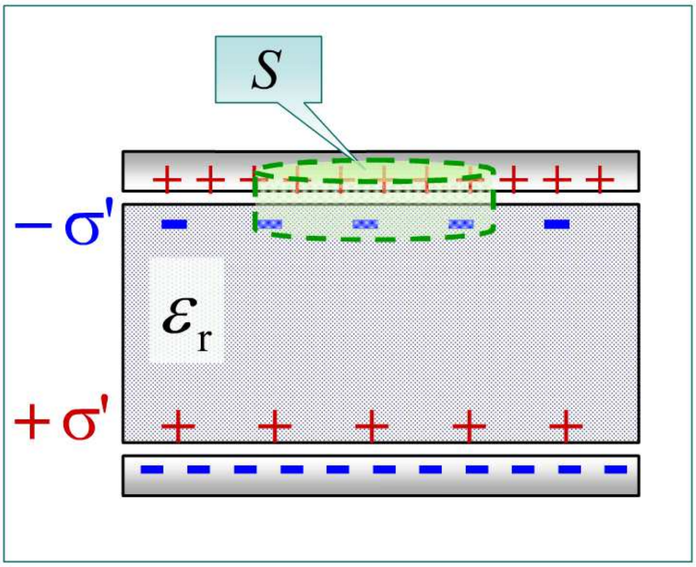
6-3 电位移 有介质时的高斯定理

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}(Q_0 - Q')$$

$$Q' = \frac{\varepsilon_r - 1}{\varepsilon_r} Q_0$$

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_0}{\varepsilon_0 \varepsilon_r}$$

$$\oint_S \varepsilon_0 \varepsilon_r \vec{E}\cdot d\vec{S} = Q_0$$



电容率 $\varepsilon = \varepsilon_0 \varepsilon_r$

$$\oint_S \varepsilon \vec{E}\cdot d\vec{S} = Q_0$$

$$\oint_S \varepsilon \vec{E} \cdot \mathrm{d}\vec{S} = Q_0$$

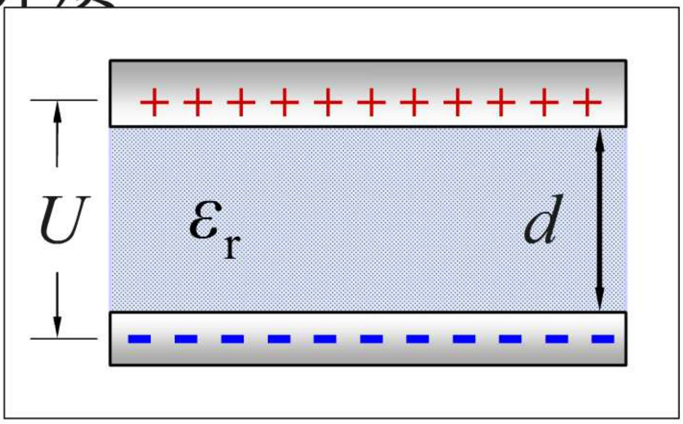
电位移矢量 $\vec{D} = \varepsilon_0 \varepsilon_{\mathrm{r}} \vec{E} = \varepsilon \vec{E}$

电位移通量 $\oint_S \vec{D} \cdot \mathrm{d}\vec{S}$

有介质时的高斯定理

$$\oint_S \vec{D} \cdot \mathrm{d}\vec{S} = \sum_{i=1}^{n} Q_{0i}$$

例1 把一块相对电容率$\varepsilon_r=3$的电介质，放在相距d=1 mm的两平行带电平板之间．放入之前，两板的电势差是1 000 V．试求两板间电介质内的电场强度E，电极化强度P，板和电介质的电荷面密度，电介质内的电位移D.

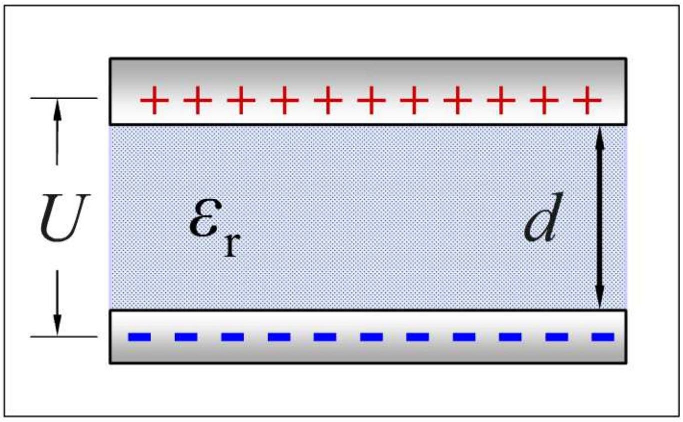


解 $E_0=\dfrac{U}{d}=10^3\ \mathrm{kV\cdot m^{-1}}$

$$E=E_0/\varepsilon_r=3.33\times10^2\ \mathrm{kV\cdot m^{-1}}$$

$$P=(\varepsilon_r-1)\varepsilon_0 E=5.89\times10^{-6}\ \mathrm{C\cdot m^{-2}}$$

$\varepsilon_r=3$，
d=1 mm，
U=1 000 V

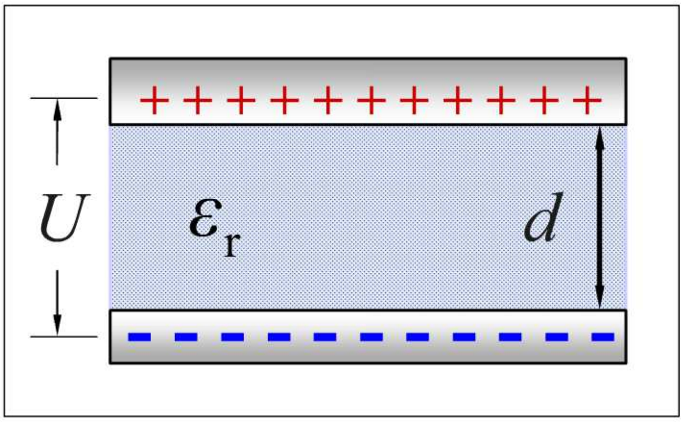


$$\sigma_0 = \varepsilon_0 E_0 = 8.85\times10^{-6}\ \mathrm{C\cdot m^{-2}}$$

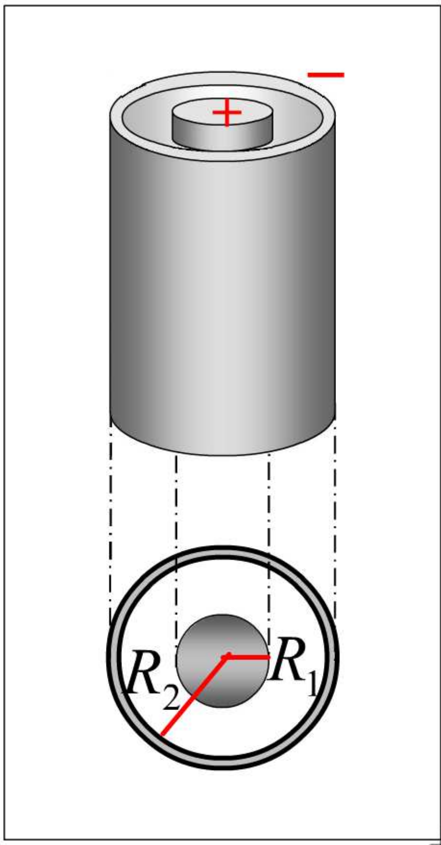
$$\sigma' = P = 5.89\times10^{-6}\ \mathrm{C\cdot m^{-2}}$$

$$D = \varepsilon_0\varepsilon_r E = \varepsilon_0 E_0 = \sigma_0 = 8.85\times10^{-6}\ \mathrm{C\cdot m^{-2}}$$

$\varepsilon_r = 3$，
d=1 mm，
U=1 000 V



例2 图中是由半径为R_1的长直圆柱导体和同轴的半径为R_2的薄导体圆筒组成，其间充以相对电容率为ε_r的电介质. 设直导体和圆筒单位长度上的电荷分别为$+\lambda$和$-\lambda$. **求(1)** 电介质中的电场强度、电位移和极化强度； **(2)** 电介质内外表面的极化电荷面密度.



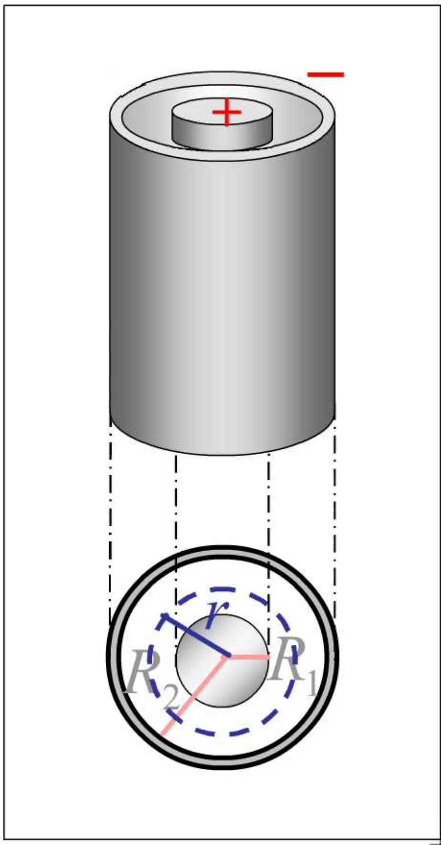
解（1） $\oint_S \vec{D}\cdot \mathrm{d}\vec{S} = \lambda l$

$D2\pi rl = \lambda l \qquad D = \dfrac{\lambda}{2\pi r}$

$E = \dfrac{D}{\varepsilon_0\varepsilon_\mathrm{r}} = \dfrac{\lambda}{2\pi\varepsilon_0\varepsilon_\mathrm{r} r}$

$(R_1 < r < R_2)$

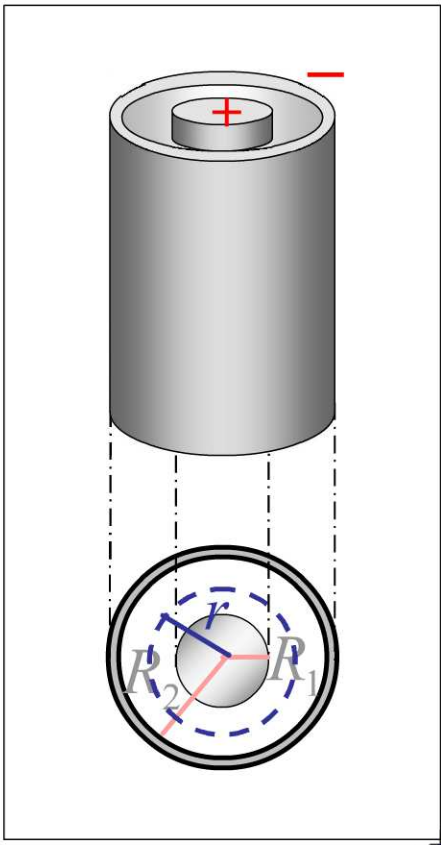
$P = (\varepsilon_\mathrm{r}-1)\varepsilon_0 E = \dfrac{\varepsilon_\mathrm{r}-1}{2\pi\varepsilon_\mathrm{r} r}\lambda$



(2) $E = \dfrac{\lambda}{2\pi\varepsilon_0\varepsilon_r r}$

$$\begin{cases} E_1 = \dfrac{\lambda}{2\pi\varepsilon_0\varepsilon_r R_1} & (r = R_1) \\ E_2 = \dfrac{\lambda}{2\pi\varepsilon_0\varepsilon_r R_2} & (r = R_2) \end{cases}$$

$$\begin{cases} \sigma_1' = -(\varepsilon_r - 1)\varepsilon_0 E_1 = -\dfrac{(\varepsilon_r - 1)\lambda}{2\pi\varepsilon_r R_1} \\ \sigma_2' = (\varepsilon_r - 1)\varepsilon_0 E_2 = \dfrac{(\varepsilon_r - 1)\lambda}{2\pi\varepsilon_r R_2} \end{cases}$$



END