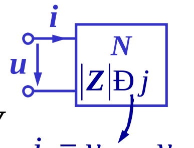
§ 9-4 正弦稳态电路的功率

一、瞬时功率 p

(*instantaneous power*)



$\varphi = \psi_u - \psi_i$

$u = \sqrt{2}U\cos(\omega t + \psi_u)\,V$

$i = \sqrt{2}I\cos(\omega t + \psi_i)\,A$

$p = ui$

$= UI\cos(\psi_u - \psi_i) + UI\cos(2\omega t + \psi_u + \psi_i)$

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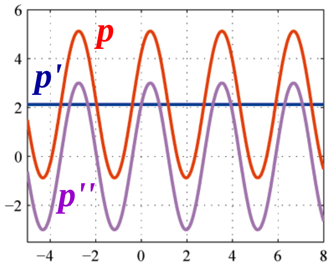
$p = UI\cos\varphi + UI\cos(2\omega t + \psi_u + \psi_i)$

$= p' + p''$ =恒定分量+正弦分量



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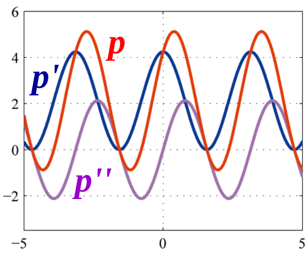
$p = UI\cos\varphi\{1 + \cos[2(\omega t + \psi_u)]\}$

$+ UI\sin\varphi\cos[2(\omega t + \psi_u)]$ 不可逆部分

$= p' + p''$ 可逆部分



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二、平均(有功)功率 P

$P = UI\cos\varphi$ 0~1

(*average power*)　　功率因数λ

感性电路：滞后

容性电路：超前

$$P = \frac{1}{T}\int_0^T p\,dt$$

$$= \frac{1}{T}\int_0^T \{UI\cos\varphi + UI\cos(2\omega t + \psi_u + \psi_i)\}dt$$

R, L, C 的P:

$P_R = UI\cos 0° = UI$

$P_L = UI\cos 90° = 0$

$P_C = UI\cos(-90°) = 0$

P为消耗在网络中各电阻上的功率之和

$P = UI\cos\varphi = \sum P_{Rk}$

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三、无功功率Q

$Q \overset{def}{=} UI\sin\varphi$ 单位：var

(*reactive power*)

R, L, C 的Q: $Q_R = UI\sin 0° = 0$

$$Q_L = U_L I_L \sin 90° = U_L I_L = I_L^2 X_L = \frac{U_L^2}{X_L}$$

$$Q_C = U_C I_C \sin(-90°) = -U_C I_C = I_C^2 X_C = \frac{U_C^2}{X_C}$$

感性电路：　　容性电路：

$0 < \varphi < 90°$，$Q > 0$　　$-90° < \varphi < 0$，$Q < 0$

$$Q = \sum Q_{Lk} + \sum Q_{Cj}$$

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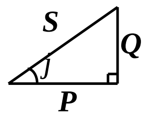
四、视在功率 S

$S \overset{def}{=} UI$ 单位：V·A

(*apparent power*)

$P = S\cos\varphi$

$Q = S\sin\varphi$



功率三角形

$$S = \sqrt{P^2 + Q^2}$$

$$\varphi = tg^{-1}\frac{Q}{P}$$

功率因数：

$$PF = \cos\varphi = \frac{P}{S}$$

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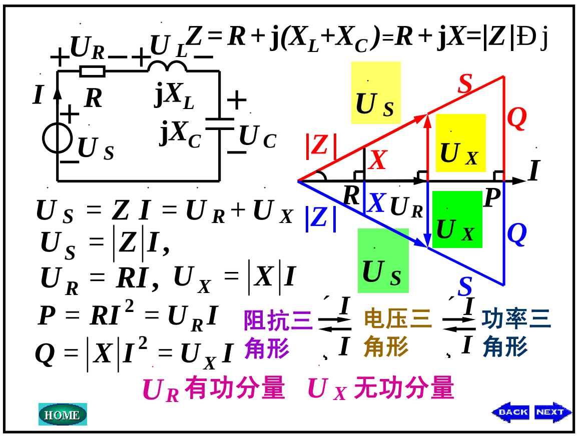
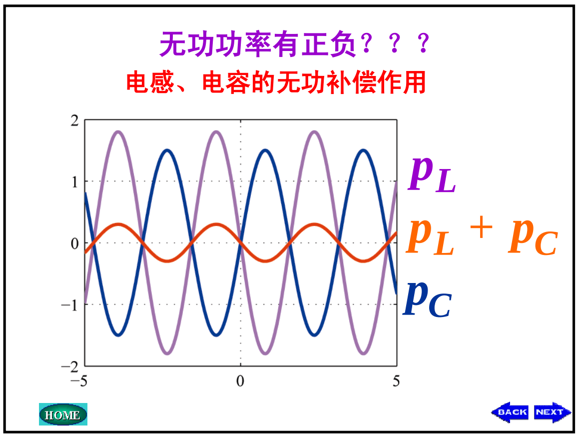
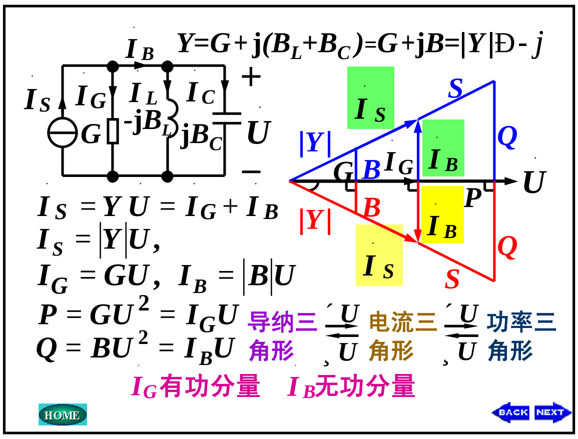


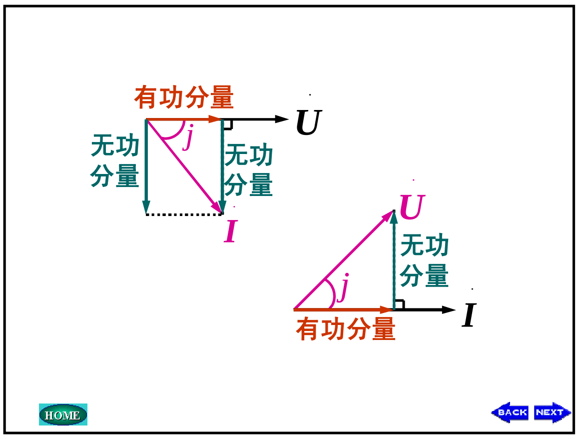
Z=R+j(X_L+X_C)=R+jX=|Z|Ðj
U_S = Z I = U_R + U_X
U_S = |Z| I,
U_R = RI, U_X = |X| I
P = RI² = U_R I
Q = |X| I² = U_X I
阻抗三角形
电压三角形
功率三角形
U_R 有功分量 U_X 无功分量



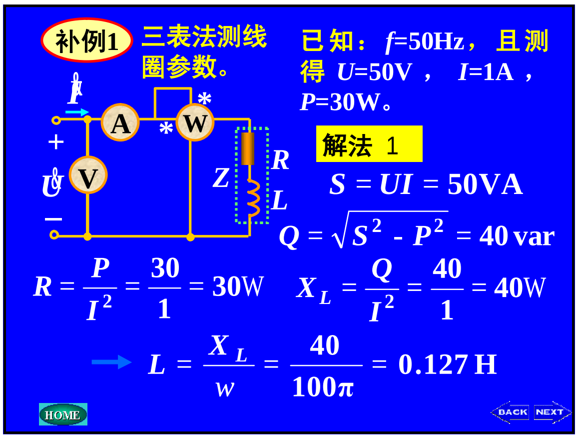
无功功率有正负？？？
电感、电容的无功补偿作用
p_L
p_L + p_C
p_C



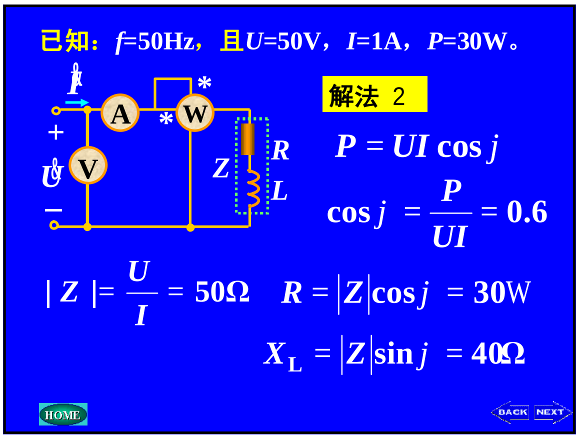
Y=G+j(B_L+B_C)=G+jB=|Y|Ð-j
I_S = Y U = I_G + I_B
I_S = |Y| U,
I_G = GU, I_B = |B| U
P = GU² = I_G U
Q = BU² = I_B U
导纳三角形
电流三角形
功率三角形
I_G有功分量 I_B无功分量



有功分量
无功分量
无功分量
U
I
U
无功分量
有功分量
I

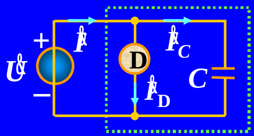


补例1 三表法测线圈参数。
已知：f=50Hz，且测得 U=50V，I=1A，P=30W。
解法 1
S = UI = 50VA
Q = √(S² - P²) = 40 var
R = P/I² = 30/1 = 30W
X_L = Q/I² = 40/1 = 40W
L = X_L/w = 40/(100π) = 0.127 H



已知：f=50Hz，且U=50V，I=1A，P=30W。
解法 2
P = UI cos j
cos j = P/UI = 0.6
|Z| = U/I = 50Ω
R = |Z| cos j = 30W
X_L = |Z| sin j = 40Ω

补例2 已知：电动机 P_D=1000W，U=220V，f=50Hz，C=30mF，$\cos\varphi_D$=0.8，求：负载电路的功率因数。



解 $I_D = \dfrac{P_D}{U\cos\varphi_D} = 5.68A$

Q $\cos\varphi_D = 0.8$(感性)，

∴ $\varphi_D = 36.8^\circ$

设 $\dot U = 220\angle 0^\circ V$

$\dot I_D = 5.68\angle -36.8^\circ A$

$\dot I_C = 220\angle 0^\circ \times j\omega C = j2.08A$

$\dot I = \dot I_D + \dot I_C = 4.73\angle -16.3^\circ A$

$\cos\varphi = \cos[0^\circ - (-16.3^\circ)] = 0.96$

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§ 9-5 复功率(Complex Power)

$$\tilde S \overset{def}{=} \dot U \dot I^* = Z\dot I \dot I^* = ZI^2 = Y^*U^2$$

$$\tilde S = U\angle\psi_u \times I\angle -\psi_i = UI\angle\psi_u - \psi_i$$

不是相量

$$= S\angle\varphi = P + jQ$$

$$P = \sum P_k \quad \tilde S = \sum \tilde S_k$$

$$Q = \sum Q_k \quad S \ne \sum S_k$$

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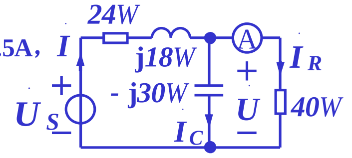
补例1 电流表读数为1.5A，求电源供出的 $\tilde S, S, P, Q$



解：$U = 1.5\times 40 = 60V$

令：$\dot U = 60\angle 0^\circ V \qquad \dot I_R = 1.5\angle 0^\circ A$

$$\dot I_C = \frac{60\angle 0^\circ}{-j30} = 2\angle 90^\circ A$$

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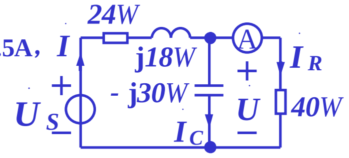
补例1 电流表读数为1.5A，求电源供出的 $\tilde S, S, P, Q$



$\dot I = \dot I_R + \dot I_C = 1.5 + j2 = 2.5\angle 53.13^\circ A$

$\dot U_S = (24 + j18)\times \dot I + \dot U$

$= 60 + j75 = 96.05\angle 51.34^\circ V$

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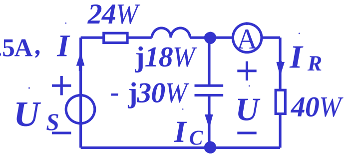
补例1 电流表读数为1.5A，求电源供出的 $\tilde S, S, P, Q$



$\tilde S = \dot U_S \dot I^* = 96.05\angle 51.34^\circ \times 2.5\angle -53.13^\circ$

$= 240.13\angle -1.79^\circ = 240 - j7.5$

$P = 240W,\ Q = -7.5\,\text{var},$

$S = 240.13V\times A$

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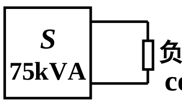
补例2 功率因数的提高

$P = UI\cos\varphi = S\cos\varphi$



负载 $\cos\varphi$ =1，P=S=75kW

$\cos\varphi$ =0.7，P=0.7S=52.5kW

一般用户： 异步电机 空载 $\cos\varphi$ =0.2~0.3

满载 $\cos\varphi$ =0.7~0.85

日光灯 $\cos\varphi$ =0.45~0.6

1、S一定，$\cos\varphi$ ↓，P ↓

2、P，U一定，$\cos\varphi$ ↓，I ↑

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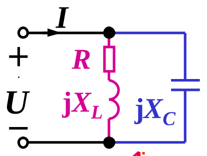
功率因数的提高 $P = UI\cos\varphi = S\cos\varphi$

$\varphi_2 < \varphi_1,\ \cos\varphi_2 > \cos\varphi_1$



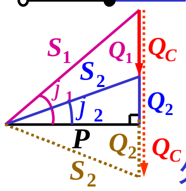
$Q_1 = P\,tg\varphi_1,\ Q_2 = P\,tg\varphi_2$

$Q_C = Q_2 - Q_1$

$= P\,tg\varphi_2 - P\,tg\varphi_1$

$= U^2 / X_C = -\omega CU^2$

$C = \dfrac{P tg\varphi_1 - P tg\varphi_2}{\omega U^2}$



欠/过补偿

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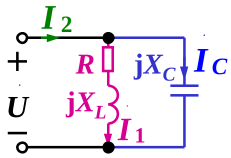

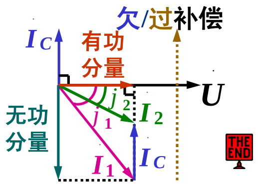

补偿电流的无功分量 $I_2\cos\varphi_2 = I_1\cos\varphi_1$

$I_1 = \dfrac{P}{U\cos\varphi_1}$　　$I_2 = \dfrac{I_1\cos\varphi_1}{\cos\varphi_2}$

$I_C = I_1\sin\varphi_1 - I_2\sin\varphi_2 = \dfrac{U}{|X_C|} = \omega CU$

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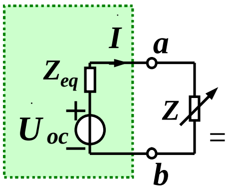
§ 9-6 最大功率传输

(*Maxium Average Power Transfer*)



$I = \dfrac{U_{oc}}{Z_{eq} + Z} = \dfrac{U_{oc}}{(R_{eq} + R) + j(X_{eq} + X)}$

$P = I^2 R = \left(\dfrac{U_{oc}}{\sqrt{(R_{eq} + R)^2 + (X_{eq} + X)^2}}\right)^2 R$

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$P = \dfrac{U_{oc}^2}{(R_{eq} + R)^2 + (X_{eq} + X)^2} R$

若R，X都可变　$P = \dfrac{U_{oc}^2}{(R_{eq} + R)^2} R$ ★

$X_{eq} + X = 0$，$R = R_{eq}$

即 $Z = R_{eq} - jX_{eq} = Z_{eq}^*$　$P_{max} = \dfrac{U_{oc}^2}{4R_{eq}}$

最佳匹配

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若为诺顿等效电路，则当 $Y = Y_{eq}^*$

$P_{max} = \dfrac{I_{sc}^2}{4G_{eq}}$

若$Z = R$为纯电阻

获得最大功率条件：　模匹配

$R = \sqrt{R_{eq}^2 + X_{eq}^2} = |Z_{eq}|$

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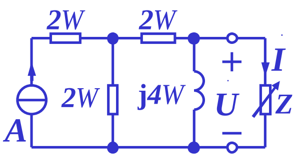
补例1　求最佳匹配时获得的最大功率

$I_S = 2\angle 0°\,A$



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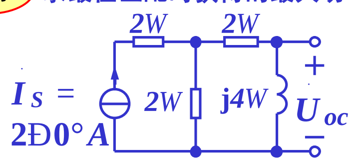
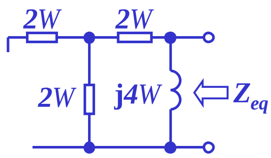
补例1 求最佳匹配时获得的最大功率



解：

$$\dot U_{oc} = \frac{2\times 2\angle 0°}{2+2+j4}\times j4 = 2\sqrt{2}\angle 45° V$$

补例1 求最佳匹配时获得的最大功率

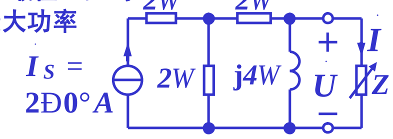


解：

$$\dot U_{oc} = \frac{2\times 2\angle 0°}{2+2+j4}\times j4 = 2\sqrt{2}\angle 45° V$$

$$Z_{eq} = (2+2)//j4 = 2+j2\Omega$$

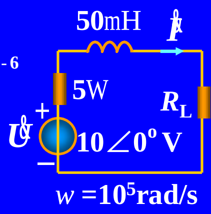
补例1 求最佳匹配时获得的最大功率



当 $Z = Z_{eq}^{*} = 2 - j2\Omega$ 时

$$P_{max} = \frac{U_{oc}^2}{4R_{eq}} = 1W$$

补例2 电路如图，求：1.R_L=5Ω时其消耗的功率；



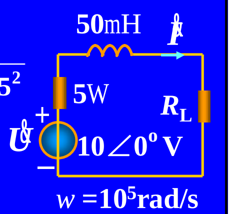
解

$$Z_{eq} = R + jX_L = 5 + j10^5\times 50\times 10^{-6} = 5+j5\ \Omega$$

$$\dot I = \frac{10\angle 0°}{5+j5+5} = 0.89\angle(-26.6°)A$$

$$P_L = I^2 R_L = 0.89^2\times 5 = 4W$$

2. R_L=?能获得最大功率，并求最大功率；



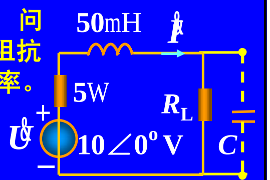
$$Z_{eq} = 5+j5\ \Omega$$

当 $R_L = \sqrt{R_{eq}^2 + X_{eq}^2} = \sqrt{5^2+5^2} = 7.07\Omega$ 获最大功率

$$\dot I = \frac{10\angle 0°}{5+j5+7.07} = 0.766\angle(-22.5°)A$$

$$P_L = I^2 R_L = 0.766^2\times 7.07 = 4.15W$$

3.在R_L两端并联一电容，问R_L和C为多大时能与内阻抗最佳匹配，并求最大功率。



$$Z_{eq} = 5+j5\ \Omega$$

$$Z_L = \frac{1}{Y} = \frac{R_L}{1+j\omega CR_L} = \frac{R_L}{1+(\omega CR_L)^2} - j\frac{\omega CR_L^2}{1+(\omega CR_L)^2} = 5 - j5$$

$\begin{cases} R_L = 10\Omega \\ C = 1\mu F \end{cases}$ 获最大功率 $P_{max} = \dfrac{U_{oc}^2}{4R_{eq}} = 5W$