non-minimum phase system.**

$$G_2(s) = \frac{K(T_1 s - 1)}{s(T_2 s + 1)(T_3 s + 1)}$$

**Generally, a non-minimum phase system is generated from two reasons: these are some non-minimum phase components contained in system, or some unstable loops.**

## (3) Minimum phase system has the minimum phase in two systems with the same magnitude.

$G_1(s)$ **and** $G_2(s)$ **have same magnitude frequency character.**

$$A(\omega) = \frac{K\sqrt{1+T_3^2\omega^2}}{\sqrt{(1+T_1^2\omega^2)(1+T_2^2\omega^2)}}$$

**Their phase frequency characters are different**

$$\varphi_1(\omega) = \arctan T_3\omega - \arctan T_1\omega - \arctan T_2\omega$$

$$\varphi_2(\omega) = \arctan \frac{T_3\omega}{-1} - \arctan T_1\omega - \arctan T_2\omega$$

**When** $\omega : 0 \to \infty$

$$\left|\varphi_2(\infty) - \varphi_2(0)\right| = \left|-90^0 - 180^0\right| = 270^0$$

**Obviously, the variety of phase in minimum phase system is minimum.**

- When $\omega = \infty$, the phase angle is -90° (n-m), the slope of log frequency character is –20(n–m)dB/dec. Minimum phase system could be determinant though this method.
- A certain relationship is between log magnitude and phase frequency character. For a minimum phase system, the phase frequency character could be confirmed when the magnitude frequency character is given.
- Non-minimum phase system has a large lag of phase angle in high frequency. The transient performance is pool and the response speed is slow.

# Section 5: Nyquist stability criterion

**1. Basic concepts: Judge stability of closed-loop system though open-loop frequency character**

**Nyquist criterion:**

$$Z = P + N$$

**P ——The number of poles of G(s)H(s) on right side of s-plane;**

**N ——When ω varies from -∞ to +∞, the time of G(jω)H(jω) circles (-1， j0) on clockwise;**

**Z ——The number of poles of closed-loop system on right side of s-plane.**

**The necessary and sufficient condition of stabile closed-loop system is: Z=0**

# Section 5: Nyquist stability criterion

**N’: When ω varies from 0 to +∞, the time of $G(j\omega)H(j\omega)$ circles (-1，j0).**

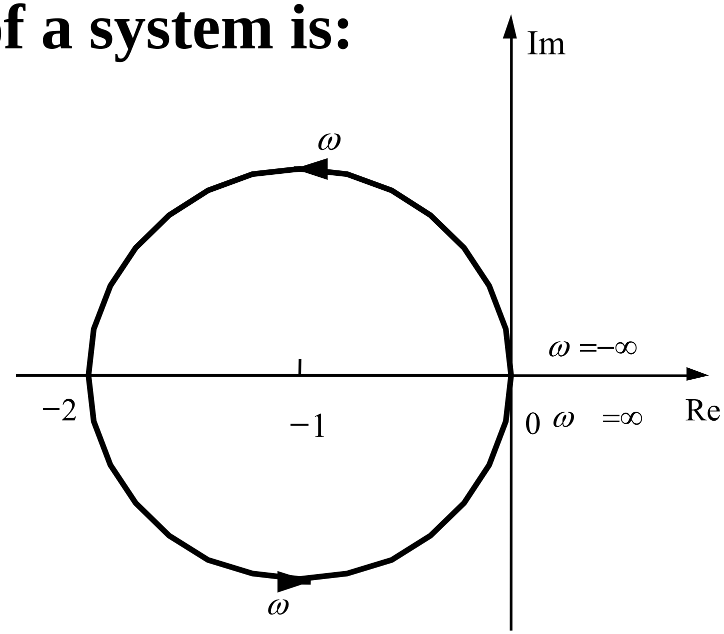
$$Z = P + 2N' = P + N$$

**If these is no poles of open-loop transfer function on the right side on s-plane, *P* = 0, the necessary and sufficient condition of stable closed-loop system is *N*=0.**

# Section 5: Nyquist stability criterion

**Example: Open-loop transfer function of a system is:**

$$G(s)H(s) = \frac{a}{s-1} \qquad (a > 0)$$




**Obtain the stability of system.**

**Open-loop frequency character is**

$$G(j\omega)H(j\omega) = \frac{a}{j\omega - 1}$$

**when** $\omega = -j\infty \to -j0 \to +j0 \to +j\infty$

**The magnitude and phase curve is shown:**

**For these is a open-loop pole on right side of s-plane, P=1.**

**The Nuquist curve circles (-1, j0) one time on counter-clockwise, N=-1.**

**From Nyquist criterion, the number of closed-loop system poles on right side of s-plane is Z=P+N=1-1=0. So the system is stable.**

# Section 5: Nyquist stability criterion

## 2. Integral element existed in open-loop transfer function

**Assistant curve: On s-plane, draw an arc with origin as the center and a infinite radius. The radius begins from the point with angle $v \times 90^0$, though $0^o$ clockwisely, ends at the point with angle $-v \times 90^\circ$ .**

# Example judge the stability of system

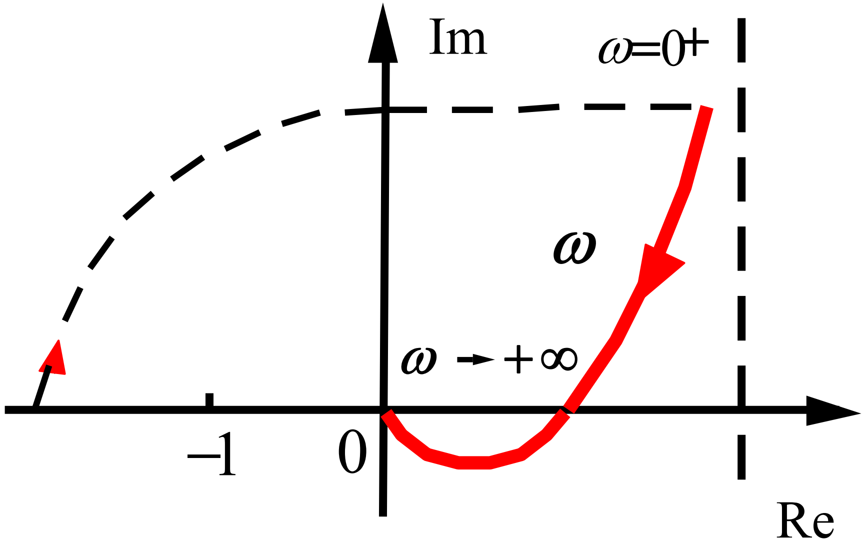
$$G(s)H(s) = \frac{K_1(s-1)}{s(s+1)} \qquad K_1 > 0$$

**Draw a curve from +*j 0* to +*j ∞*. Based on symmetry, draw *G*(*j ω*)*H*(*j ω*) curve from -*j 0* to -*j ∞*.**

$$G(j\omega)H(j\omega) = \frac{K_1(j\omega-1)}{j\omega(j\omega+1)}$$

**From figure, the Nyquist curve of *G*(*s*)*H*(*s*) circles (-1, j0) one time clockwisely, *N* =1.**
**For *P* =0,**

$$Z = P + N = 1$$




**It is a unstable system with a closed-loop pole on right side of s-plane.**

# 3 A kind of simple Nyquist stability criterion

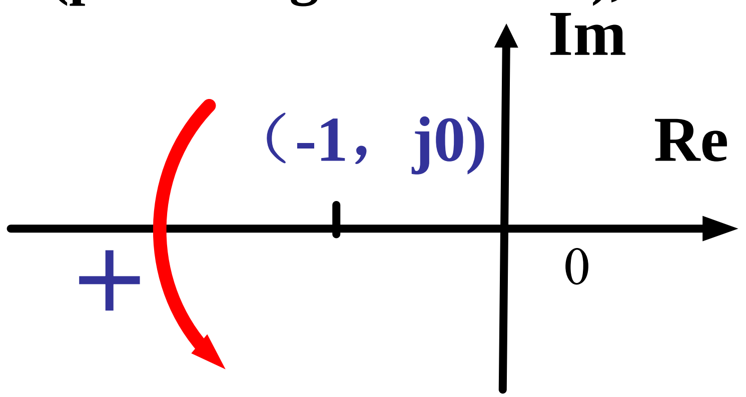
## (1) Concepts of positive and negative cross

**$G(j\omega)H(j\omega)$ curve is symmetrical with real axis. Draw $\omega = 0 \to \infty$ only.**

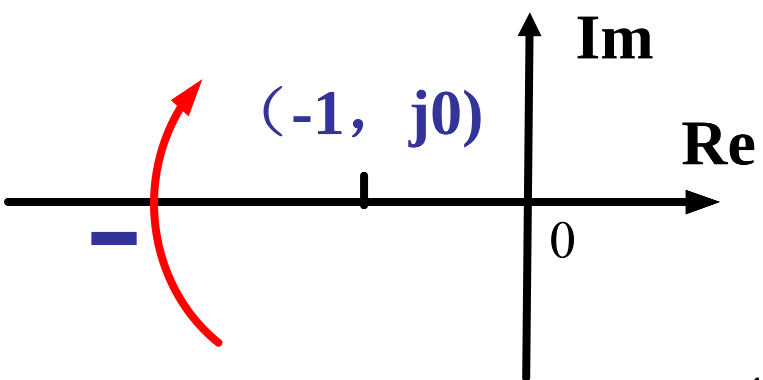
**The "cross" means cross the section of $(-1, -\infty)$ .**

**Positive cross: Cross this section from upside to downside one time (phase angle increase), demoted with $N_-$ .**

**Negative cross: Cross this section from downside to upside one time (phase angle decrease), demoted with $N_+$ .**

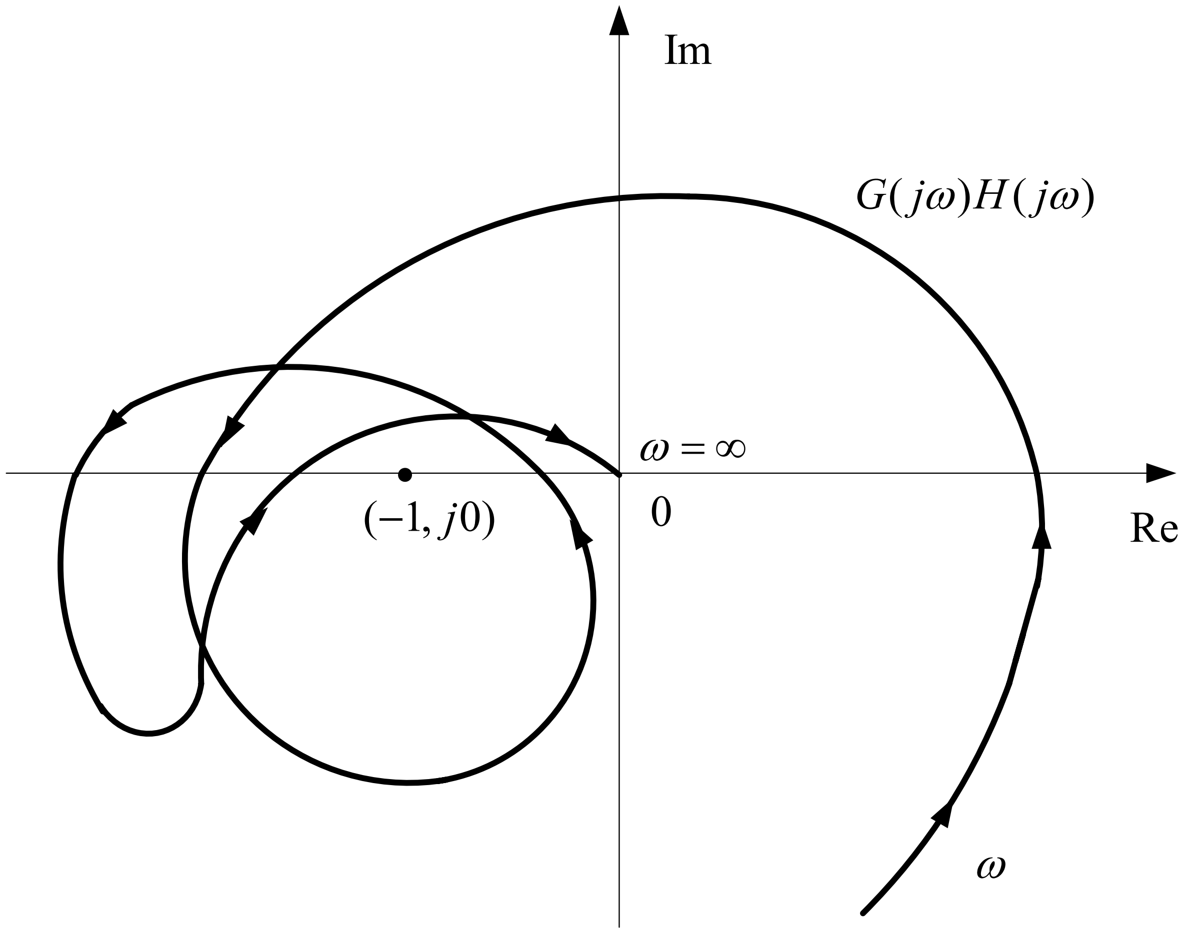



**Positive cross**




**Negative cross**

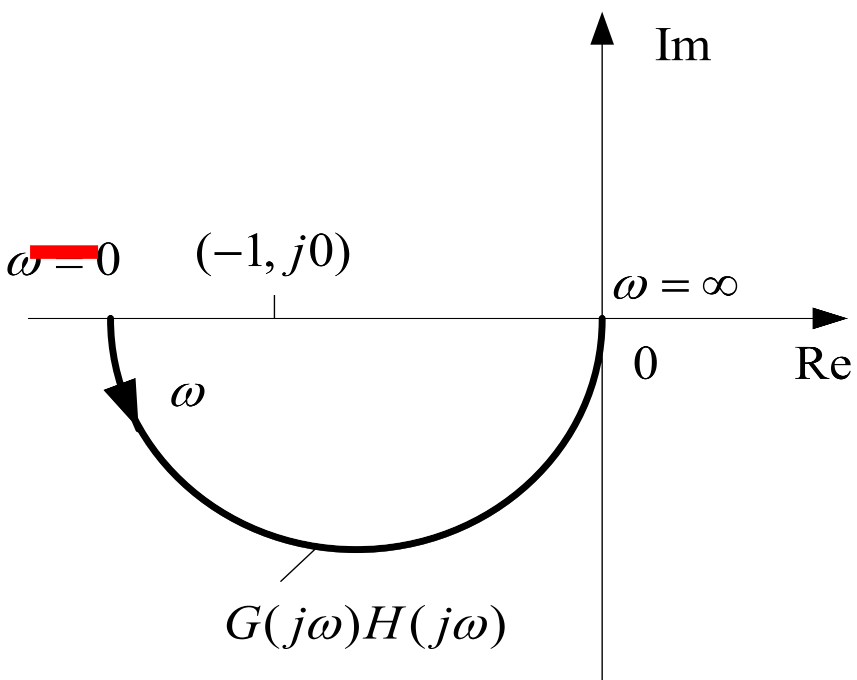
# Section 5: Nyquist stability criterion

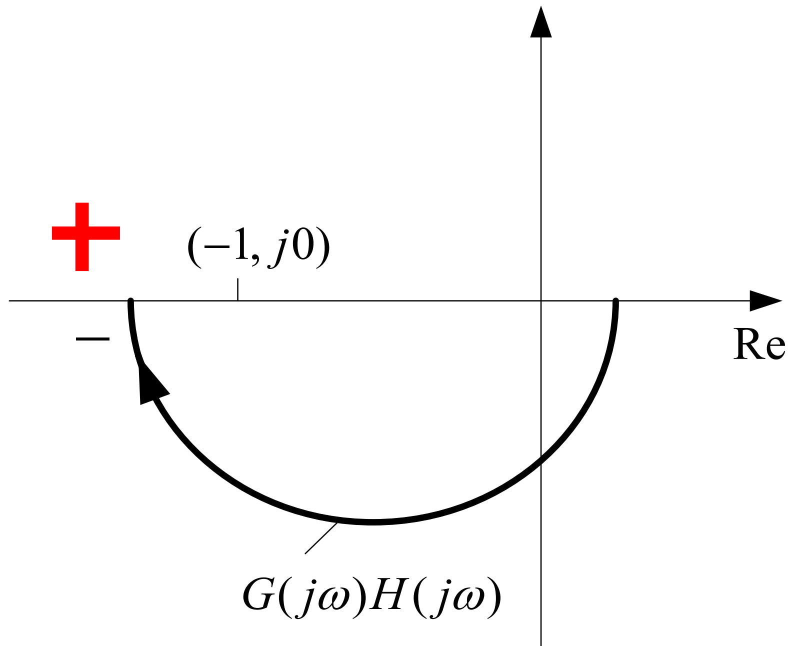



$$N_+ = 1 \qquad N_- = 2$$

# Section 5: Nyquist stability criterion

**If G(j ω )H(j ω ) begins or ends at negative axis on the left of (-1,j0), and the cross time is a half (a half positive cross or a half negative cross).**



## Example: The curve of G(jω)H(jω) is shown. These are 2 open-loop poles on right side of s-plane, judge the stability of system.

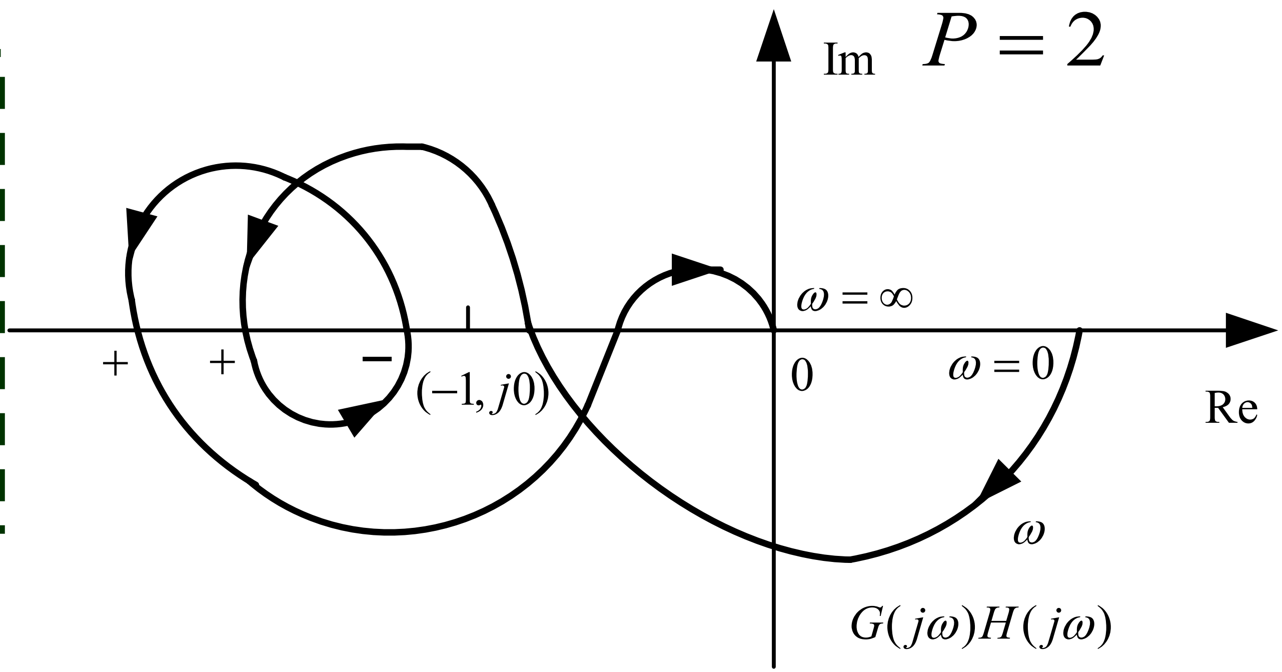
**These are 2 open-loop poles on the right side of s-plane (P=2), curve of *G*(*j ω*)*H*(*j ω*) have 2 positive crosses and a negative cross on the negative real axis on the left side of (-1, j0).**

**For:** $N' = N_+ - N_- = 1 - 2 = -1$

**Obtain:**

**Z=P+N=2-2=0**

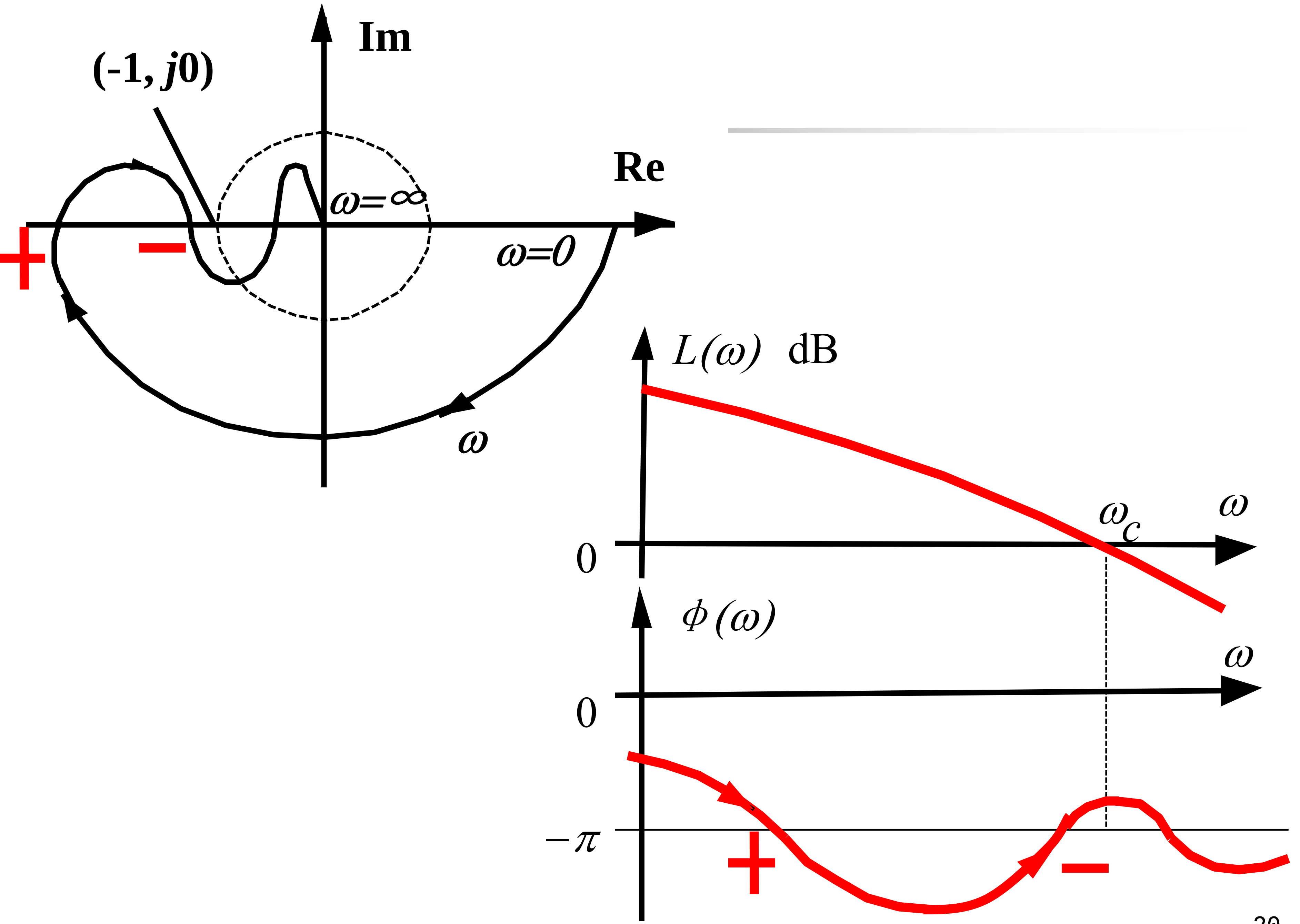
**The system is stable.**



# 4. Nyquist criterion on Bode plot

| Pole coordinate plot | Bode plot |
|---|---|
| Unit circle | 0db line |
| Inside of unit circle | Under 0db line |
| Outside of unit circle | Above 0db line |
| Negative real axis | $-180^0$ line |

**The positive cross (or negative cross) of Nyquist plot is equal to when L( ω )>0db, phase frequency character cross $-180^o$ line from upside to downside (or downside to upside).**


Im
(-1, j0)
Re
ω=∞
ω=0
ω
+
−
L(ω) dB
ω
ω_c
0
φ(ω)
ω
0
−π
+
−

# Section 5: Nyquist stability criterion

**According the definition of Nyquist criterion in pole coordinate, the Nyquist criterion in log coordinate is:**

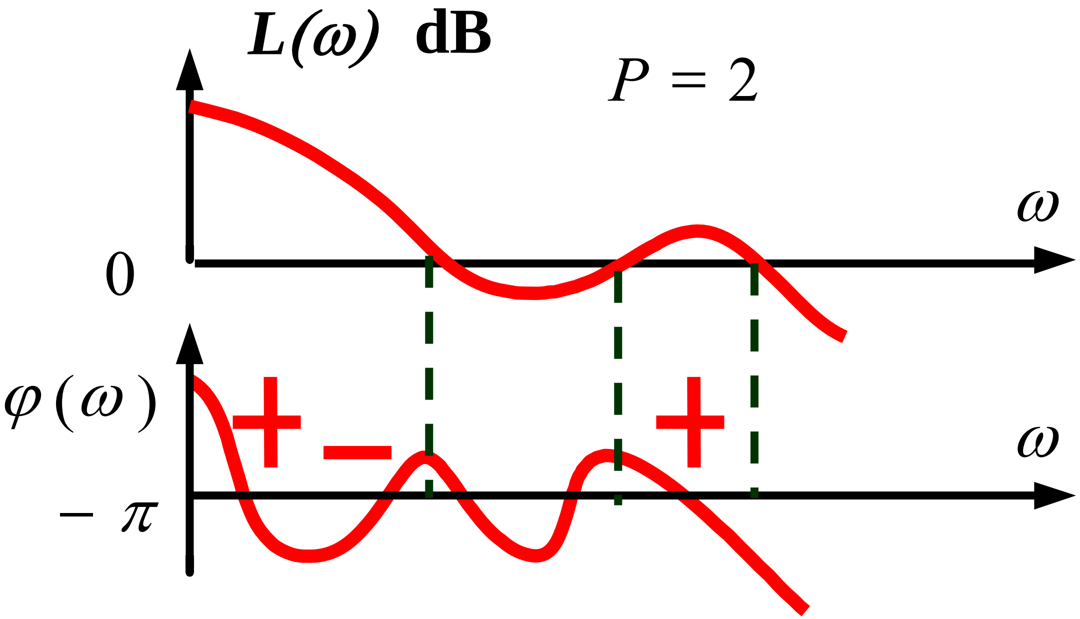
$$Z = P + N$$

**When $\omega$ varies from 0 to $\infty$ and the open-loop log magnitude frequency character $L(\omega)>0$, the cross time of phase frequency character $\phi(\omega)$ on $-180^0$ line:**

**(Positive cross N+) – (Negative cross N-) = N/2**

# Section 5: Nyquist stability criterion

**Example: These are two open-loop poles on right side of s-plane (P=2)**




**$N_+$- $N_-$=2-1= 1 not equals P/2 (=1)**

**The system is unstable.**

# 5. Application of Nyquist criterion

**Example: Analyze stability of system though Nyquist criterion. The open-loop transfer function is**

$$G(s)H(s) = \frac{K(T_2 s + 1)}{s(T_1 s + 1)} \qquad (T_1 > T_2)$$

**The frequency character is**

$$G(j\omega)H(j\omega) = \frac{K(jT_2\omega + 1)}{j\omega(jT_1\omega + 1)}$$

$$\begin{cases} |G(j\omega)H(j\omega)| = \dfrac{K\sqrt{(T_2\omega)^2 + 1}}{\omega\sqrt{(T_1\omega)^2 + 1}} \\ \varphi(\omega) = \tan^{-1} T_2\omega - 90^0 - \tan^{-1} T_1\omega \end{cases}$$

**Confirm the start and end points of magnitude and phase curve. It is very important to draw the correct curve for judging system stability.**

**Analysis on blackboard**

# Section 6: Relative stability of control system

## Concept of relative stability

**In engineering application, the system parameters would change, and the stability of system would be disturbed. When we design the system and set the components and parameters, we should consider not only stability, but also the degree of stability. This is the relative stability of automatic control system.**

# Section 6: Relative stability of control system

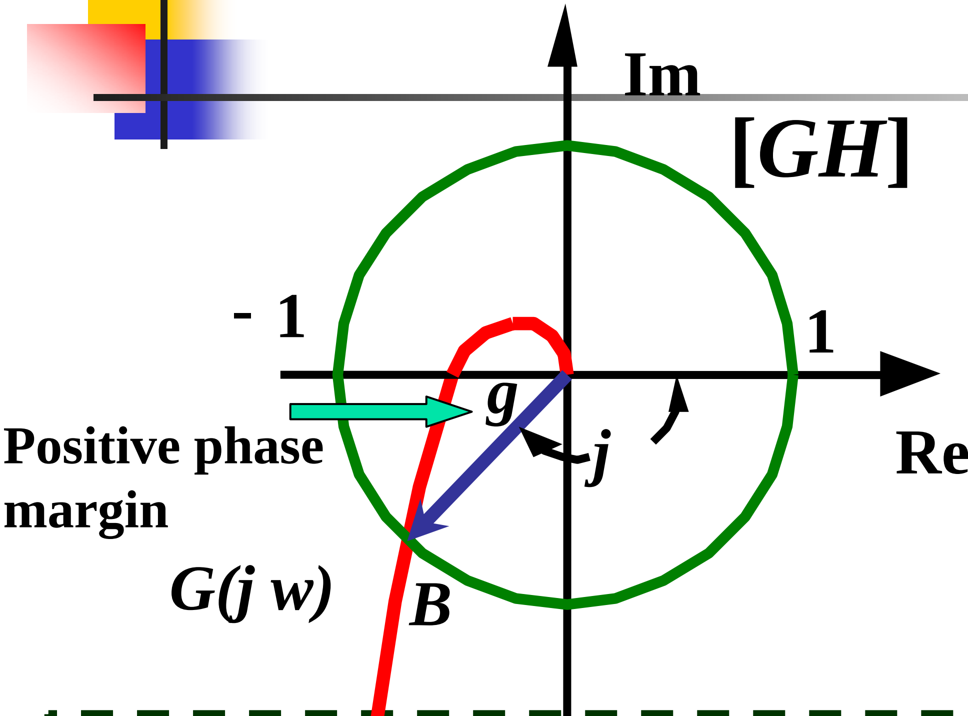
**The distance of open-loop frequency character $G(j\omega)H(j\omega)$ and the (-1,j0) on GH plane is use to describe the stability degree of close-loop system. Generally, the distance is bigger, the system is more stabile.**

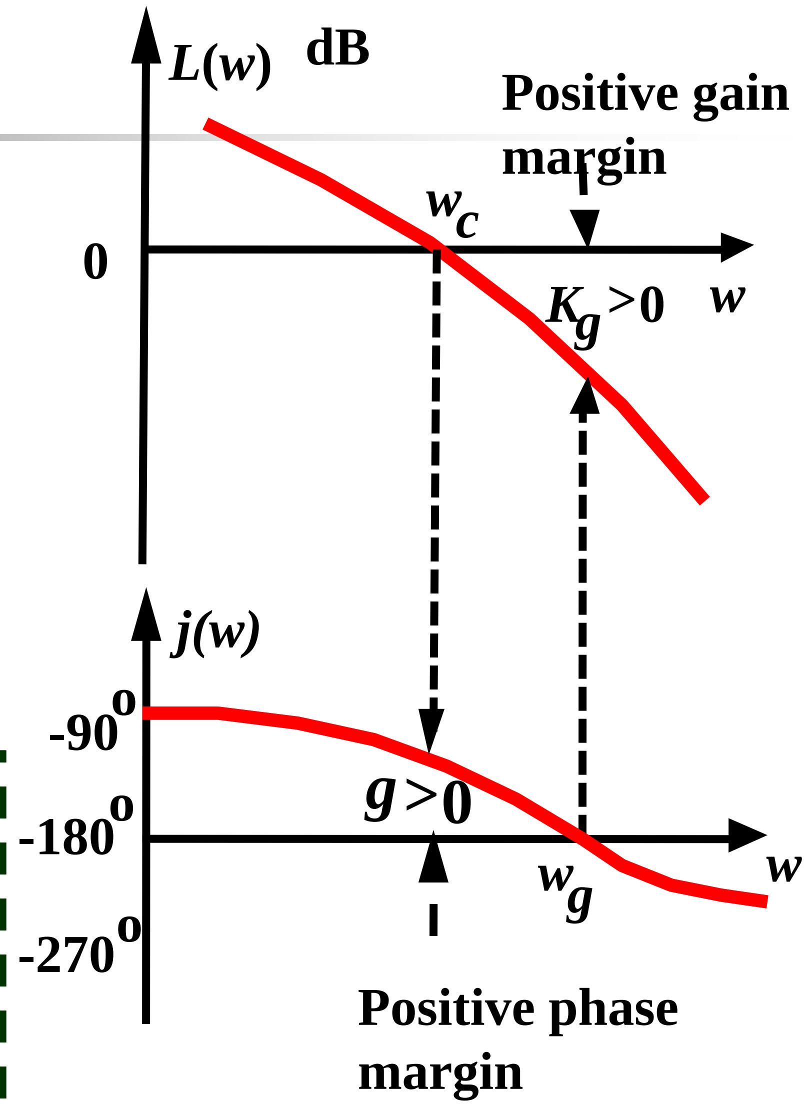
1. Phase margin

**Unit gain frequency $\omega_c$ : The frequency with magnitude of open-loop frequency character $G(j\omega)H(j\omega)$ equals 1.**

$$\left|G(j\omega_c)H(j\omega_c)\right| = 1$$

**Phase margin: The phase angle through which the G(j$\omega$)H(j$\omega$) locus must be rotated so that the unity magnitude point will pass through (-1,0) point in the GH plane, denoted as γ .**

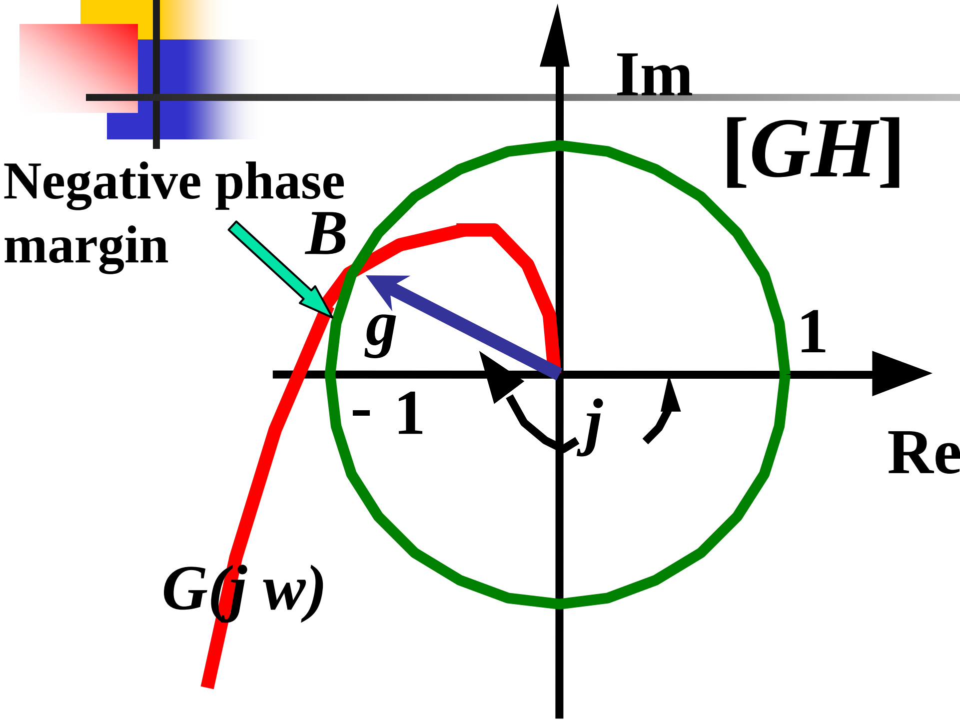


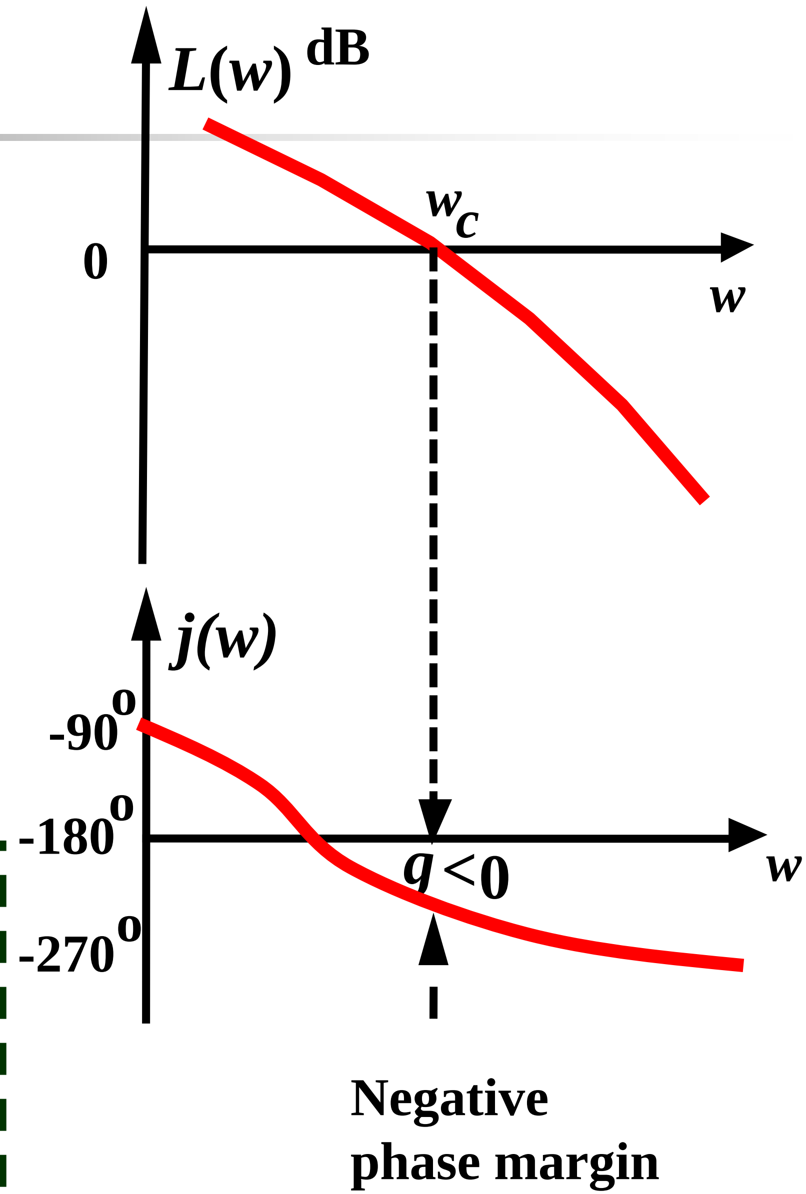




**Phase margin:**

**When γ >0, phase margin is positive, and the system is stable.**

$$\gamma = \varphi(\omega_c) - \left(-180^0\right) = 180^0 + \varphi(\omega_c)$$




**Phase margin:**

**When γ <0, phase margin is negative, and the system is unstable.**

# 2. Gain margin

**On phase cutoff frequency $\omega_g$ ($\omega_g$>0), the reciprocal number of open-loop frequency character is the gain margin of control system, denoted as $K_g$**

$$K_g = \frac{1}{\left|G(j\omega_g)H(j\omega_g)\right|}$$

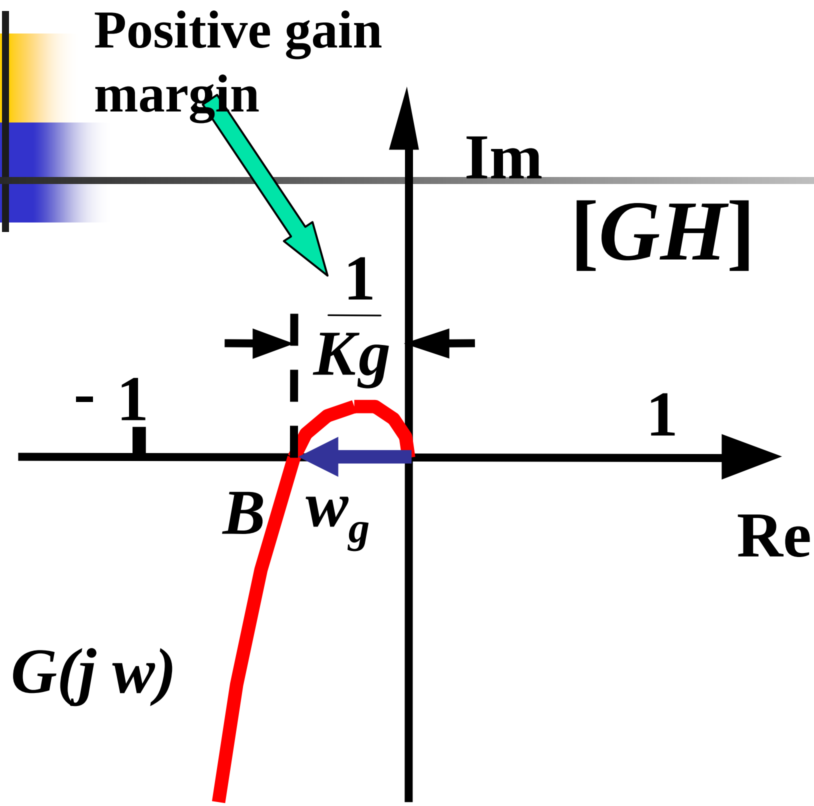
**Described in dB**

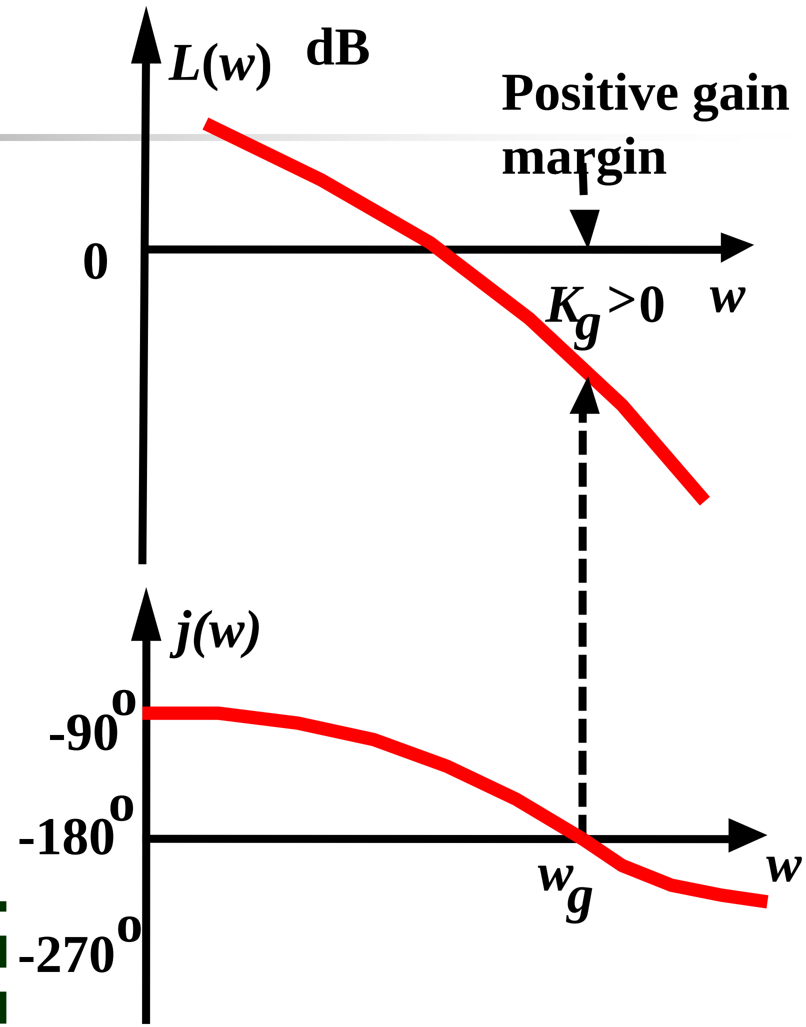
$$K_g(\mathrm{dB}) = 20\lg K_g = -20\lg\left|G(\mathrm{j}\omega_g)H(\mathrm{j}\omega_g)\right| \ (\mathrm{dB})$$

**When $K_g$>1, gain margin is positive, the system is stable.**

**When $K_g$<1, gain margin is negative, the system is unstable.**

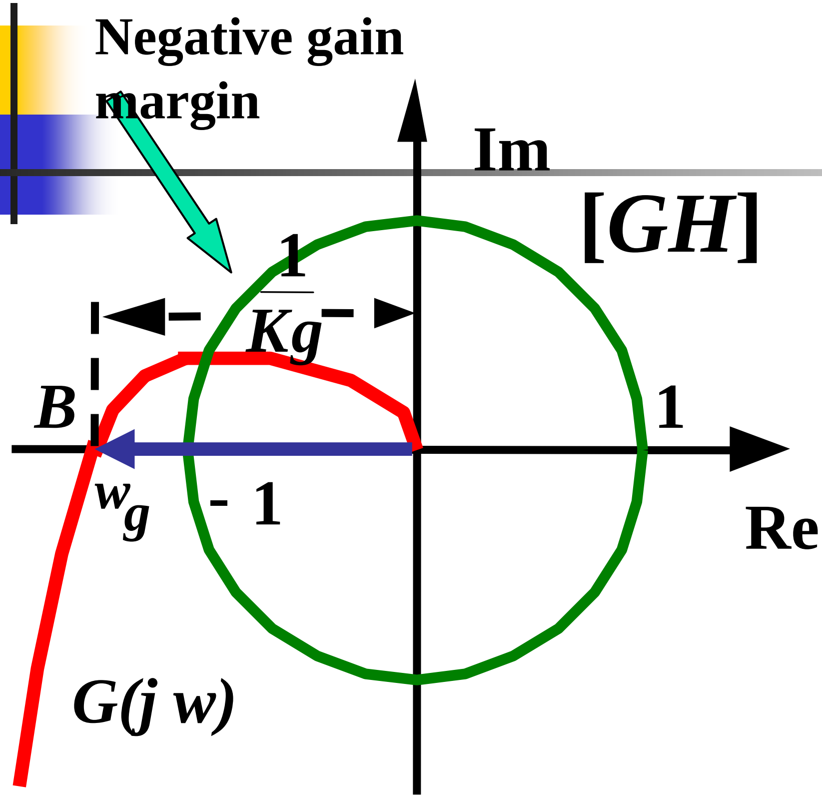
**Generally, phase margin is 30° to 60°, gain margin is bigger than 6dB.**

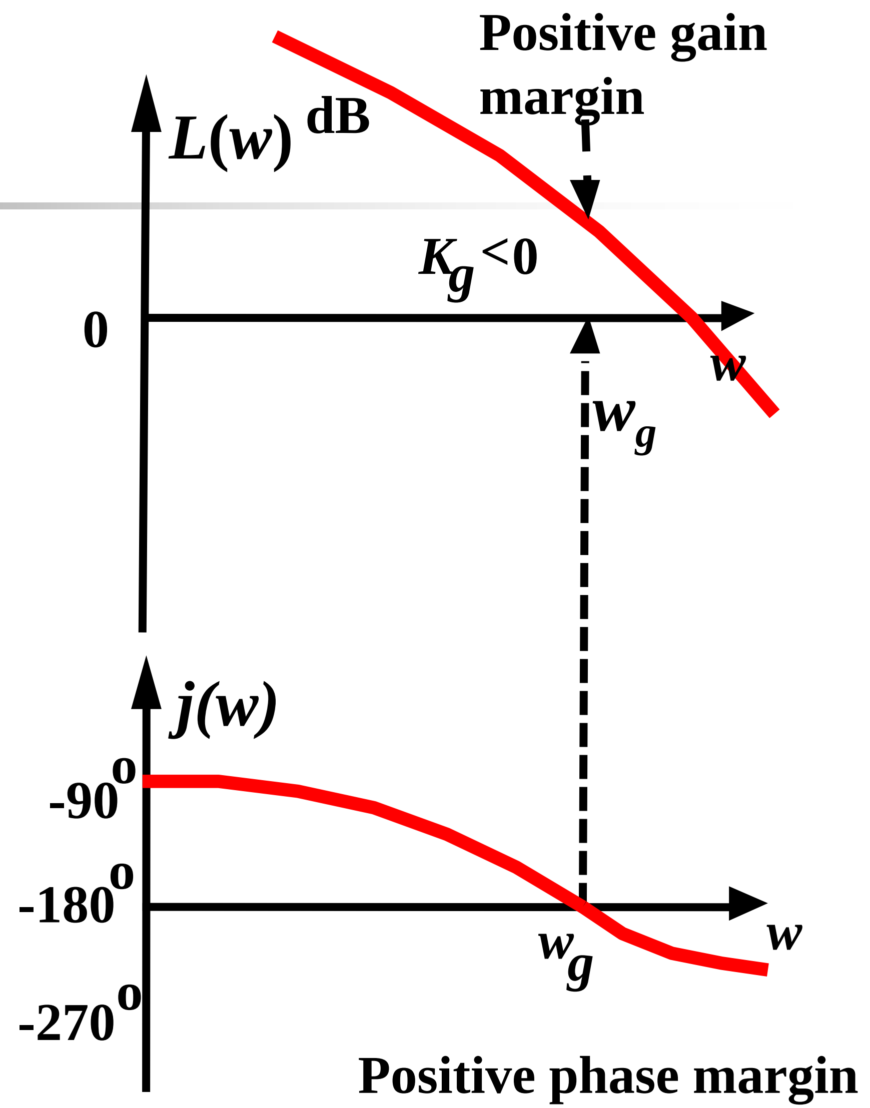



**Gain margin:**

**When *K*g >0, gain margin is positive, and the system is stable.**




## Phase margin:

**When γ >0, phase margin is positive, and the system is stabile.**

$$\gamma = \varphi(\omega_c) - (-180^0) = 180^0 + \varphi(\omega_c)$$

# Section 6: Relative stability of control system

**Conclusions: Generally**

**When the slope of L($\omega_c$) is -20dB/dec, the system is stable.**

**When the slope of L($\omega_c$) is -40dB/dec, the system could be stable or unstable. If stable, $\gamma$ is small.**

**When the slope of L($\omega_c$) is -60dB/dec, the system is unstable.**

**For the requirements of relative stability of system, the slope of L($\omega$) on $\omega_c$ would be -20dB/dec.**

# Section 7: Analysis system performance though frequency character

## Review: steady-state error of system

| System type | Error coefficient $K_p, K_v, K_a$ | Unit step input $r(t)=u(t)$ | Unit ramp input $r(t)=t$ | Unit parabolic input $r(t)=\frac{1}{2}t^2$ |
|---|---|---|---|---|
| O | $K$ 0 0 | $\frac{1}{1}$ | | |
| $I$ | $K$ 0 | 0 | $\frac{1}{K}$ | |
| $II$ | $K$ | 0 | 0 | $\frac{1}{}$ |

1. Steady-state error is related with input and system structure.

2. Decrease or eliminate steady-state eror:

*a.* Increase open-loop amplifier coefficient *K*; *b.* Increase the type of system.

# 1. Low frequency section

**Key point: Obtain steady-state error coefficient of log frequency character**

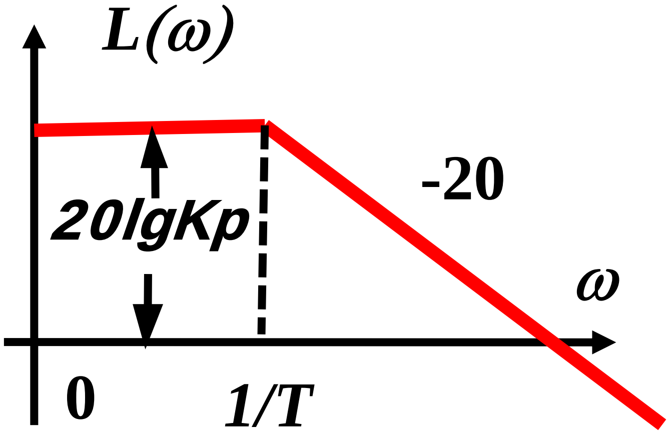
## 1. Type 0 system

**The open-loop frequency character of a system is**

$$G(j\omega) = \frac{K}{jT\omega + 1}$$

**At low frequency section ($\omega \to 0$), the magnitude is**

$$L(\omega) = 20K = 20lgK_P \quad K = K_P$$



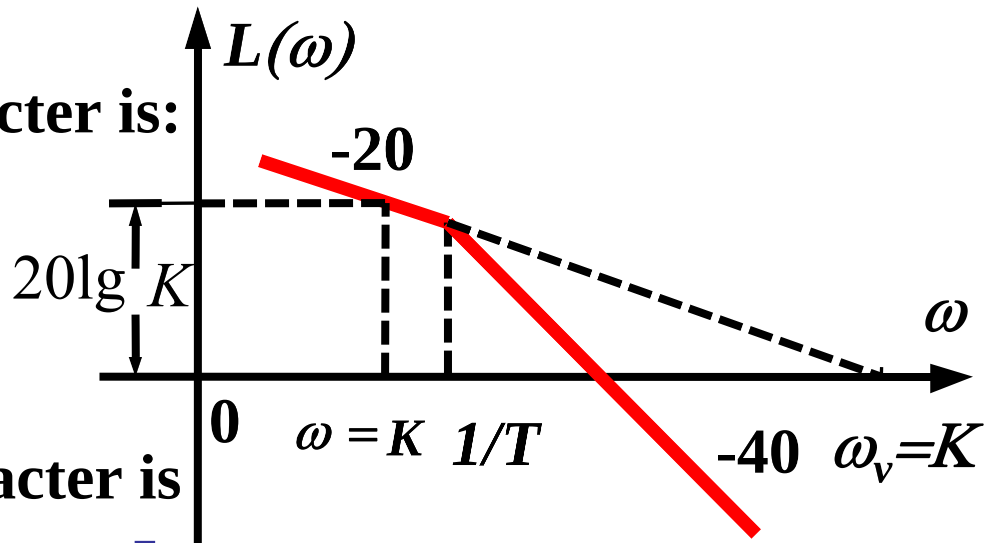

**Conclusion: The slope of type 0 system in low frequency section is 0, with the height 20lg$K_P$. $K_P$ is the steady-state position error coefficient.**

# 1. Low frequency section

## 2. Type I system

**The open-loop frequency character is:**

$$G(j\omega)=\frac{K}{j\omega(j\omega T+1)}$$




**The magnitude frequency character is**

**1) Log magnitude frequency of type I system**

**Slope of character curve in low frequency section is -20dB/dec. The magnitude of cross point of the curve (or its extension) with $\omega$=1 is 20lg$K_V$. Prove:**

**When ω =1** $$20\lg\left|\frac{K_V}{j\omega}\right|_{\omega=1}=20\lg K_V$$

## 1. Low frequency section

**2) The cross point frequency $\omega_V$ of the start section (or its extension) with slope -20dB/dec and 0dB equals *K*. Prove:**

$$20\lg\left|\frac{K}{j\omega}\right|_{\omega=\omega_{\mathrm{v}}} = 0(\mathrm{dB}) \qquad \left|\frac{K}{j\omega_v}\right| = 1$$

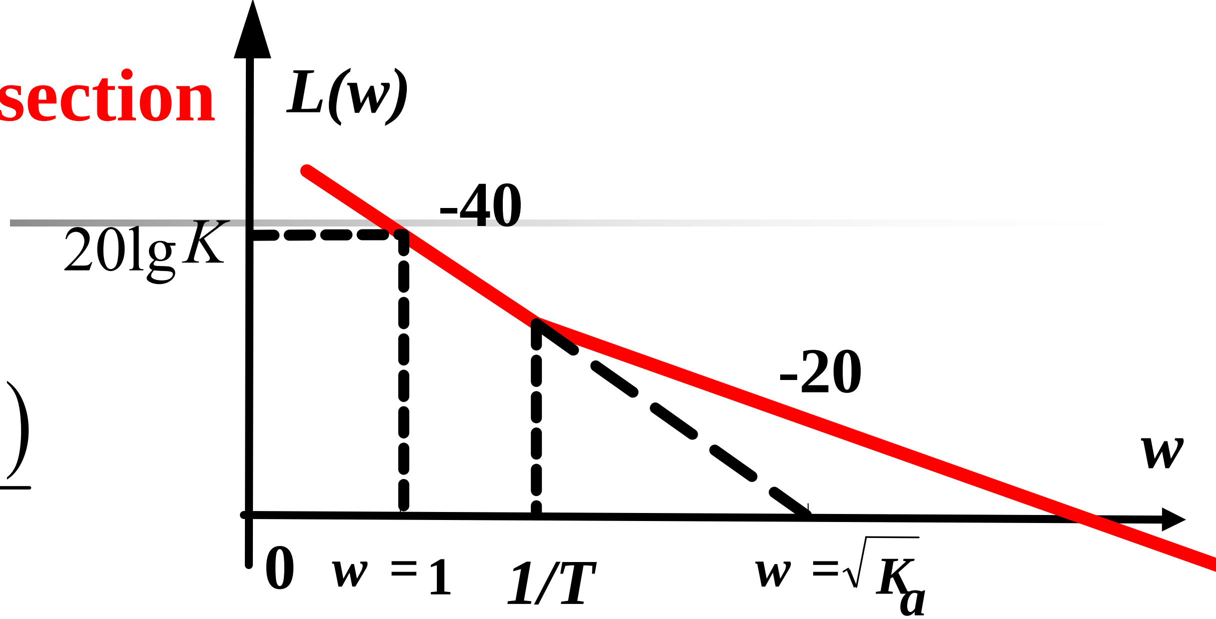
**Then**

$$K = \omega_{\mathrm{v}}$$

## 1. Low frequency section

### (3) Type II system

$$G(j\omega)=\frac{K(j\omega T+1)}{(j\omega)^2}$$




**1) For type II system, the magnitude of cross point of low frequency section (or its extension) and ω =1 equals 20lg*K*. Prove:**

**The frequency character of type II system in low frequency character is**

$$G(j\omega)=\frac{K}{(j\omega)^2}\quad(\omega<<1)$$

**When ω =1**

$$20\lg\left|\frac{K}{(j\omega)^2}\right|_{\omega=1}=20\lg K$$

# 1. Low frequency section

**2) If the frequency of cross point of start section (or its extension) with slope -40dB/dec and 0dB is $\omega_a$, the value equals sqrt of *K*. Prov**

**For**

$$20\lg\left|\frac{K}{(j\omega)^2}\right|_{\omega=\omega_a} = 0 \quad (\mathrm{dB})$$

$$K = \omega_a^2 \qquad\qquad \omega_a = \sqrt{K_a}$$

- **Obtain**

**Increase the magnitude of low frequency section of open-loop frequency character or increase the absolute value of slope of low frequency section (increase system type) could decrease system steady-state error.**

**Conclusion: Generally, low frequency section of open-loop frequency character dominate the steady-state performance of closed-loop system.**

- **Thanks!**