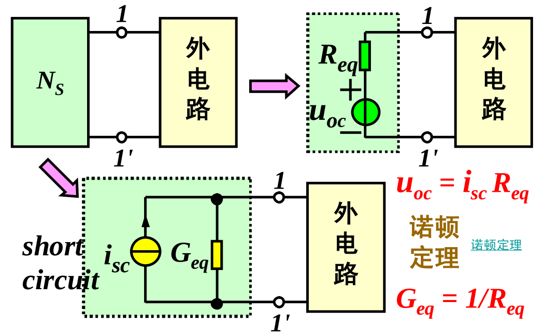
§4-3 戴维宁定理和诺顿定理

等效电源定理的概念

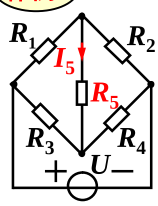
有源二端网络用电源模型替代，称为等效电源定理。

用电压源模型替代
---- 戴维宁定理(Thevenin’s Theorem)
用电流源模型替代
---- 诺顿定理(Norton’s Theorem)

open circuit 戴维宁定理 证明



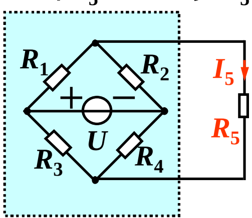
补例1

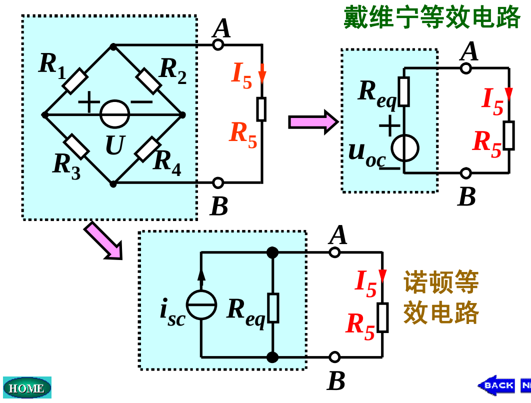


已知：R_1=20 W、R_2=30 W
R_3=30 W、R_4=20 W
U=10V

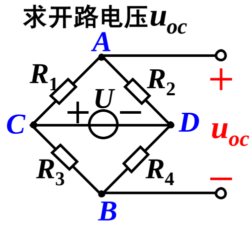
求：当 R_5=10 W 时，I_5=?

解：等效电路



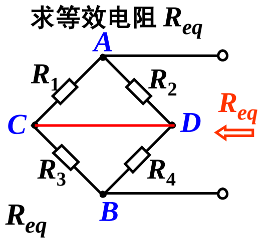


求开路电压 u_{oc}



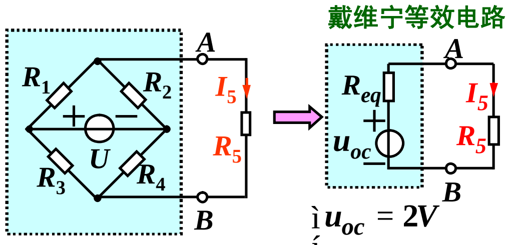
$$u_{oc} = u_{AD} + u_{DB} = \frac{R_2 \times U}{R_1 + R_2} - \frac{R_4 \times U}{R_3 + R_4} = 2V$$

求等效电阻 R_{eq}



$$R_{eq} = R_1 // R_2 + R_3 // R_4 = 20 // 30 + 30 // 20 = 24\text{W}$$

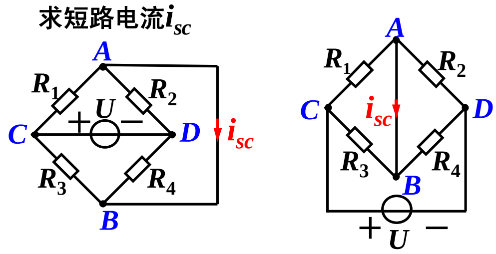
戴维宁等效电路



$$\begin{cases} u_{oc} = 2V \\ R_{eq} = 24\text{W} \end{cases}$$

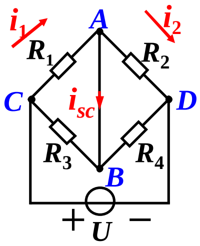
$$I_5 = \frac{u_{oc}}{R_{eq} + R_5} = 0.059\,A$$

求短路电流 i_{sc}



$$R_1 // R_3 = R_2 // R_4$$

$$u_{CA} = u_{AD} = 5V$$



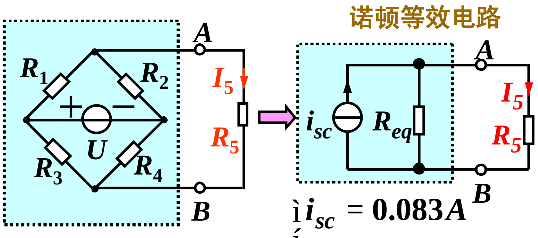
$$\begin{cases} i_1 = \dfrac{u_{CA}}{R_1} = 0.25A \\ i_2 = \dfrac{u_{AD}}{R_2} = 0.167A \end{cases}$$

$$i_{sc} = i_1 - i_2 = 0.083A$$

验证：$R_{eq} = 24 = \dfrac{u_{oc}}{i_{sc}} = \dfrac{2}{0.083}$

求等效电阻可采用这种方法（短路断路法）

诺顿等效电路

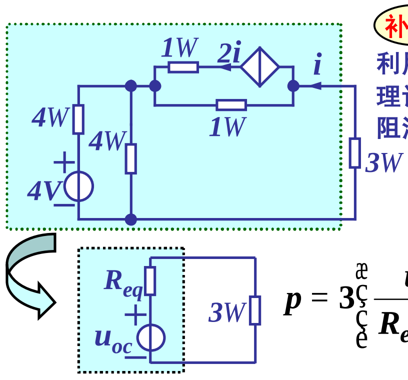


$$\begin{cases} i_{sc} = 0.083A \\ R_{eq} = 24\Omega \end{cases}$$

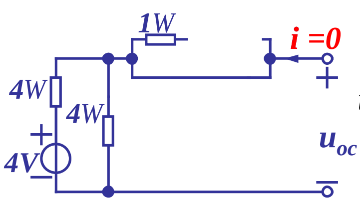
$$I_5 = \frac{R_{eq} \times i_{sc}}{R_{eq} + R_5} = 0.059A$$

补例2

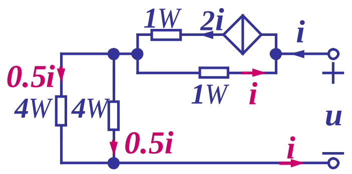
利用戴维宁定理计算3Ω电阻消耗的功率



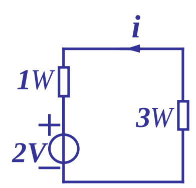
$$p = 3\left(\frac{u_{oc}}{R_{eq} + 3}\right)^2$$



$$u_{oc} = \frac{4 \times 4}{4 + 4} = 2V$$

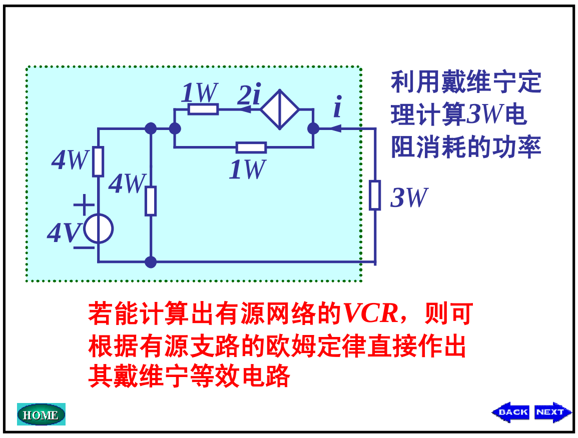


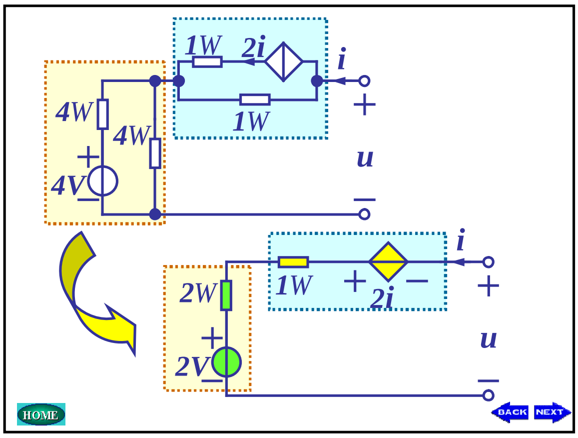
$$R_{eq} = u / i = 1\Omega$$

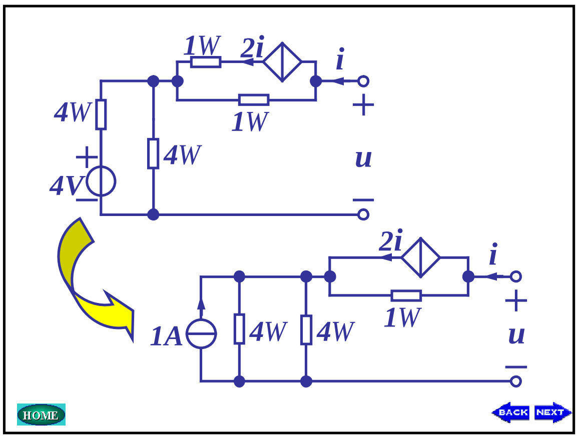


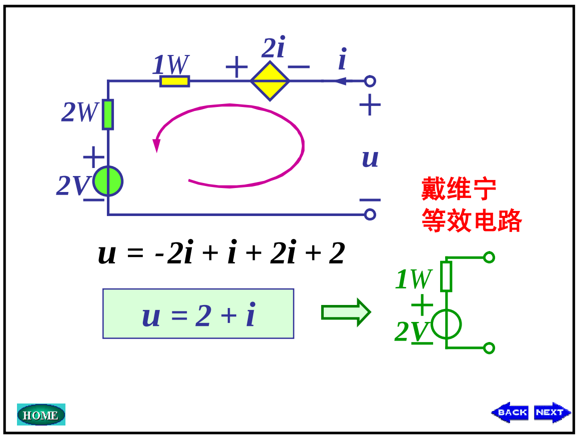
3Ω电阻消耗的功率为：

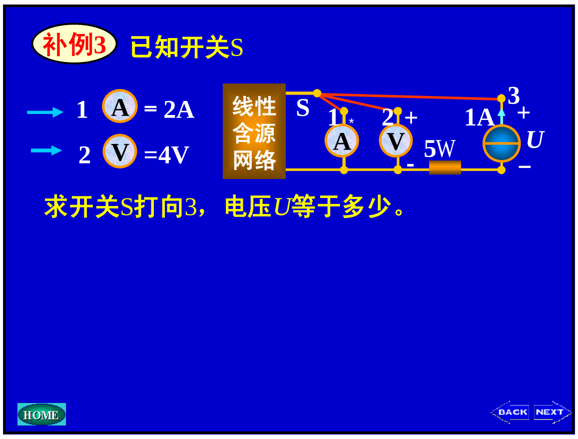
$$p = 3\left(\frac{u_{oc}}{R_{eq} + 3}\right)^2 = 0.75W$$

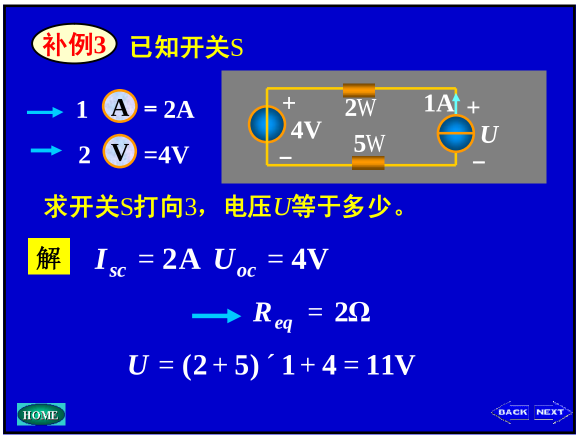

利用戴维宁定理计算3W电阻消耗的功率
1W 2i i
4W 4W 1W 3W
4V
若能计算出有源网络的VCR，则可根据有源支路的欧姆定律直接作出其戴维宁等效电路
HOME BACK NEXT


1W 2i i
4W 4W 1W
4V u
2W 1W 2i i
2V u
HOME BACK NEXT


1W 2i i
4W 4W 1W u
4V
2i i
1A 4W 4W 1W u
HOME BACK NEXT


1W 2i i
2W u
2V
戴维宁等效电路
u = -2i + i + 2i + 2
u = 2 + i
1W
2V
HOME BACK NEXT


补例3 已知开关S
1 A = 2A
2 V =4V
线性含源网络
S 1 2 3 1A U 5W
求开关S打向3，电压U等于多少。
HOME BACK NEXT


补例3 已知开关S
1 A = 2A
2 V =4V
4V 2W 5W 1A U
求开关S打向3，电压U等于多少。
解 $I_{sc} = 2A \quad U_{oc} = 4V$
$R_{eq} = 2\Omega$
$U = (2+5) \times 1 + 4 = 11V$
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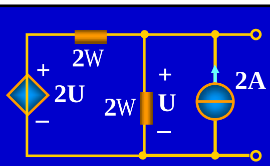
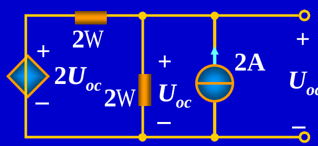
补例4

求图示电路的戴维宁和诺顿等效电路。

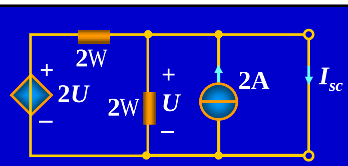


解

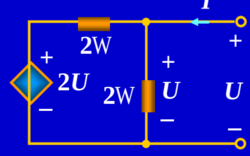


$$\left(\frac{1}{2}+\frac{1}{2}\right)U_{oc} = \frac{2U_{oc}}{2} + 2$$

U_{oc}不存在



I_{sc}=2A



$$\left(\frac{1}{2}+\frac{1}{2}\right)U = \frac{2U}{2} + I$$

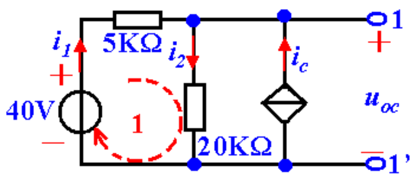
$$I = 0$$

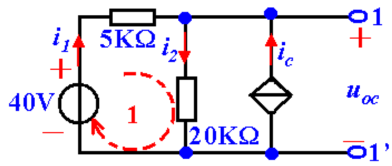
$$R_{eq} \to \infty$$

该网络只有诺顿等效电路，无戴维宁等效电路。

例4—7

求下图所示含一端口的戴维宁等效电路和诺顿等效电路。一端口内部有电流控制电流源，$i_c=0.75i_1$。





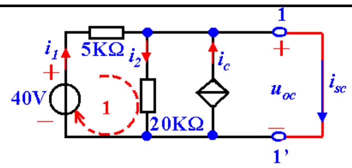
解：先求开路电压u_{oc}。当端口1—1'开路时有：

$$i_2 = i_1 + i_c = 1.75i_1$$

对网孔1列KVL方程得 $5\times10^3\times i_1 + 20\times10^3 i_2 = 40$

解得：i_1=10mA

而开路电压 $u_{oc} = 20\times10^3\times i_2 = 35V$



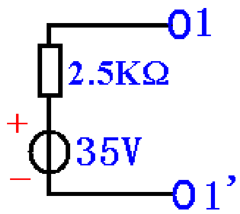
解：当端口1—1'短路时电流i_{sc}，此时

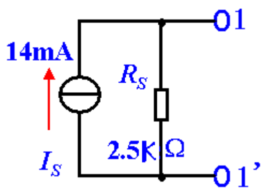
$$i_1 = \frac{40}{5\times10^3}A = 8mA$$

$$i_{sc} = i_1 + i_c = 1.75i_1 = 14mA$$

故得 $R_s = \dfrac{u_{oc}}{i_{sc}} = 2.5K\Omega$

对应的戴维宁和诺顿等效电路如下图所示：





注意

①若一端口网络的等效电阻 $R_{eq}=0$，该一端口网络只有戴维宁等效电路，无诺顿等效电路。

②若一端口网络的等效电阻 R_{eq}=¥，该一端口网络只有诺顿等效电路，无戴维宁等效电路。

THE END

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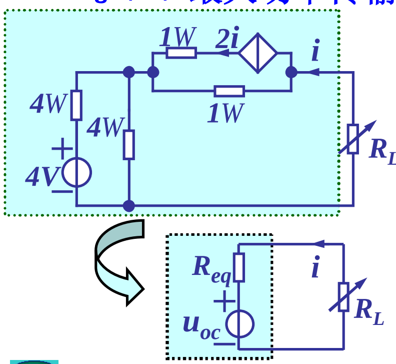
§4-4 最大功率传输定理



问R_L为何值时获得最大功率，并求此最大功率

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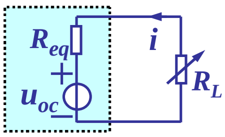

$$P_L = i^2 R_L = \left(\frac{u_{oc}}{R_{eq}+R_L}\right)^2 R_L = \frac{u_{oc}^2 R_L}{(R_{eq}+R_L)^2}$$

当 $R_L = R_{eq}$ 时获得最大功率为：

$$P_{max} = \frac{u_{oc}^2}{4R_{eq}} = 1\text{W}$$

最大功率传输定理

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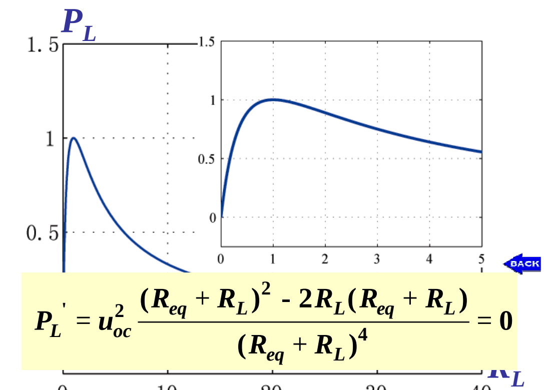

$$P_L' = u_{oc}^2 \frac{(R_{eq}+R_L)^2 - 2R_L(R_{eq}+R_L)}{(R_{eq}+R_L)^4} = 0$$

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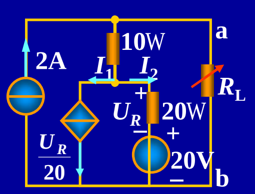
补例1 R_L为何值时能获得最大功率，并求最大功率



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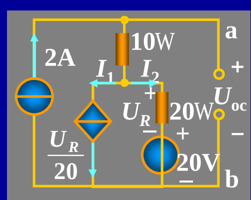
补例1 R_L为何值时能获得最大功率，并求最大功率

解 ①求开路电压U_{oc}

$$I_1 = I_2 = U_R/20$$

$$I_1 + I_2 = 2\text{A}$$

$$\rightarrow I_1 = I_2 = 1\text{A}$$



$$U_{oc} = 2 \times 10 + 20I_2 + 20 = 60\text{V}$$

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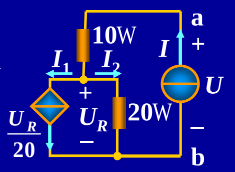
②求等效电阻R_{eq}

$I_1 = I_2 = I/2$

$U = 10I + 20 \times I/2 = 20I$

$R_{eq} = \dfrac{U}{I} = 20\Omega$



③由最大功率传输定理得:

$R_L = R_{eq} = 20\Omega$ 时其上可获得最大功率

$P_{max} = \dfrac{U_{oc}^2}{4R_{eq}} = \dfrac{60^2}{4 \times 20} = 45\,W$

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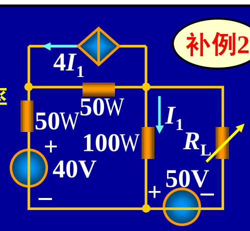
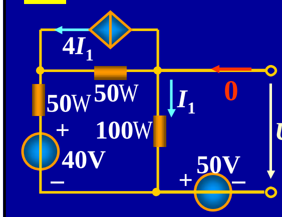
补例2

R_L为何值时能获得最大功率，并求最大功率



解 ①求开路电压U_{oc}



$50I_1 + 250I_1 + 100I_1 = 40$

$\Rightarrow I_1 = 0.1A$

$U_{oc} = 100I_1 + 50 = 60V$

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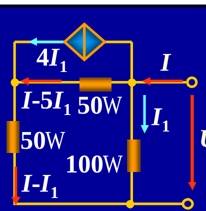

②求等效电阻R_{eq}

$50(I - 5I_1) + 50(I - I_1) = 100I_1$

$\Rightarrow I = 4I_1$

$U = 100I_1 = 25I$

$R_{eq} = U/I = 25\Omega$

③由最大功率传输定理得:

$R_L = R_{eq} = 25\Omega$ 时其上可获得最大功率

$P_{max} = \dfrac{U_{oc}^2}{4R_{eq}} = \dfrac{60^2}{4 \times 25} = 36\,W$

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注意

①最大功率传输定理用于一端口电路给定，负载电阻可调的情况；

②当负载获取最大功率时，电路的传输效率并不一定是50%；

③结合戴维宁定理或诺顿定理最方便.

THE END

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§4-5 特勒根定理

一、特勒根定理1

含b条支路的电路中：$\displaystyle\sum_{k=1}^{b} u_k i_k = 0$

适用范围：集总电路

实质是功率守恒

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二、特勒根定理2

拓扑结构相同

友网络： N_1 N_2

$\displaystyle\sum_{k=1}^{b} u_k \hat{i}_k = 0 \qquad \sum_{k=1}^{b} \hat{u}_k i_k = 0$

适用范围：集总电路

“拟功率守恒”

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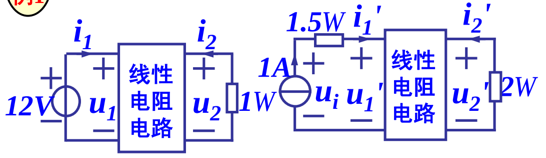
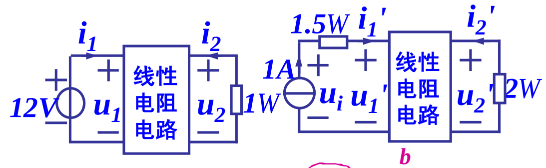
补例1 已知 $i_1 = 2A,\ u_2 = 5V,\ u_i = 5V$，求：$u_2' = ?$



解：由特勒根定理

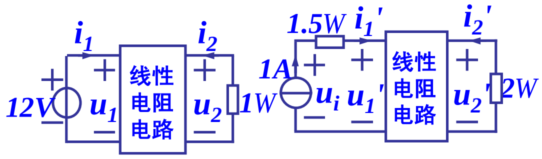
$$\sum_{k=1}^{b} u_k i_k' = 0 = \sum_{k=1}^{b} u_k' i_k$$



$$u_1 i_1' + u_2 i_2' + \sum_{k=3}^{b} u_k i_k' = \sum_{k=3}^{b} R_k i_k i_k'$$

$$\| $$

$$= u_1' i_1 + u_2' i_2 + \sum_{k=3}^{b} u_k' i_k = \sum_{k=3}^{b} R_k' i_k' i_k$$



$$u_1 i_1' + u_2 i_2' = u_1' i_1 + u_2' i_2$$

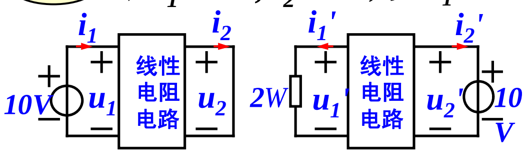
经典公式 只考虑变化支路 注意参考方向

$$u_1' = u_i - 1.5\times 1 = 3.5V \qquad i_2' = -0.5u_2'$$

$$12\times 1 + 5\times(-0.5u_2') = 3.5\times 2 + u_2'\times(-5)$$

$$u_2' = -2V$$

补例2 已知 $i_1 = 5A,\ i_2 = 1A$，求：$i_1' = ?$



解：由特勒根定理

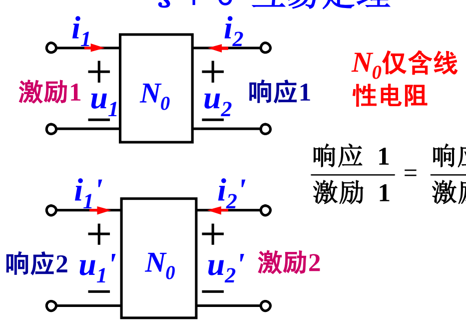
$$u_1 i_1' + u_2 i_2' = -u_1' i_1 + u_2' i_2$$

$$10\times i_1' + 0 = -2i_1'\times 5 + 10\times 1$$

$$u_1' = 2i_1' \qquad \therefore \quad i_1' = 0.5A$$

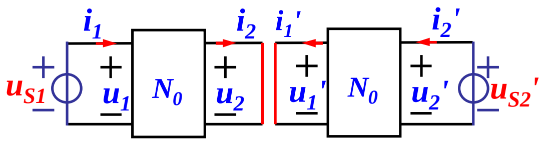
THE END

§4-6 互易定理



$$\frac{\text{响应 } 1}{\text{激励 } 1} = \frac{\text{响应 } 2}{\text{激励 } 2}$$

互易定理 第一种形式：电压源和短路电流



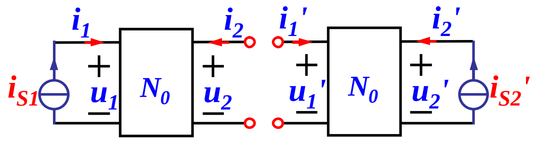
$$u_1 i_1' - \cancel{u_2 i_2'} = \cancel{-u_1' i_1} + u_2' i_2$$

$u_1 = u_{S1}$，$u_2' = u_{S2}'$

$$\frac{i_2}{u_{S1}} = \frac{i_1'}{u_{S2}'}$$

激励和响应关联关系一致取正

互易定理 第二种形式：电流源和开路电压



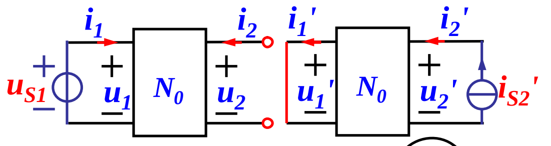
$$\cancel{u_1 i_1'} + u_2 i_2' = u_1' i_1 + \cancel{u_2' i_2}$$

$i_2' = i_{S2}'$ $i_1 = i_{S1}$

$$\frac{u_2}{i_{S1}} = \frac{u_1'}{i_{S2}'}$$

激励和响应关联
关系一致取正

第三种形式：电压源开路电压，电流源短路电流



$$-u_1 i_1' + u_2 i_2' = \cancel{u_1' i_1} + \cancel{u_2' i_2}$$

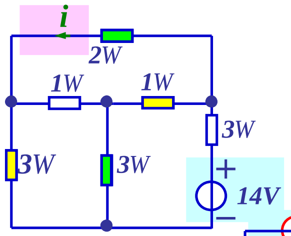
$u_1 = u_{S1}$ $i_2' = i_{S2}'$

$$\frac{u_2}{u_{S1}} = \frac{i_1'}{i_{S2}'}$$

激励和响应关联
关系不一致取正

互易定理有3种不同的形式，但在互换位置前后，如把电源置零，则电路保持不变。互易定理可归纳如下：

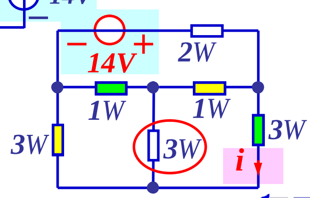
“对一个仅含线性电阻的电路，在单一激励下而产生的响应，当激励和响应互换位置时，其比值保持不变”。



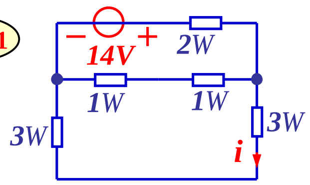
补例1

求：$i = ?$

解：由互易定理



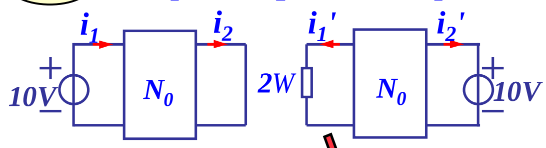
补例1



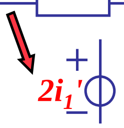
$$R_{eq} = 2 + 2 // 6 = 3.5W$$

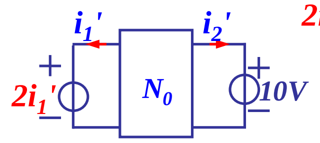
$$i = \frac{14}{R_{eq}} \times \frac{2}{2+6} = 1A$$

补例2 已知 $i_1 = 5A$, $i_2 = 1A$, 求：$i_1' = ?$

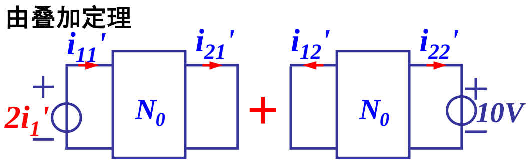


解：由替代定理





由叠加定理



齐次性 $\frac{i_1}{10} = \frac{i_{11}'}{2i_1'}$

互易定理

$i_{12}' = 1A$

$i_{11}' = i_1'$

$i_1' = -i_{11}' + i_{12}' = -i_1' + 1$

\ $i_1' = 0.5A$