一　等势面

电场中电势相等的点所构成的面.

- 电荷沿等势面移动时，电场力做功为零.

$$W_{AB}=q(V_A-V_B)=\int_a^b q\vec{E}\cdot \mathrm{d}\vec{l}=0$$

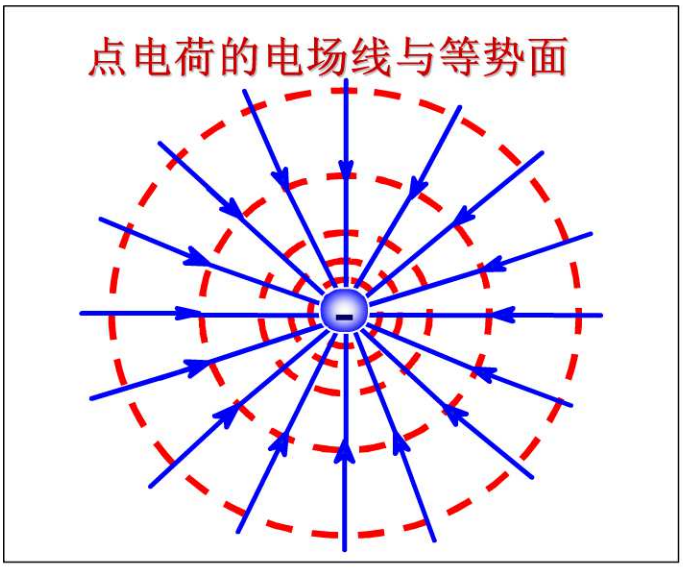
$$\therefore \quad \vec{E}\perp \mathrm{d}\vec{l}$$

- 某点的电场强度与通过该点的等势面垂直.

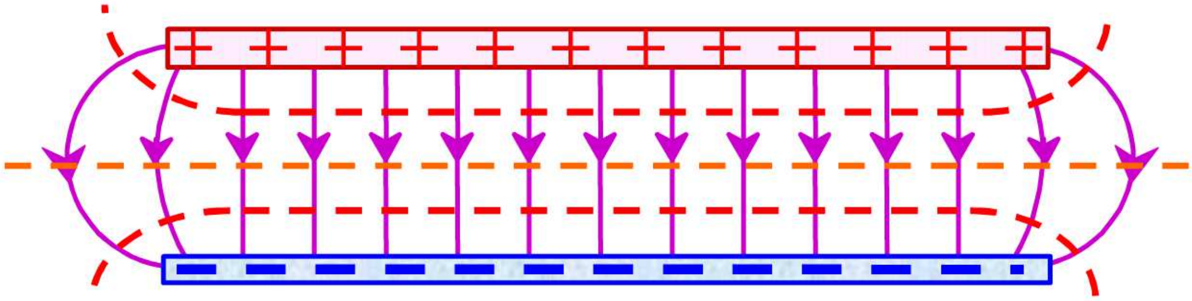
◆ 用等势面的疏密表示电场的强弱.

任意两相邻等势面间的电势差相等.

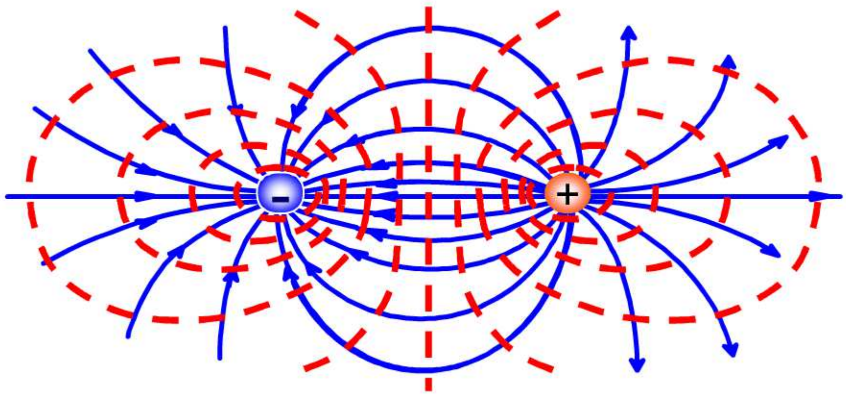
等势面越密的地方，电场强度越大.



两平行带电平板的电场线和等势面



一对等量异号点电荷的电场线和等势面



二 电场强度与电势梯度

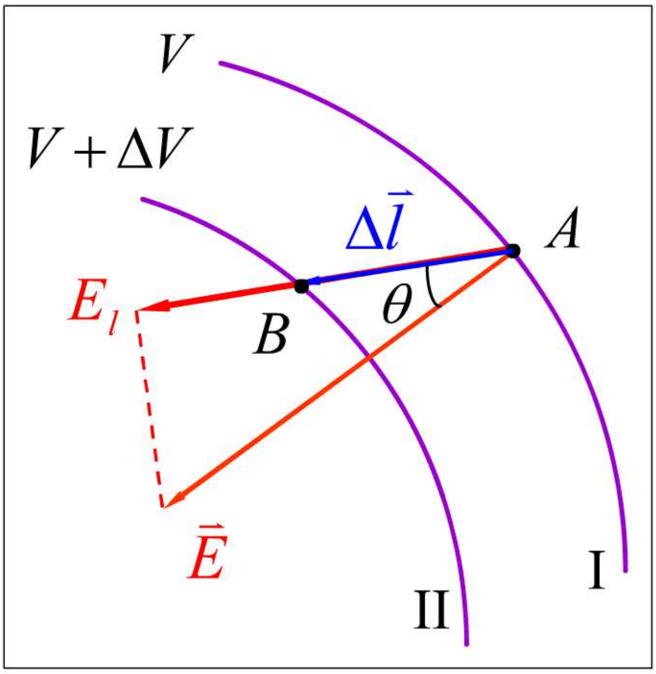
$$-\Delta V = \vec{E}\cdot\Delta\vec{l}$$

$$= E\Delta l\cos\theta$$

$$E\cos\theta = E_l$$

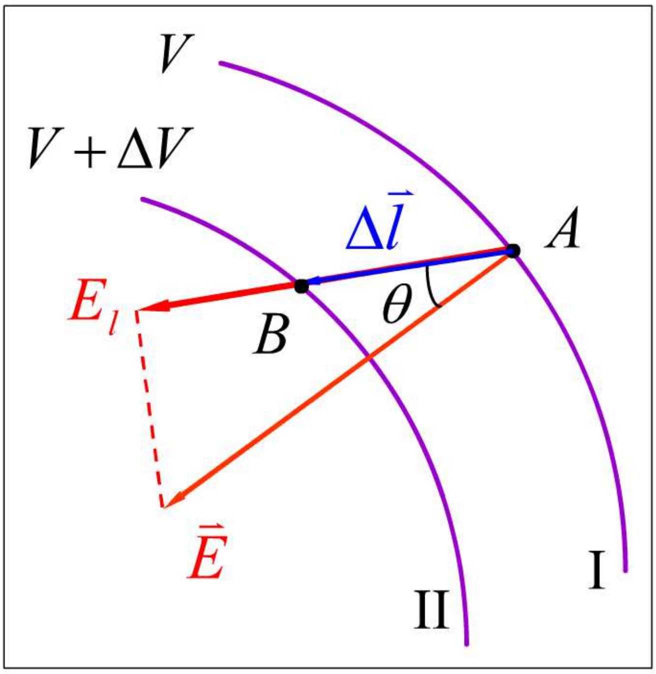
$$E_l = -\frac{\Delta V}{\Delta l}$$

$$E_l = -\lim_{\Delta l\to 0}\frac{\Delta V}{\Delta l} = -\frac{\mathrm{d}V}{\mathrm{d}l}$$



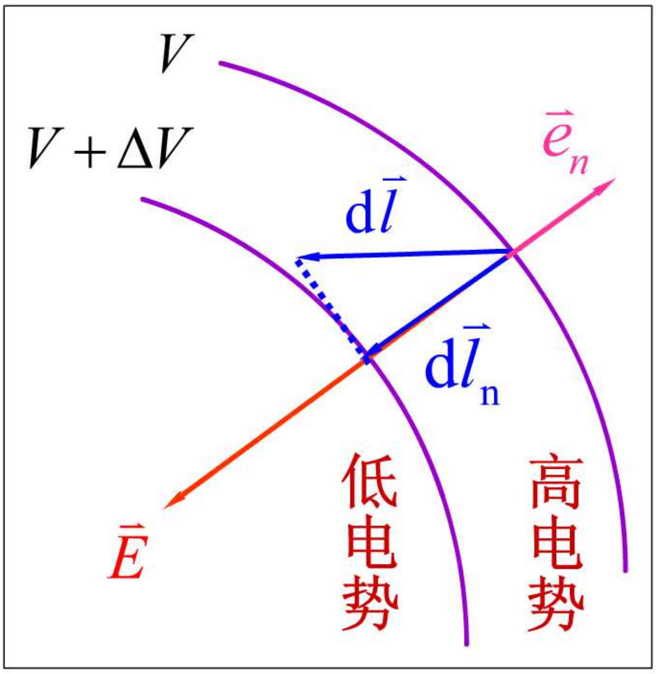
$$E_l = -\frac{\mathrm{d}V}{\mathrm{d}l}$$

电场中某一点的电场强度沿任一方向的分量，等于这一点的电势沿该方向单位长度上电势变化率的负值.



$$E_l = -\frac{\mathrm{d}V}{\mathrm{d}l} \qquad E_\mathrm{n} = -\frac{\mathrm{d}V}{\mathrm{d}l_\mathrm{n}}$$

$$\because \mathrm{d}l > \mathrm{d}l_\mathrm{n} \qquad \therefore E_\mathrm{n} > E_l$$



$$\vec{E} = -\frac{\mathrm{d}V}{\mathrm{d}l_\mathrm{n}}\vec{e}_\mathrm{n}$$

大小 $|\vec{E}| = \left|\frac{\mathrm{d}V}{\mathrm{d}l_\mathrm{n}}\right|$

方向 由高电势处指向低电势处

电场强度等于**电势梯度**的负值

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right) = -\mathrm{grad}V = -\nabla V$$

求电场强度的三种方法

- 利用电场强度叠加原理
- 利用高斯定理
- 利用电势与电场强度的关系

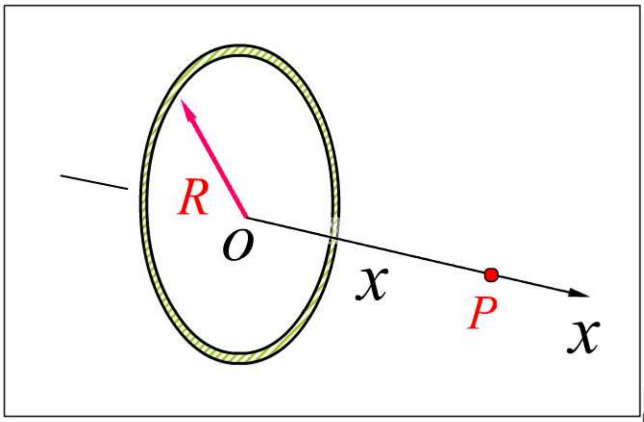
例1 用电场强度与电势的关系，求均匀带电细圆环轴线上一点的电场强度.

解
$$V=\frac{q}{4\pi\varepsilon_0(x^2+R^2)^{1/2}}$$

$$E=E_x=-\frac{\partial V}{\partial x}$$

$$=-\frac{\partial}{\partial x}\left[\frac{q}{4\pi\varepsilon_0(x^2+R^2)^{1/2}}\right]$$

$$=\frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

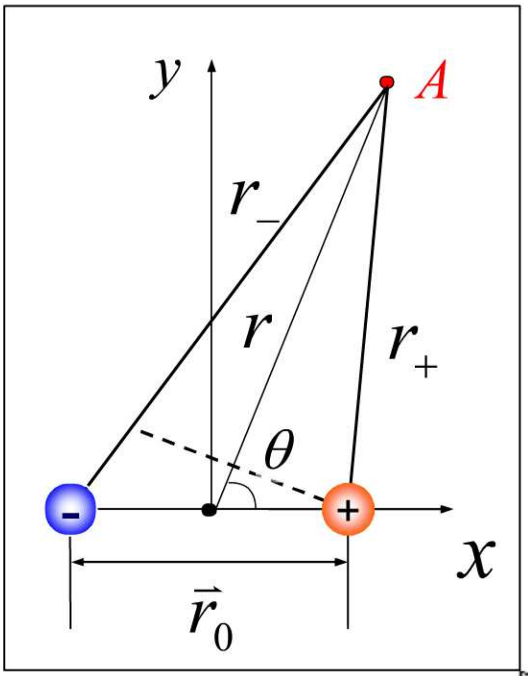


例2 求电偶极子电场中任意一点A的电势和电场强度.

解

$$\begin{cases} V_{+} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_{+}} \\ V_{-} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_{-}} \end{cases}$$

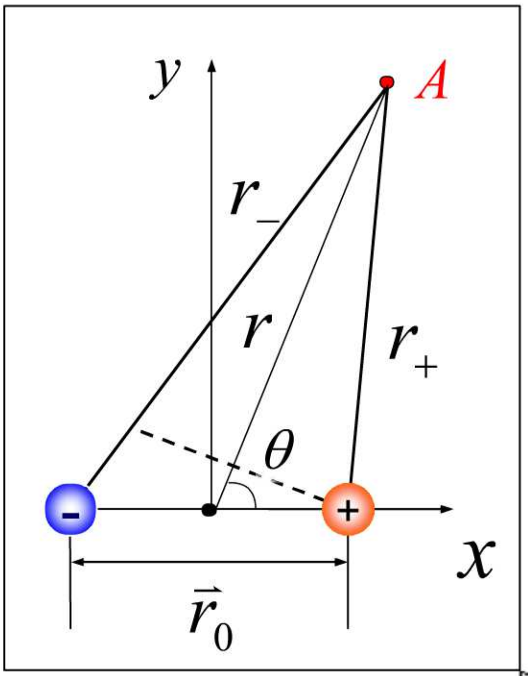
$$V = V_{+} + V_{-} = \frac{q}{4\pi\varepsilon_0}\frac{r_{-} - r_{+}}{r_{+}r_{-}}$$



$\because r_0 << r \quad \therefore r_- - r_+ \approx r_0 \cos\theta \qquad r_- r_+ \approx r^2$

$$V = V_+ + V_- = \frac{q}{4\pi\varepsilon_0} \frac{r_- - r_+}{r_+ r_-}$$

$$\approx \frac{q}{4\pi\varepsilon_0} \frac{r_0 \cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$$



$$\begin{cases} \theta = 0 & V \approx \dfrac{1}{4\pi\varepsilon_0} \dfrac{p}{r^2} \\ \theta = \pi & V \approx -\dfrac{1}{4\pi\varepsilon_0} \dfrac{p}{r^2} \\ \theta = \dfrac{\pi}{2} & V = 0 \end{cases}$$

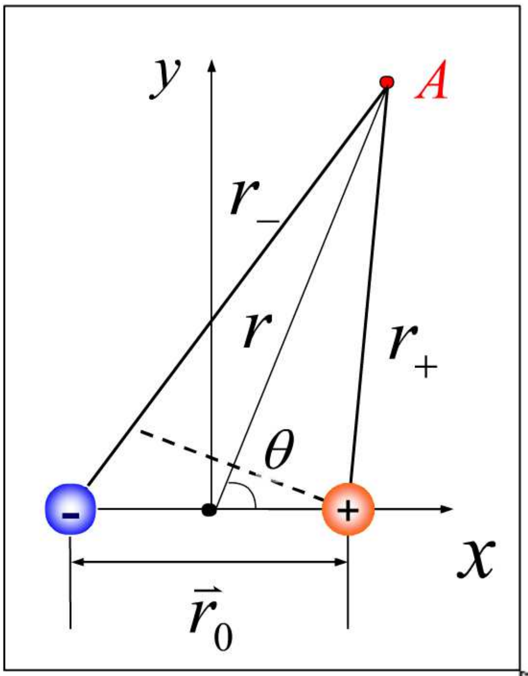
$$V=\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}=\frac{p}{4\pi\varepsilon_0}\frac{x}{(x^2+y^2)^{3/2}}$$

$$E_x=-\frac{\partial V}{\partial x}=-\frac{p}{4\pi\varepsilon_0}\frac{y^2-2x^2}{(x^2+y^2)^{5/2}}$$

$$E_y=-\frac{\partial V}{\partial y}=\frac{p}{4\pi\varepsilon_0}\frac{3xy}{(x^2+y^2)^{5/2}}$$

$$E=\sqrt{E_x^2+E_y^2}$$

$$=\frac{p}{4\pi\varepsilon_0}\frac{(4x^2+y^2)^{1/2}}{(x^2+y^2)^2}$$



$$E = \frac{p}{4\pi\varepsilon_0}\frac{(4x^2+y^2)^{1/2}}{(x^2+y^2)^2}$$

$$\begin{cases} y=0 & E = \dfrac{2p}{4\pi\varepsilon_0}\dfrac{1}{x^3} \\ x=0 & E = \dfrac{p}{4\pi\varepsilon_0}\dfrac{1}{y^3} \end{cases}$$