电动力学
Electrodynamics

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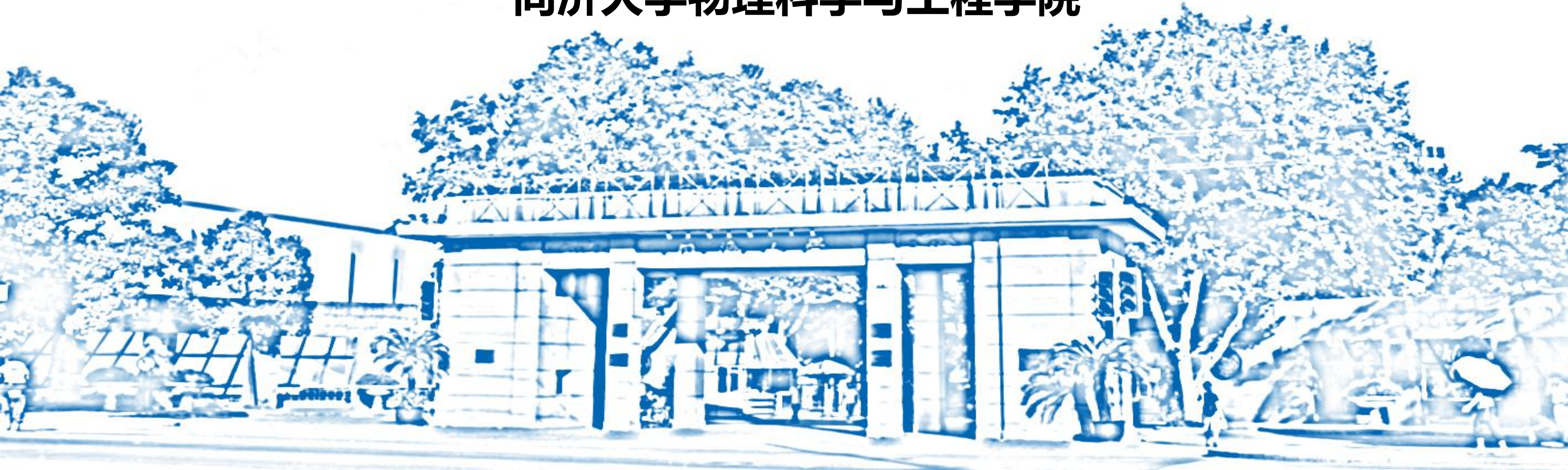
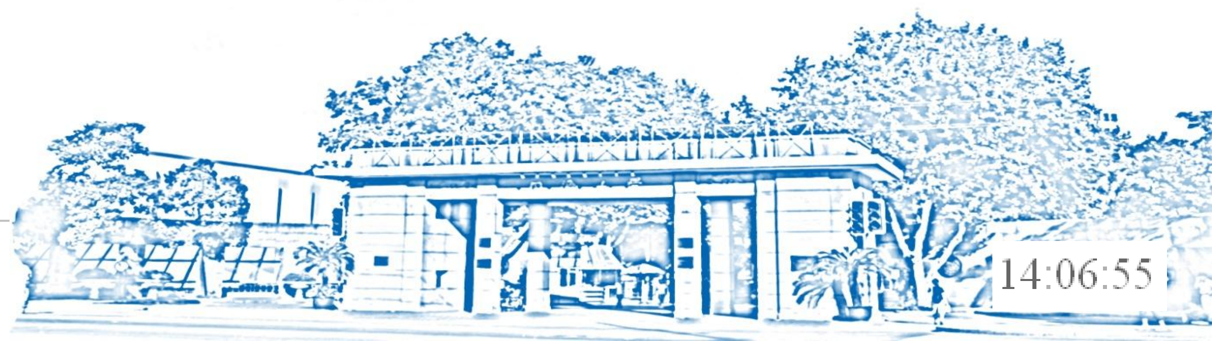


第四章 电磁波的传播



第四章 电磁波的传播

§2 电磁波在介质界面上的反射和折射

内 容 概 要

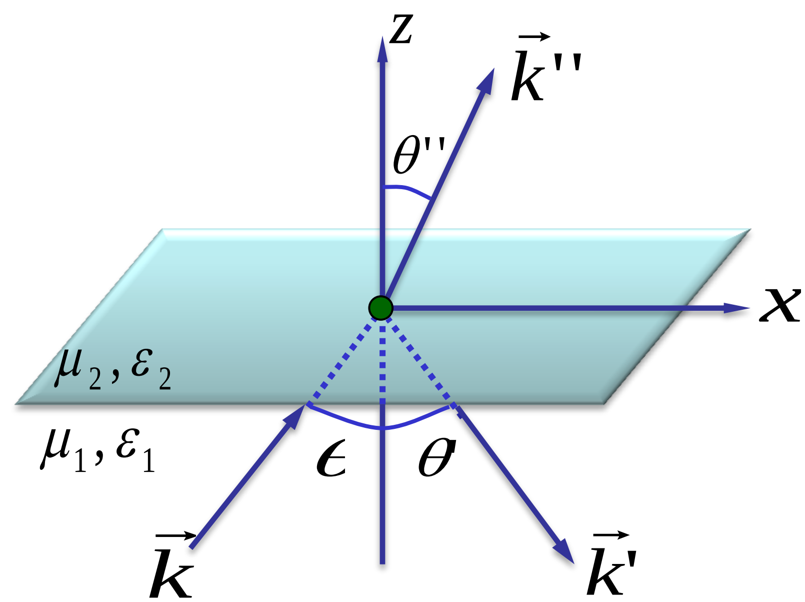
1. 反射和折射定律
2. 振幅公式 菲涅耳公式
3. 全反射

介质界面上的边值关系

	一般情况	无源、介质情况	时谐电磁场
方程	$\begin{cases}\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{H}=\vec{J}+\dfrac{\partial\vec{D}}{\partial t}\\ \nabla\cdot\vec{D}=\rho\\ \nabla\cdot\vec{B}=0\end{cases}$	$\begin{cases}\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{H}=\dfrac{\partial\vec{D}}{\partial t}\\ \nabla\cdot\vec{D}=0\\ \nabla\cdot\vec{B}=0\end{cases}$	$\begin{cases}\nabla\times\vec{E}=i\omega\vec{B}\\ \nabla\times\vec{H}=-i\omega\vec{D}\end{cases}$ $\vec{B}$、$\vec{D}$由$\vec{E}$、$\vec{H}$表示
边界条件	$\begin{cases}\vec{n}\times(\vec{E}_2-\vec{E}_1)=0\\ \vec{n}\times(\vec{H}_2-\vec{H}_1)=\vec{\alpha}\\ \vec{n}\cdot(\vec{D}_2-\vec{D}_1)=\sigma\\ \vec{n}\cdot(\vec{B}_2-\vec{B}_1)=0\end{cases}$	$\begin{cases}\vec{n}\times(\vec{E}_2-\vec{E}_1)=0\\ \vec{n}\times(\vec{H}_2-\vec{H}_1)=0\\ \vec{n}\cdot(\vec{D}_2-\vec{D}_1)=0\\ \vec{n}\cdot(\vec{B}_2-\vec{B}_1)=0\end{cases}$	$\begin{cases}\vec{n}\times(\vec{E}_2-\vec{E}_1)=0\\ \vec{n}\times(\vec{H}_2-\vec{H}_1)=0\end{cases}$

一、反射和折射定律

考察两介质界面为无限大平面



（1）入射波（介质 1 内）：

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\omega = \omega(k) \qquad \vec{B} = \frac{\vec{k}\times\vec{E}_0}{\omega} e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

（2）反射波（介质 1 内）：$\vec{E}' = \vec{E}'_0 e^{i(\vec{k}'\cdot\vec{x}-\omega t)}$

$$\vec{E}' = \vec{R}_0 e^{i(\vec{k}'\cdot\vec{x}-\omega' t)} \qquad \omega' = \omega'(k') \qquad \vec{B}' = \frac{\vec{k}'\times\vec{R}_0}{\omega'} e^{i(\vec{k}'\cdot\vec{x}-\omega' t)}$$

（3）折射波（介质 2 内）：$\vec{E}'' = \vec{E}''_0 e^{i(\vec{k}''\cdot\vec{x}-\omega t)}$

$$\vec{E}'' = \vec{T}_0 e^{i(\vec{k}''\cdot\vec{x}-\omega'' t)} \qquad \omega'' = \omega''(k'') \qquad \vec{B}'' = \frac{\vec{k}''\times\vec{T}_0}{\omega''} e^{i(\vec{k}''\cdot\vec{x}-\omega'' t)}$$

由电场边界条件

$$\vec{n}\times\left(\vec{E}_0 e^{i\vec{k}\cdot\vec{x}}+\vec{E}_0' e^{i\vec{k}'\cdot\vec{x}}-\vec{E}_0'' e^{i\vec{k}''\cdot\vec{x}}\right)=0 \qquad (z=0)$$

$$\vec{n}\times\left[\vec{E}_0 e^{i(k_x x+k_y y)}+\vec{E}_0' e^{i(k_x' x+k_y' y)}-\vec{E}_0'' e^{i(k_x'' x+k_y'' y)}\right]=0$$

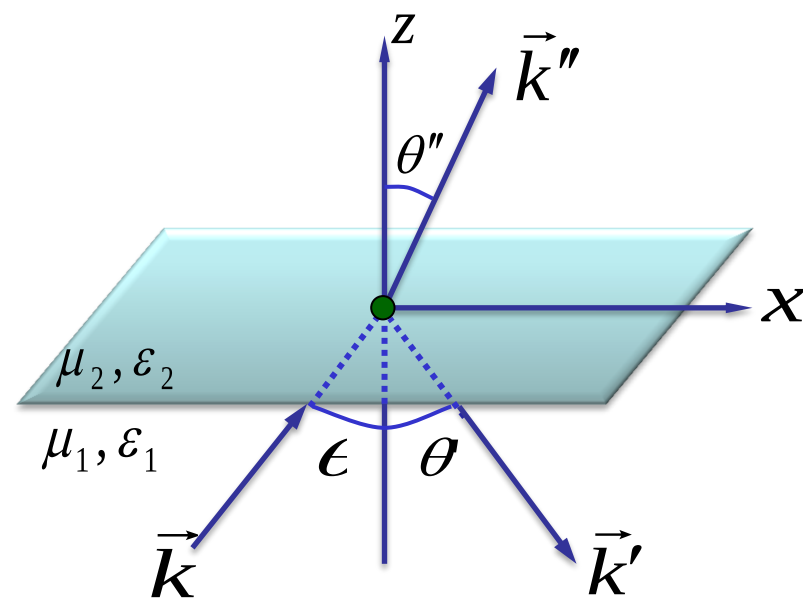
由于对任意(x, y)成立，有：

$$\begin{cases} k_x=k_x'=k_x'' \\ k_y=k_y'=k_y'' \end{cases}$$

取入射波波矢在(x, z)平面：

$$\vec{k}=k_x\vec{e}_x+k_z\vec{e}_z$$

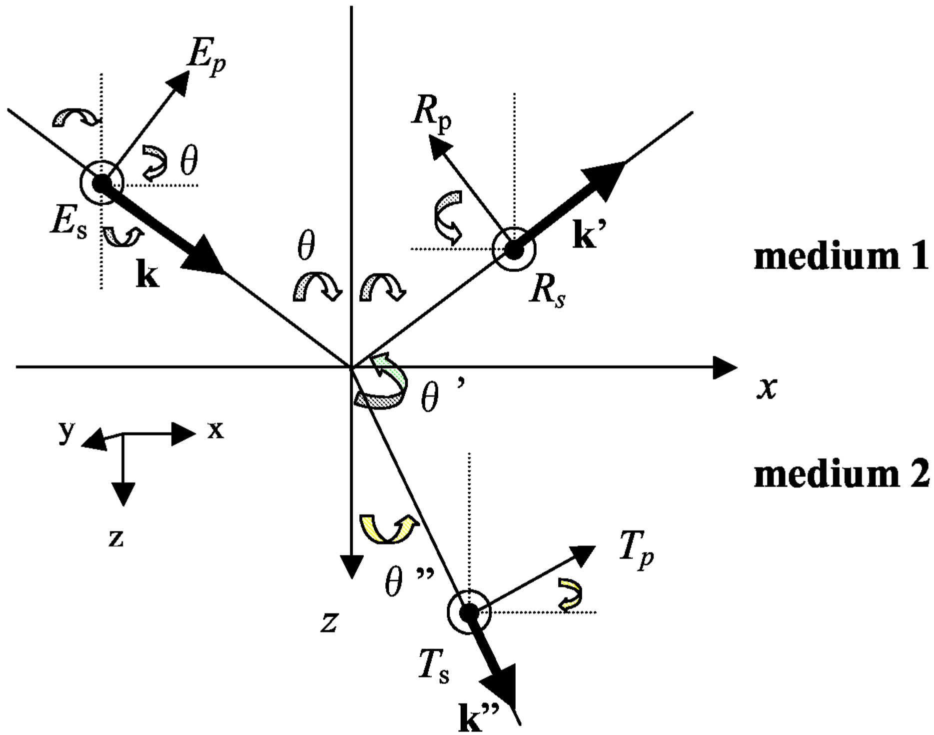
$$k_y=k_y'=k_y''=0$$



反射波矢、折射波矢与入射波矢在同一平面上(入射面)

三波矢共面！

$$\vec{k}=\left(k\sin\theta, 0, k\cos\theta\right)$$
$$\vec{k}'=\left(k'\sin\theta', 0, k'\cos\theta'\right)$$
$$\vec{k}''=\left(k''\sin\theta'', 0, k''\cos\theta''\right)$$



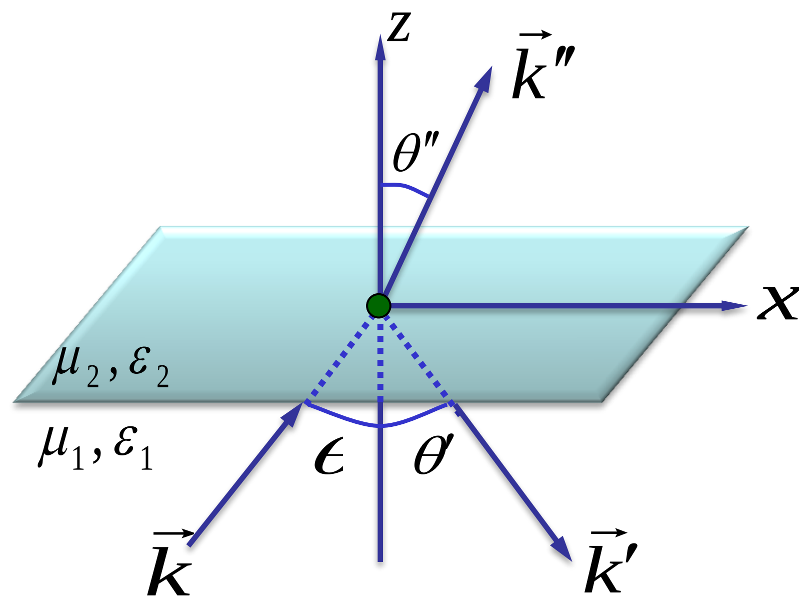
*P*波**电场**部分由*x*和*z*分量组成, 但*S*波只有*y*分量. 或者说*S*波**电场**部分垂直于入射面, *P*波平行于入射面.

波矢关系： $k_x = k'_x = k''_x$

$$k\sin\theta = k'\sin\theta' = k''\sin\theta''$$

$$k = k' = \omega\sqrt{\mu_1\varepsilon_1} \qquad k'' = \omega\sqrt{\mu_2\varepsilon_2}$$

$$\begin{cases} \theta' = \theta \\ \dfrac{\sin\theta}{\sin\theta''} = \sqrt{\dfrac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} \approx \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = n_{21} \end{cases}$$



反射定率

折射定律

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{v} = \frac{n\omega}{c}$$

$$k\sin\theta = n_1\frac{\omega}{c}\sin\theta = k''\sin\theta'' = n_2\frac{\omega}{c}\sin\theta''$$

$$n_1\sin\theta = n_2\sin\theta''$$

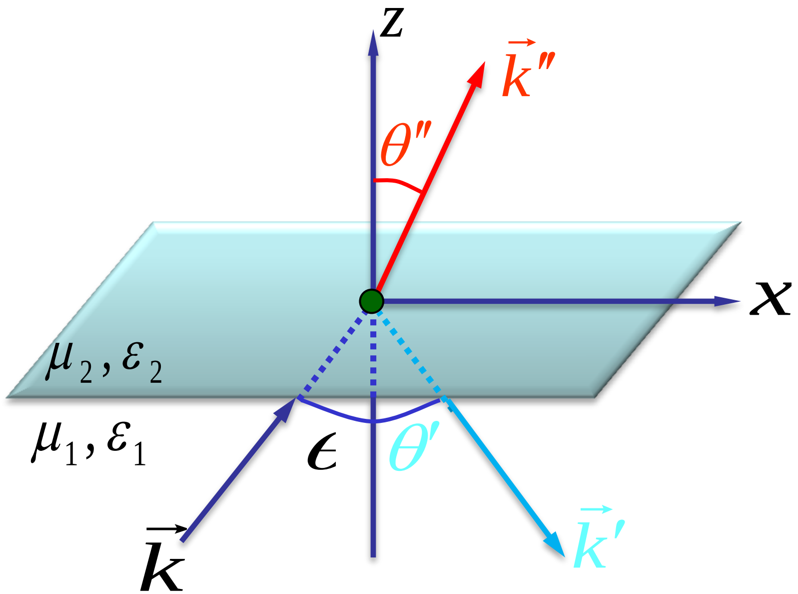
（1）入射波： $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

（2）反射波： $\vec{E}' = \vec{E}_0' e^{i(\vec{k}'\cdot\vec{r}-\omega t)}$

（3）折射波： $\vec{E}'' = \vec{E}_0'' e^{i(\vec{k}''\cdot\vec{r}-\omega t)}$

$z = 0$

$$\vec{n}\times(\vec{E}_0 e^{i\vec{k}\cdot\vec{r}} + \vec{E}_0' e^{i\vec{k}'\cdot\vec{r}} - \vec{E}_0'' e^{i\vec{k}''\cdot\vec{r}}) = 0$$

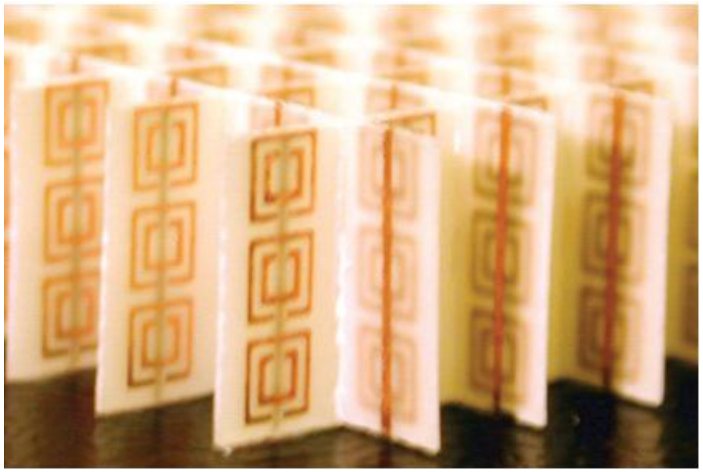


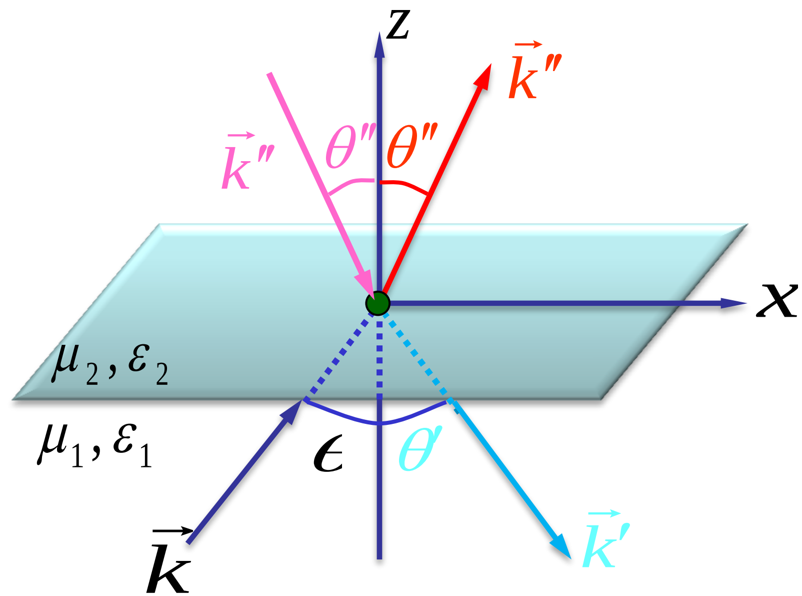
由于对任意(x, y)成立，有：

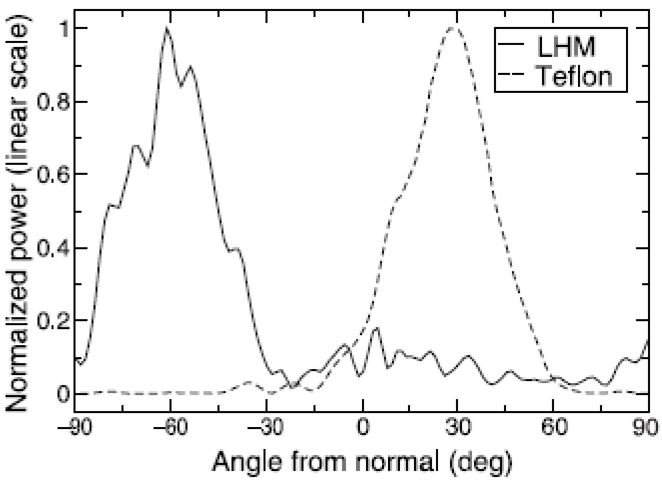
$$k_x = k_x' = k_x'', \quad k_y = k_y' = k_y'', \quad \omega = \omega' = \omega''$$

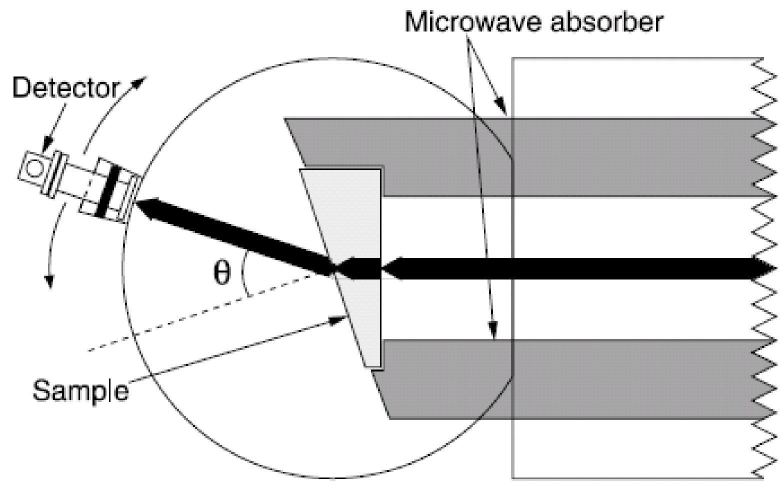
$$\begin{cases} \theta' = \theta \\ \dfrac{\sin\theta}{\sin\theta''} = \sqrt{\dfrac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} \approx \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = n_{21} \end{cases}$$

负折射





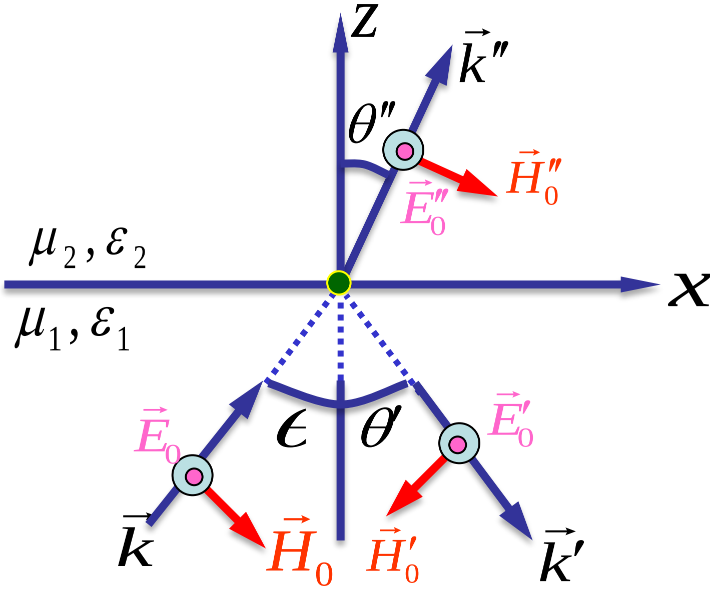


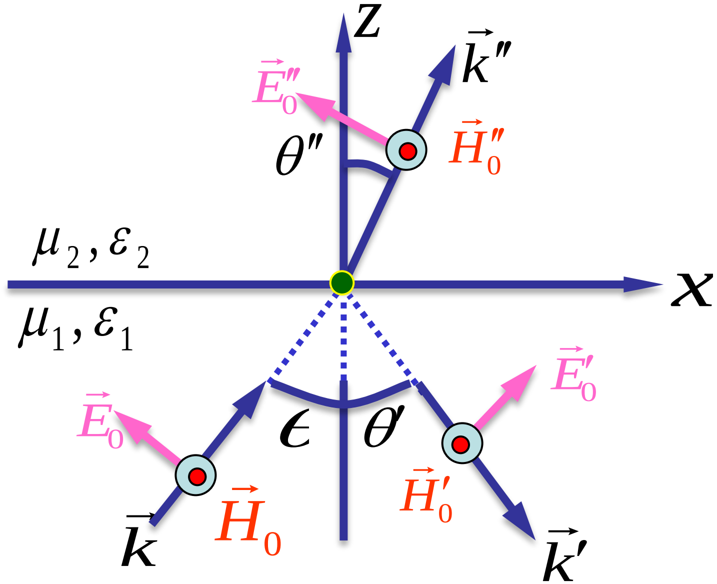


R. A. Shelby, *et al*. Science 292 77(2001)

二、振幅关系 菲涅耳公式

电磁波有两种偏振态，这里划分：
（1）垂直偏振，电场矢量垂直入射面（S分量）
（2）平行偏振，电场矢量在入射面内（P分量）





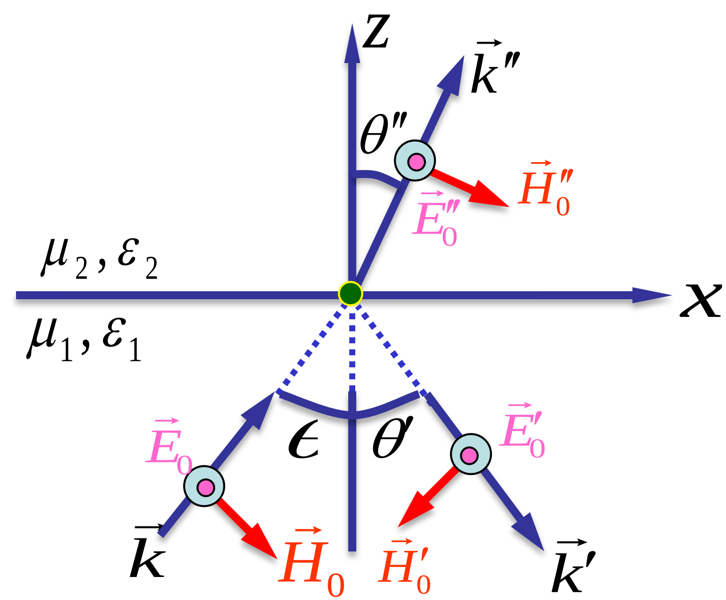
若入射波是垂直偏振，则反射、折射波也是垂直偏振
若入射波是平行偏振，则反射、折射波也是平行偏振

垂直偏振入射时振幅关系

$$\begin{cases} E_0 + E_0' - E_0'' = 0 \\ H_0 \cos\theta - H_0' \cos\theta' - H_0'' \cos\theta'' = 0 \end{cases}$$

$$H_0 = \sqrt{\varepsilon/\mu}\, E_0 \approx \sqrt{\varepsilon/\mu_0}\, E_0$$

$$\sqrt{\varepsilon_1}(E_0 - E_0')\cos\theta - \sqrt{\varepsilon_2}\, E_0''\cos\theta'' = 0$$



$$\begin{cases} \dfrac{E_0'}{E_0} = \dfrac{\sqrt{\varepsilon_1}\cos\theta - \sqrt{\varepsilon_2}\cos\theta''}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''} = -\dfrac{\sin(\theta - \theta'')}{\sin(\theta + \theta'')} \\ \dfrac{E_0''}{E_0} = \dfrac{2\sqrt{\varepsilon_1}\cos\theta}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2}\cos\theta''} = \dfrac{2\cos\theta\sin\theta''}{\sin(\theta + \theta'')} \end{cases}$$

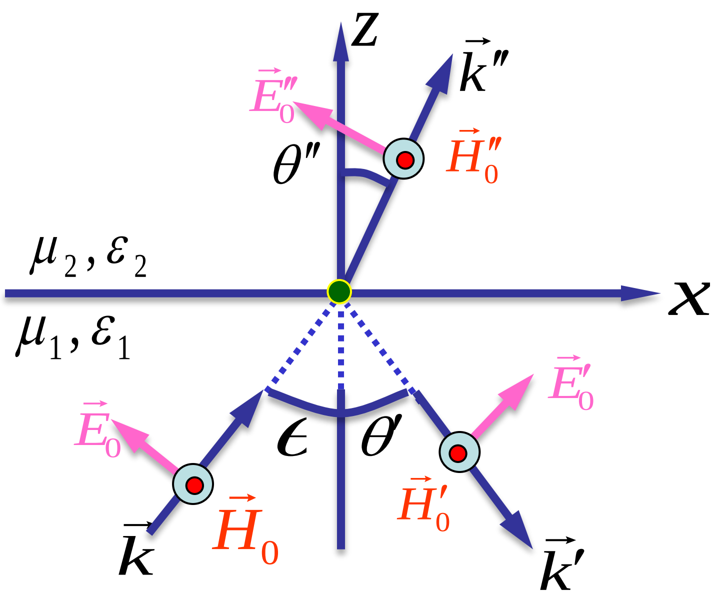
$$\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{\sin\theta''}{\sin\theta}$$

平行偏振入射时振幅关系

$$\begin{cases} -E_0\cos\theta + E_0'\cos\theta' + E_0''\cos\theta'' = 0 \\ H_0 + H_0' - H_0'' = 0 \end{cases}$$

$$H_0 = \sqrt{\varepsilon/\mu}\,E_0 \approx \sqrt{\varepsilon/\mu_0}\,E_0$$

$$\sqrt{\varepsilon_1}(E_0 + E_0') - \sqrt{\varepsilon_2}E_0'' = 0$$



$$\begin{cases} \dfrac{E_0'}{E_0} = \dfrac{\sqrt{\varepsilon_2}\cos\theta - \sqrt{\varepsilon_1}\cos\theta''}{\sqrt{\varepsilon_2}\cos\theta + \sqrt{\varepsilon_1}\cos\theta''} = \dfrac{\sin\theta\cos\theta - \sin\theta''\cos\theta''}{\sin\theta\cos\theta + \sin\theta''\cos\theta''} = \dfrac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')} \\ \dfrac{E_0''}{E_0} = \dfrac{2\sqrt{\varepsilon_1}\cos\theta}{\sqrt{\varepsilon_2}\cos\theta + \sqrt{\varepsilon_1}\cos\theta''} = \dfrac{2\cos\theta\sin\theta''}{\sin\theta\cos\theta + \sin\theta''\cos\theta''} = \dfrac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')\cos(\theta-\theta'')} \end{cases}$$

$$\sin\theta\cos\theta \pm \sin\theta''\cos\theta'' = \sin(\theta \pm \theta'')\cos(\theta \mp \theta'')$$

菲涅耳（Fresnel）公式

$$\begin{cases} \dfrac{E'_\perp}{E_\perp} = -\dfrac{\sin(\theta-\theta'')}{\sin(\theta+\theta'')} \\ \dfrac{E''_\perp}{E_\perp} = \dfrac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')} \end{cases} \qquad \begin{cases} \dfrac{E'_{//}}{E_{//}} = \dfrac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')} \\ \dfrac{E''_{//}}{E_{//}} = \dfrac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')\cos(\theta-\theta'')} \end{cases}$$

Brewster定律：

$$\theta+\theta'' = \frac{\pi}{2} \qquad \theta_{Brewster} = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

当入射为布儒斯特角时, E只有S波反射, 是完全偏振光.

半波损失：

当 $\theta > \theta''$ 时，$E'_{0\perp}/E_{0\perp} < 0$ 反射波与入射波相位相反，或相当于半个波长的光程差。

定义

$$r_p=\frac{R_p}{E_p}\qquad t_p=\frac{T_p}{E_p}\qquad r_s=\frac{R_s}{E_s}\qquad t_s=\frac{T_s}{E_s}$$

S波

$$\Rightarrow\begin{cases}r_s=\dfrac{n_1\cos\theta-n_2\cos\theta''}{n_1\cos\theta+n_2\cos\theta''}=-\dfrac{\sin(\theta-\theta'')}{\sin(\theta+\theta'')}\\[2ex] t_s=\dfrac{2n_1\cos\theta}{n_1\cos\theta+n_2\cos\theta''}=\dfrac{\sin 2\theta''}{\sin(\theta+\theta'')}\end{cases}$$

P波

$$\Rightarrow\begin{cases}r_p=\dfrac{-n_1\cos\theta''+n_2\cos\theta}{n_1\cos\theta''+n_2\cos\theta}=\dfrac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')}\\[2ex] t_p=\dfrac{2n_1\cos\theta}{n_1\cos\theta''+n_2\cos\theta}=\dfrac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')\cos(\theta-\theta'')}\end{cases}$$

菲涅耳公式

平均玻印亭矢量已由流矢量给出

$$\left\langle \vec{E}\times\vec{H}\right\rangle = nE^2/(2c\mu_0)$$

定义**反射率（Reflectance）**

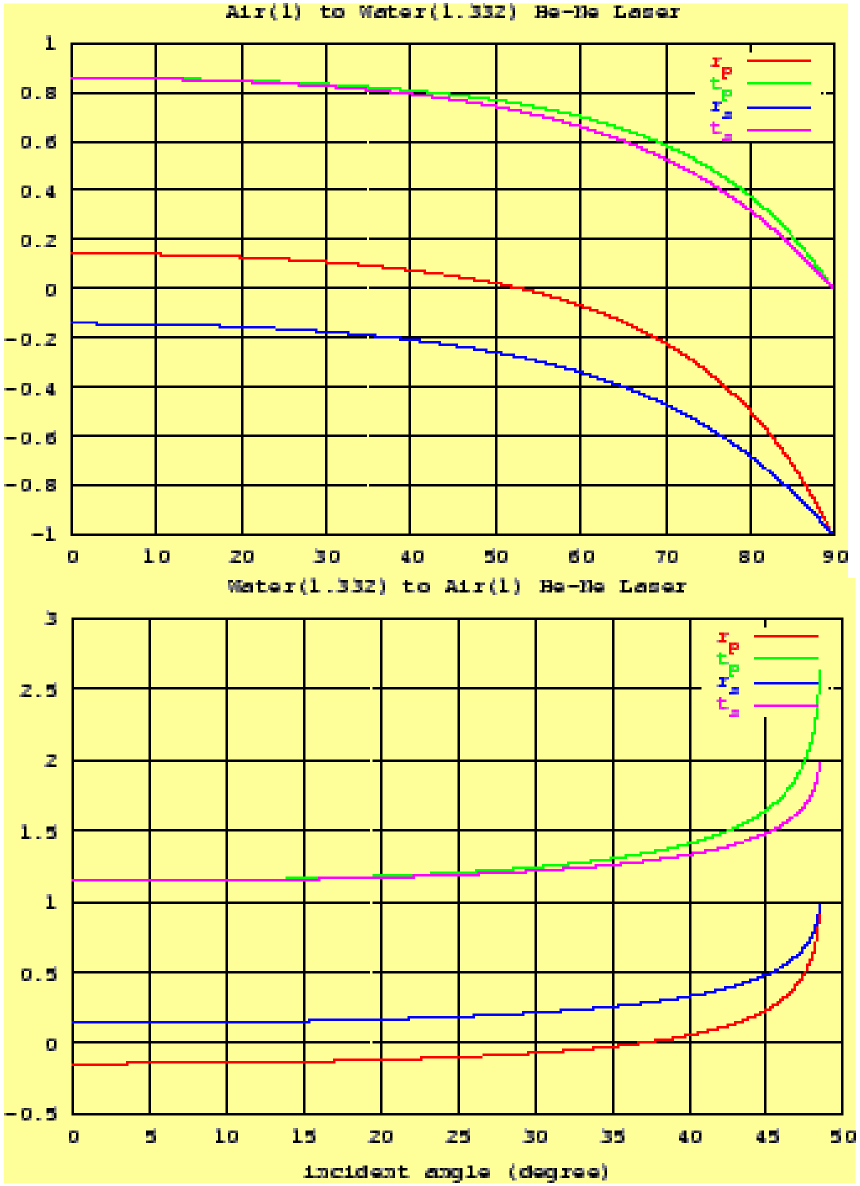
$$R=\frac{I'\cos\theta'}{I\cos\theta}=\frac{I'}{I}=\frac{n_1R^2/(2c\mu_0)}{n_1E^2/(2c\mu_0)}=r^2$$

反射系数

定义**透射率**

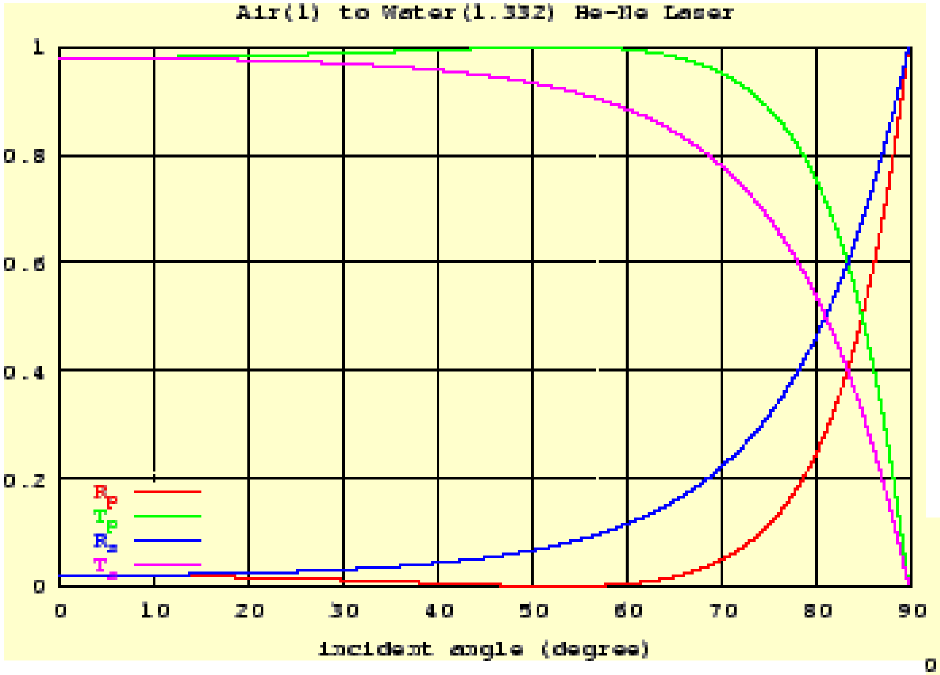
$$T=\frac{I''\cos\theta''}{I\cos\theta}=\frac{n_2T^2\cos\theta''}{n_1E^2\cos\theta}=\frac{n_2\cos\theta''}{n_1\cos\theta}t^2$$

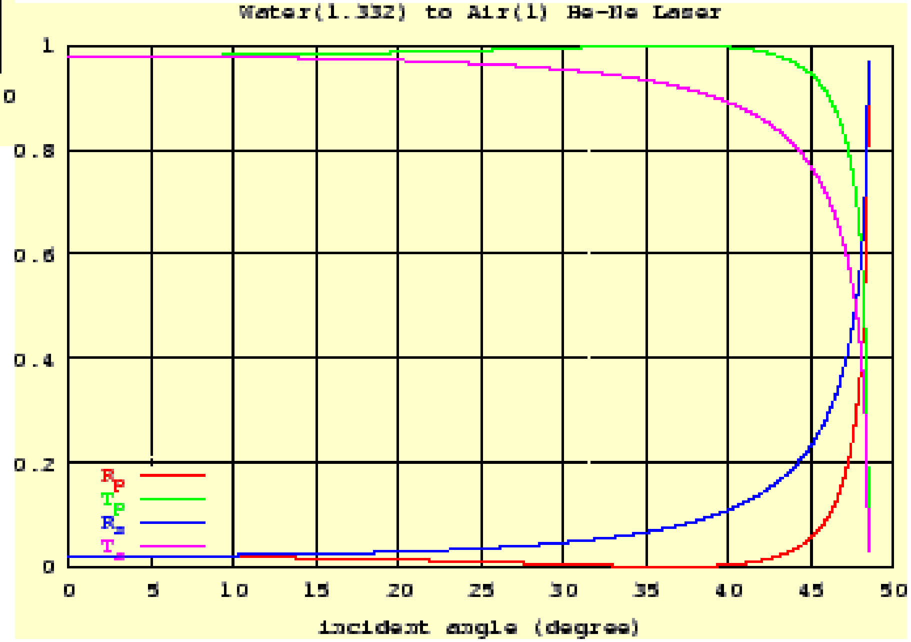
透射系数



$$\begin{cases} \dfrac{E'_\perp}{E_\perp} = -\dfrac{\sin(\theta-\theta'')}{\sin(\theta+\theta'')} \\ \dfrac{E''_\perp}{E_\perp} = \dfrac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')} \end{cases}$$

$$\begin{cases} \dfrac{E'_{//}}{E_{//}} = \dfrac{\tan(\theta-\theta'')}{\tan(\theta+\theta'')} \\ \dfrac{E''_{//}}{E_{//}} = \dfrac{2\cos\theta\sin\theta''}{\sin(\theta+\theta'')\cos(\theta-\theta'')} \end{cases}$$


Air(1) to Water(1.332) He-Ne Laser
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
incident angle (degree)


Water(1.332) to Air(1) He-Ne Laser
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35
40
45
50
incident angle (degree)

三、全反射

如果从光密到光疏, 入射角满足

$$\theta_c = \sin^{-1}\frac{n_2}{n_1} \qquad (n_2 < n_1)$$

则折射角为90度,透射波沿界面掠过.

如果 $\theta > \theta_c$

$$\cos\theta'' = \sqrt{1-\sin^2\theta''} = i\sqrt{\sin^2\theta'' - 1}$$

$$= i\sqrt{(n_1/n_2)^2\sin^2\theta - 1}$$

$$n_1 \sin\theta = n_2 \sin\theta''$$

折射波：

$$\vec{E}'' = \vec{T}_0 e^{i\left(\vec{k}''\cdot\vec{r}-\omega''t\right)} = \vec{T}_0 e^{i\left(\vec{k}''\sin\theta''x+k''\cos\theta''z-\omega''t\right)}$$

$$= \vec{T}_0 e^{i\left(\vec{k}''\sin\theta''x-\omega''t\right)} e^{-k''\sqrt{(n_1/n_2)^2\sin^2\theta-1}z}$$

$$= \vec{T}_0 e^{i\frac{n_2\omega\sin\theta''}{c}x} e^{-i\omega''t} e^{-\frac{n_2\omega\sqrt{(n_1/n_2)^2\sin^2\theta-1}}{c}z}$$

这是沿着x轴方向传播的电磁波, 它的场强沿z轴方向指数衰减. 因此只能存在于界面附近一薄层内,该层厚度大约为

全反射现象

$$\kappa^{-1} \approx \frac{c}{n_2\omega\sqrt{(n_1/n_2)^2\sin^2\theta-1}} = \frac{\lambda_1}{2\pi\sqrt{\sin^2\theta-(n_2/n_1)^2}}$$

玻印亭矢量S在介质2中沿z方向

$$\vec{z}\cdot\overline{\vec{S}}=\frac{1}{2\mu}\mathrm{Re}\left(\vec{E}\times\vec{B}''^{*}\right)\cdot\vec{z}$$

$$=\frac{1}{2\mu\omega}\mathrm{Re}\left[\vec{E}\times\left(\vec{k}''\times\vec{E}''\right)^{*}\right]\cdot\vec{z}$$

$$=\frac{1}{2\mu\omega}\mathrm{Re}\left[\vec{k}''\left|\vec{E}''\right|^{2}-\vec{E}''^{*}\left(\vec{k}''\cdot\vec{E}''\right)\right]\cdot\vec{z}$$

$$=\frac{1}{2\mu\omega}\mathrm{Re}\left(\vec{z}\cdot\vec{k}''\left|\vec{E}''\right|^{2}\right)=0$$

(因为k''在z方向上投影为虚数)

这种波叫做**消散波**,因为不能传播能量.

对于全反射, 入射和反射波振幅相同但相位发生了变化

$$\cos\theta'' = i\sqrt{\frac{\sin^2\theta}{n_{21}^2}-1}$$

$$S波:\ \frac{E_0'}{E_0} = \frac{n_1\cos\theta - n_2\cos\theta''}{n_1\cos\theta + n_2\cos\theta''}$$

$$\frac{E_0'}{E_0} = \frac{\cos\theta - i\sqrt{\sin^2\theta - n_{21}^2}}{\cos\theta + i\sqrt{\sin^2\theta - n_{21}^2}} = e^{-2i\phi}$$

$$\mathrm{tg}\,\phi = \frac{\sqrt{\sin^2\theta - n_{21}^2}}{\cos\theta}$$

E' 和E振幅相等，但相位不同，因此反射波与入射波的瞬时能流值是不同的。只是 S_z'' 的平均值为零，其瞬时值不为零。由此可见，在全反射过程中第二介质是起作用的。在半周内，电磁能量透入第二介质，在界面附近薄层内储存起来，在另一半周内，该能量释放出来变为反射波能量。