§8-4 电路定律的相量形式

一、KCL、KVL的相量形式

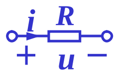
$\sum i = 0 \Rightarrow \sum \dot{I} = 0$

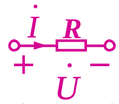
$\sum u = 0 \Rightarrow \sum \dot{U} = 0$

二、VCR的相量形式

$\dot{U} = U\angle\psi_u \qquad \dot{I} = I\angle\psi_i$

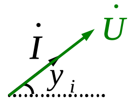
1、R



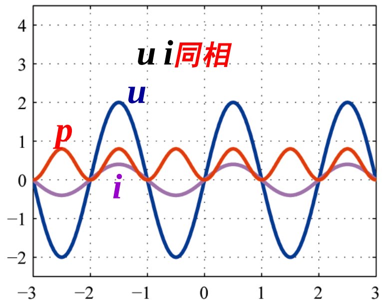


$u = Ri \Rightarrow \dot{U} = R\dot{I}$

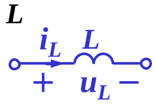
向量图
相量图

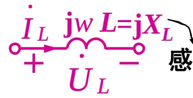


$\begin{cases} U = RI \\ \psi_u = \psi_i \end{cases}$



2、L



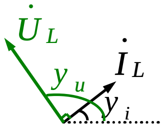


感抗

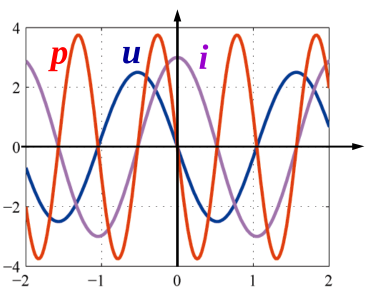
$u_L = L\dfrac{di_L}{dt} \qquad \dot{U}_L = j\omega L\dot{I}_L$

感纳

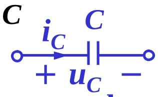
$\dot{U}_L = jX_L\dot{I}_L, \quad \dot{I}_L = jB_L\dot{U}_L$

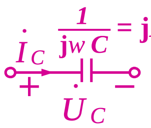


$\begin{cases} U_L = \omega L I_L = X_L I_L \\ \psi_u = \psi_i + 90° \end{cases}$



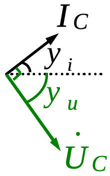
3、C





容抗
容纳

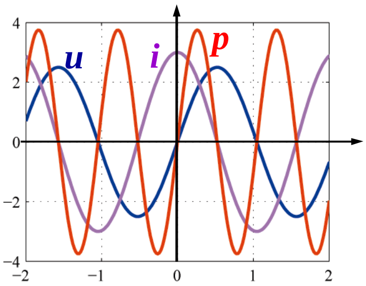
$i_C = C\dfrac{du_C}{dt} \qquad \dot{I}_C = j\omega C\dot{U}_C = jB_C\dot{U}_C$



$\dot{U}_C = \dfrac{1}{j\omega C}\dot{I}_C = jX_C\dot{I}_C$

$\begin{cases} U_C = \dfrac{1}{\omega C}I_C = |X_C| I_C \\ \psi_u = \psi_i - 90° \end{cases}$

u滞后于i　正交



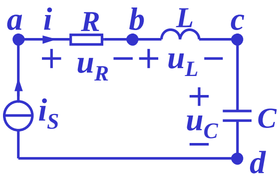
补例1　试判断下列表达式的正、误。

1. $u = \omega L \dot{I}$ ✗
2. $u = 5\cos\omega t \neq 5\angle 0^0$
3. $\dot{I}_m = j\omega C U_m$ ✗
4. $X_L = \dfrac{\dot{U}_L}{\dot{I}_L}$ ✗
5. $\dfrac{\dot{U}_L}{\dot{I}_L} = j\omega C$ ✗
6. $\dot{I}_C = j\omega C \dot{U}_C$ ✓
7. $i = L\dfrac{du}{dt}$ ✗

补例1　试判断下列表达式的正、误。

1. $U = \omega L I$
2. $u = 5\cos\omega t \neq 5\angle 0^0$
3. $\dot{I}_m = j\omega C \dot{U}_m$
4. $X_L = \dfrac{U}{I} = \dfrac{U_m}{I_m}$
5. $\dfrac{\dot{U}_L}{\dot{I}_L} = j\omega L$
6. $\dot{I}_C = j\omega C \dot{U}_C$
7. $i = C\dfrac{du}{dt}$

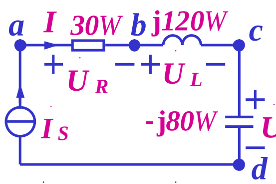
例8-4　已知：R=30Ω, L=0.12H, C=12.5μF, $i_S = 5\sqrt{2}\cos(10^3 t + 30°)A$,求电压 u_{ad}和 u_{bd}。



基本步骤

作相量模型 ⇒ 求出待求量的相量 ⇒ 求出其它量

例8-4　已知：R=30Ω, L=0.12H, C=12.5μF, $i_S = 5\sqrt{2}\cos(10^3 t + 30°)A$,求电压 u_{ad}和 u_{bd}。

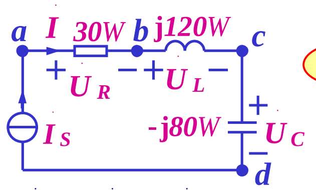


解：$\dot{I}_S = 5\angle 30° A$

$\dot{U}_R = R\dot{I} = 150\angle 30°V$

$\dot{U}_L = j\omega L\dot{I} = 600\angle 120°V$

$\dot{U}_C = \dfrac{1}{j\omega C}\dot{I} = 400\angle -60°V$



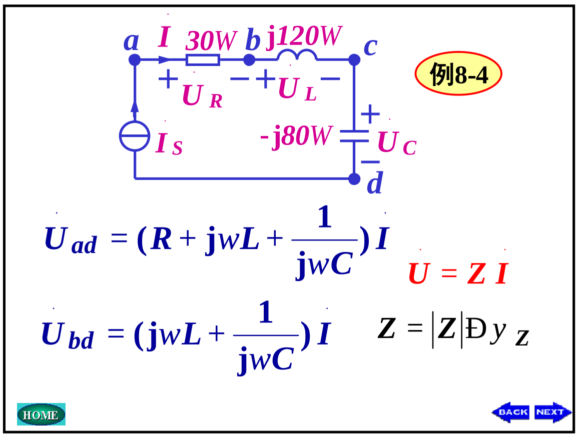
例8-4

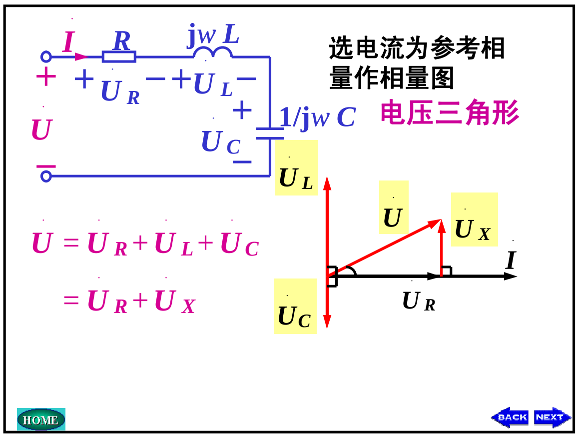
$\dot{U}_{ad} = \dot{U}_R + \dot{U}_L + \dot{U}_C = 250\angle 83.13°V$

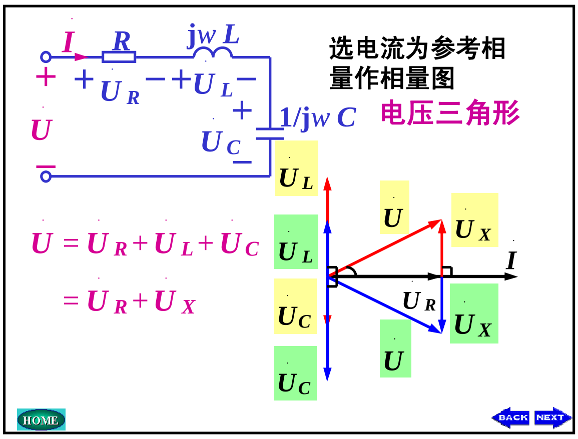
$\dot{U}_{bd} = \dot{U}_L + \dot{U}_C = 200\angle 120°V$

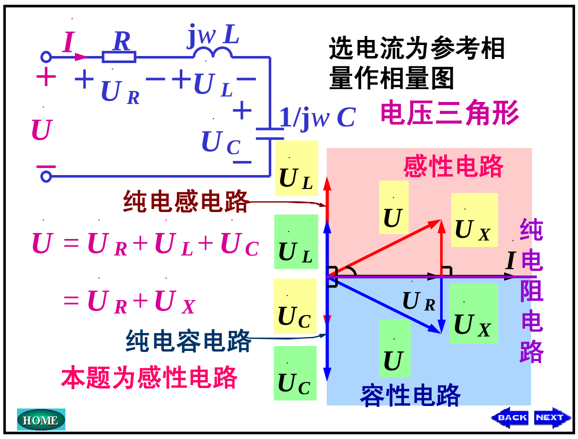
$u_{ad} = 250\sqrt{2}\cos(1000t + 83.13°)V$

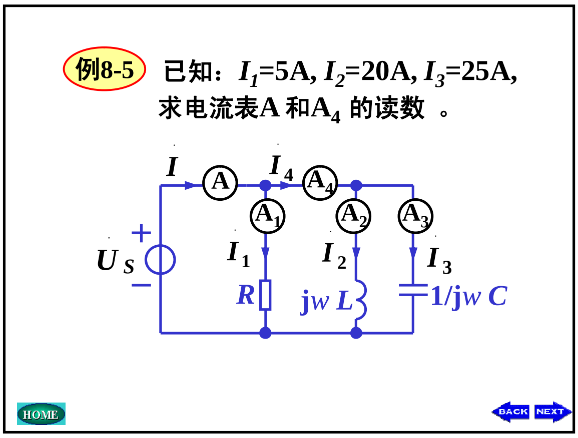
$u_{bd} = 200\sqrt{2}\cos(1000t + 120°)V$

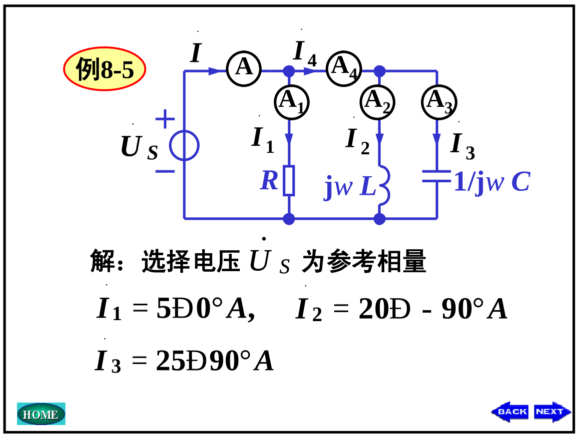

例8-4
a I 30Ω b j120Ω c
+ U_R − + U_L −
I_S -j80Ω + U_C − d
U_ad = (R + jωL + 1/jωC) I
U = Z I
U_bd = (jωL + 1/jωC) I
Z = |Z|∠ψ_Z
HOME BACK NEXT


I R jωL
+ U_R − + U_L −
U 1/jωC U_C
选电流为参考相量作相量图
电压三角形
U = U_R + U_L + U_C
= U_R + U_X
U_L U U_X I U_R U_C
HOME BACK NEXT

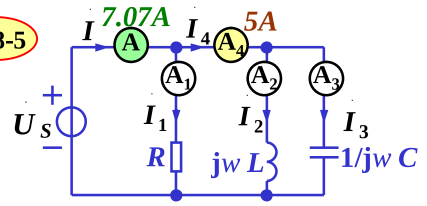

I R jωL
+ U_R − + U_L −
U 1/jωC U_C
选电流为参考相量作相量图
电压三角形
U = U_R + U_L + U_C
= U_R + U_X
U_L U U_X I U_R U_C
HOME BACK NEXT


I R jωL
+ U_R − + U_L −
U 1/jωC U_C
选电流为参考相量作相量图
电压三角形
感性电路
纯电感电路
U = U_R + U_L + U_C
= U_R + U_X
纯电容电路
本题为感性电路
纯电阻电路
容性电路
HOME BACK NEXT


例8-5
已知：I1=5A, I2=20A, I3=25A,
求电流表A 和A4 的读数 。
I A I4 A4 A1 A2 A3
U_S I1 I2 I3 R jωL 1/jωC
HOME BACK NEXT


例8-5
I A I4 A4 A1 A2 A3
U_S I1 I2 I3 R jωL 1/jωC
解：选择电压 U_S 为参考相量
I1 = 5∠0°A, I2 = 20∠-90°A
I3 = 25∠90°A
HOME BACK NEXT

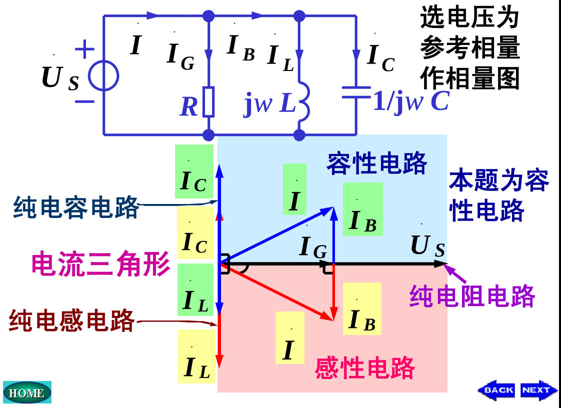
例8-5

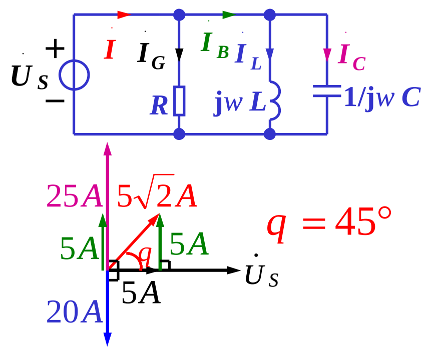


$$\dot I_4 = \dot I_2 + \dot I_3 = 5\angle 90^\circ A$$

$$\dot I = \dot I_1 + \dot I_4 = 5 + j5 = 5\sqrt{2}\angle 45^\circ A$$

选电压为参考相量作相量图

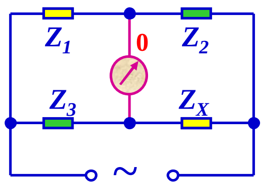




补例2 已知平衡电桥 $Z_1=1k\Omega$, $Z_2=4.2k\Omega$, $Z_3=1.43-j0.32M\Omega$。求：Z_X。

解： 平衡条件：

$$Z_1 Z_X = Z_2 Z_3$$



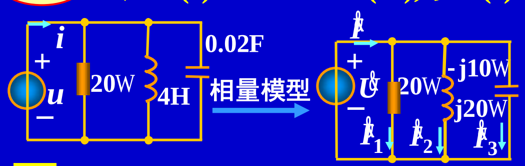
$$Z_X = \frac{Z_2 Z_3}{Z_1} = 5.99 - j1.36\,M\Omega$$

$f = 2kHz$ 时并联参数：

Z_3:1.5MΩ电阻和12pF电容

Z_X:2.1MΩ电阻和4pF电容

补例3 已知 $u(t) = 120\sqrt{2}\cos(5t)$，求：$i(t)$



解

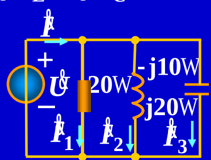
$$\dot U = 120\angle 0^\circ V$$

$$jX_L = j4 \times 5 = j20\Omega$$

$$jX_C = -j\frac{1}{5 \times 0.02} = -j10\Omega$$

$$\dot I = \dot I_R + \dot I_L + \dot I_C = \frac{\dot U}{R} + \frac{\dot U}{jX_L} + \frac{\dot U}{jX_C}$$

$$= 120\left(\frac{1}{20} + \frac{1}{j20} - \frac{1}{j10}\right)$$



$$= 6 - j6 + j12 = 6 + j6$$

$$= 6\sqrt{2}\angle 45^\circ A$$

$$i(t) = 6\sqrt{2}\cos(5t + 45^\circ)A$$