《自动控制理论》
Automatic Control Theory

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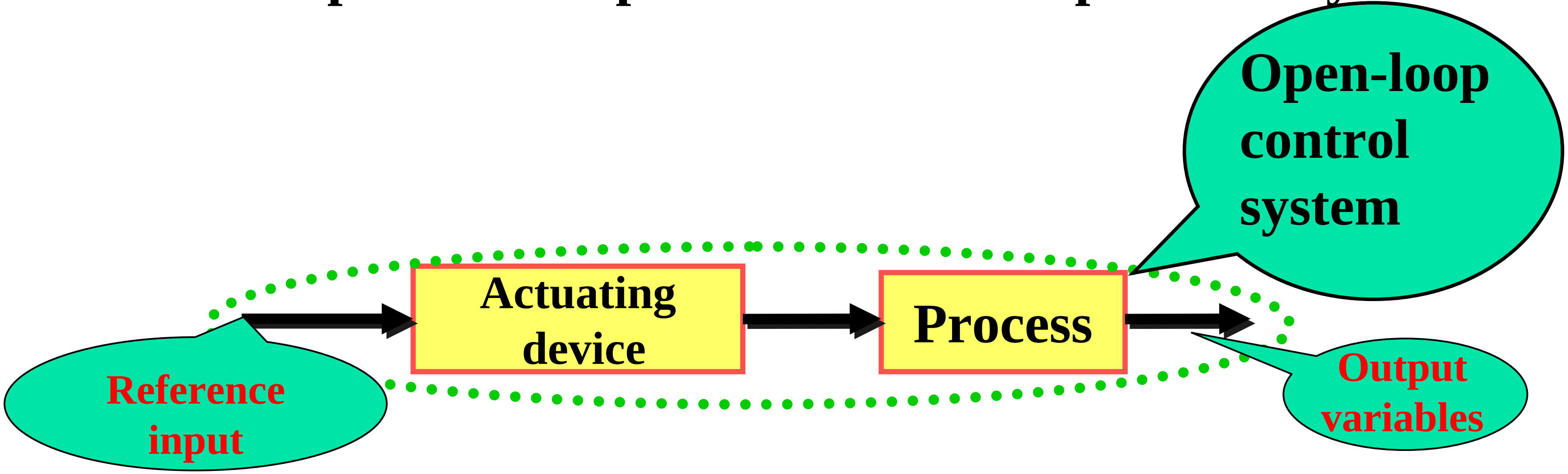
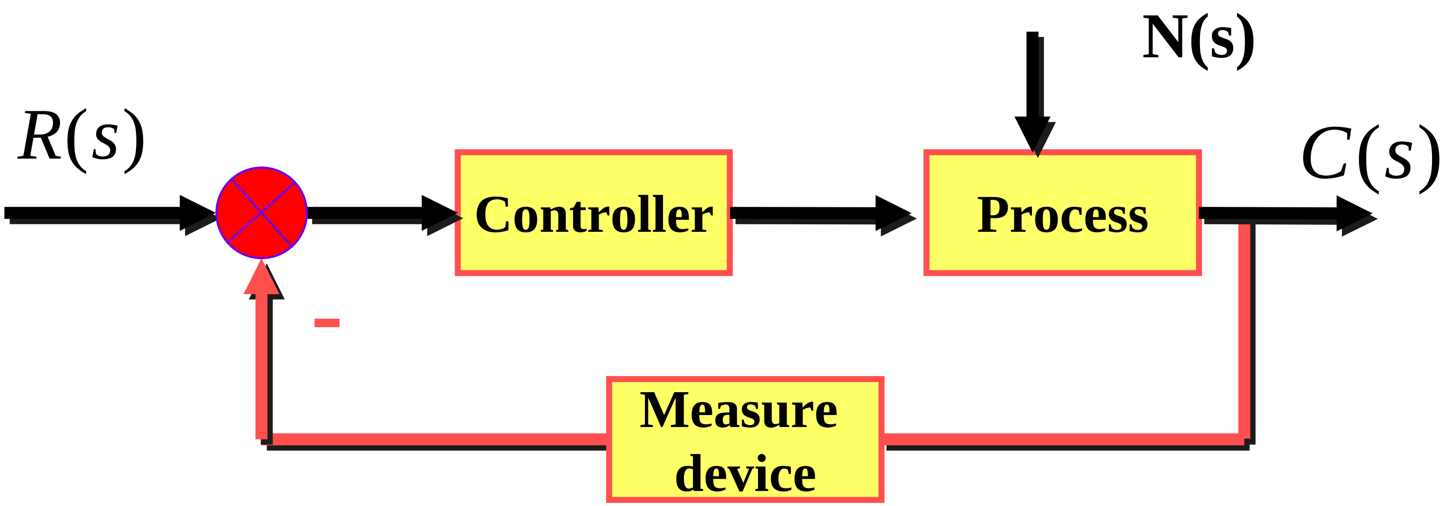
Reviews

- **1. General concept of automatic control system**
- **Automatic control is using controllers to operate some physical quantities (such as: temperature, pressure, PH value) of a process without direct human interventions, and make these physical quantities changing in accordance with the law.**

Reviews

- 2. Components of open- and closed-loop control system





Section 4: Classification of automatic control systems

1. Based on mathematical model

(1) Linear system

Features: the system is composed by linear components, The mathematical model is linear differential equations. The dynamic system represented by the differential equations

$$a_0 \frac{d_{n}^{\ n} c(t)}{dt^{n}} + a_1 \frac{d_{n-1}^{\ n-1} c(t)}{dt^{n-1}} + \cdots + a_{n-1} \frac{dc(t)}{dt} + a_n c(t)$$

$$= b_0 \frac{d_{m}^{\ m} r(t)}{dt^{m}} + b_1 \frac{d_{m-1}^{\ m-1} r(t)}{dt^{m-1}} + \cdots + b_{m-1} \frac{dr(t)}{dt} + b_m r(t)$$

s.t. r(t)——system input； c(t)——system output

The major features include principle of superposition and homogeneity.

Section 4: Classification of automatic control systems

(2) Nonlinear system

Features: the system is composed by one or more nonlinear components.

The research of nonlinear system is much more incomplete than linear system. These is no general methods to solute nonlinear systems now.

Section 4: Classification of automatic control systems

2. Based on control signal

(1) Constant control system

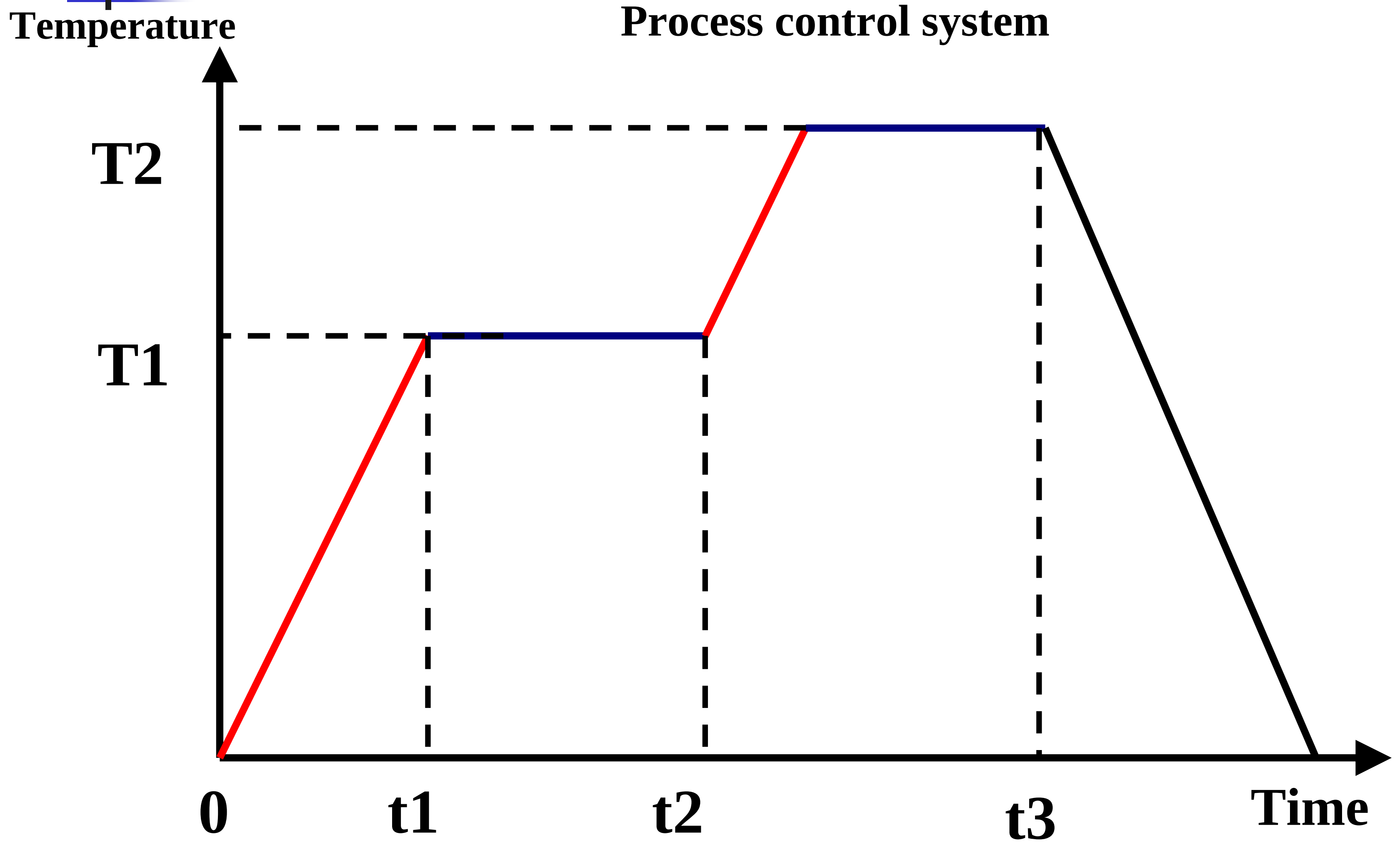
Features: the input is constant. Constant control system include Constant Temperature Controlling System, Constant pressure Controlling System.

(2) Process control system

Features: the input is a known function. The control process accords to scheduled program, and the output could affect input quickly and accurately, such as pressure, temperature, flow in chemical industry.

Constant control system could be considered as a process control system with a constant input.

Process control system



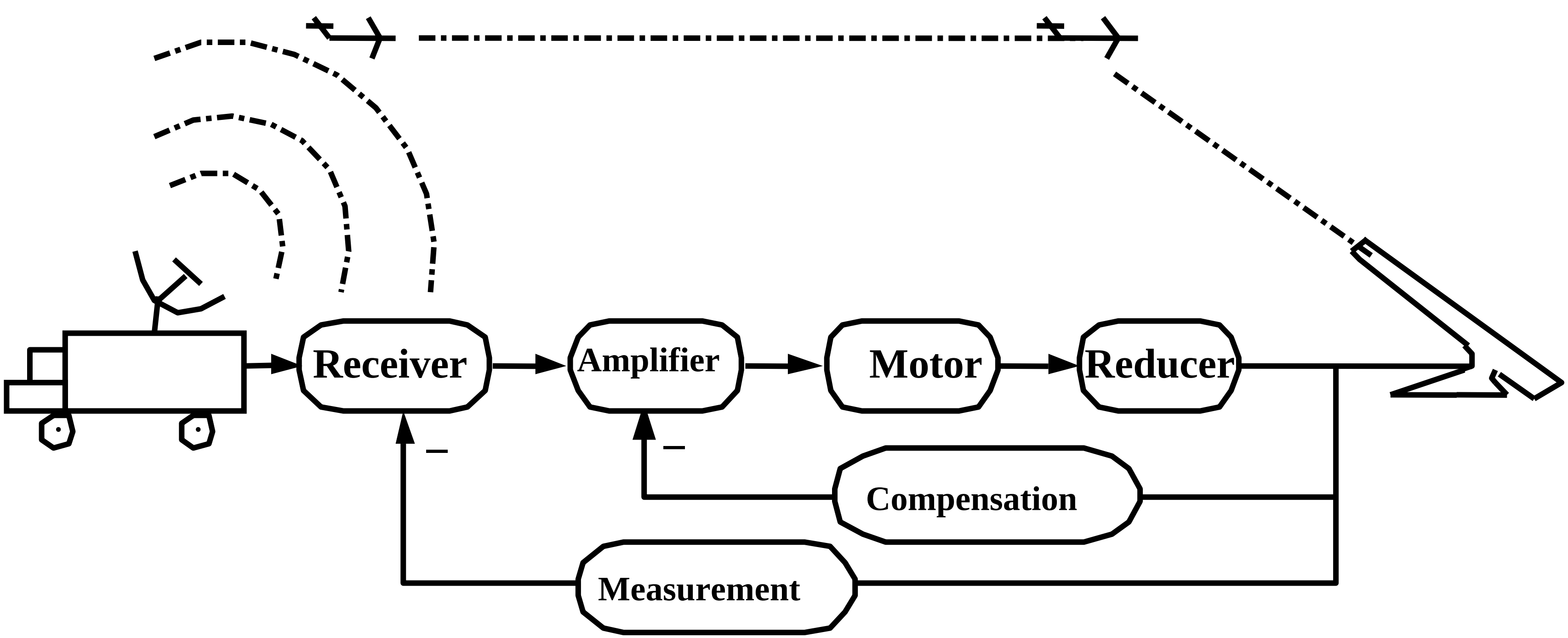
Control curve of temperature in stove

Section 4: Classification of automatic control systems

(3)Servo system

Features: the input is an unknown function. The output is required to follow the changing of input.

Artillery Servo system requires a strong tracking ability.



Section 4: Classification of automatic control systems

3. Based on signal properties
(1) Continuous system

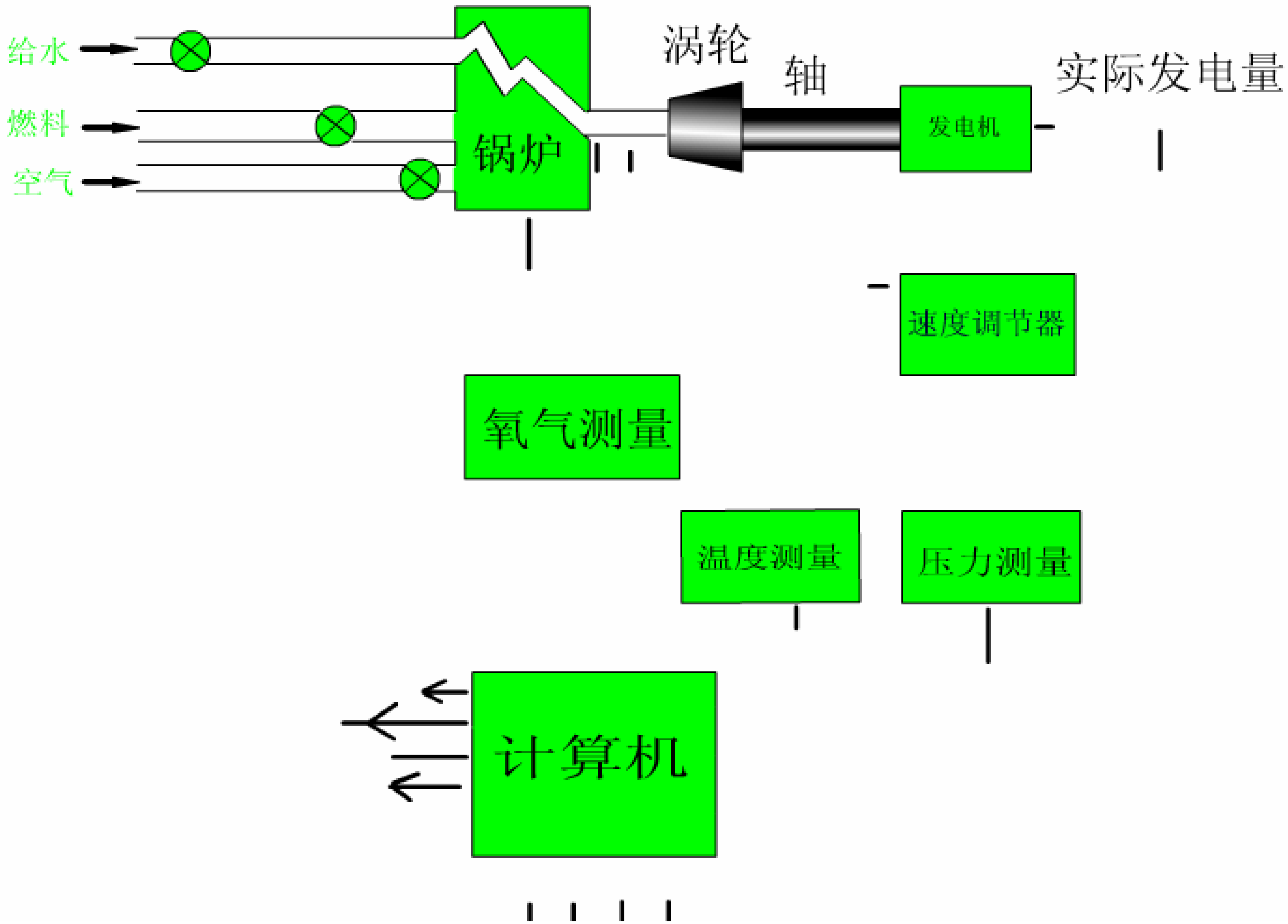
Features: all signal in system is analog continuous signal. **PID regulator is a kind of popular continuous system adopted in industry.**

(2) Discrete system

Features: one or more parts of signal in system is pulse sequence or digital signal. **Pulse or sampling switches are adopted to transform continuous signal into digital signal. The system with pulse signal is called pulse control system, and the system with digital signal is called digital control system.**

Steam generator controller (Process control)

- Water
- Fuel
- Air



- Desired oxygen content, temperature, pressure and electric energy production

Section 4: Classification of automatic control systems

Other classifications

Based on functions: **temperature control system, speed control system, position control system, etc.**

Base on components: **electro-mechanical system, hydraulic system, biological system, etc.**

Section 5: Performance requirements of control system

In order to realize the control objectives, some specifications should be proposed. Generally, it is tested by some specific input, such as some eigenvalues in transient response and steady-state characteristics using unit step signal as input.

Performance requirements of control system:

1. Stability Necessary conditions, dominates the regular operation of system.

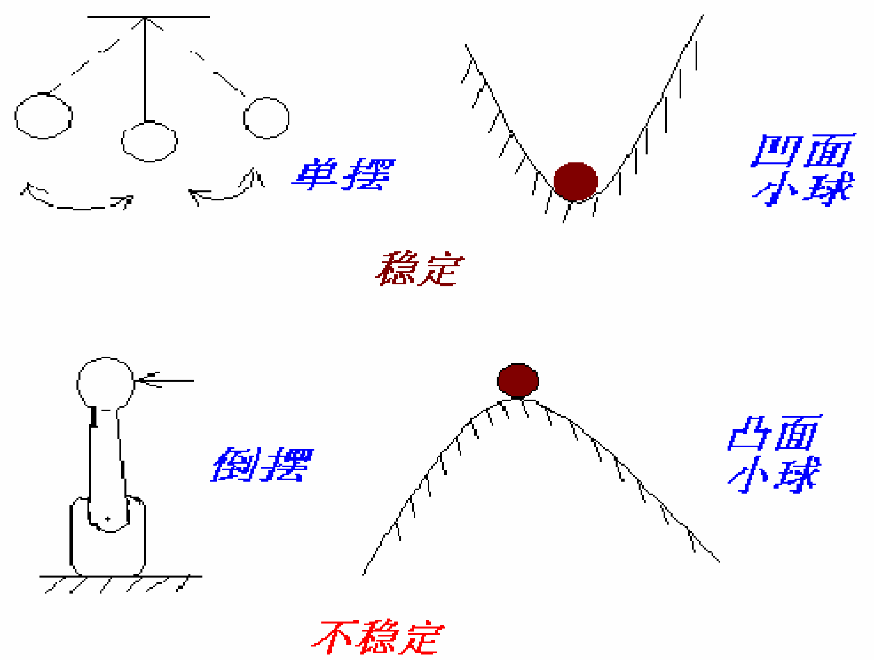
2. Steady-state performance After transition process, the level of control accuracy of steady-state performance.

3. Transient performance The rapidity of dynamic response, and the transition process should be as short as possible.

Section 5: Performance requirements of control system

•1. Stability

•A stability system is a dynamic system with a bounded response to a bounded input. If a system is subjected to a disturbance and could return equilibrium state automatically, the system is said to be stable. The response is bounded if the input is bounded. Otherwise, the system is a instability system.



Section 5: Performance requirements of control system

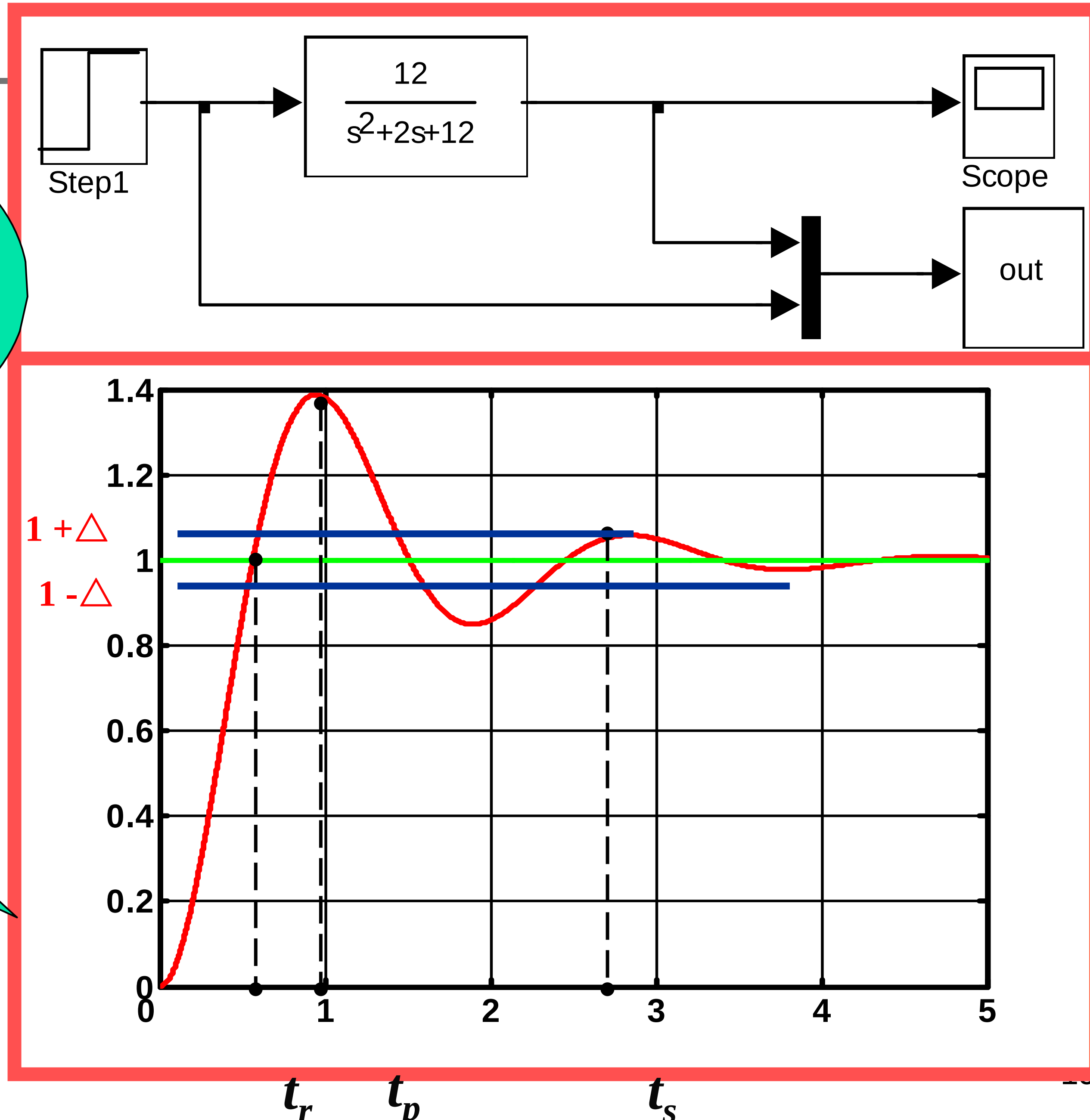
•2. Steady-state error

•The steady-state error is the error after the transient response has decayed, leaving only the continuous response. The steady-state error of open-loop system is related to its gain or magnification. The steady-state error of **feedback control system (closed-loop control system) is related to the degree of its feedback. A system has a higher accuracy if it has a smaller steady-state error, and vice versa. It is reflected by steady-state response of the system.**

Steady-state error = Desired output – Actual output

3. Transient performance

Unit step response simulated in MATLAB



Simulation output

Section 5: Performance requirements of control system

Percent overshoot

Percent overshoot relates with the damping ratio. In stable system, the percent overshoot is adopted as a performance parameter.

$$\sigma_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

Settling time (t_s)

Settling time is defined as the time required for the system to settle within a certain percentage, Δ, of the input amplitude.

Chapter 2 Mathematical models of control systems

- **1 Dynamic differential equations of control systems**
- **2 Linear Approximations of differential equations**
- **3 Transfer function**
- **4 Block diagram and transformations**
- **5 Signal-flow graph and Mason gain formula**

Summarization

1. What's mathematical model?

2. Why should we obtain mathematical model?

3. Key points in obtaining mathematical model

Summarization

- **1. Why should we discuss and obtain mathematical model?**
- **Automatic control system could be electrical, mechanical, hydraulic and pneumatic. But their mathematical model could be same. So, researching automatic control system though mathematical model could cast off the external relationship of different kinds of system, but get the common laws.**
- **If the mathematical models are linear differential equations, the system is a linear system. And if the coefficients are constants, the system is a linear constant, stationary system. We discuss linear, stationary system mainly in this chapter, and learn Laplace transform, transfer function, block diagram, signal-flow graph, etc.**

Since most physical systems are nonlinear, we discuss linearization approximations. All the systems discussed in this book are linear system, but chapter 7.

Mathematical model is the math description of dynamic characteristics of systems, and they have many description forms. In classical theory, differential equations, block diagram, signal-flow graph are adopted; and in modern control theory, we use state differential equation. Differential equations, block diagram and signal-flow graph are graph descriptions of mathematical model.

Summarization

2. Key points in obtaining mathematical model

It's very important work to obtain a reasonable mathematical model in analyzing the systems. The key points include:

(1) describe the components and systems correctly;

(2) the description should be concisely.

These are two kinds of method of obtaining mathematical model: analytical method and experimentation. Analytical method obtains mathematical models though related laws of the systems and components. The main laws should be described and the subordination law would be ignored. Experimentation obtains mathematical models though experiment data.

The principle of superposition is the main property of linear system. And a linear system also satisfies the property of homogeneity.

Section 1 Dynamic differential equations of control systems

1. Differential equations

Differential equations describe the time-response dynamic property of the automatic control system. It is the motion equation of system in time-response.

2. The approach to obtain motion equation is listed as follows:

(1) Confirm the input (certain value and disturbance) and output (system response) of the system and components.

(2) List the differential equations of every parts of system.

(3) Eliminate the intervening variables, then obtain the differential equations with system input and output.

Section 1 Dynamic differential equations of control systems

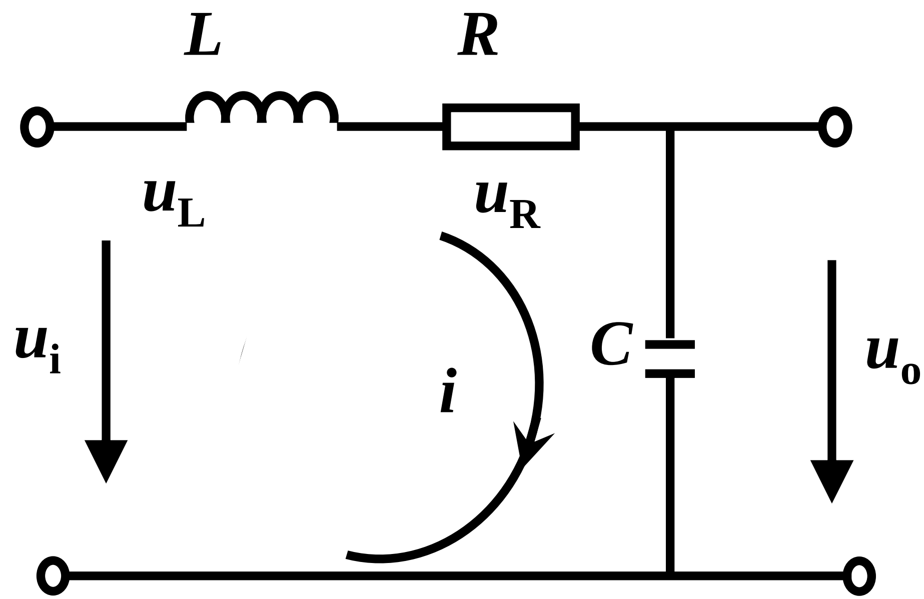
$$a_0 \frac{d^n c(t)}{dt^n} + a_1 \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_{n-1} \frac{dc(t)}{dt} + a_n c(t)$$

$$= b_0 \frac{d^m r(t)}{dt^m} + b_1 \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_{m-1} \frac{dr(t)}{dt} + b_m r(t)$$

- **Three parts of contents are included in standard equation: ①put the variables related with input to right side of equation; and put the variables related with output to left side of equation; ②arrange the equation in lower-order term of derivative; ③transfer the equation parameters into coefficients with physical meaning.**

Section 1 Dynamic differential equations of control systems

Example 1 RLC circuit. (1) input u_i , *output* u_0; list the differential equation of every component



(2)

$$L\frac{di}{dt}+Ri+u_o=u_i$$

$$i=C\frac{du_o}{dt}$$

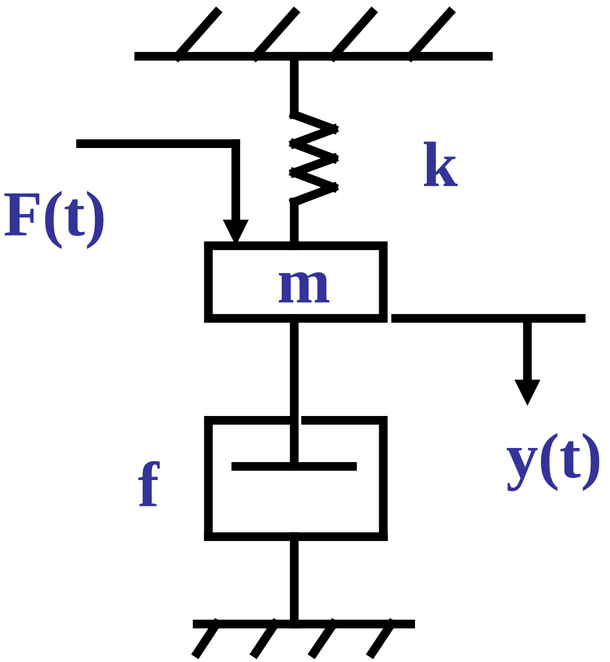
(3) Eliminate intervening variables

$$LC\frac{d^2u_o}{dt}+RC\frac{du_o}{dt}+u_o=u_i$$

Section 1 Dynamic differential equations of control systems

Example 2 **A spring-mass-damper mechanical system is shown, input is F(t). Obtain the differential equations with the input force F(t) and output displacement y(t).**

(1) Input: F(t)
output: y(t)



(2) $F_1(t) = -f\frac{dy(t)}{dt}$ $F_2(t) = -ky(t)$

Where: f- damping coefficient, k-elasticity

$$F(t) + F_1(t) + F_2(t) = m\frac{d^2y(t)}{dt^2}$$

(3) Eliminate the intervening variables

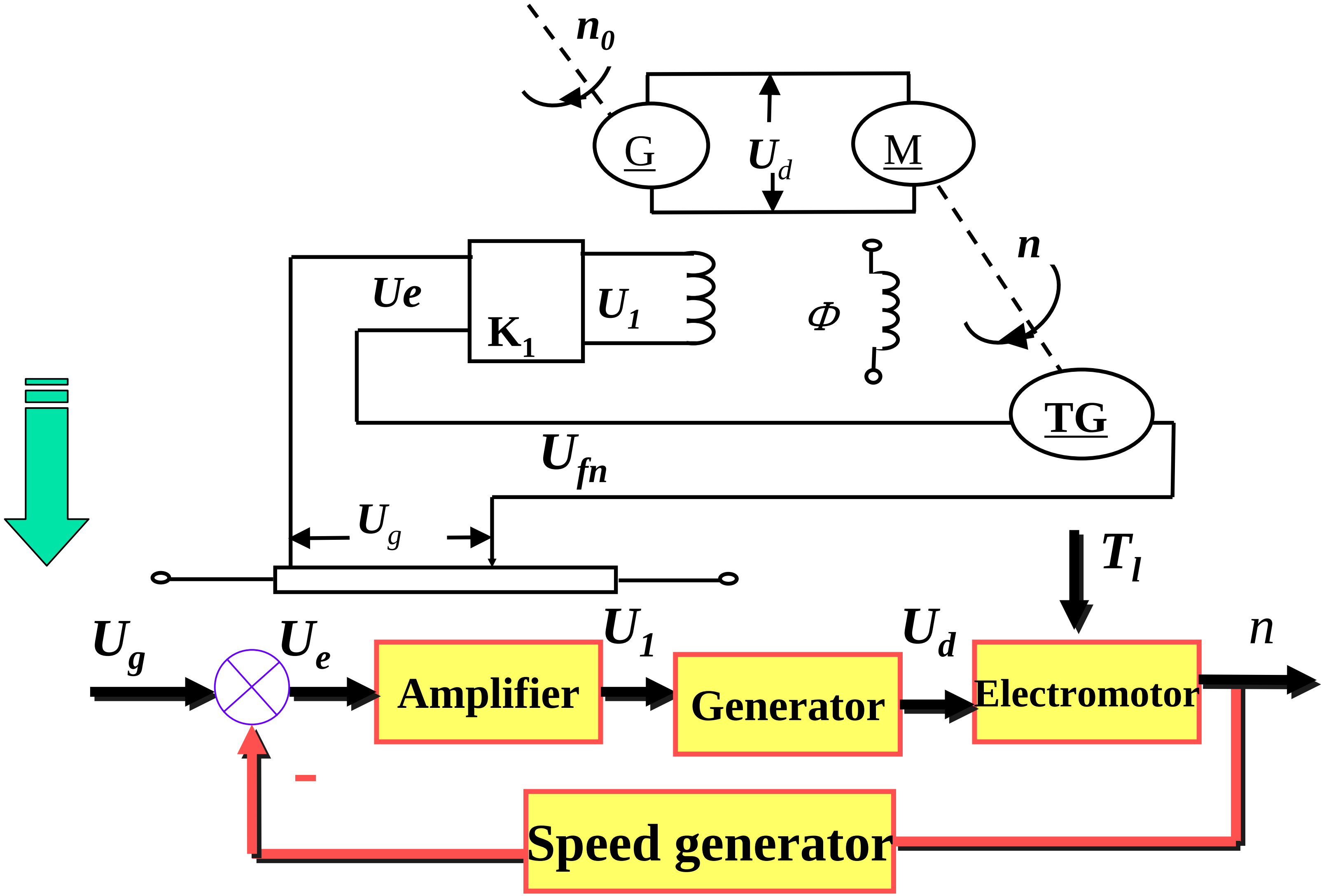
$$\frac{m}{k}\frac{d^2 y(t)}{dt^2}+\frac{f}{k}\frac{dy(t)}{dt}+y(t)=\frac{1}{k}F(t)$$

$$Let \quad T=\sqrt{\frac{m}{k}},\zeta=\frac{f}{2\sqrt{mk}},K=\frac{1}{k}$$

$$T^2\frac{d^2 y(t)}{dt^2}+2\zeta T\frac{dy(t)}{dt}+y(t)=KF(t)$$

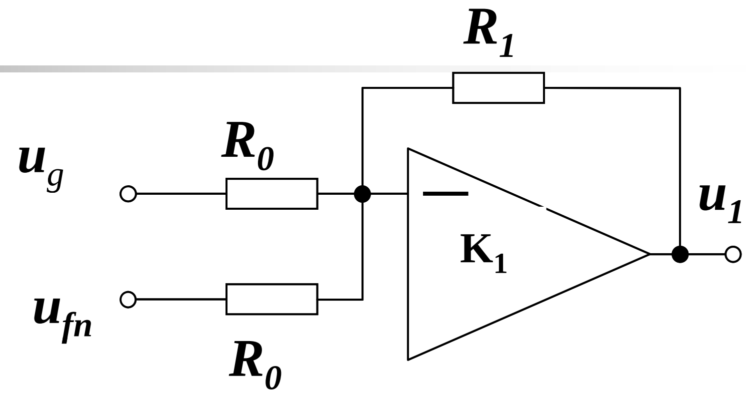
Section 1 Dynamic differential equations of control systems

Example 3 **Direct current timing system (P15)**



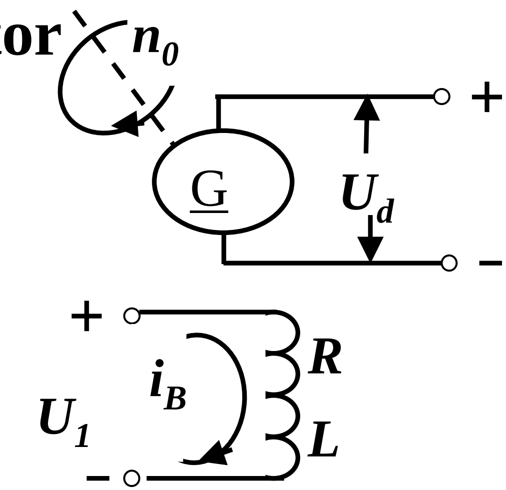
■ Example 3 Direct current timing system (P15)

- **(1) Operation amplifier:**



$$\frac{u_1}{u_e} = -\frac{R_1}{R_0} = K_1 \qquad u_e = u_g - u_{fn}$$

- **(2) Direct separately excited generator**



$$L\frac{di_B}{dt} + i_B R = u_1$$

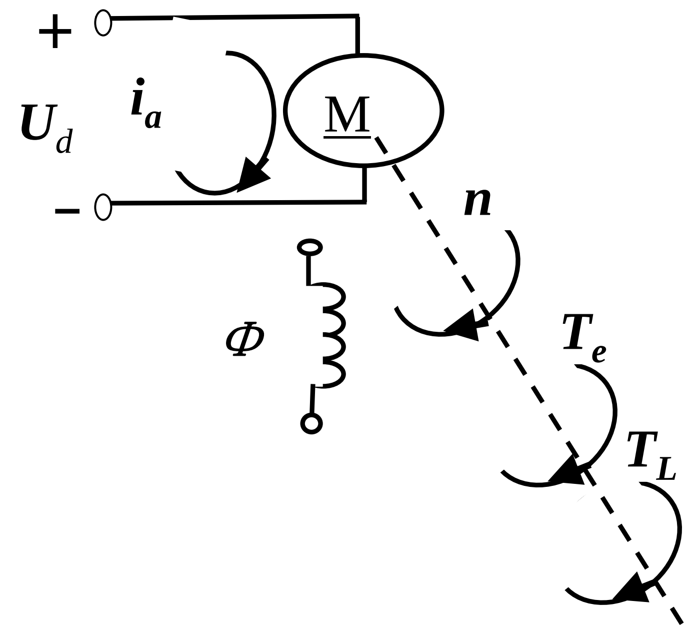
$$u_d = C_1\phi = C_2 i_B$$

$$\Rightarrow \quad \tau_G \frac{du_d}{dt} + u_d = K_2 u_1$$

其中 $\tau_G = L/R$, $K_2 = C_1 L/R$

▪ Example 3 Direct current timing system (P15)

- (3) Direct separately excited electromotor:



$$i_a R + L\frac{di_a}{dt} + C_e n = u_d$$

$$T_e - T_L = \frac{GD^2}{375}\frac{dn}{dt}$$

$$T_e = C_u i_a$$

$$\tau_m \tau_a \frac{d^2 n}{dt^2} + \tau_m \frac{dn}{dt} + n = \frac{1}{C_e} u_d - \frac{R}{C_e C_u}\left(T_L + \tau_a \frac{dT_L}{dt}\right)$$

其中 $\tau_m = (GD^2 R)/(375 C_e C_u)$, $\tau_a = L/R$

Example 3 Direct current timing system (P15)

- **(4) Speed generator:**

$$u_{fn} = an$$

- **Eliminate the intervening variables**

$$\tau_m \tau_a \tau_G \frac{d^3 n}{dt^3} + \tau_m(\tau_a + \tau_G)\frac{d^2 n}{dt^2} + (\tau_m + \tau_G)\frac{dn}{dt} + (1 + \frac{K_a}{C_e})n$$

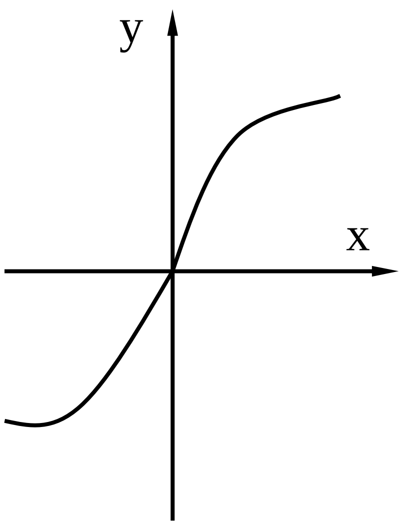
$$= \frac{K}{C_e}U_g - \frac{R}{C_e C_u}[T_L + (\tau_a + \tau_G)\frac{dT_L}{dt} + \tau_a \tau_G \frac{d^2 T_L}{dt^2}]$$

其中 $K = K_1 K_2,\quad R = R_G + R_M$

Section 2: Linear approximations of differential equations

Under some assumptions, nonlinear system could be linearized as a linear system, it is called linear approximation.

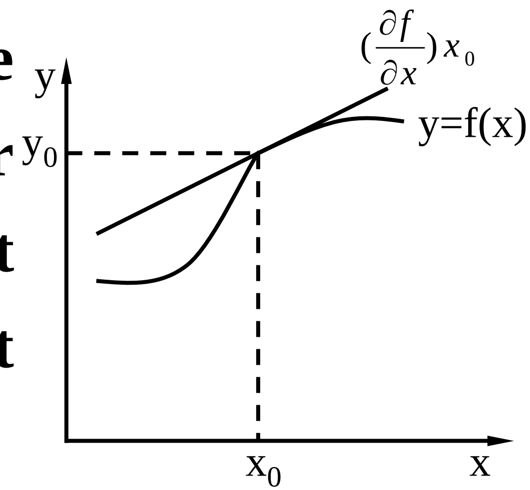
In actual project, control system has its own rated operating point. When the variables change around the rated operating point, it could be written in Taylor series. The high order indefinite small terms are eliminated. This linearized method is called micro-deflection method.



Saturation and nonlinear

Section 2: Linear approximations of differential equations

A nonlinear function $y=f(x)$ is shown in the figure, the input is x, output is y, if each order derivatives of $y_0=f(x_0)$ in the operating point are existed, the Taylor series expansion about $y_0=f(x_0)$ is:



$$y = f(x) = f(x_0) + \left[\frac{\partial f(x)}{\partial x}\right]_{x_0} (x - x_0) + \frac{1}{2!}\left[\frac{\partial^2 f(x)}{\partial x^2}\right]_{x_0} (x - x_0)^2 + \ldots\ldots$$

Then, as a reasonable approximation, the equation can be rewritten as the linear equation:

$$y = f(x) \approx f(x_0) + \left[\frac{\partial f(x)}{\partial x}\right]_{x_0} (x - x_0)$$

$$\Delta y = y - y_0 = f(x) - f(x_0) = \left[\frac{\partial f(x)}{\partial x}\right]_{x_0} (x - x_0) = K\Delta x$$

K is the slope of the tangent of $y=f(x)$ at (x_0, y_0).

- **Homework: exercises 2-2(b), 2-3(a)**

- **Thanks!**