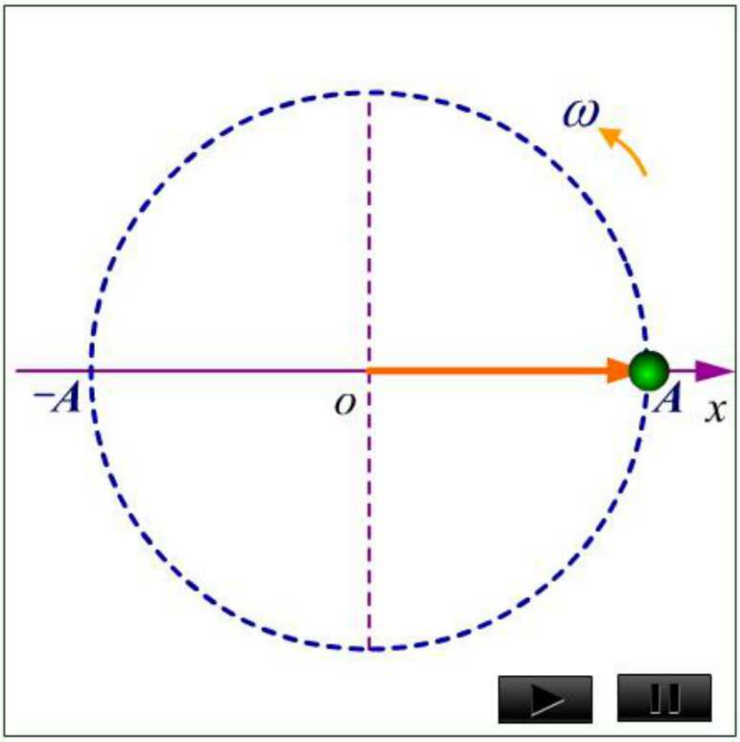
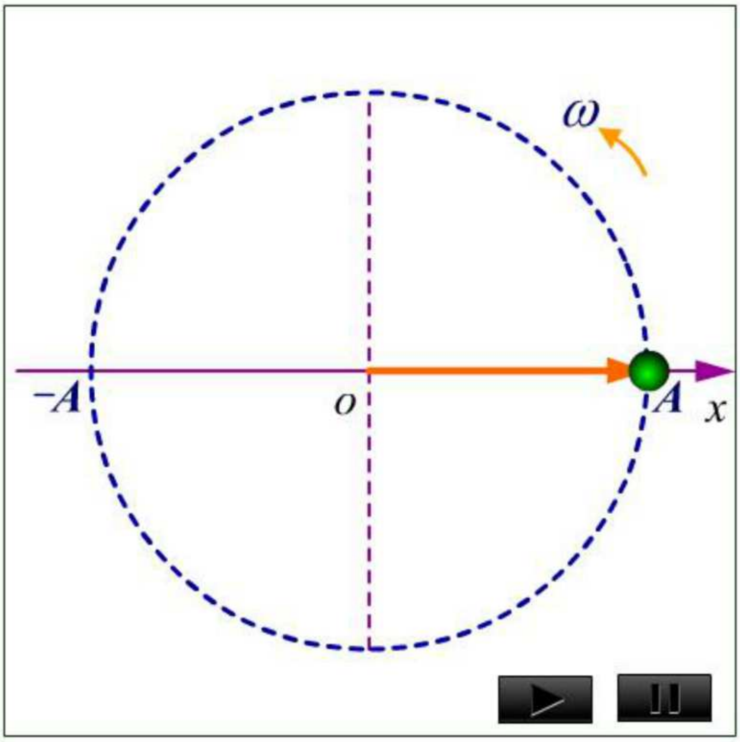
旋转矢量

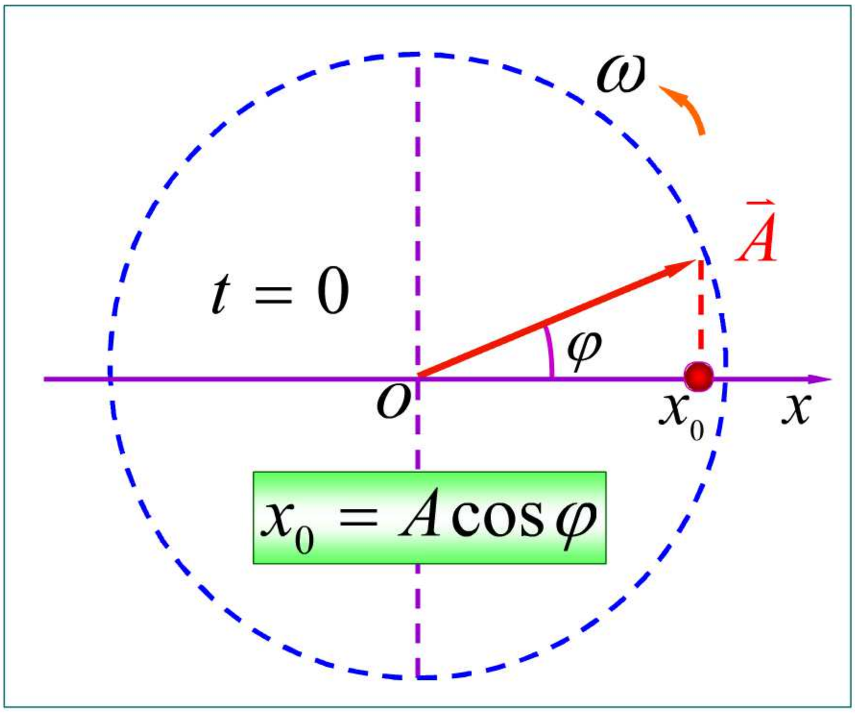


自Ox轴的原点O作一矢量 $\vec{A}$，使它的模等于振动的振幅A，并使矢量$\vec{A}$在 Oxy平面内绕点O作逆时针方向的匀角速转动，其角速度 ω与振动频率相等，这个矢量就叫做旋转矢量.

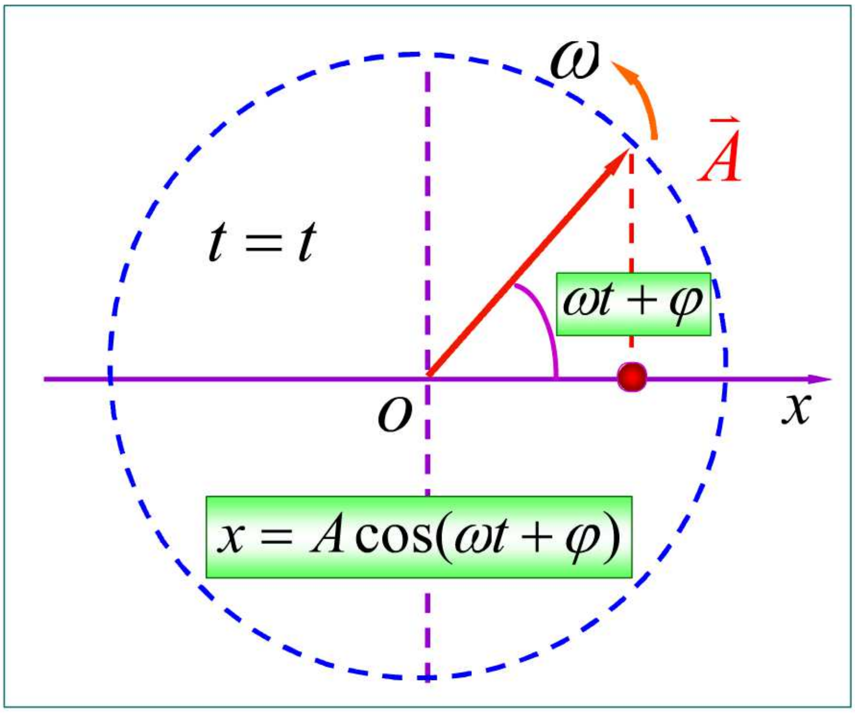
$$x = A\cos(\omega t + \varphi)$$



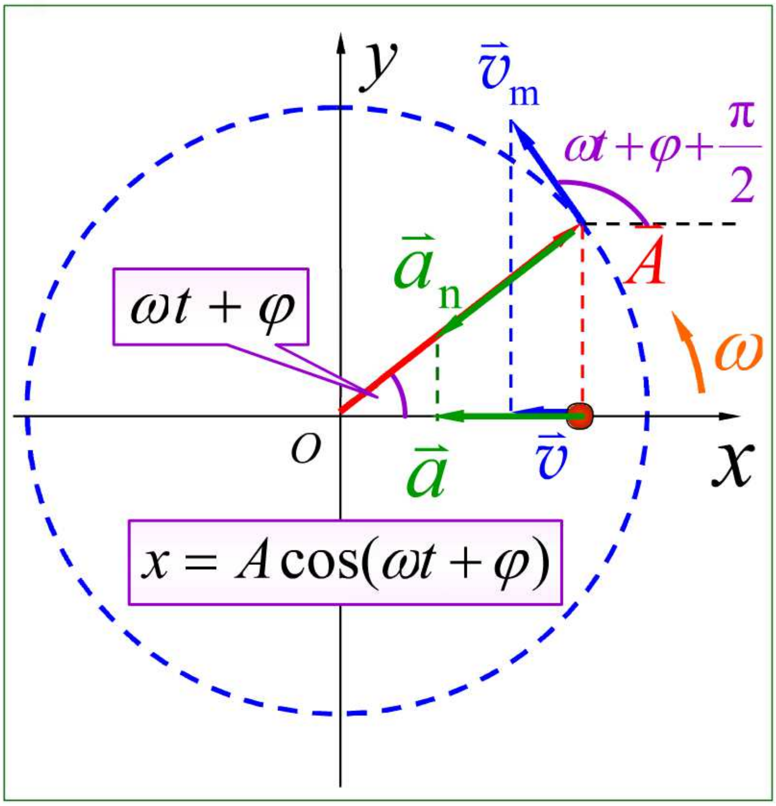
以 o 为原点旋转矢量 $\vec{A}$ 的端点在 x 轴上的投影点的运动为简谐运动.



以 o 为原点旋转矢量 $\vec{A}$ 的端点在 x 轴上的投影点的运动为简谐运动.



以 o 为原点旋转矢量 $\vec{A}$ 的端点在 x 轴上的投影点的运动为简谐运动.



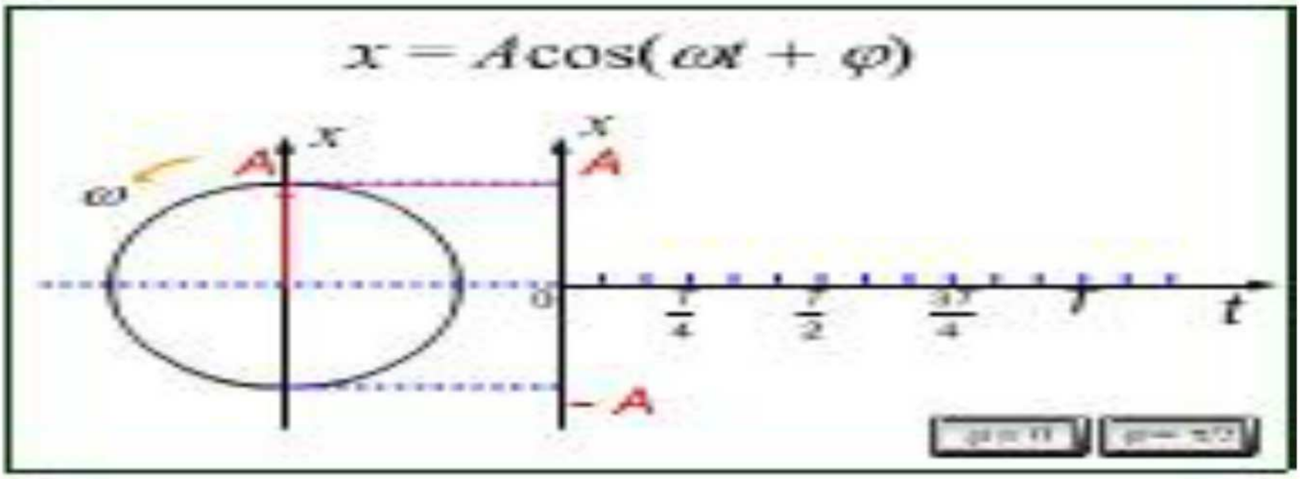
$$v_m = A\omega$$

$$\boxed{v=-A\omega\cos(\omega t+\varphi)}$$

$$a_n = A\omega^2$$

$$\boxed{a=-A\omega^2\cos(\omega t+\varphi)}$$

用旋转矢量图画简谐运动的$x-t$图



讨论

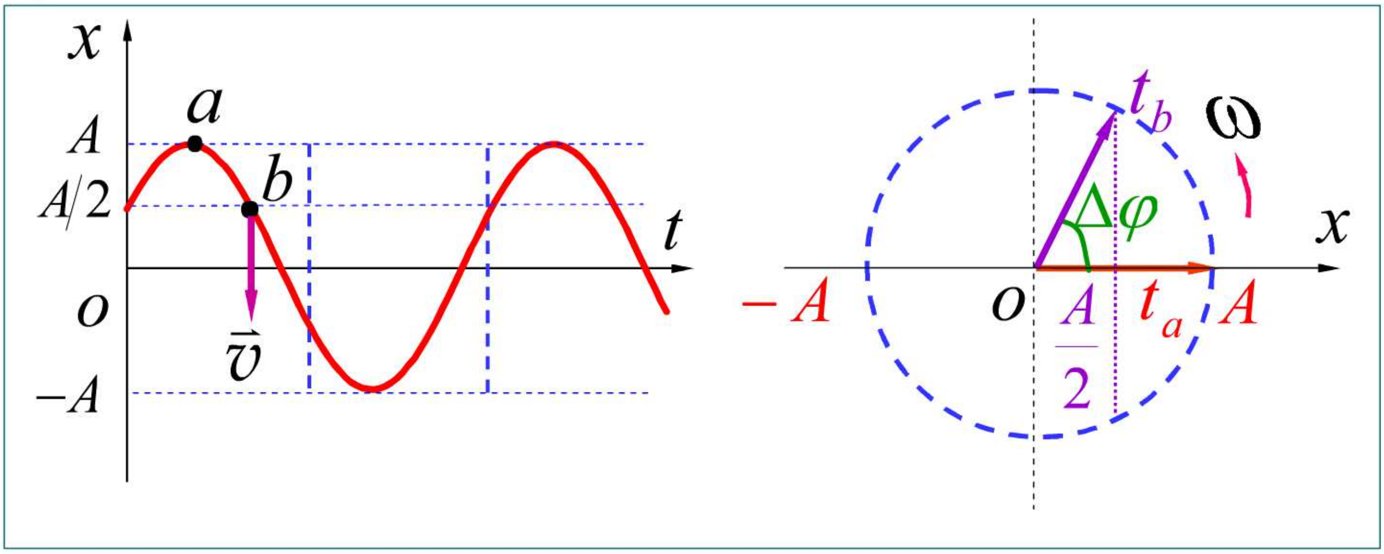
➤ 相位差：表示两个相位之差

（1）对同一简谐运动，相位差可以给出两运动状态间变化所需的时间.

$$x_1 = A\cos(\omega t_1 + \varphi) \qquad x_2 = A\cos(\omega t_2 + \varphi)$$

$$\Delta\varphi = (\omega t_2 + \varphi) - (\omega t_1 + \varphi)$$

$$\Delta t = t_2 - t_1 = \frac{\Delta\varphi}{\omega}$$



$$\Delta\varphi=\frac{\pi}{3}\qquad \Delta t=\frac{\pi/3}{2\pi}T=\frac{1}{6}T$$

（2）对于两个同频率的简谐运动，相位差表示它们间步调上的差异（解决振动合成问题）．

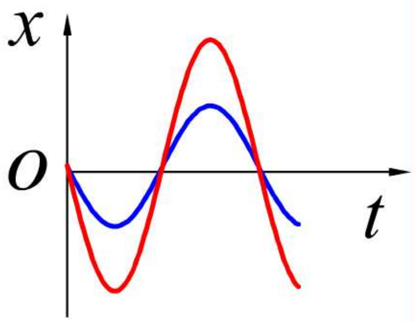
$$x_1 = A_1\cos(\omega t+\varphi_1) \quad x_2 = A_2\cos(\omega t+\varphi_2)$$

$$\Delta\varphi = (\omega t+\varphi_2)-(\omega t+\varphi_1)$$

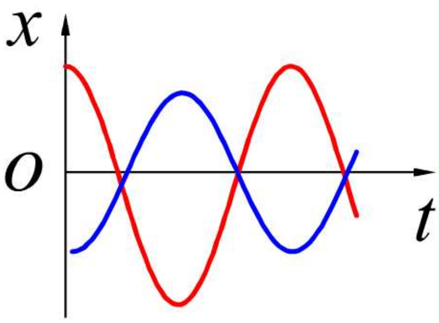
$$\boxed{\Delta\varphi = \varphi_2-\varphi_1}$$

$$\Delta\varphi = \varphi_2 - \varphi_1$$

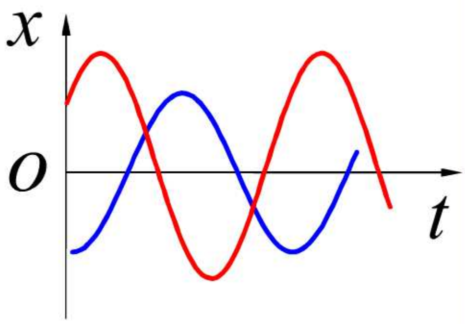
$\Delta\varphi = 0$ 同步



$\Delta\varphi = \pm\pi$ 反相

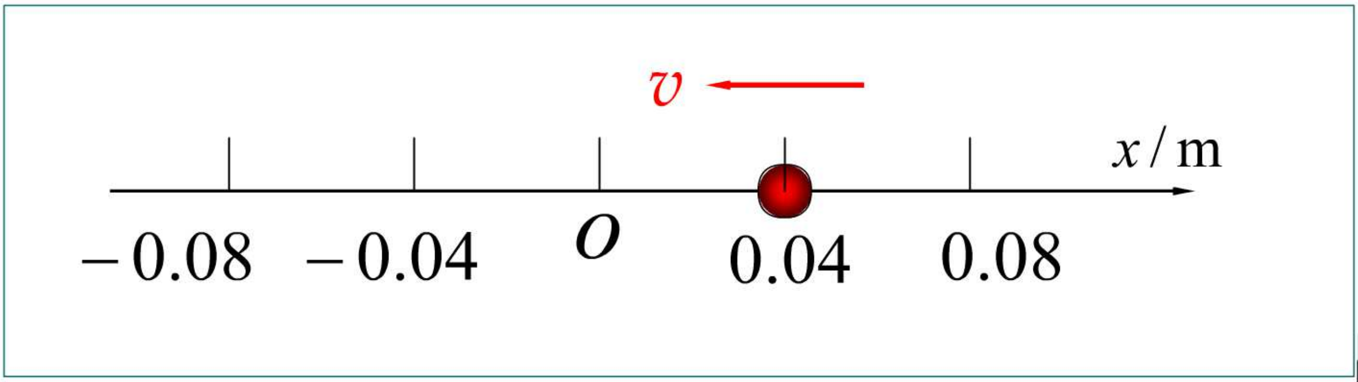


$\Delta\varphi$为其它 $\begin{cases}\text{超前}\\ \text{落后}\end{cases}$



例 一质量为0.01 kg的物体作简谐运动，其振幅为0.08 m，周期为4 s，起始时刻物体在x=0.04 m处，向ox轴负方向运动（如图）．试求

（1）t=1.0 s时，物体所处的位置和所受的力；



已知 $m = 0.01\,\mathrm{kg}, A = 0.08\,\mathrm{m}, T = 4\,\mathrm{s}$

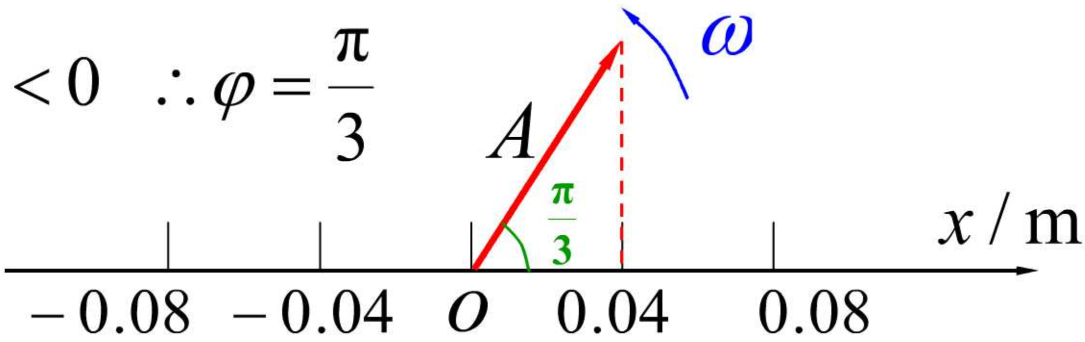
$t = 0, x = 0.04\,\mathrm{m}, v_0 < 0$ 求（1）$t = 1.0\,\mathrm{s}, x, F$

解 $A = 0.08\,\mathrm{m}\quad \omega = \dfrac{2\pi}{T} = \dfrac{\pi}{2}\,\mathrm{s}^{-1}$

$t = 0,\quad x = 0.04\ \mathrm{m}$

代入 $x = A\cos(\omega t + \varphi) \Longrightarrow \varphi = \pm\dfrac{\pi}{3}$

$\because v_0 < 0 \quad \therefore \varphi = \dfrac{\pi}{3}$



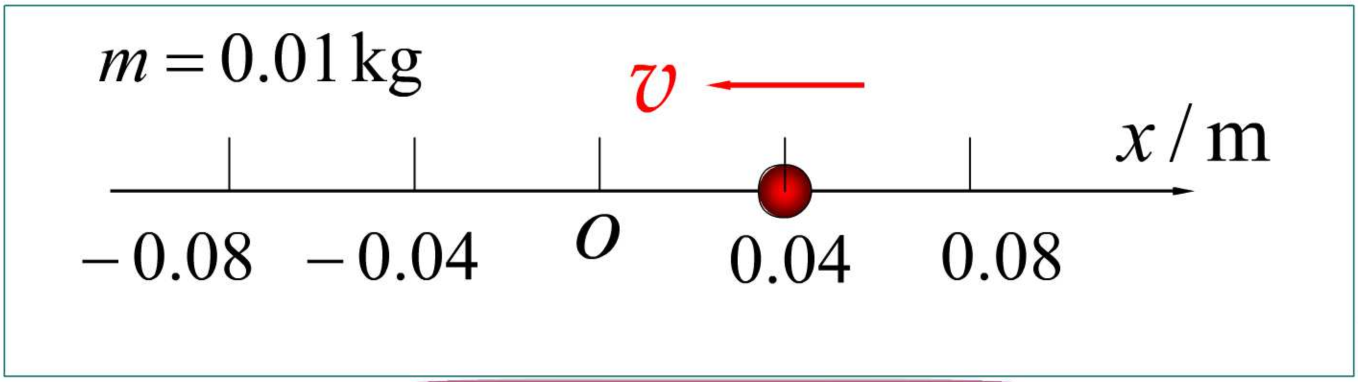
$$\because \varphi = \frac{\pi}{3}$$

$$\therefore x = 0.08\cos(\frac{\pi}{2}t + \frac{\pi}{3})$$

可求（1）$t = 1.0\,\mathrm{s}, x, F$

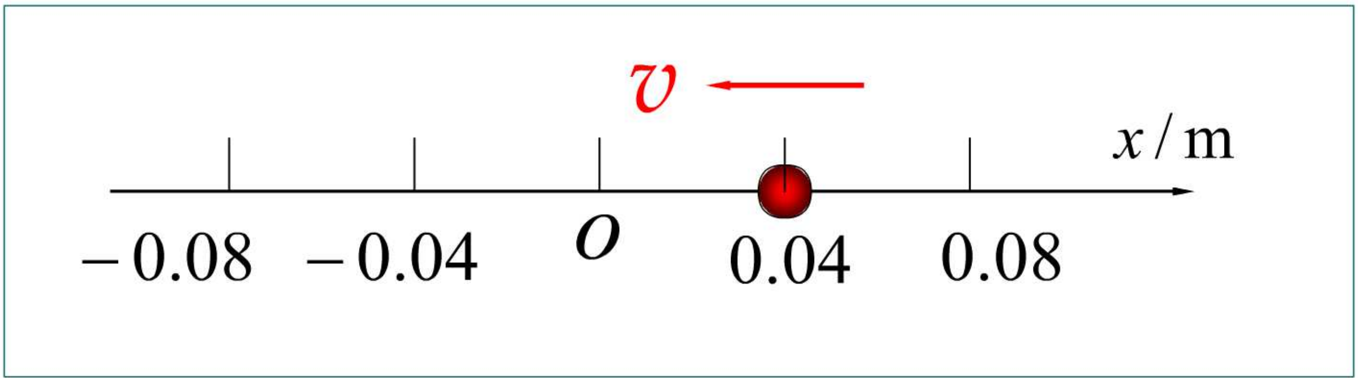
$t = 1.0\,\mathrm{s}$ 代入上式得 $x = -0.069\,\mathrm{m}$

$$F = -kx = -m\omega^2 x = 1.70\times10^{-3}\ \mathrm{N}$$



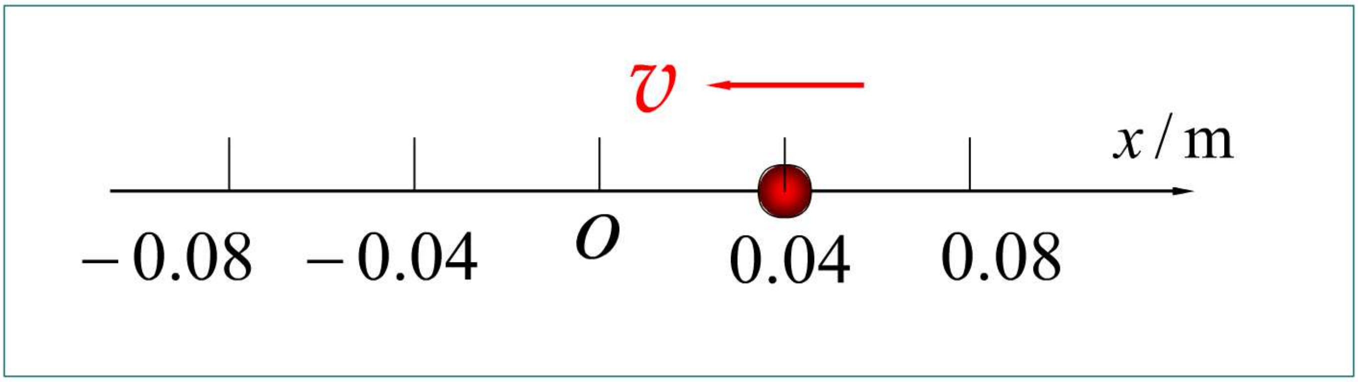
（2）由起始位置运动到x = -0.04 m处所需要的最短时间.

法一 设由起始位置运动到x= -0.04 m处所需要的最短时间为t

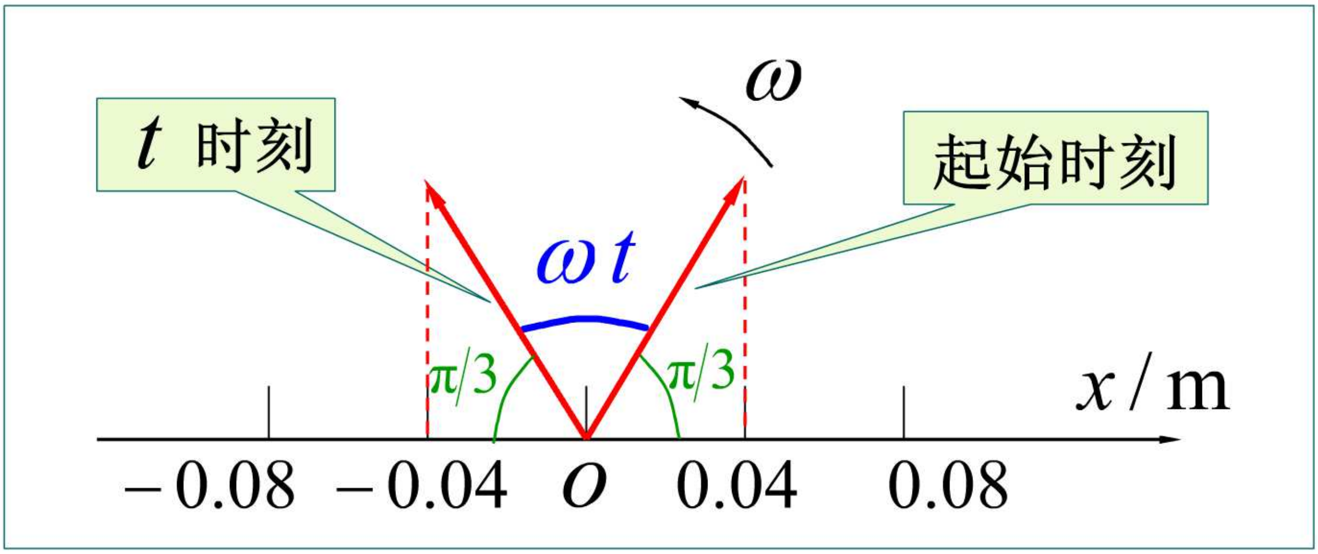


$$x = 0.08\cos(\frac{\pi}{2}t + \frac{\pi}{3}) \Rightarrow -0.04 = 0.08\cos(\frac{\pi}{2}t + \frac{\pi}{3})$$

$$t = \frac{\arccos(-\frac{1}{2}) - \frac{\pi}{3}}{\pi/2} = \frac{2}{3} = 0.667\ \text{s}$$



法二



$$\omega t=\frac{\pi}{3}\qquad \omega=\frac{\pi}{2}\,\mathrm{rad}\cdot\mathrm{s}^{-1}\qquad t=\frac{2}{3}=0.667\,\mathrm{s}$$