补例4　已知：电感无初始储能$t=0$时合S_1，$t=0.2s$时合S_2，求两次换路后的电感电流$i(t)$。

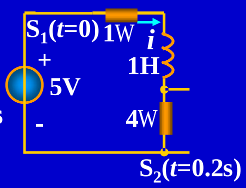
解　$0<t<0.2s$

$i(0_+)=i(0_-)=0$

$t_1=L/R=1/5=0.2s$

$i(\infty)=5/5=1A$

$i(t)=1-e^{-5t}\ A$



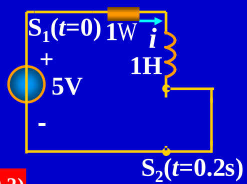
$t>0.2s$　$i(0.2_-)=1-e^{-5\times0.2}=0.63$

$i(0.2_+)=0.63A$

$t_2=L/R=1/1=1$

$i(\infty)=5/1=5A$

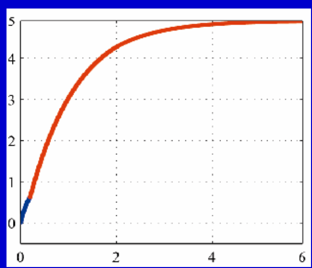
$i(t)=5-4.37e^{-(t-0.2)}\ A$



注意

$i=1-e^{-5t}$　$(0<t\le0.2s)$

$i=5-4.37e^{-(t-0.2)}$　$(t\ge0.2s)$

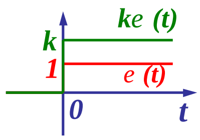


§7-7　一阶电路的阶跃响应

一、单位阶跃函数$\varepsilon(t)$

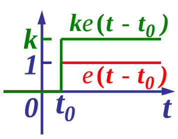
1、定义

$$\varepsilon(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$$



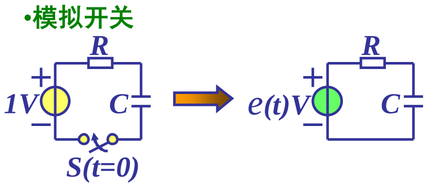
2、延迟的单位阶跃函数

$$\varepsilon(t-t_0)=\begin{cases}0, & t<t_0\\ 1, & t>t_0\end{cases}$$



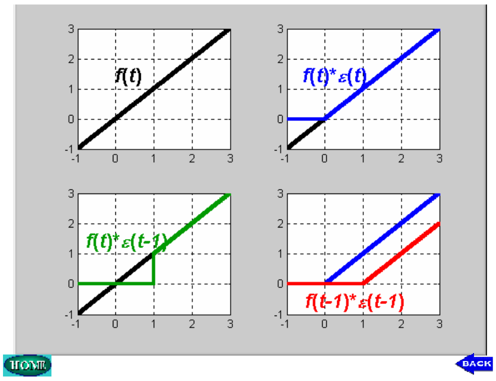
3、单位阶跃函数的作用

•模拟开关



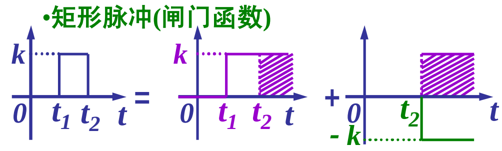

```
clear; t=-1:0.01:3; f1=t;  f2=t.*stepfun(t,0);
f3=t.*stepfun(t-1,0); f4=(t-1).*stepfun(t-1,0);
subplot 221;  plot(t,f1,'k','LineWidth',3);
axis([-1 3 -1 3]);grid on
subplot 222; plot(t,f1,'LineWidth',3);
hold on; plot(t,f2,'b','LineWidth',3);
axis([-1 3 -1 3]);grid on
subplot 223; plot(t,f1,'k','LineWidth',3);
hold on; plot(t,f3,'g','LineWidth',3);
axis([-1 3 -1 3]);grid on
subplot 224; plot(t,f2,'b','LineWidth',3);
hold on; plot(t,f4,'r','LineWidth',3);
axis([-1 3 -1 3]);grid on
```

•起始特性和延时作用

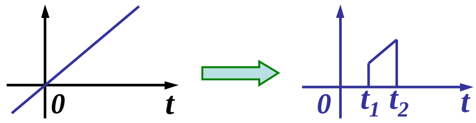


4、用单位阶跃函数表示复杂信号

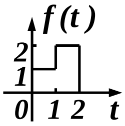
•矩形脉冲(闸门函数)



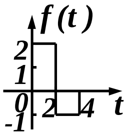
$$f(t) = k\varepsilon(t - t_1) - k\varepsilon(t - t_2)$$



补例1 求 $f(t)$ 的表达式



$$f(t) = \varepsilon(t) + \varepsilon(t-1) - 2\varepsilon(t-2)$$



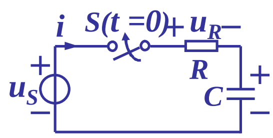
$$f(t) = 2\varepsilon(t) - 3\varepsilon(t-2) + \varepsilon(t-4)$$

二、一阶电路的阶跃响应

单位阶跃响应 $s(t)$：激励为 $\varepsilon(t)$ 时的零状态响应

$\varepsilon(t) \rightarrow N_0 \rightarrow s(t) = f(t)\varepsilon(t)$

N_0为线性非时变电路



注意

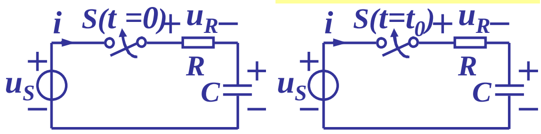
$i(t) = f(t)\varepsilon(t)$

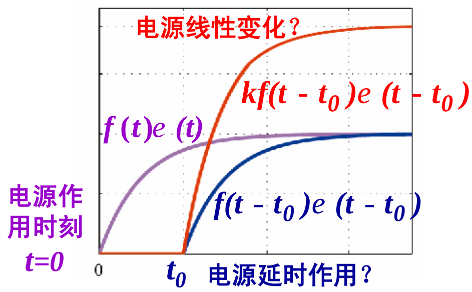
$i(t) = f(t),\ t > 0$

$\varepsilon(t) \rightarrow N_0 \rightarrow s(t) = f(t)\varepsilon(t)$

N_0为线性非时变电路

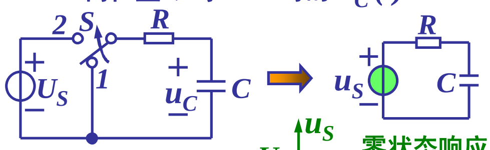
$k\varepsilon(t-t_0) \rightarrow N_0 \rightarrow ks(t-t_0) = kf(t-t_0)\varepsilon(t-t_0)$



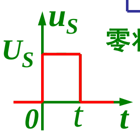


电源作用时刻不同，得到的响应波形相同，且响应和激励满足齐次性

例7-11 电路原已达稳态，在$t=0$时开关由位置1合向位置2，在$t=\tau$ 时又由位置2合向位置1，求 $t\geq 0$时的 $u_C(t)$



解：电路可等效为



零状态响应

$$u_S(t)=U_S\varepsilon(t)-U_S\varepsilon(t-\tau)V$$

1、先求单位阶跃响应

$$u_C=u_C(\infty)(1-e^{-\frac{t}{RC}})V$$

$$u_S=\varepsilon(t)\ \Rightarrow\ s(t)=(1-e^{-\frac{t}{\tau}})\varepsilon(t)V$$

2、根据线性时不变关系写出零状态响应

$$u_S(t)=U_S\varepsilon(t)-U_S\varepsilon(t-\tau)V$$

$$u_C(t)=U_S(1-e^{-\frac{t}{\tau}})\varepsilon(t)-U_S(1-e^{-\frac{t-\tau}{\tau}})\varepsilon(t-\tau)V$$

•分段表示为：

$$u_C(t)=U_S(1-e^{-\frac{t}{\tau}})\varepsilon(t)-U_S(1-e^{-\frac{t-\tau}{\tau}})\varepsilon(t-\tau)V$$

$0<t<\tau \qquad u_C(t)=U_S(1-e^{-\frac{t}{\tau}})V$

$t>\tau$

$$u_C(t)=U_S(1-e^{-\frac{t}{\tau}})-U_S(1-e^{-\frac{t-\tau}{\tau}})$$

$$=U_S(e^{-\frac{t-\tau}{\tau}}-e^{-\frac{t}{\tau}})=U_Se^{-\frac{t-\tau}{\tau}}(1-e^{-1})$$

$$=0.632U_Se^{-\frac{t-\tau}{\tau}}V$$

•分段表示为：

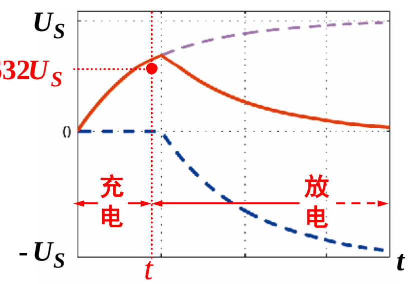
$$u_C(t)=U_S(1-e^{-\frac{t}{\tau}})\varepsilon(t)-U_S(1-e^{-\frac{t-\tau}{\tau}})\varepsilon(t-\tau)V$$

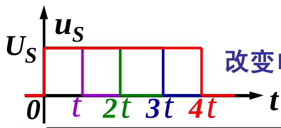
$0<t<\tau \qquad u_C(t)=U_S(1-e^{-\frac{t}{\tau}})V$

$t>\tau \qquad u_C(t)=0.632U_Se^{-\frac{t-\tau}{\tau}}V$

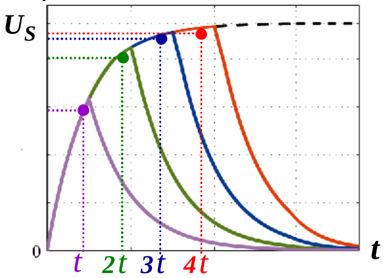
$$u_C(t)=U_S(1-e^{-\frac{t}{\tau}})[\varepsilon(t)-\varepsilon(t-\tau)]$$
$$-0.632U_Se^{-\frac{t-\tau}{\tau}}\varepsilon(t-\tau)V$$

$$u_C(t)=U_S(1-e^{-\frac{t}{\tau}})\varepsilon(t)-U_S(1-e^{-\frac{t-\tau}{\tau}})\varepsilon(t-\tau)V$$

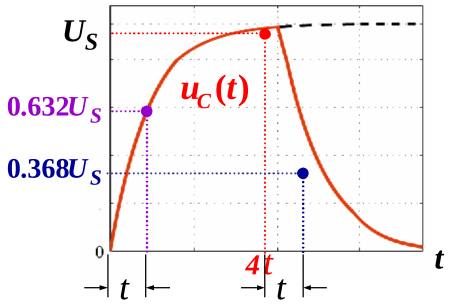




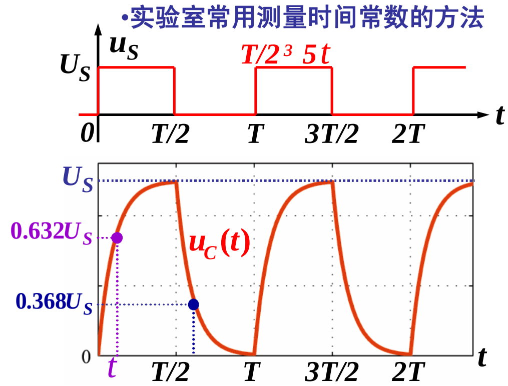
改变电源作用时间



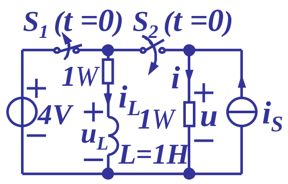
•RC一阶电路时间常数的测量

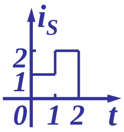


•实验室常用测量时间常数的方法



补例2 求：$t \geq 0$时的 $i_L(t)$





解：全响应分为 零输入响应 + 零状态响应

$$i_L(t) = i_L'(t) + i_L''(t)$$

$$i_L'(t) = 4e^{-2t}\varepsilon(t)A$$

具体计算过程
见7-4补例1

先求单位阶跃响应 能否由下式推出?

i_S =2A作用，零状态响应 $i_L''= 1 - e^{-2t} A$

$i_S = \varepsilon(t) \Rightarrow s(t) = (0.5 - 0.5e^{-2t})\varepsilon(t)A$

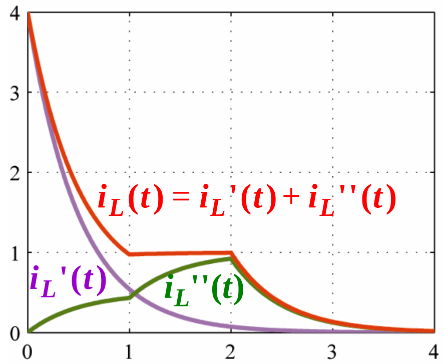
根据线性非时变关系写出零状态响应 ☼

$i_S(t) = \varepsilon(t) + \varepsilon(t-1) - 2\varepsilon(t-2)A$

$i_L''(t) = s(t) + s(t-1) - 2s(t-2)$

$= (0.5 - 0.5e^{-2t})\varepsilon(t) + (0.5 - 0.5e^{-2(t-1)})\varepsilon(t-1)$

$- 2\times(0.5 - 0.5e^{-2(t-2)})\varepsilon(t-2)A$

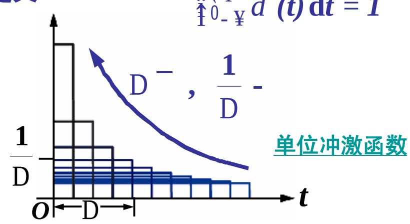


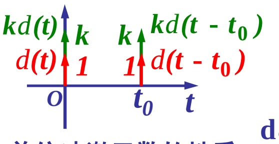
§7-8 一阶电路的冲激响应

一、单位冲激函数$\delta(t)$

1、定义

$$\begin{cases} \delta(t) = 0, & t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)\,dt = 1 \end{cases}$$





2、延迟的单位冲激函数

3、单位冲激函数的性质 $\frac{d\varepsilon(t)}{dt}=\delta(t)$;

$\int_{-\infty}^{\infty} f(t)\delta(t)dt=\int_{0_-}^{0_+} f(0)\delta(t)dt=f(0)$

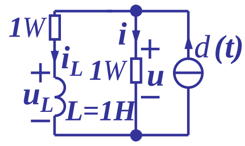
$f(t)\delta(t-t_0)=f(t_0)\delta(t-t_0)$

$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt=f(t_0)$ 积分=?

二、一阶电路的冲激响应

单位冲激响应$h(t)$：激励为$\delta(t)$时的**零状态**响应

补例 $i_L(0_-)=0$，求：$t\geq 0_-$时的 $i_L(t), u_L(t)$



$i=u=i_L+u_L$

$=i_L+L\frac{di_L}{dt}$

解：$i+i_L=\delta(t)$

$$\begin{cases}\frac{di_L}{dt}+2i_L=\delta(t)\\ i_L(0_-)=0\end{cases}$$

$$\begin{cases}\frac{di_L}{dt}+2i_L=\delta(t)\\ i_L(0_-)=0\end{cases}$$

$i_L=k\delta(t)$

$i=(1-k)\delta(t)$

$u=i\neq k\delta(t)+\delta'(t)$

$\int_{0_-}^{0_+}\frac{di_L}{dt}dt+\int_{0_-}^{0_+}2i_L dt=\int_{0_-}^{0_+}\delta(t)dt$ 积分=0

$i_L\neq k\delta(t)$

$\int_{0_-}^{0_+}di_L=1$

$\Rightarrow i_L(0_+)-i_L(0_-)=1$

$$\begin{cases}\frac{di_L}{dt}+2i_L=0\\ i_L(0_+)=1\end{cases}$$

$$\begin{cases}\frac{di_L}{dt}+2i_L=0\\ i_L(0_+)=1\end{cases}$$

零输入响应

$i_L(t)=e^{-\frac{t}{\tau}}\varepsilon(t)=e^{-2t}\varepsilon(t)A$

$u_L(t)=L\frac{di_L}{dt}=-2e^{-2t}\varepsilon(t)+e^{-2t}\delta(t)$

$=-2e^{-2t}\varepsilon(t)+\delta(t)V$

•**实际计算时可采用下面的方法：**

1、$0_-\rightarrow 0_+$时刻，$\delta(t)$作用，

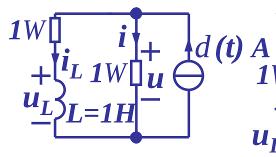
$u_C(0_+)$、$i_L(0_+)$变化

$$\begin{cases}u_C(0_+)=u_C(0_-)+\frac{1}{C}\int_{0_-}^{0_+}i_C(t)dt\\ i_L(0_+)=i_L(0_-)+\frac{1}{L}\int_{0_-}^{0_+}u_L(t)dt\end{cases}$$

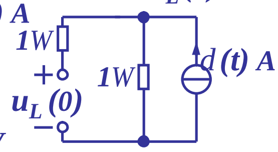
2、$t>0_+$，$\delta(t)=0$

$u_C(0_+)$、$i_L(0_+)$ 引起零输入响应

补例 $i_L(0_-)=0$，求：$t\geq 0_-$时的 $i_L(t), u_L(t)$



解：1、求$u_L(0)$



$u_L(0)=\delta(t)$ V

$t=0$

$i_L(0_+)=i_L(0_-)+\frac{1}{L}\int_{0_-}^{0_+}u_L(0)dt$

$=0+\int_{0_-}^{0_+}\delta(t)dt=1A$

$t \geq 0_-$ 时:

$$i_L(t) = e^{-\frac{t}{\tau}}\varepsilon(t) = e^{-2t}\varepsilon(t)A$$

$$u_L(t) = -2e^{-2t}\varepsilon(t) + \delta(t)V$$

单位阶跃响应见7-7补例2

$$\frac{ds(t)}{dt} = \frac{d\left[(0.5 - 0.5e^{-2t})\varepsilon(t)\right]}{dt}$$

$$= e^{-2t}\varepsilon(t)A = h(t)$$

单位冲激响应和单位阶跃响应的关系