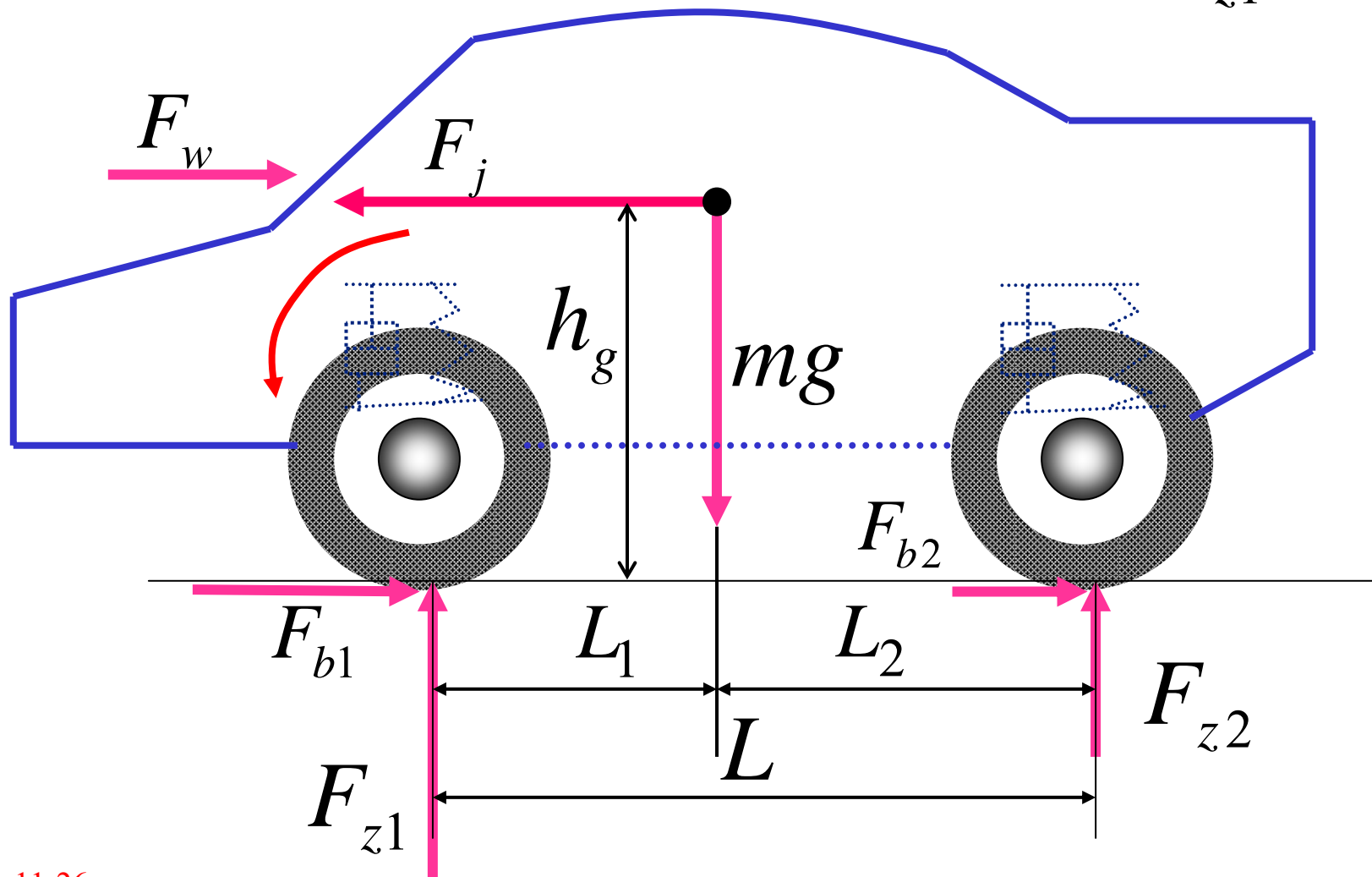


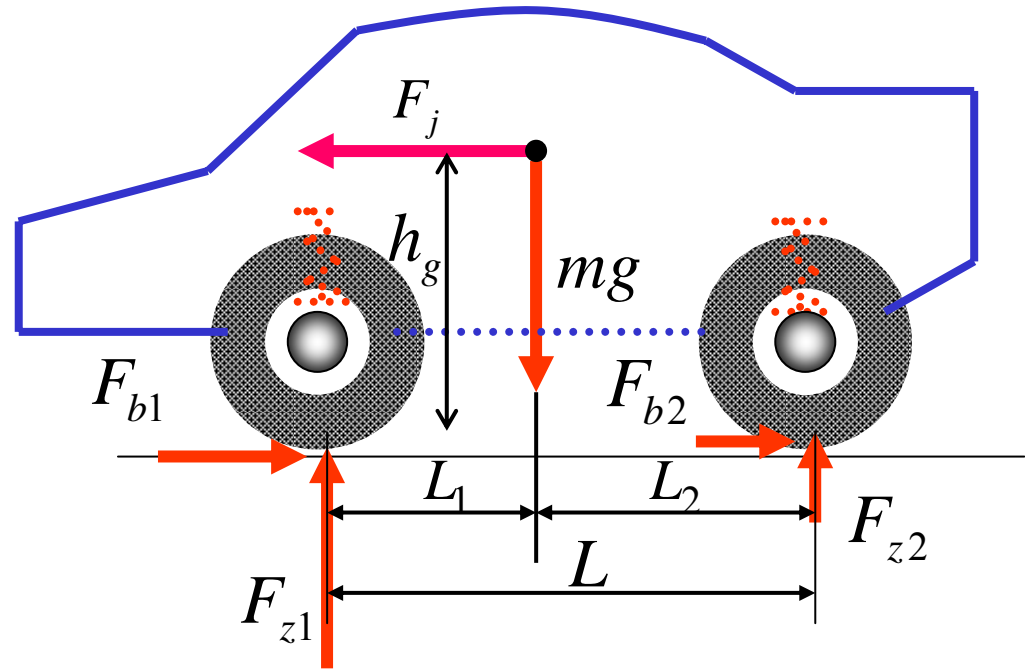
# 4.5 前后制动器制动力分配

1. 地面对前后车轮的法向反作用力  $mg = F_{z1} + F_{z2}$



$$F_{z1}L = mgL_2 + m \frac{du}{dt} h_g$$

$$F_{z2}L = mgL_1 - m \frac{du}{dt} h_g$$



$$F_{z1} = \frac{mg}{L} \left( L_2 + \frac{h_g}{g} \frac{du}{dt} \right) = \frac{mg}{L} (L_2 + h_g \varphi_b) = F_{z10} + \Delta F$$

$$F_{z2} = \frac{mg}{L} \left( L_1 - \frac{h_g}{g} \frac{du}{dt} \right) = \frac{mg}{L} (L_1 - h_g \varphi_b) = F_{z20} - \Delta F$$

$$\left( \frac{du}{dt} = g \varphi_b \Leftrightarrow \varphi_b = \frac{1}{g} \frac{du}{dt} \right)$$

## 2 理想的前、后制动器制动力分配曲线

制动时前、后车轮同时抱死时，前后制动器制动力分配关系曲线。（自行车模型）前后车轮同时抱死的条件为

$$\begin{cases} F_{\mu 1} + F_{\mu 2} = \varphi mg \\ F_{\mu 1} = \varphi F_{z1} \\ F_{\mu 2} = \varphi F_{z2} \end{cases}$$

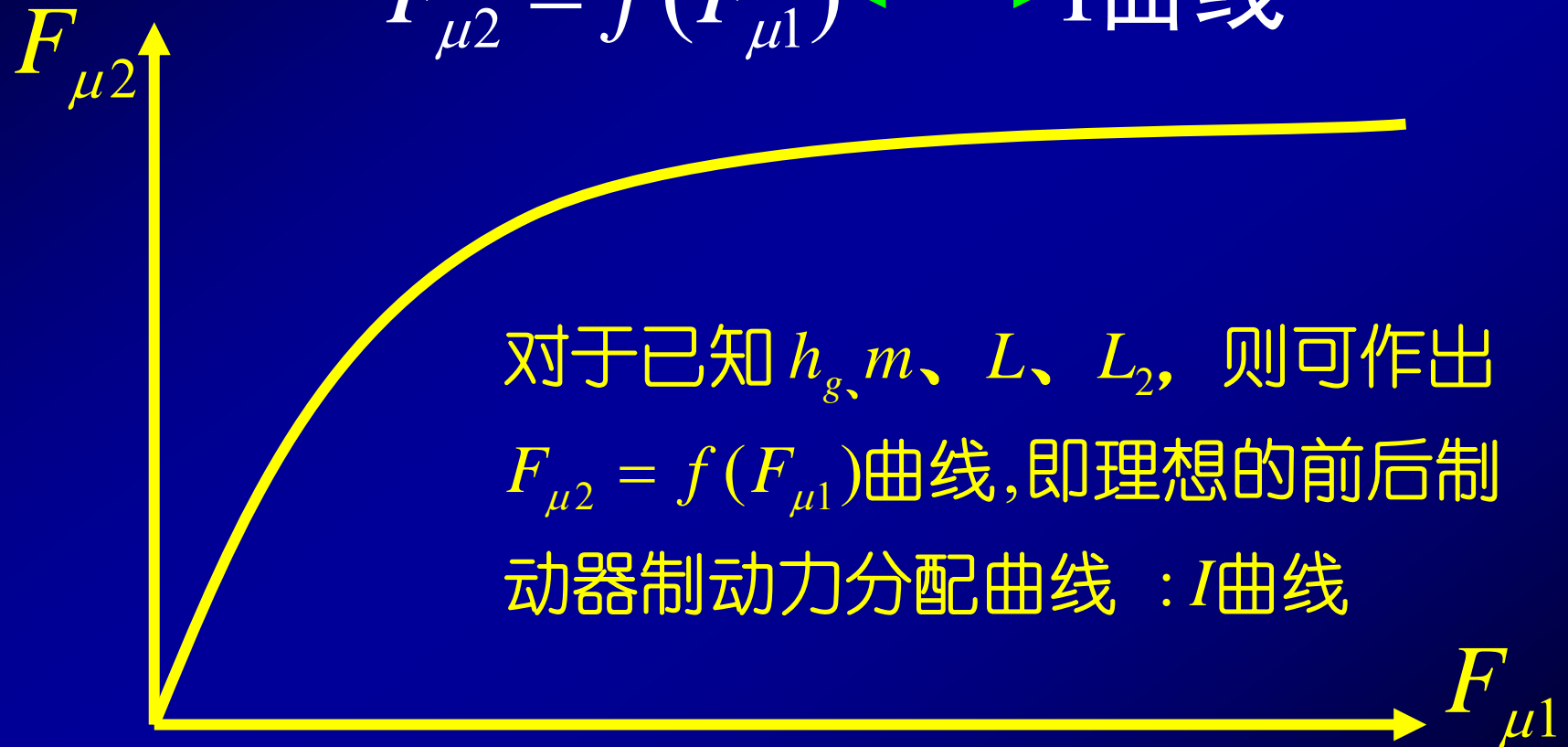
$$\begin{cases} F_{\mu 1} + F_{\mu 2} = \varphi mg \\ \frac{F_{\mu 1}}{F_{\mu 2}} = \frac{F_{z1}}{F_{z2}} = \frac{L_2 + \varphi h_g}{L_1 - \varphi h_g} \end{cases}$$

$$F_{\mu 2} = f(F_{\mu 1})$$

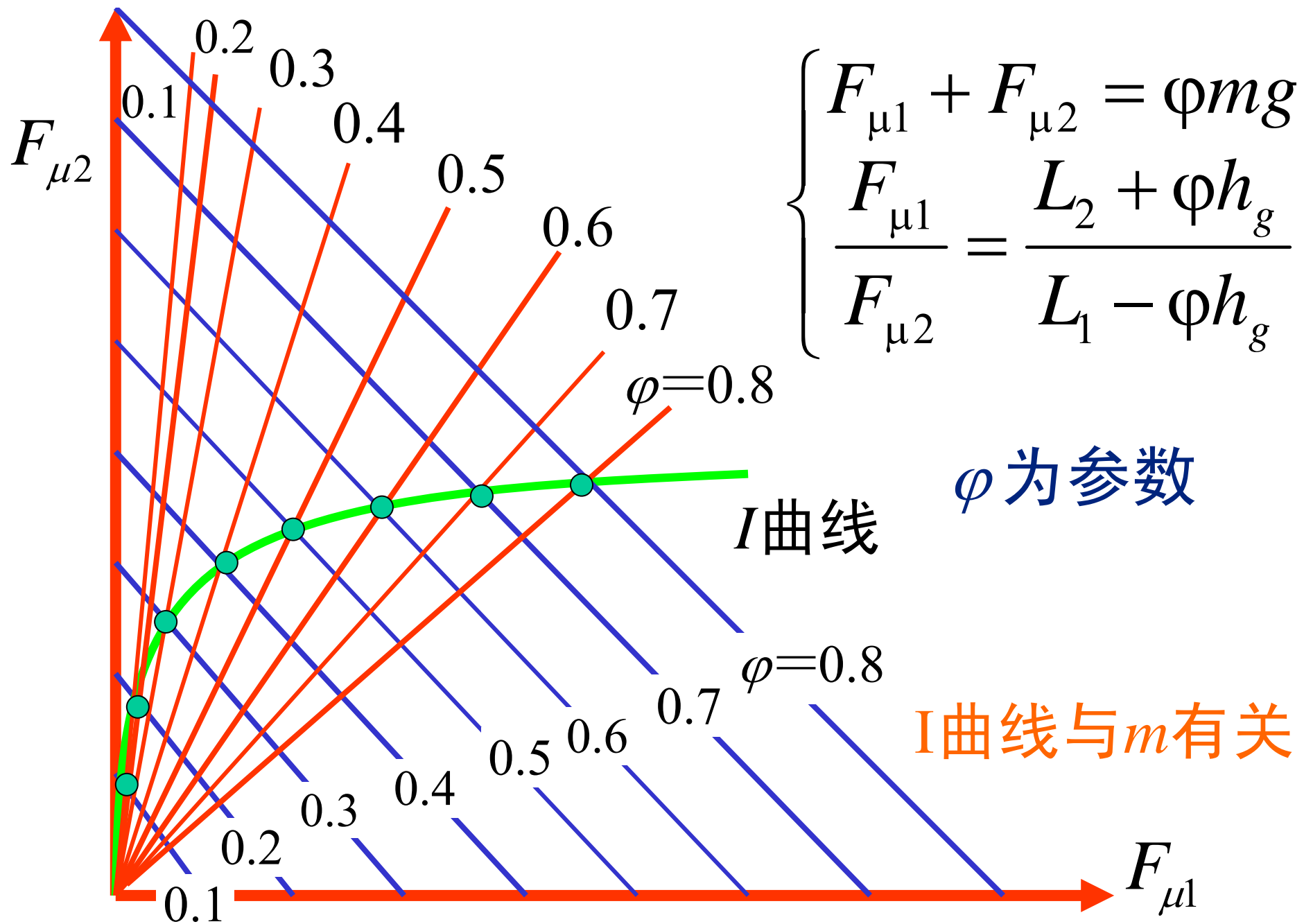
$$F_{\mu 2} = \frac{1}{2} \left[ \frac{mg}{h_g} \sqrt{L_2^2 + \frac{4h_g L}{mg} F_{\mu 1}} - \left( \frac{mgL_2}{h_g} + 2F_{\mu 1} \right) \right]$$

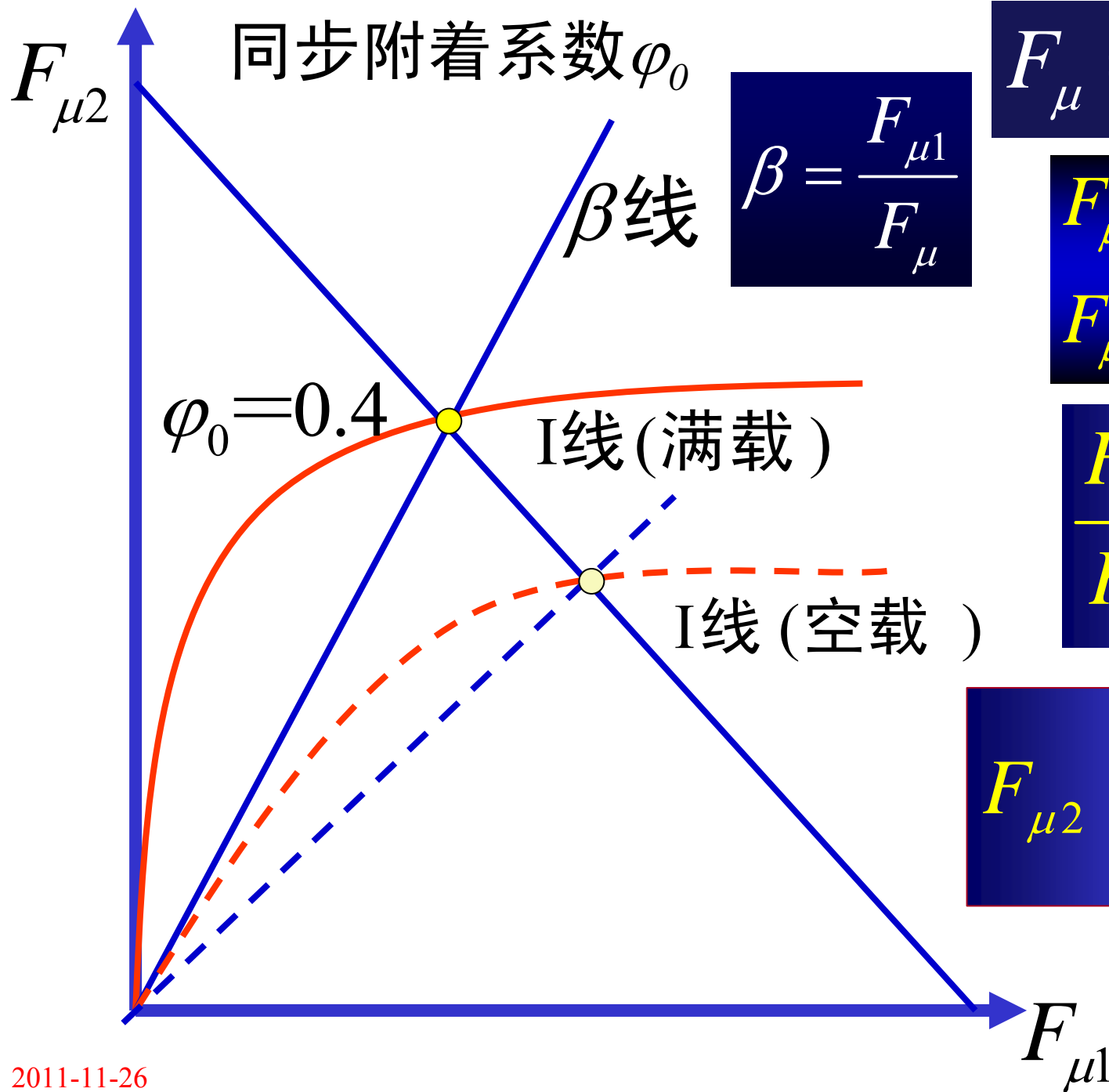
$$F_{\mu 2} = \frac{1}{2} \left[ \frac{mg}{h_g} \sqrt{L_2^2 + \frac{4h_g L}{mg} F_{\mu 1}} - \left( \frac{mgL_2}{h_g} + 2F_{\mu 1} \right) \right]$$

$F_{\mu 2} = f(F_{\mu 1}) \longleftrightarrow I$  曲线



Curve of ideal distribution of braking forces





$$\beta = \frac{F_{\mu 1}}{F_{\mu}}$$

$$F_{\mu} = F_{\mu 1} + F_{\mu 2}$$

$$F_{\mu 1} = \beta F_{\mu}$$

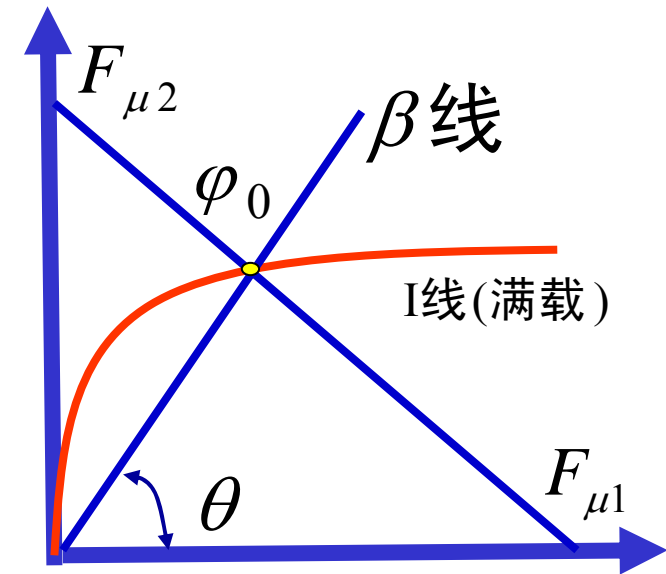
$$F_{\mu 2} = (1 - \beta) F_{\mu}$$

$$\frac{F_{\mu 2}}{F_{\mu 1}} = \frac{1 - \beta}{\beta}$$

$$F_{\mu 2} = \frac{1 - \beta}{\beta} F_{\mu 1}$$

$$F_{\mu 2} = \frac{1-\beta}{\beta} F_{\mu 1} \Rightarrow F_{\mu 2} = f(F_{\mu 1})$$

$$\tan \theta = \frac{1-\beta}{\beta} \Rightarrow \beta \text{线}$$



$\beta$  线与  $I$  线交点处的  $\varphi$  为同步附着系数  $\varphi_0$ 。

同步附着系数说明：

前后制动器制动力为固定比值的汽车，只能在一种路面上，即在同步附着系数的路面上才能保证前、后轮同时抱死。

同步附着系数也可用解析方法求出。

# 用解析方法求同步附着系数

$$\begin{cases} F_{\mu 1} + F_{\mu 2} = \varphi mg \\ \frac{F_{\mu 1}}{F_{z1}} = \frac{L_2 + \varphi h_g}{L_1 - \varphi h_g} \end{cases}$$

$$\varphi = \varphi_0$$

$$\frac{F_{\mu 1}}{F_{\mu 2}} = \frac{\beta}{1 - \beta}$$

$$\frac{F_{\mu 1}}{F_{\mu 2}} = \frac{L_2 + h_g \varphi}{L_1 - h_g \varphi} = \frac{\beta}{1 - \beta}$$

$$\tan \theta = \frac{1 - \beta}{\beta}$$



$$(L_2 + h_g \varphi)(1 - \beta) = \beta(L_1 - h_g \varphi)$$

$$L_2 + h_g \varphi - L_2 \beta - \cancel{h_g \varphi \beta} = L_1 \beta - \cancel{h_g \varphi \beta}$$

$$L_2 + h_g \varphi - \underline{L_2 \beta} = L_1 \beta \quad L\beta = L_1 \beta + L_2 \beta$$

$$L_2 + h_g \varphi - L\beta = 0$$

$$\varphi_0 = \frac{L\beta - L_2}{h_g}$$

## 4 固定 $\beta$ , 在不同路面上的制动过程分析

求  $f$  线组:  $F_{xb} = mg\varphi$

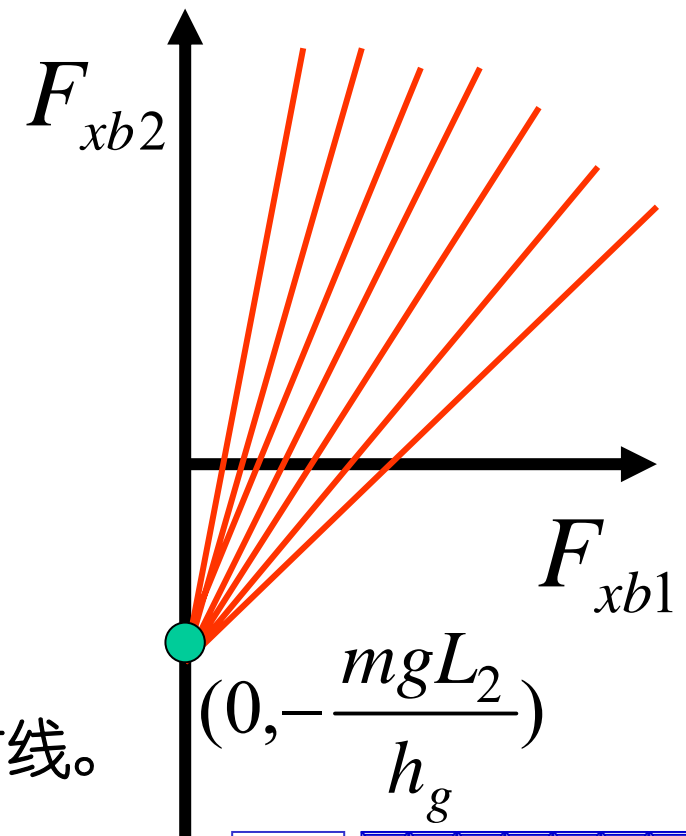
$$F_{xb1} = \varphi F_{z1} = \varphi \frac{mg}{L} (L_2 + \varphi h_g) = \varphi \left( \frac{mgL_2}{L} + \frac{F_{xb} h_g}{L} \right)$$

$$F_{xb} = F_{xb1} + F_{xb2}$$

$$F_{xb1} = \varphi \left( \frac{mgL_2}{L} + \frac{(F_{xb1} + F_{xb2}) h_g}{L} \right)$$

$$F_{xb2} = \frac{L - \varphi h_g}{\varphi h_g} F_{xb1} - \frac{mgL_2}{h_g}$$

$F_{xb2} = 0$ , 固定  $\varphi$ , 求得  $F_{xb1}$ , 过两点得一条射线。



$$F_{xb2} = \frac{L - \varphi h_g}{\varphi h_g} F_{xb1} - \frac{mgL_2}{h_g}$$

如果后轮无制动力(因堵塞或者间隙过大), 前轮有足够的制动力

若  $F_{xb2} = 0$ , 则  $\frac{L - \varphi h_g}{\varphi h_g} F_{xb1} = \frac{mgL_2}{h_g}$

$$\frac{L - \varphi h_g}{\varphi} F_{xb1} = \frac{mgL_2}{h_g}$$

$$F_{xb1} = \frac{\varphi mgL_2}{L - \varphi h_g} > \frac{\varphi mgL_2}{L}$$

前轮动态制动力比静态的大!

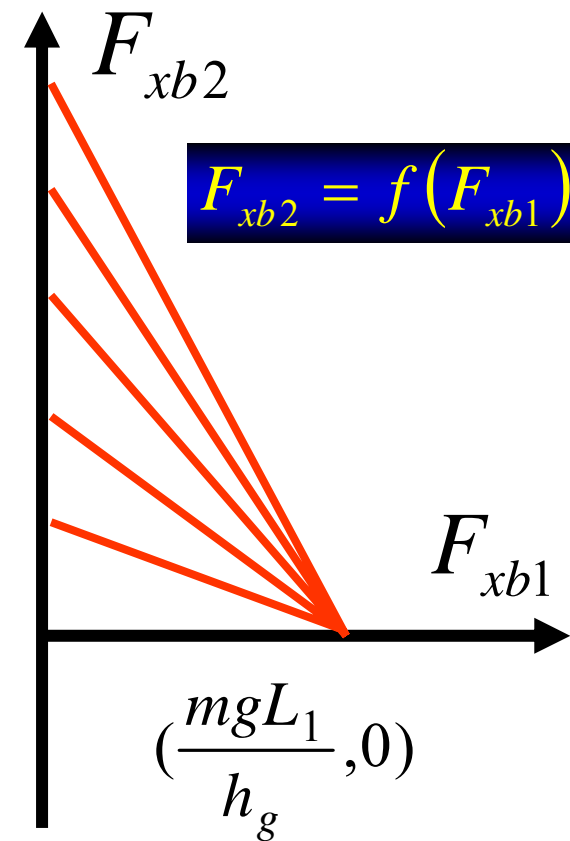
求  $r$  线组:  $F_{xb} = mg\varphi$

$$F_{xb2} = \varphi F_{z2} = \varphi \frac{mg}{L} (L_1 - \varphi h_g) = \varphi \left( \frac{mgL_1}{L} - \frac{F_{xb} h_g}{L} \right)$$

$$F_{xb} = F_{xb1} + F_{xb2}$$

$$F_{xb2} = \varphi \left( \frac{mgL_1}{L} - \frac{(F_{xb1} + F_{xb2}) h_g}{L} \right)$$

$$F_{xb2} = \frac{\varphi mgL_1}{L + \varphi h_g} - \frac{\varphi h_g}{L + \varphi h_g} F_{xb1}$$



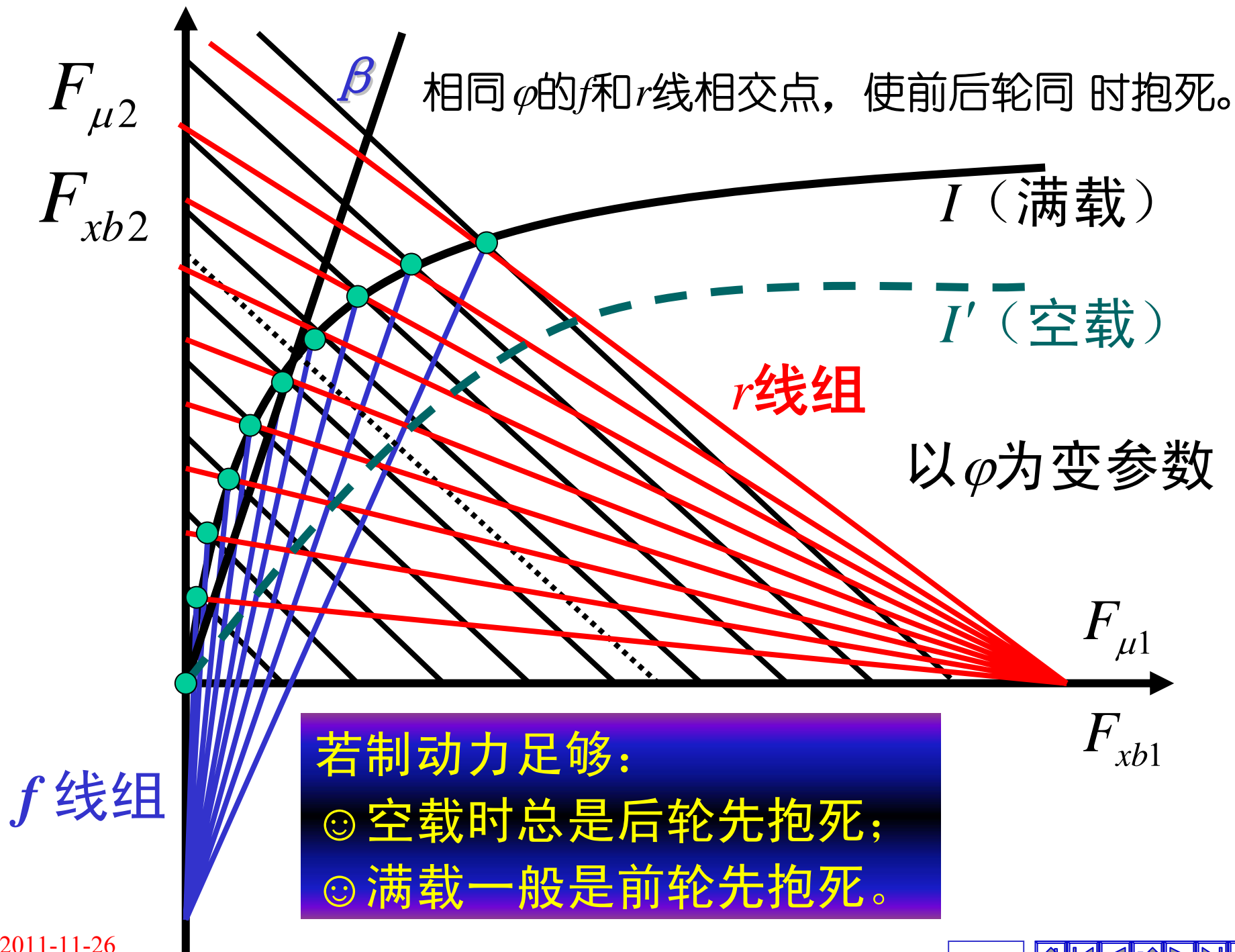
$$F_{xb2} = \frac{\varphi mgL_1}{L + \varphi h_g} - \frac{\varphi h_g}{L + \varphi h_g} F_{xb1}$$

如果前轮无制动力(因堵塞或者间隙过大),后轮有足够的制动力

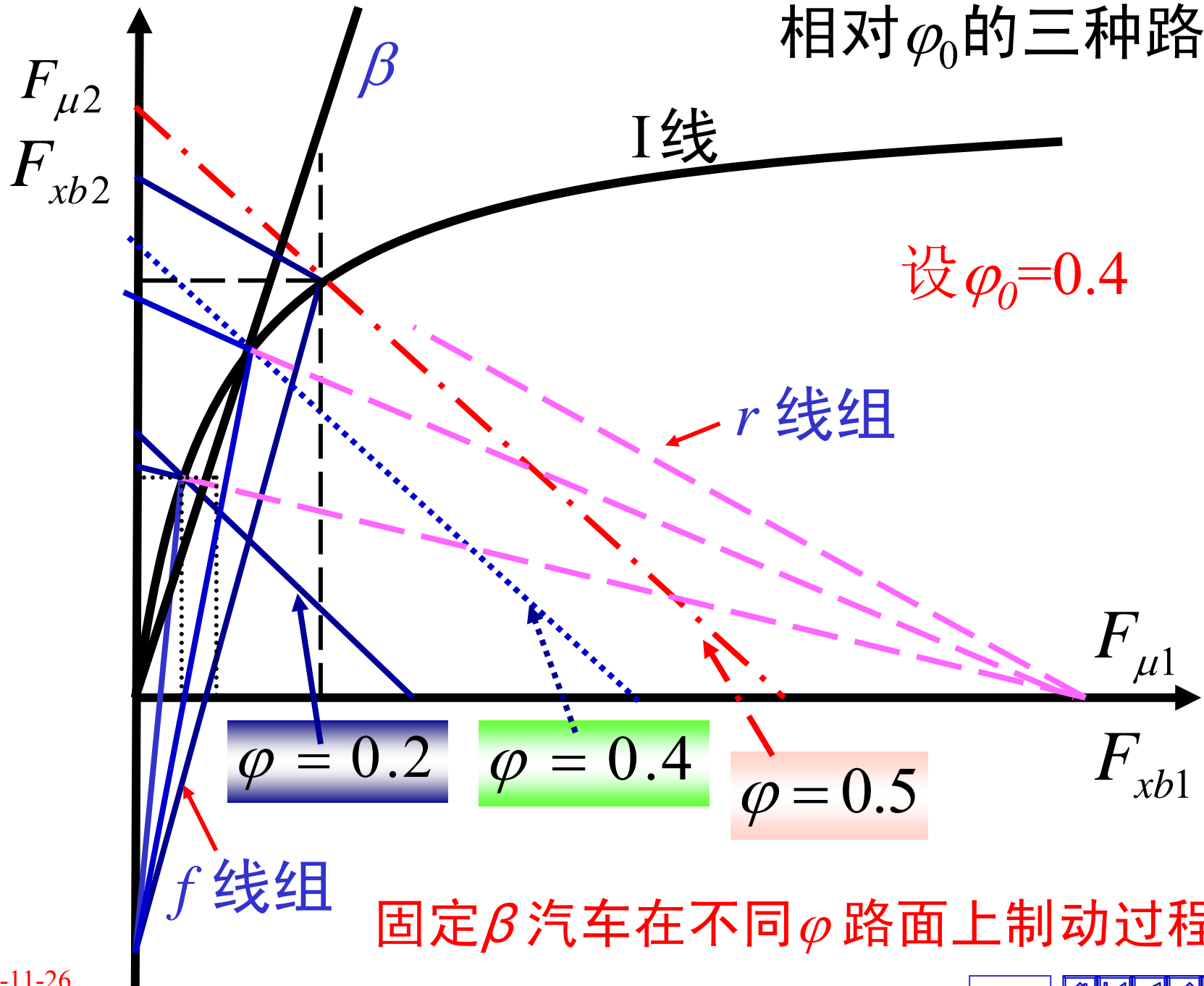
即 $F_{xb1} = 0$ , 则

$$F_{xb2} = \frac{\varphi mgL_1}{L + \varphi h_g} < \frac{\varphi mgL_1}{L}$$

**后轮动态制动力比静态的小!**



# 相对 $\varphi_0$ 的三种路面



固定  $\beta$  汽车在不同  $\varphi$  路面上制动过程

# 利用附着系数和附着效率

$\frac{du}{dt} = zg \Rightarrow z$ 制动强度,  $z = \varphi_b$ 。前轮和后轮均不抱死时  $z < \varphi_0$ 。只有  $z = \varphi_0$ , 附着条件才能得到充分的利用。汽车以一定减速度制动时, 除了  $z = \varphi_0$  以外, 不发生车轮抱死的地面附着系数总是大于制动强度  $z$ 。这个要求的路面附着系数为汽车在该制动强度时的利用附着系数。



# 利用附着系数 $E$ 的解析计算

$$\frac{du}{dt} = zg \iff \frac{1}{g} \frac{du}{dt} = z$$

$$F_{\mu 1} = F_{b1} = \beta m \frac{du}{dt} = \underline{\beta m z g}$$

$$\underline{F_{z1}} = \frac{mg}{L} (L_2 + zh_g) \Rightarrow \varphi_f = \frac{F_{b1}}{F_{z1}} = \frac{\beta z}{\frac{1}{L} (L_2 + zh_g)}$$

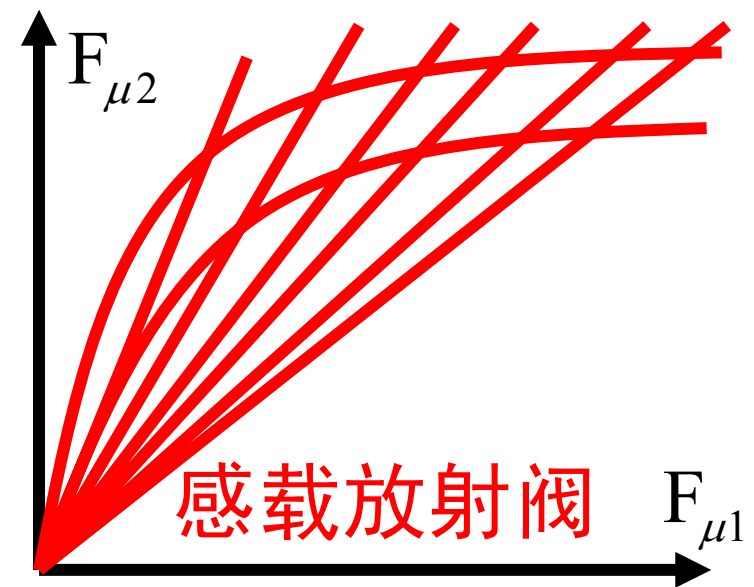
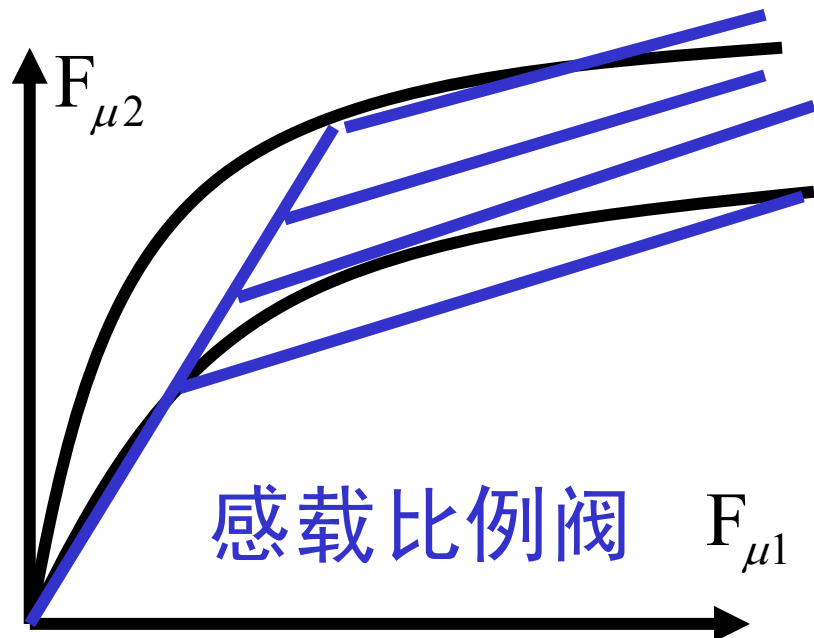
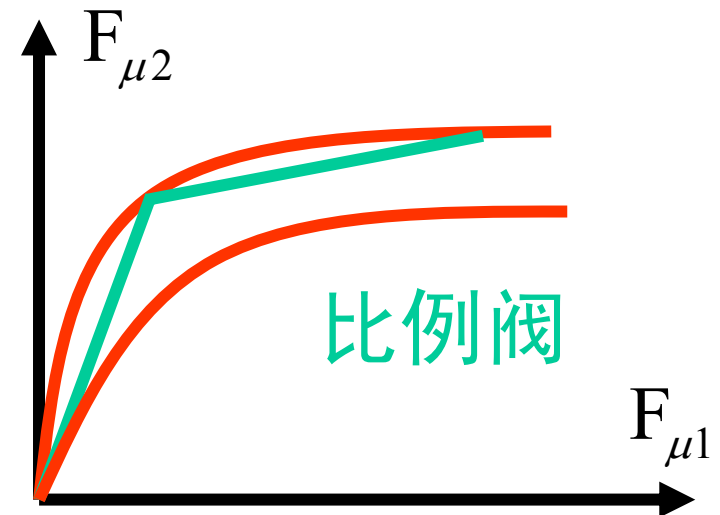
$$F_{\mu 2} = F_{b2} = (1 - \beta) m \frac{du}{dt} = \underline{(1 - \beta) m g z}$$

$$\underline{F_{z2}} = \frac{mg}{L} (L_1 - zh_g) \Rightarrow \varphi_r = \frac{F_{b2}}{F_{z2}} = \frac{(1 - \beta) z}{\frac{1}{L} (L_1 - zh_g)}$$

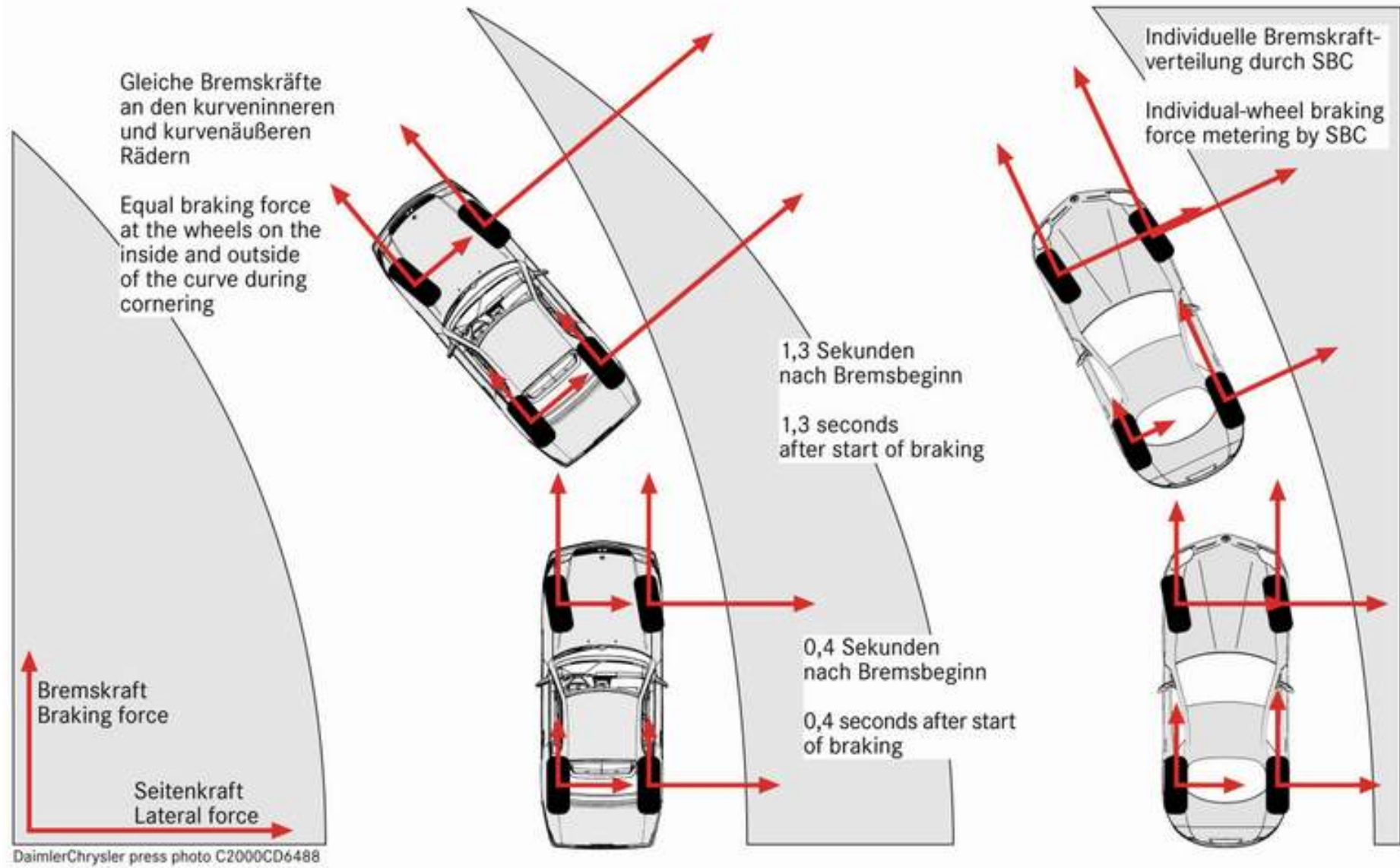
# 附着效率：制动强度与利用附着系数之比。

$$E_f = \frac{z}{\varphi_f} = \frac{L_2}{\beta L - \varphi_f h_g}$$

$$E_r = \frac{z}{\varphi_r} = \frac{L_2}{(1 - \beta)L - \varphi_r h_g}$$



**Sensotronic Brake Control - Höhere Fahrsicherheit beim Bremsen in der Kurve**  
**Sensotronic Brake Control - Greater safety when braking on bends**



**SBC根据车轮负荷 $F_z$ 确定制动力**