Chapter 2
Mathematical Background

Some Concepts-1

Set: A set is a collection of distinguishable members or elements. A set has no duplicates.

Bag: A collection of elements with no order (like a set), but with duplicate-valued elements.

Sequence: a collection of elements with an order, and which may contain duplicate-valued elements.

Some Concepts-2

- Recursion
  - An algorithm is recursive if it calls itself to do part of its work.
  - In general, a recursive algorithm must have two parts: the base case and the recursive part.
- Logarithms
  - A logarithm of base b for value y is the power to which b is raised to get y. $\log_b y = x$
  - We almost always use log to base 2. That is our default base.

Some Concepts-3

- Summations
  - Analyze running time costs for programs with loops
  - Equation (2.1)~(2.9)

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

$$\sum_{i=1}^{n} i^2 = \frac{2n^3+3n^2+n}{6} = \frac{n(2n+1)(n+1)}{6}.$$

$$\sum_{i=1}^{\log n} n = n \log n.$$

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \text{ for } 0 < a < 1.$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1} \text{ for } a \neq 1.$$

$$\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$$

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

$$\sum_{i=0}^{\log n} 2^i = 2^{\log n+1} - 1 = 2n - 1$$

$$\sum_{i=1}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$$

Estimation Techniques

Known as “back of the envelope” or “back of the napkin” calculation

1. Determine the major parameters that affect the problem.
2. Derive an equation that relates the parameters to the problem.
3. Select values for the parameters, and apply the equation to yield an estimated solution.

Estimation Example

- How many library bookcases does it take to store books totaling one million pages?

Estimate:

- Pages/inch: Guess 500
- Feet/shelf: Guess 4 (actually 3)
- Shelves/bookcase: Guess 5 (actually 7)

Units check: pages/in * ft/shelf * shelf/bookcase
$\Rightarrow$pages/bookcase

•Another Example: Is it more economical to buy a car that gets 20 miles per gallon, or one that gets 30 miles per gallon but costs $2000 more?

Homework

- 课后习题：2.14