第十六章　二端口网络

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本章重点：

- 二端口网络四种参数方程及参数矩阵的求解。
- 二端口网络的等效电路。
- 二端口网络的级联及其参数的求解。
- 二端口网络的转移函数和阻抗变换。

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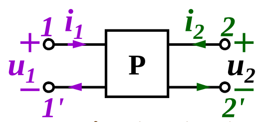
§16-1 二端口网络



已知：	u_1 u_2	i_1 i_2	i_1 u_1	i_2 u_2	i_1 u_2	i_2 u_1
未知：	i_1 i_2	u_1 u_2	i_2 u_2	i_1 u_1	u_1 i_2	u_2 i_1
对应参数：	Y	Z	T'	T	H	G

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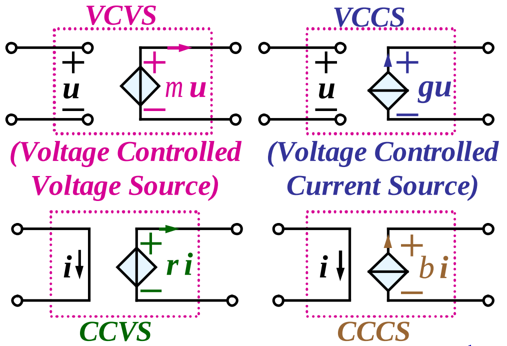
学习过的二端口元件：



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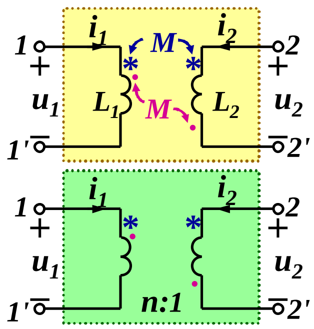
学习过的二端口元件：



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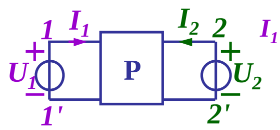
§16-2 二端口的方程和参数

一、Y参数　　叠加定理：



$\dot I_1 = \dot I_1' + \dot I_1''$　$\dot I_2 = \dot I_2' + \dot I_2''$

$\dot U_1$ 作用的响应分量　　$\dot U_2$ 作用的响应分量

Y参数方程
$$\begin{cases} \dot I_1 = Y_{11}\dot U_1 + Y_{12}\dot U_2 \\ \dot I_2 = Y_{21}\dot U_1 + Y_{22}\dot U_2 \end{cases}$$

Y参数矩阵：

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\begin{bmatrix} \dot I_1 \\ \dot I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot U_1 \\ \dot U_2 \end{bmatrix}$$

$$\mathbf{I} = \mathbf{Y}\mathbf{U}$$

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$$Y_{11}=\frac{I_1}{U_1}\Big|_{U_2=0}\qquad Y_{21}=\frac{I_2}{U_1}\Big|_{U_2=0}$$

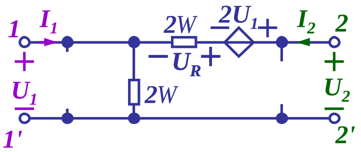
$$Y_{12}=\frac{I_1}{U_2}\Big|_{U_1=0}\qquad Y_{22}=\frac{I_2}{U_2}\Big|_{U_1=0}$$

短路导纳矩阵

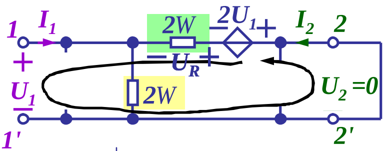
若P不含受控源 $Y_{12}=Y_{21}$ 网络互易 三个独立参数

若P电气对称 & $Y_{11}=Y_{22}$ 两个独立参数

补例1 求出图示电路的Y参数矩阵



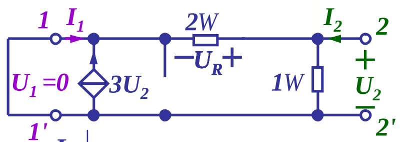
补例1 求出图示电路的Y参数矩阵



解：

$$Y_{11}=\frac{I_1}{U_1}\Big|_{U_2=0}=\frac{0.5U_1-0.5U_R}{U_1}=2S$$

$$Y_{21}=\frac{I_2}{U_1}\Big|_{U_2=0}=\frac{0.5U_R}{U_1}=-1.5S$$

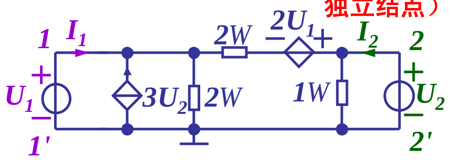


$$Y_{12}=\frac{I_1}{U_2}\Big|_{U_1=0}=\frac{-3U_2-0.5U_R}{U_2}=-3.5S$$

$$Y_{22}=\frac{I_2}{U_2}\Big|_{U_1=0}=\frac{U_2+0.5U_R}{U_2}=1.5S$$

$$\mathbf{Y}=\begin{bmatrix}2 & -3.5\\ -1.5 & 1.5\end{bmatrix}$$

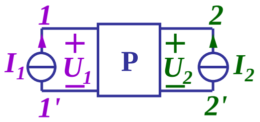
可列写结点方程求Y参数矩阵（电路只有两个独立结点）



$$\begin{cases}(0.5+0.5)U_1-0.5U_2=I_1+3U_2-U_1\\ -0.5U_1+(1+0.5)U_2=I_2+U_1\end{cases}$$

$$\begin{cases}2U_1-3.5U_2=I_1\\ -1.5U_1+1.5U_2=I_2\end{cases}\qquad \mathbf{YU}=\mathbf{I}$$

二、Z参数



Z参数方程

$$\begin{cases}U_1=Z_{11}I_1+Z_{12}I_2\\ U_2=Z_{21}I_1+Z_{22}I_2\end{cases}$$

Z参数矩阵：

$$\mathbf{Z}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\qquad \begin{bmatrix}U_1\\ U_2\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}$$

$$\mathbf{U}=\mathbf{ZI}\qquad \mathbf{I}=\mathbf{YU}\ \Rightarrow\ \mathbf{Z}=\mathbf{Y}^{-1}$$

$$Z_{11}=\frac{U_1}{I_1}\Big|_{I_2=0} \qquad Z_{21}=\frac{U_2}{I_1}\Big|_{I_2=0}$$

$$Z_{12}=\frac{U_1}{I_2}\Big|_{I_1=0} \qquad Z_{22}=\frac{U_2}{I_2}\Big|_{I_1=0}$$

开路阻抗矩阵

网络互易

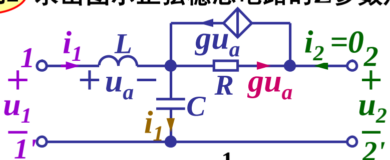
若P不含受控源 $Z_{12}=Z_{21}$ 三个独立参数

& $Z_{11}=Z_{22}$

若P电气对称

两个独立参数

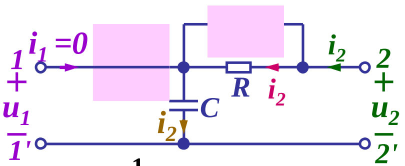
补例2 求出图示正弦稳态电路的Z参数矩阵



解：

$$Z_{11}=\frac{\dot{U}_1}{\dot{I}_1}\Big|_{\dot{I}_2=0}=\frac{j\omega L\dot{I}_1+\frac{1}{j\omega C}\dot{I}_1}{\dot{I}_1}=j\omega L+\frac{1}{j\omega C}$$

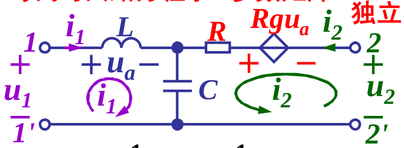
$$Z_{21}=\frac{\dot{U}_2}{\dot{I}_1}\Big|_{\dot{I}_2=0}=\frac{-R\times g\dot{U}_a+\frac{1}{j\omega C}\dot{I}_1}{\dot{I}_1}=-j\omega LRg+\frac{1}{j\omega C}$$



$$Z_{12}=\frac{\dot{U}_1}{\dot{I}_2}\Big|_{\dot{I}_1=0}=\frac{\frac{1}{j\omega C}\dot{I}_2}{\dot{I}_2}=\frac{1}{j\omega C} \qquad Z=\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}$$

$$Z_{22}=\frac{\dot{U}_2}{\dot{I}_2}\Big|_{\dot{I}_1=0}=\frac{R\dot{I}_2+\frac{1}{j\omega C}\dot{I}_2}{\dot{I}_2}=R+\frac{1}{j\omega C}$$

可列写回路方程求Z参数矩阵 （电路只有两个独立回路）

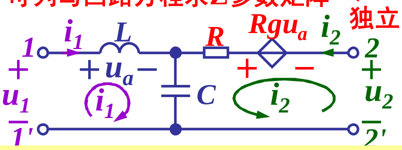


$$(j\omega L+\frac{1}{j\omega C})\dot{I}_1+\frac{1}{j\omega C}\dot{I}_2=\dot{U}_1$$

$$\frac{1}{j\omega C}\dot{I}_1+(R+\frac{1}{j\omega C})\dot{I}_2=\dot{U}_2+Rg\dot{U}_a$$

$$\dot{U}_a=j\omega L\dot{I}_1$$

可列写回路方程求Z参数矩阵 （电路只有两个独立回路）

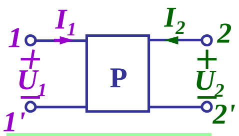


$$(j\omega L+\frac{1}{j\omega C})\dot{I}_1+\frac{1}{j\omega C}\dot{I}_2=\dot{U}_1$$

$$(-j\omega LRg+\frac{1}{j\omega C})\dot{I}_1+(R+\frac{1}{j\omega C})\dot{I}_2=\dot{U}_2$$

ZI = U

三、T参数(传输参数, A参数)



T参数方程

$$\begin{cases}U_1=A\,U_2+B\,(-I_2)\\ I_1=C\,U_2+D\,(-I_2)\end{cases}$$

$$\begin{bmatrix}U_1\\ I_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}U_2\\ -I_2\end{bmatrix}$$

T参数矩阵

$$\begin{cases}I_1=Y_{11}U_1+Y_{12}U_2\\ I_2=Y_{21}U_1+Y_{22}U_2\end{cases}$$

$$\begin{cases}U_1=-\frac{Y_{22}}{Y_{21}}U_2+\frac{1}{Y_{21}}I_2\\ I_1=(Y_{12}-\frac{Y_{11}Y_{22}}{Y_{21}})U_2+\frac{Y_{11}}{Y_{21}}I_2\end{cases}$$

$$T=\begin{bmatrix}-\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}}\\ Y_{12}-\frac{Y_{11}Y_{22}}{Y_{21}} & -\frac{Y_{11}}{Y_{21}}\end{bmatrix}$$

$$A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \qquad B = \left.\frac{U_1}{-I_2}\right|_{U_2=0}$$

$$C = \left.\frac{I_1}{U_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{U_2=0}$$

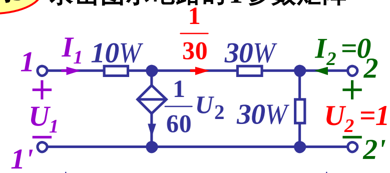
网络互易 $B \neq C$ $Y_{12} = Y_{21}$

$$AD - BC = \frac{Y_{11}Y_{22}}{Y_{21}^2} - \left(\frac{Y_{11}Y_{22}}{Y_{21}^2} - \frac{Y_{12}}{Y_{21}}\right) = \frac{Y_{12}}{Y_{21}}$$

$$AD - BC = 1$$

网络对称 $\because Y_{11} = Y_{22}$ $\therefore A = D$

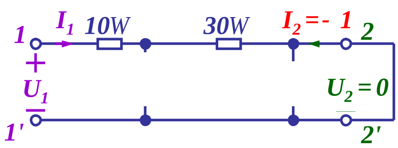
补例3 求出图示电路的T参数矩阵



解:

$$A = \left.\frac{U_1}{U_2}\right|_{I_2=0} = 2.5 \qquad C = \left.\frac{I_1}{U_2}\right|_{I_2=0} = \frac{1}{20}$$

$$I_1 = \frac{1}{30} + \frac{1}{60} = \frac{1}{20} \qquad U_1 = 10I_1 + 2U_2 = 2.5$$



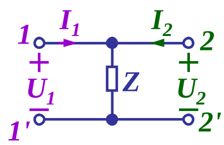
$$B = \left.\frac{U_1}{-I_2}\right|_{U_2=0} = 40 \qquad D = \left.\frac{I_1}{-I_2}\right|_{U_2=0} = 1$$

$$U_1 = -40I_2 = 40$$

$$I_1 = -I_2 = 1$$

$$\mathbf{T} = \begin{bmatrix} 2.5 & 40 \\ 1/20 & 1 \end{bmatrix}$$

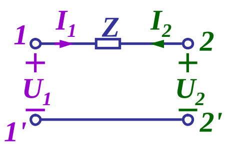
简单二端口网络的参数



Y不存在

$$\mathbf{Z} = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$

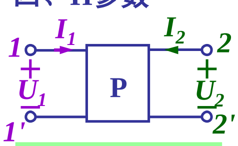
简单二端口网络的参数



Z不存在

$$\mathbf{Y} = \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

四、H参数



$$\begin{cases} I_1 = Y_{11}U_1 + Y_{12}U_2 \\ I_2 = Y_{21}U_1 + Y_{22}U_2 \end{cases}$$

$$\begin{cases} U_1 = \frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}U_2 \\ I_2 = \frac{Y_{21}}{Y_{11}}I_1 + \left(Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}}\right)U_2 \end{cases}$$

H参数方程

$$\begin{cases} U_1 = H_{11}I_1 + H_{12}U_2 \\ I_2 = H_{21}I_1 + H_{22}U_2 \end{cases}$$

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

H参数矩阵

$$\mathbf{H} = \begin{bmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}} \end{bmatrix}$$

$$H_{11}=\frac{U_1}{I_1}\Big|_{U_2=0}\qquad H_{12}=\frac{U_1}{U_2}\Big|_{I_1=0}$$

$$H_{21}=\frac{I_2}{I_1}\Big|_{U_2=0}\qquad H_{22}=\frac{I_2}{U_2}\Big|_{I_1=0}$$

网络互易 $Y_{12}=Y_{21}$ $H_{12}=-H_{21}$

网络对称 $\Rightarrow Y_{11}=Y_{22}$

$\Rightarrow H_{11}H_{22}-H_{12}H_{21}=1$

$$H_{11}H_{22}-H_{12}H_{21}=\frac{Y_{22}}{Y_{11}}-\frac{Y_{12}Y_{21}}{Y_{11}^2}-(-\frac{Y_{11}Y_{22}}{Y_{21}^2})=\frac{Y_{22}}{Y_{11}}$$

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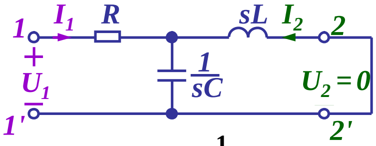
补例4 求出图示电路的H参数矩阵



解:

$$H_{11}=\frac{U_1}{I_1}\Big|_{U_2=0}=\frac{(R+sL//\frac{1}{sC})I_1}{I_1}=R+\frac{sL}{1+s^2LC}$$

$$H_{21}=\frac{I_2}{I_1}\Big|_{U_2=0}=\frac{-\frac{1/sC}{sL+1/sC}I_1}{I_1}=\frac{-1}{1+s^2LC}$$

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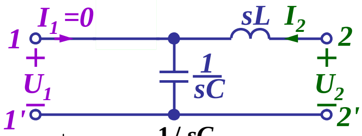

$$H_{12}=\frac{U_1}{U_2}\Big|_{I_1=0}=\frac{\frac{1/sC}{sL+1/sC}U_2}{U_2}=\frac{1}{1+s^2LC}$$

$$H_{22}=\frac{I_2}{U_2}\Big|_{I_1=0}=\frac{\frac{1}{sL+1/sC}U_2}{U_2}=\frac{sC}{1+s^2LC}$$

$$H=\begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}$$

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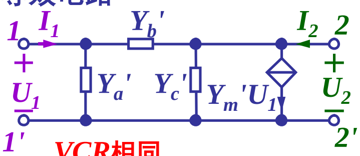
§ 16-3 二端口的等效电路

一、 p型等效电路



VCR相同

$$\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}=\begin{bmatrix}Y_a'+Y_b' & -Y_b'\\ -Y_b'+Y_m' & Y_b'+Y_c'\end{bmatrix}$$

$$\begin{cases}Y_a'=Y_{11}+Y_{12}\\ Y_b'=-Y_{12}\\ Y_c'=Y_{22}+Y_{12}\\ Y_m'=Y_{21}-Y_{12}\end{cases}$$

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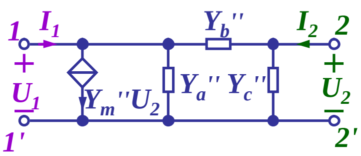

$$\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}=\begin{bmatrix}Y_a''+Y_b'' & -Y_b''+Y_m''\\ -Y_b'' & Y_b''+Y_c''\end{bmatrix}$$

$$\begin{cases}Y_a''=Y_{11}+Y_{21}\\ Y_b''=-Y_{21}\\ Y_c''=Y_{22}+Y_{21}\\ Y_m''=Y_{12}-Y_{21}\end{cases}$$

网络互易 $Y_{12}=Y_{21}$

网络对称 无受控源

$\Rightarrow Y_{11}=Y_{22}$ $Y_a=Y_c$

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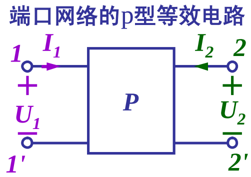
补例1

当 U_1=1V, U_2=0 时, I_1=5A, I_2=- 3A;当 U_1=2V, U_2=1V 时, I_1=8A, I_2=- 3A;问当 U_1=4V, U_2=4V 时, I_1=?, I_2=?并作出二端口网络的p型等效电路



解:

$$I_1=Y_{11}U_1+Y_{12}U_2$$

$$I_2=Y_{21}U_1+Y_{22}U_2$$

由已知条件得: $Y=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}=\begin{bmatrix}5 & -2\\ -3 & 3\end{bmatrix}$

当 U_1=4V, U_2=4V 时,

$I_1=5U_1-2U_2$=12A $I_2=-3U_1+3U_2=0$

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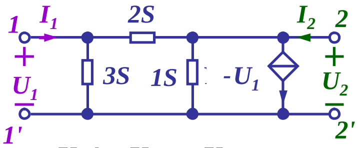
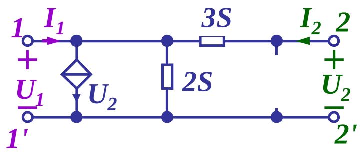
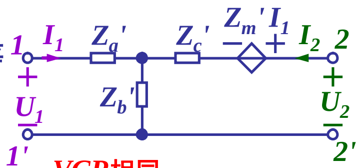


$$\begin{cases} Y_a' = Y_{11} + Y_{12} = 3S \\ Y_b' = -Y_{12} = 2S \\ Y_c' = Y_{22} + Y_{12} = 1S \\ Y_m' = Y_{21} - Y_{12} = -1S \end{cases}$$



$$\begin{cases} Y_a'' = Y_{11} + Y_{21} = 2S \\ Y_b'' = -Y_{21} = 3S \\ Y_c'' = Y_{22} + Y_{21} = 0 \\ Y_m'' = Y_{12} - Y_{21} = 1S \end{cases}$$

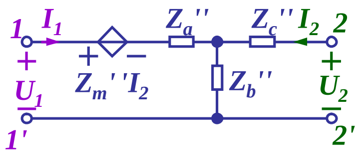
二、T型等效电路



VCR相同

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_a'+Z_b' & Z_b' \\ Z_b'+Z_m' & Z_b'+Z_c' \end{bmatrix}$$

$$\begin{cases} Z_a' = Z_{11} - Z_{12} \\ Z_b' = Z_{12} \\ Z_c' = Z_{22} - Z_{12} \\ Z_m' = Z_{21} - Z_{12} \end{cases}$$



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_a''+Z_b'' & Z_b''+Z_m'' \\ Z_b'' & Z_b''+Z_c'' \end{bmatrix}$$

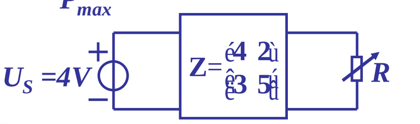
$$\begin{cases} Z_a'' = Z_{11} - Z_{21} \\ Z_b'' = Z_{21} \\ Z_c'' = Z_{22} - Z_{21} \\ Z_m'' = Z_{12} - Z_{21} \end{cases}$$

网络互易 $Z_{12} = Z_{21}$

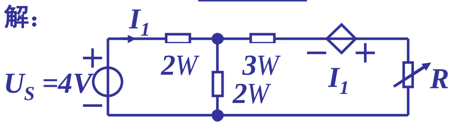
网络对称 无受控源

& $Z_{11} = Z_{22}$ $Z_a = Z_c$

补例2 问R为何值时取得最大功率，并求出P_{max}

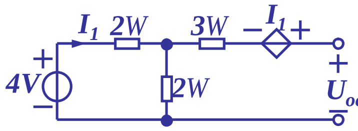


解:



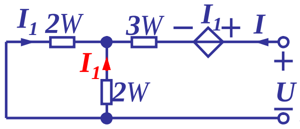
$Z_b' = Z_{12} = 2\Omega$ $Z_a' = Z_{11} - Z_{12} = 2\Omega$

$Z_c' = Z_{22} - Z_{12} = 3\Omega$ $Z_m' = Z_{21} - Z_{12} = 1\Omega$



$U_{oc} = I_1 + 2I_1$

$= 3V$



$I = -2I_1$

$U = I_1 + 3I - 2I_1$

$= -7I_1$

$R_{eq} = U/I = 3.5\Omega$

当 $R = R_{eq} = 3.5\Omega$ 时，获最大功率：$P_{max} = \dfrac{U_{oc}^2}{4R_{eq}} = \dfrac{9}{14} W$

End}