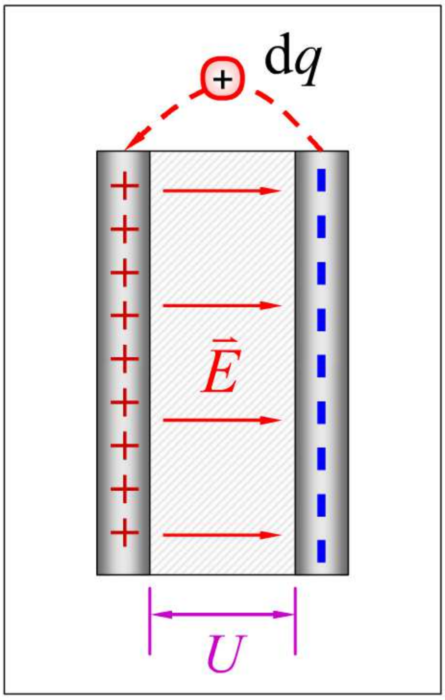
一　电容器的电能

$$\mathrm{d}W = U\mathrm{d}q = \frac{q}{C}\mathrm{d}q$$

$$W = \frac{1}{C}\int_0^Q q\mathrm{d}q = \frac{Q^2}{2C}$$

$$C = \frac{Q}{U}$$

$$W_\mathrm{e} = \frac{Q^2}{2C} = \frac{1}{2}QU = \frac{1}{2}CU^2$$



二 静电场的能量 能量密度

$$W_{\mathrm{e}}=\frac{1}{2}CU^{2}=\frac{1}{2}\frac{\varepsilon S}{d}(Ed)^{2}=\frac{1}{2}\varepsilon E^{2}Sd$$

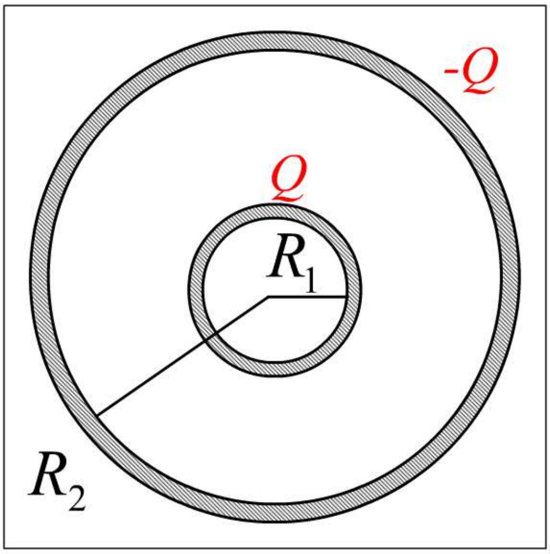
电场能量密度

$$w_{\mathrm{e}}=\frac{1}{2}\varepsilon E^{2}=\frac{1}{2}ED$$

电场空间所存储的能量

$$W_{\mathrm{e}}=\int_{V}w_{\mathrm{e}}\mathrm{d}V=\int_{V}\frac{1}{2}\varepsilon E^{2}\mathrm{d}V$$

例1 如图所示, 球形电容器的内、外半径分别为R_1和R_2，所带电荷为$\pm Q$. 若在两球壳间充以电容率为ε的电介质，问此电容器贮存的电场能量为多少?



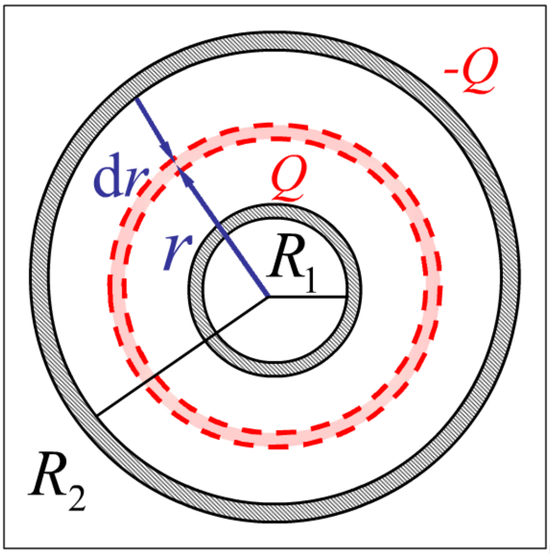
解 $E=\dfrac{1}{4\pi\varepsilon}\dfrac{Q}{r^2}$

$$w_e=\frac{1}{2}\varepsilon E^2=\frac{Q^2}{32\pi^2\varepsilon r^4}$$

$$\mathrm{d}W_e=w_e\mathrm{d}V=\frac{Q^2}{8\pi\varepsilon r^2}\mathrm{d}r$$

$$W_e=\int\mathrm{d}W_e=\frac{Q^2}{8\pi\varepsilon}\int_{R_1}^{R_2}\frac{\mathrm{d}r}{r^2}$$

$$=\frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$



讨 论

$$W_e = \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

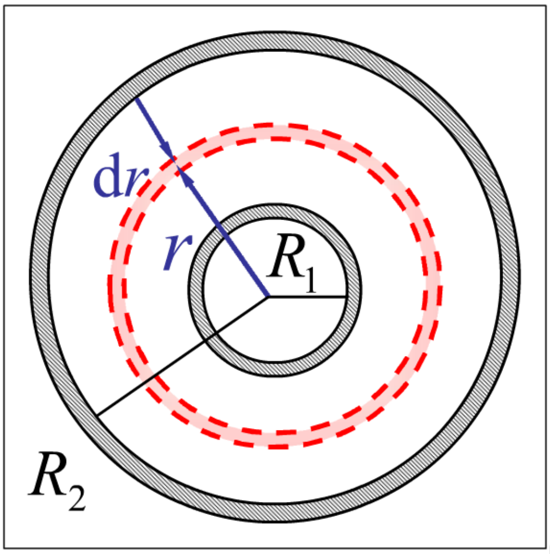
（1） $W_e = \frac{Q^2}{2C}$

$$C = 4\pi\varepsilon \frac{R_2 R_1}{R_2 - R_1}$$

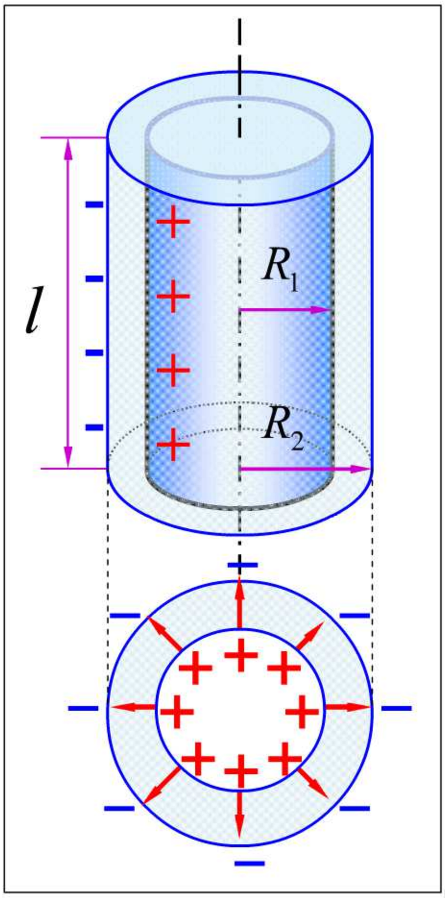
（球形电容器）

（2） $R_2 \to \infty \quad W_e = \frac{Q^2}{8\pi\varepsilon R_1}$

（孤立导体球）



例2 圆柱形空气电容器中，空气的击穿场强是 $E_b=3\times10^6\ \mathrm{V\cdot m^{-1}}$，设导体圆筒的外半径 $R_2=10^{-2}\ \mathrm{m}$．在空气不被击穿的情况下，长圆柱导体的半径 R_1 取多大值可使电容器存储能量最多？

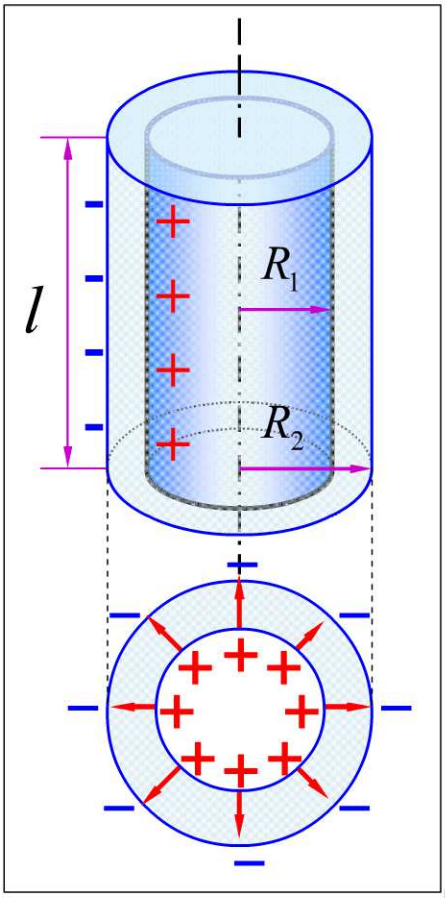


解 $E=\dfrac{\lambda}{2\pi\varepsilon_0 r}\quad (R_1<r<R_2)$

$$E_b=\frac{\lambda_{max}}{2\pi\varepsilon_0 R_1}$$

$$U=\frac{\lambda}{2\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$$



$$U=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$$

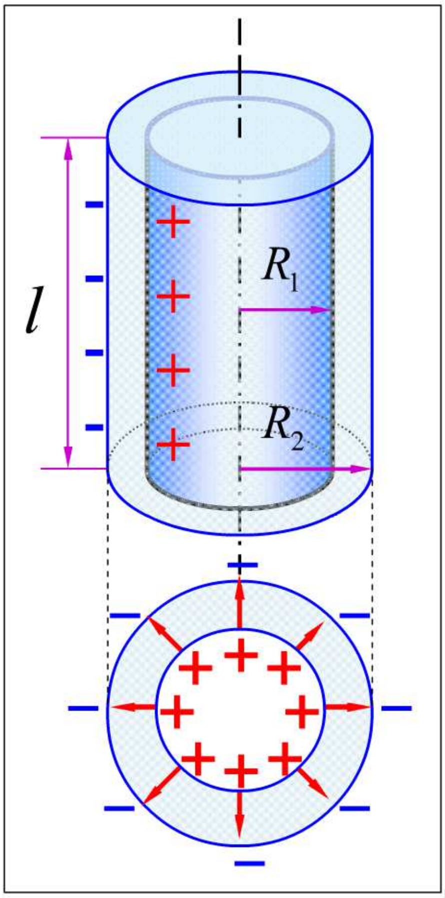
单位长度的电场能量

$$W_e=\frac{1}{2}\lambda U=\frac{\lambda^2}{4\pi\varepsilon_0}\ln\frac{R_2}{R_1}$$

$$E_b=\frac{\lambda_{max}}{2\pi\varepsilon_0 R_1}$$

$$\lambda=\lambda_{max}=2\pi\varepsilon_0 E_b R_1$$

$$W_e=\pi\varepsilon_0 E_b^2 R_1^2\ln\frac{R_2}{R_1}$$



$$W_e = \pi\varepsilon_0 E_b^2 R_1^2 \ln\frac{R_2}{R_1}$$

$$\frac{dW_e}{dR_1} = \pi\varepsilon_0 E_b^2 R_1 \left(2\ln\frac{R_2}{R_1} - 1\right) = 0$$

$$R_1 = \frac{R_2}{\sqrt{e}} \approx 6.07\times10^{-3}\ \text{m}$$

$$U_{max} = E_b R_1 \ln\frac{R_2}{R_1} = \frac{E_b R_2}{2\sqrt{e}}$$

$$= 9.10\times10^{3}\ \text{V}$$

$E_b = 3\times10^6\ \text{V}\cdot\text{m}^{-1}$ ， $R_2 = 10^{-2}\ \text{m}$