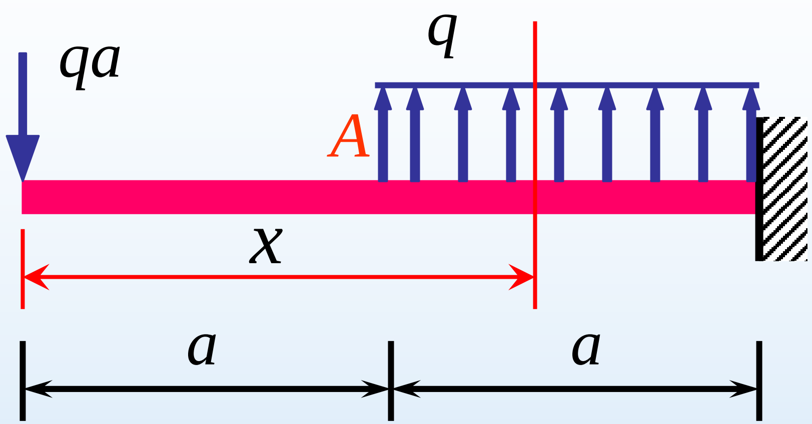
例：讨论微分关系



$$F_S(x) = -qa + q(x-a)$$

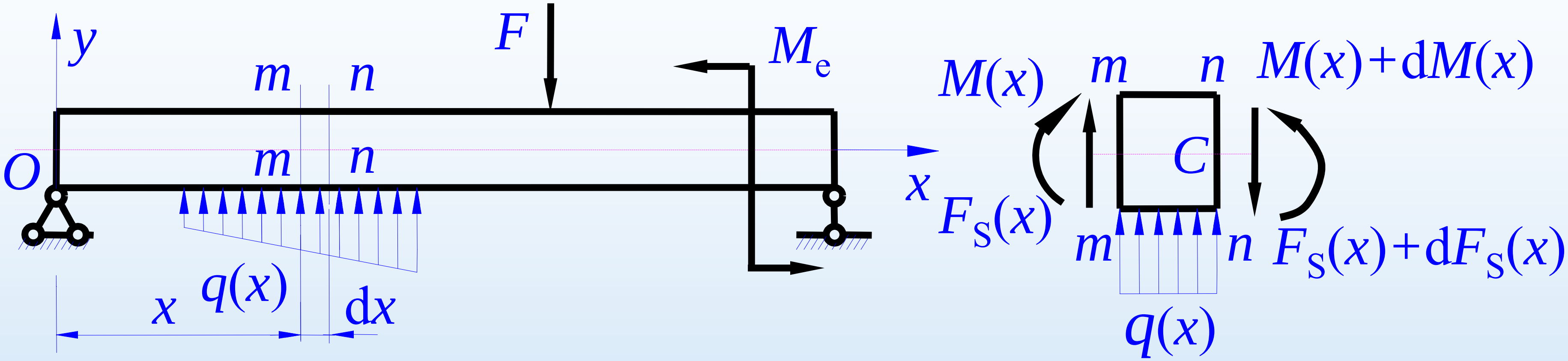
$$M(x) = -qax + \frac{1}{2}q(x-a)^2$$

$$\frac{dF_S(x)}{dx} = q$$

$$\frac{dM(x)}{dx} = -qa + q(x-a) = F_S(x)$$

$$\frac{d^2M(x)}{dx^2} = q$$

III、弯矩、剪力与分布荷载集度之间的关系及其应用



$$\sum F_y = 0 \qquad F_S(x) - [F_S(x) + dF_S(x)] + q(x)dx = 0$$

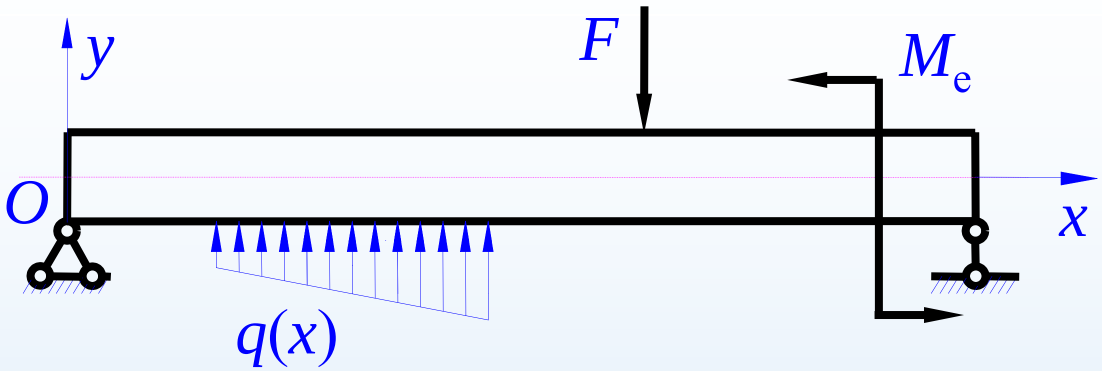
$$dF_S(x) = q(x)dx \Rightarrow \frac{dF_S(x)}{dx} = q(x)$$

$$\sum M_C = 0$$

略去

$$[M(x) + dM(x)] - M(x) - F_S(x)dx - q(x)dx \cdot \frac{dx}{2} = 0$$

$$dM(x) = F_S(x)dx \Rightarrow \frac{dM(x)}{dx} = F_S(x)$$



$q(x)$、$F_S(x)$、$M(x)$间的微分关系

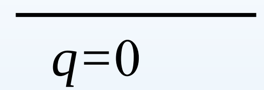
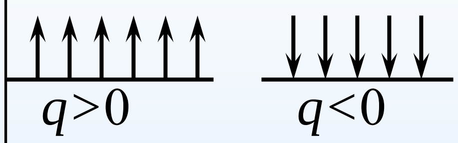
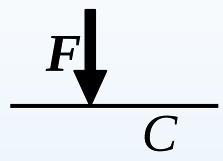
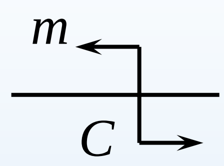
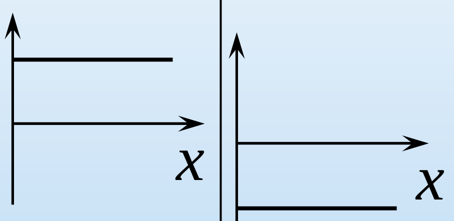
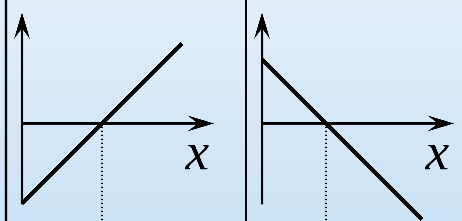
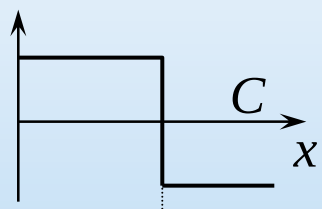
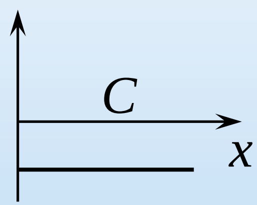
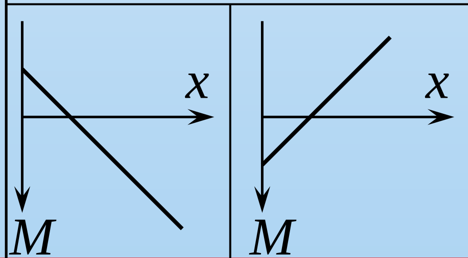
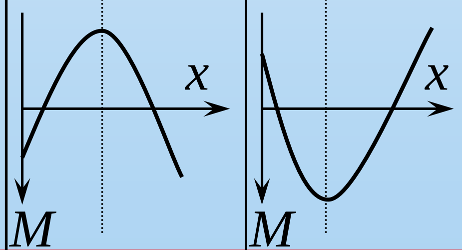
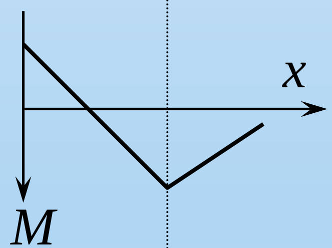
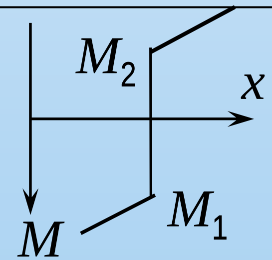
$$\frac{\mathrm{d}F_S(x)}{\mathrm{d}x}=q(x)$$

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x}=F_S(x)$$

$$\Longrightarrow \quad \frac{\mathrm{d}^2M(x)}{\mathrm{d}x^2}=q(x)$$

其中分布荷载集度 $q(x)$ 以向上为正，向下为负。

剪力、弯矩与外力间的关系

外力	无外力段	均布载荷段	集中力	集中力偶
	 $q=0$	 $q>0$　$q<0$		
F_S图特征	水平直线	斜直线	自左向右突变	无变化
	 $F_S>0$　$F_S<0$	 增函数　降函数		
M图特征	斜直线	曲线	自左向右折角	自左向右突变
	 增函数　降函数	 坟状　盆状	 折向与F反向	与m反  $M_1-M_2=m$

利用以上特征

1、可以校核已作出的剪力图和弯矩图是否正确；

2、可以不建立剪力方程和弯矩方程，利用微分关系直接绘制剪力图和弯矩图。

利用微分关系直接绘制剪力图和弯矩图的步骤：

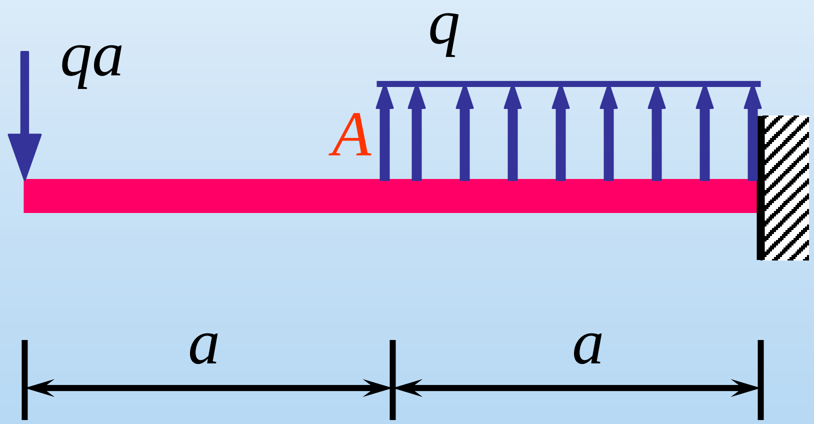
1．求支座反力；

2．分段确定剪力图和弯矩图的形状；

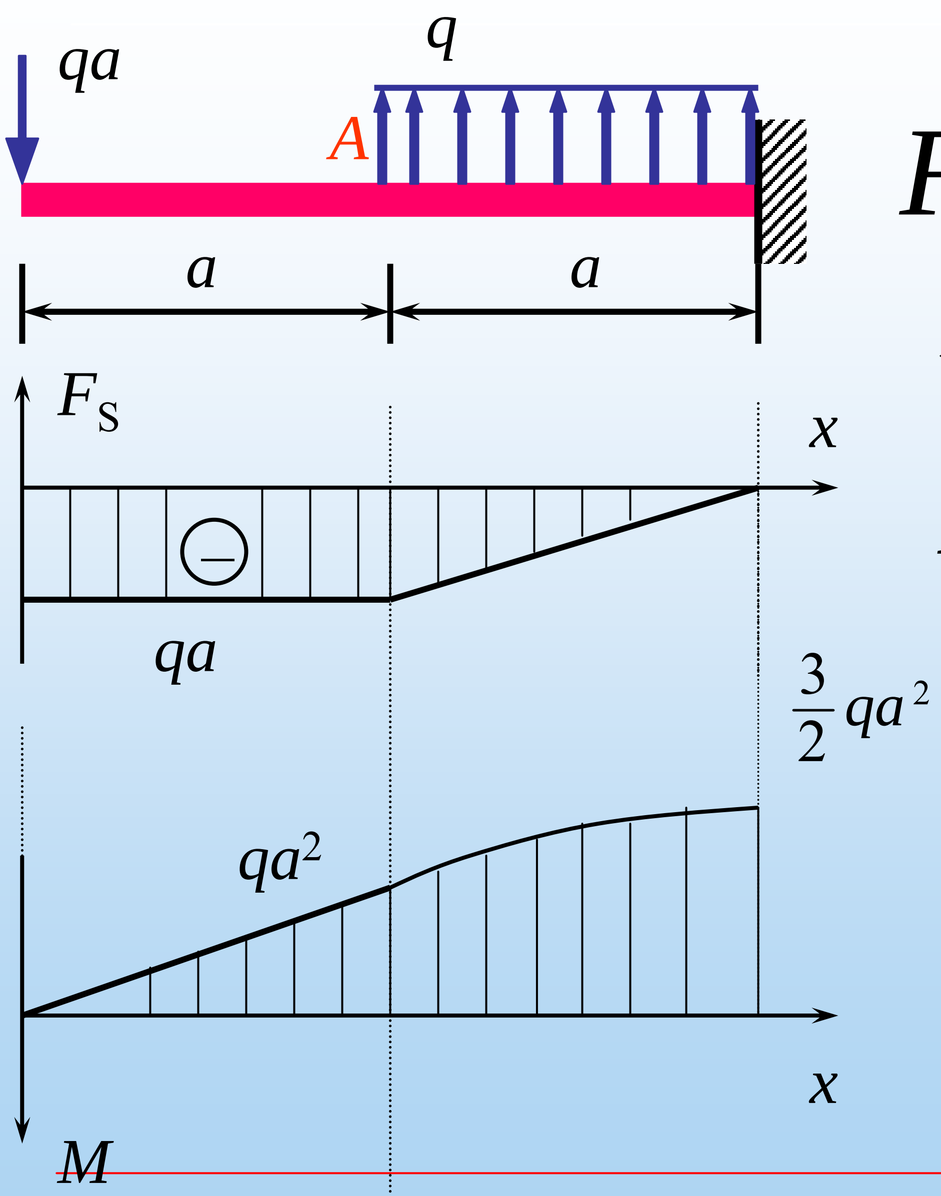
3．计算控制截面内力值，根据微分关系绘剪力图和弯矩图；

4．确定 $\left|F_S\right|_{max}$ 和 $\left|M\right|_{max}$。

例4-8 试利用弯矩、剪力与分布荷载集度间的微分关系作图示梁的剪力图和弯矩图。



解：利用内力和外力的关系及特殊点的内力值来作图。



左端点：

$$F_S = -qa, M = 0$$

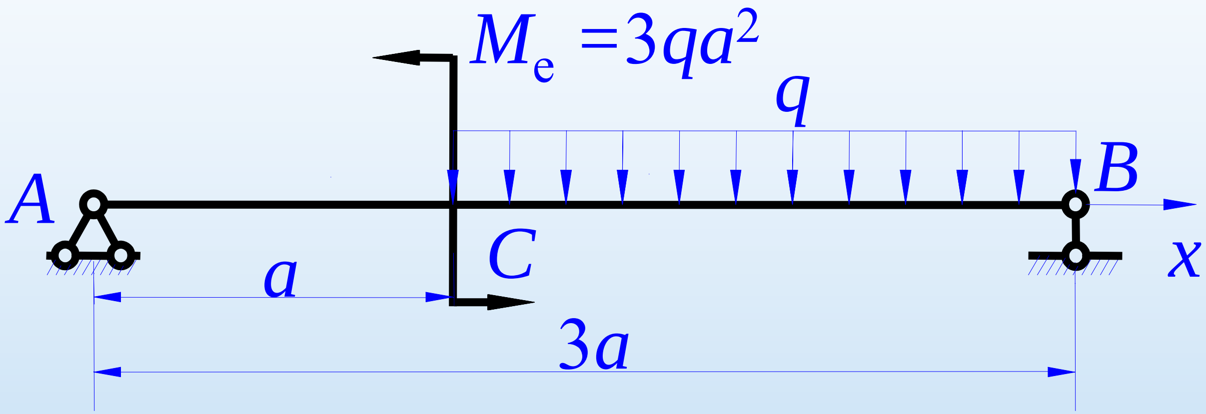
分区点A：

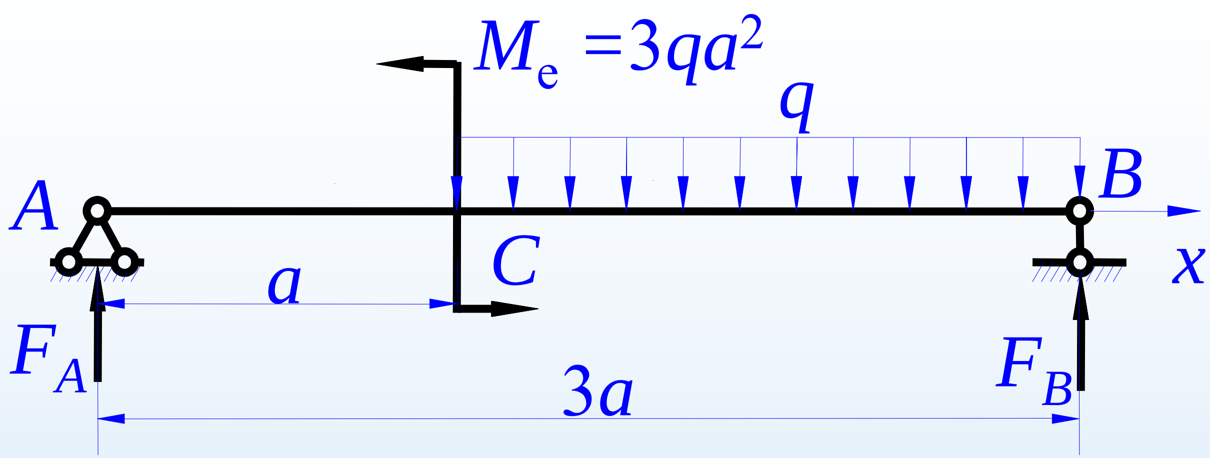
$$F_S = -qa, M = -qa^2$$

右端点：

$$F_S = 0, M = -\frac{3}{2}qa^2$$

例4-9试利用弯矩、剪力与分布荷载集度间的微分关系作图示梁的剪力图和弯矩图。





解：1、支反力为

$$\sum M_B = 0 \qquad 3qa^2 + q\times 2a\times a - F_A\times 3a = 0$$

$$\sum M_B = 0 \qquad F_A\times 3a + 3qa^2 - q\times 2a\times 2a = 0$$

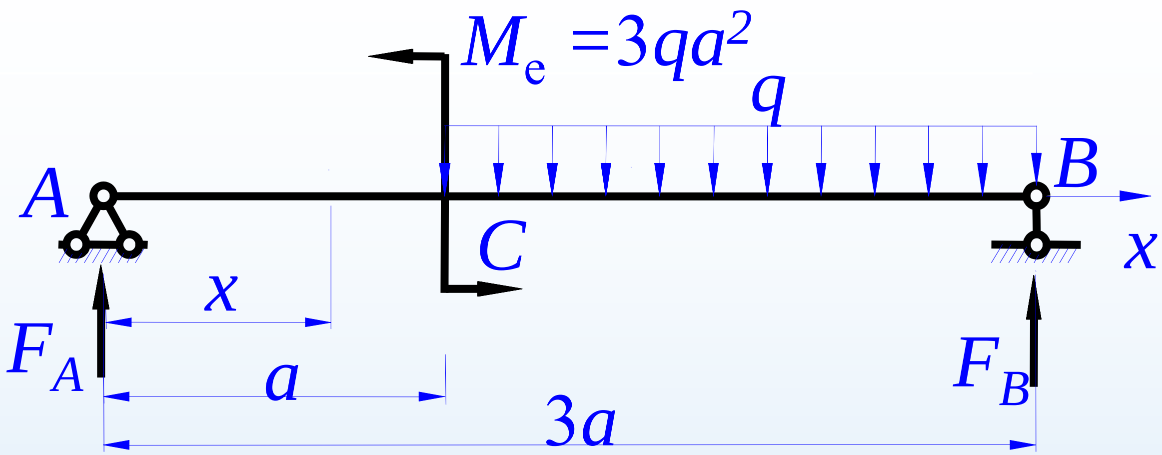
$$F_A = \frac{5}{3}qa \qquad F_B = \frac{1}{3}qa$$

2、作剪力图

AC段： q=0

剪力图为水平直线

剪力值 $F_S=\frac{5}{3}qa$

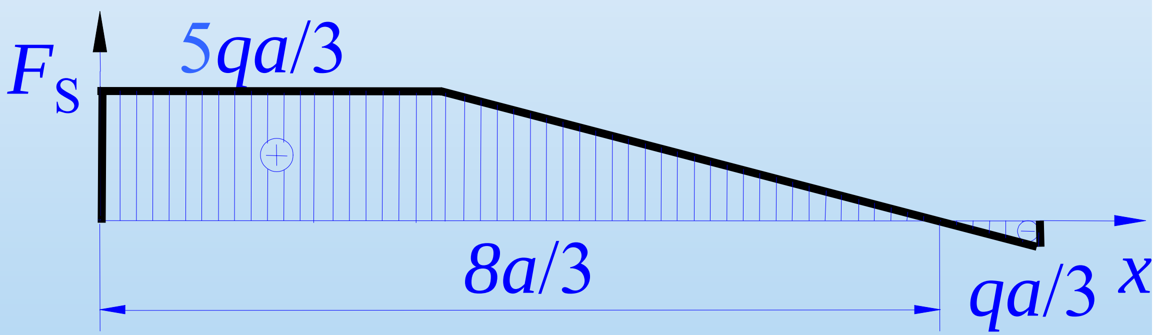


CB段：q=常量<0

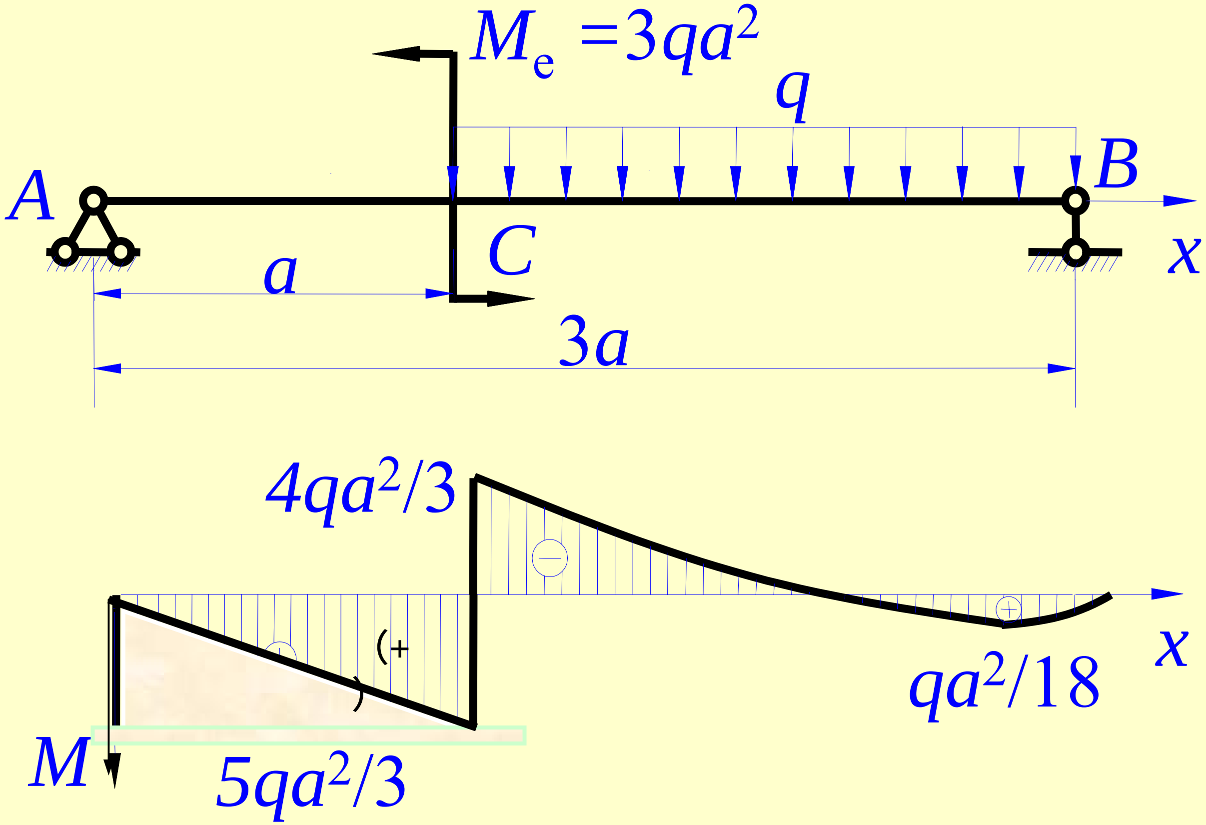
剪力图为向右下方倾斜的斜直线

$$F_{SC}=\frac{5}{3}qa$$

$$F_{SB}=-\frac{1}{3}qa$$



3、作弯矩图

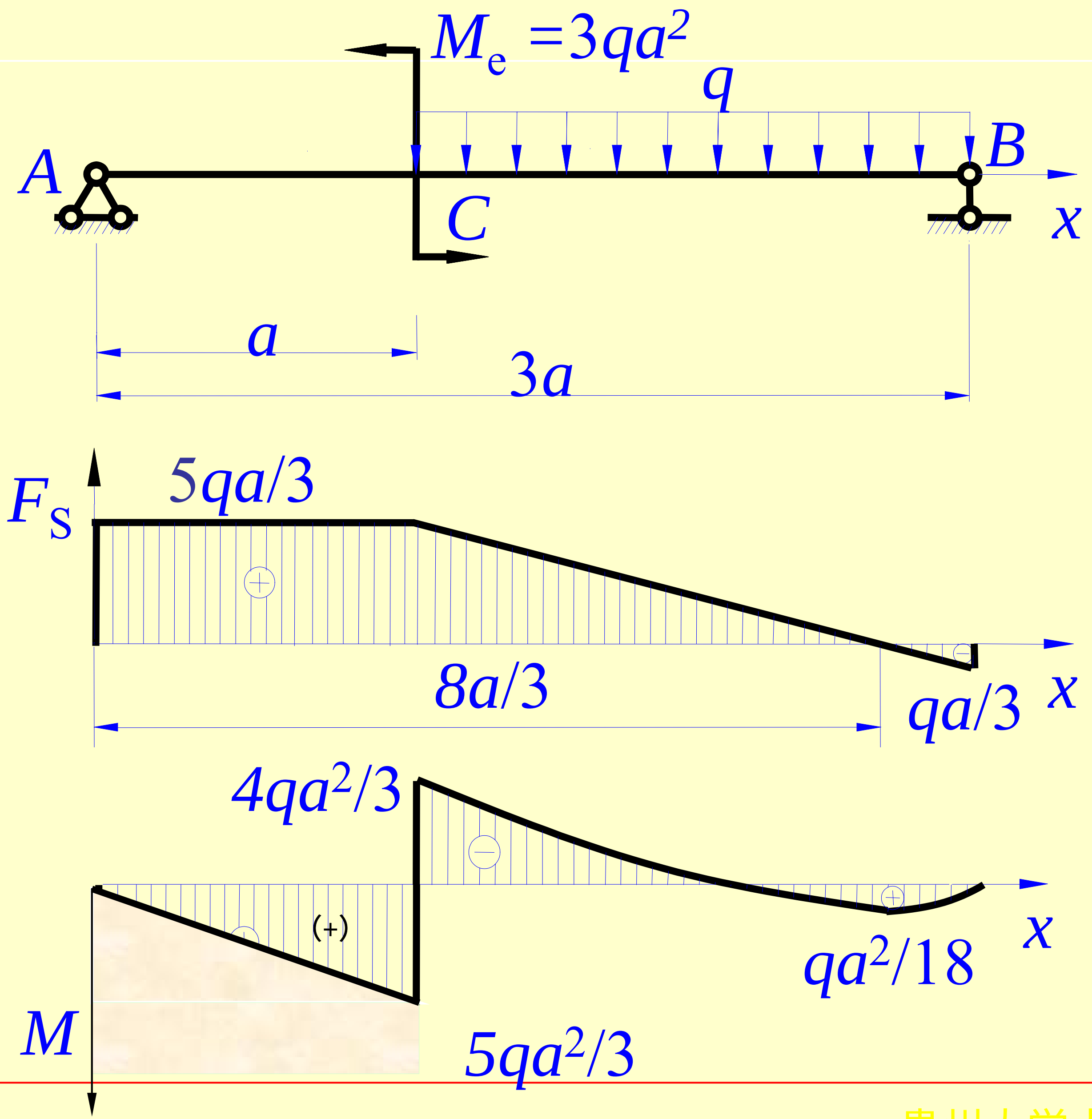


*AC*段

弯矩图→斜直线

CB段

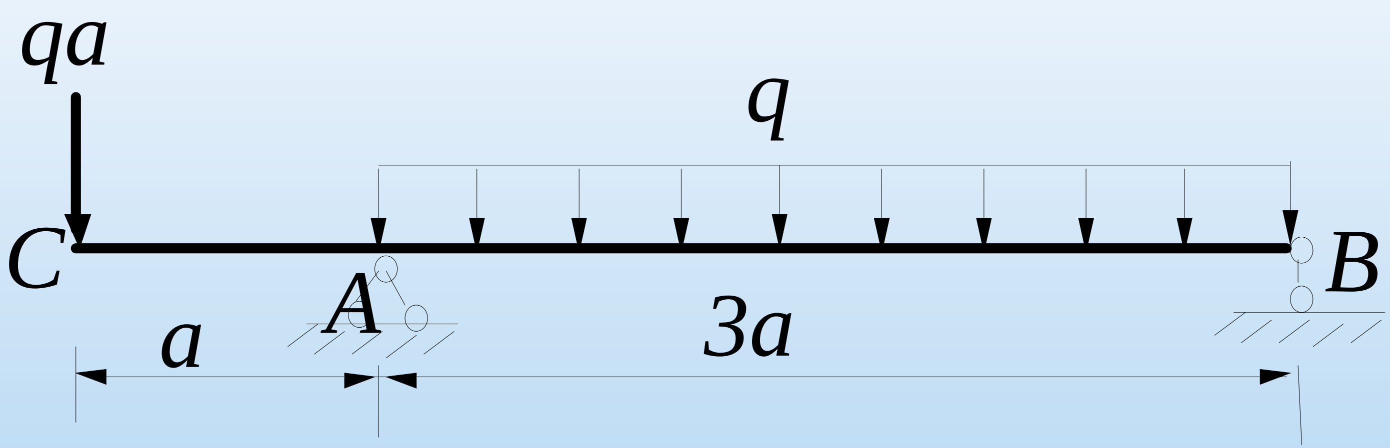
弯矩图→二次抛物线

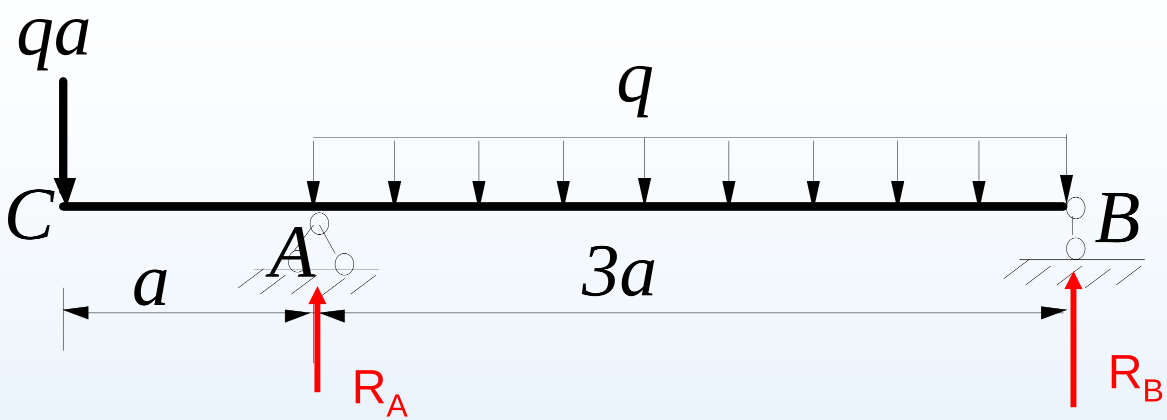


$M_e = 3qa^2$
q
A
B
C
x
a
$3a$
F_S
$5qa/3$
$8a/3$
$qa/3$
x
$4qa^2/3$
(+)
$qa^2/18$
x
M
$5qa^2/3$

课堂练习

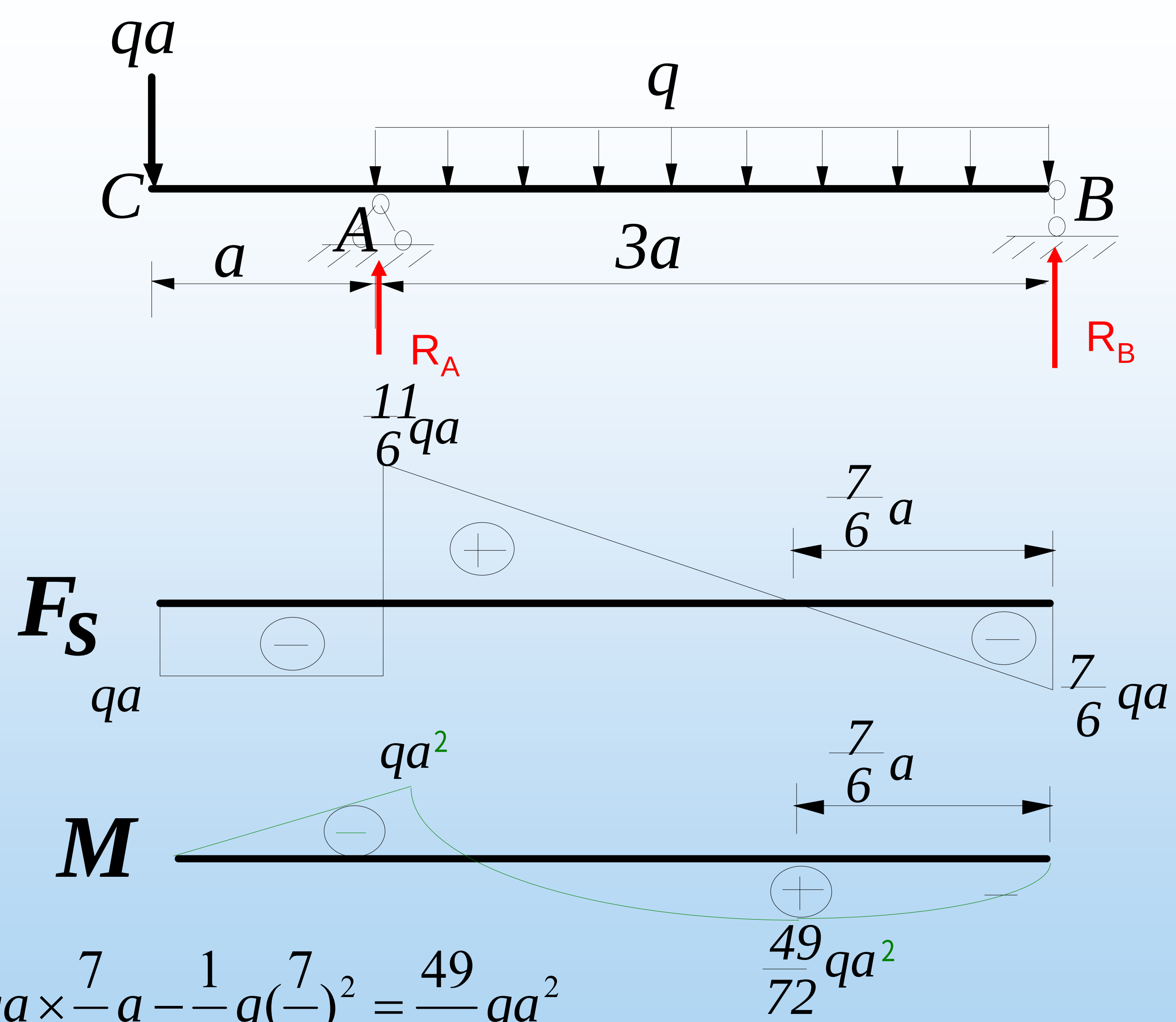
P143 4-3 (j) 作Q、M图





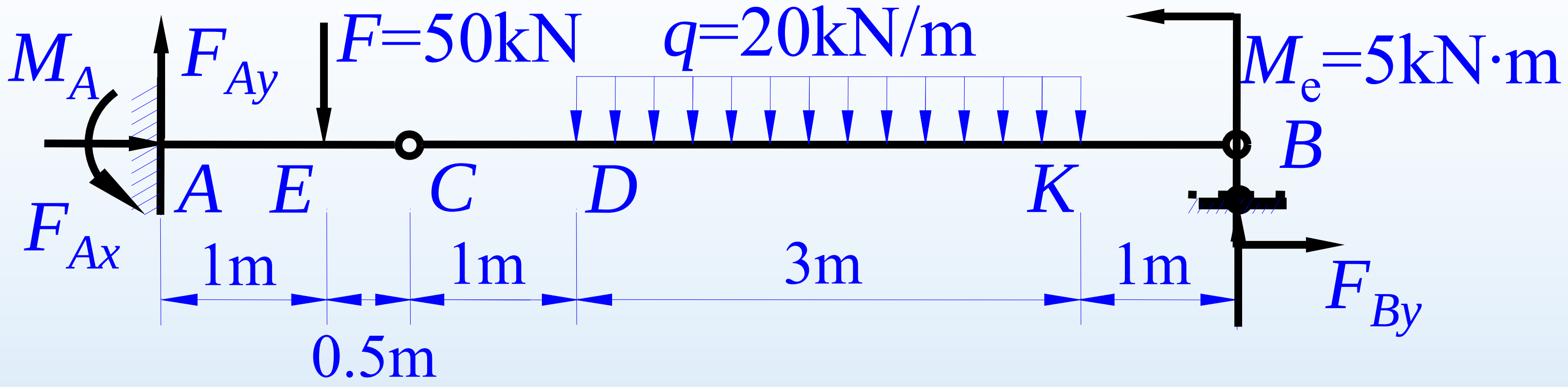
1)求约束力：

$$R_A = \frac{17}{6} qa \qquad R_B = \frac{7}{6} qa$$



$$M_{\max} = \frac{7}{6}qa \times \frac{7}{6}a - \frac{1}{2}q(\frac{7}{6})^2 = \frac{49}{72}qa^2$$

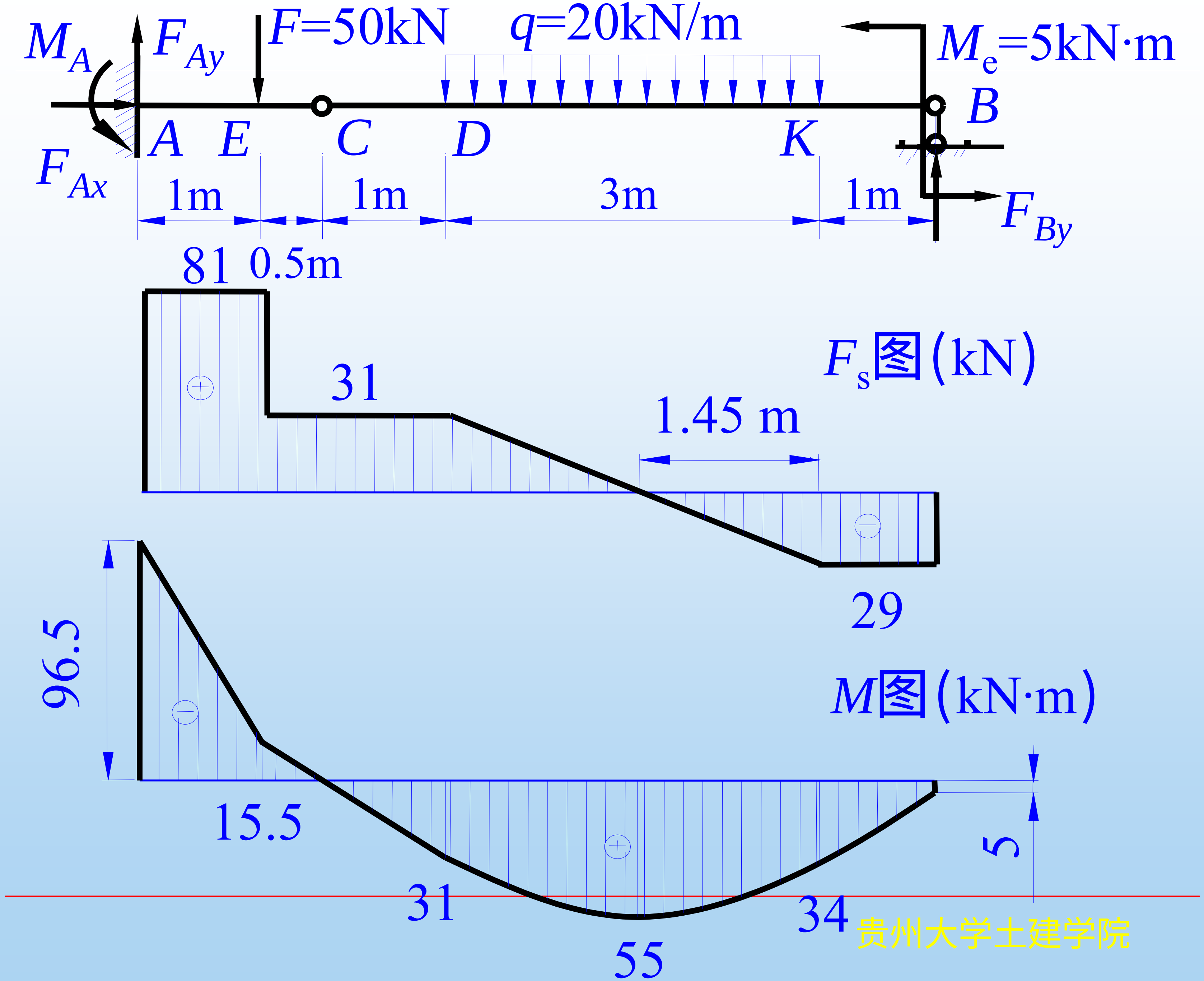
例4-10 试绘出图示有中间铰的静定梁的剪力弯矩图。



已知：$F_{Ay}=81\text{kN}(\uparrow)$

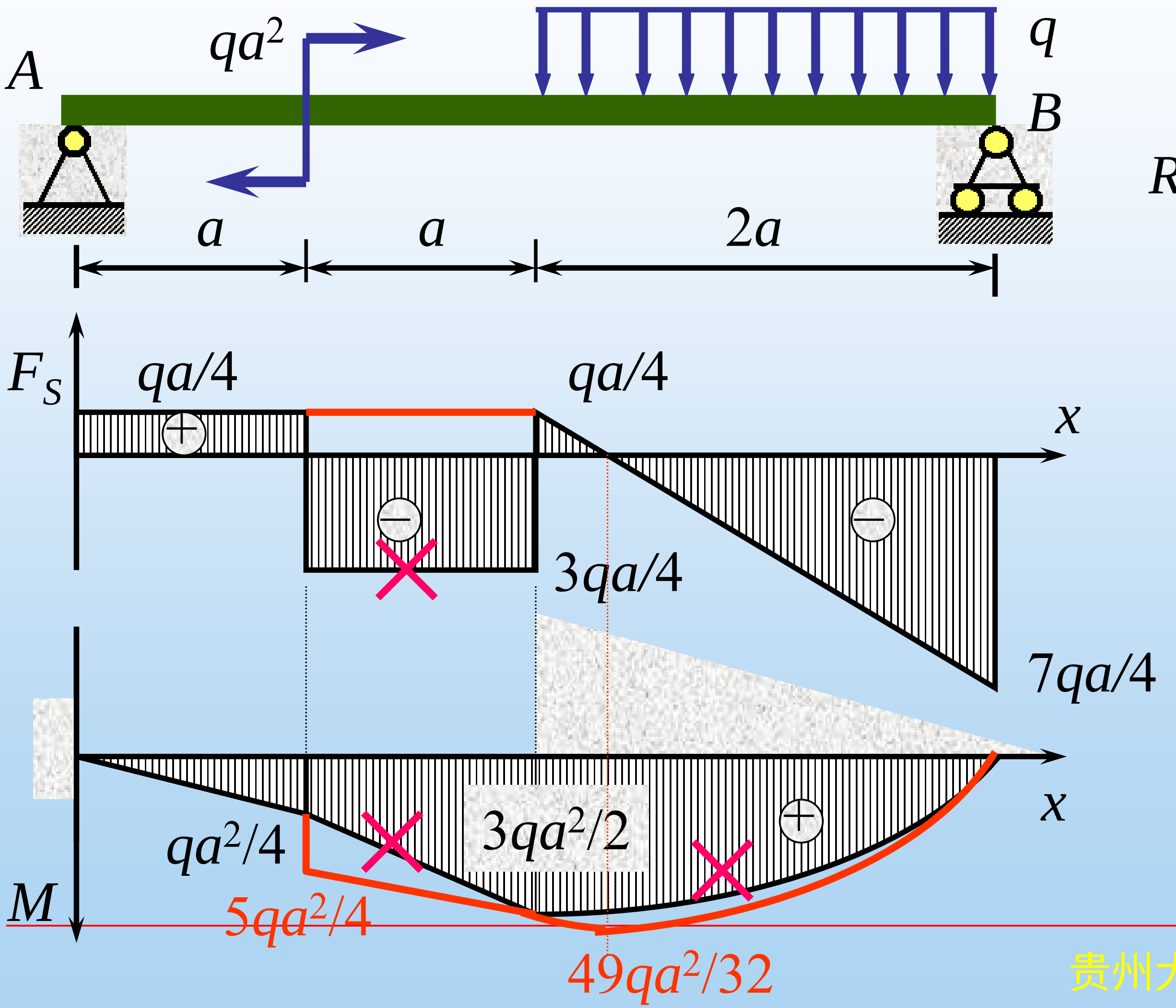
$$F_{By}=29\text{kN}$$

$$M_A=96.5\text{kN}\cdot\text{m}$$ (逆时针)



M_A
F_{Ay}
F=50kN
q=20kN/m
M_e=5kN·m
F_{Ax}
A E C D K B
1m
1m
3m
1m
0.5m
F_{By}
81
31
1.45 m
29
F_s图(kN)
96.5
M图(kN·m)
15.5
31
55
34
5
贵州大学土建学院

例4-11 改内力图之错。



$$R_A = \frac{qa}{4}; R_B = \frac{7qa}{4}$$

IV. 按叠加原理作弯矩图

叠加原理：多个载荷同时作用于结构而引起的内力等于每个载荷单独作用于结构而引起的内力的代数和。

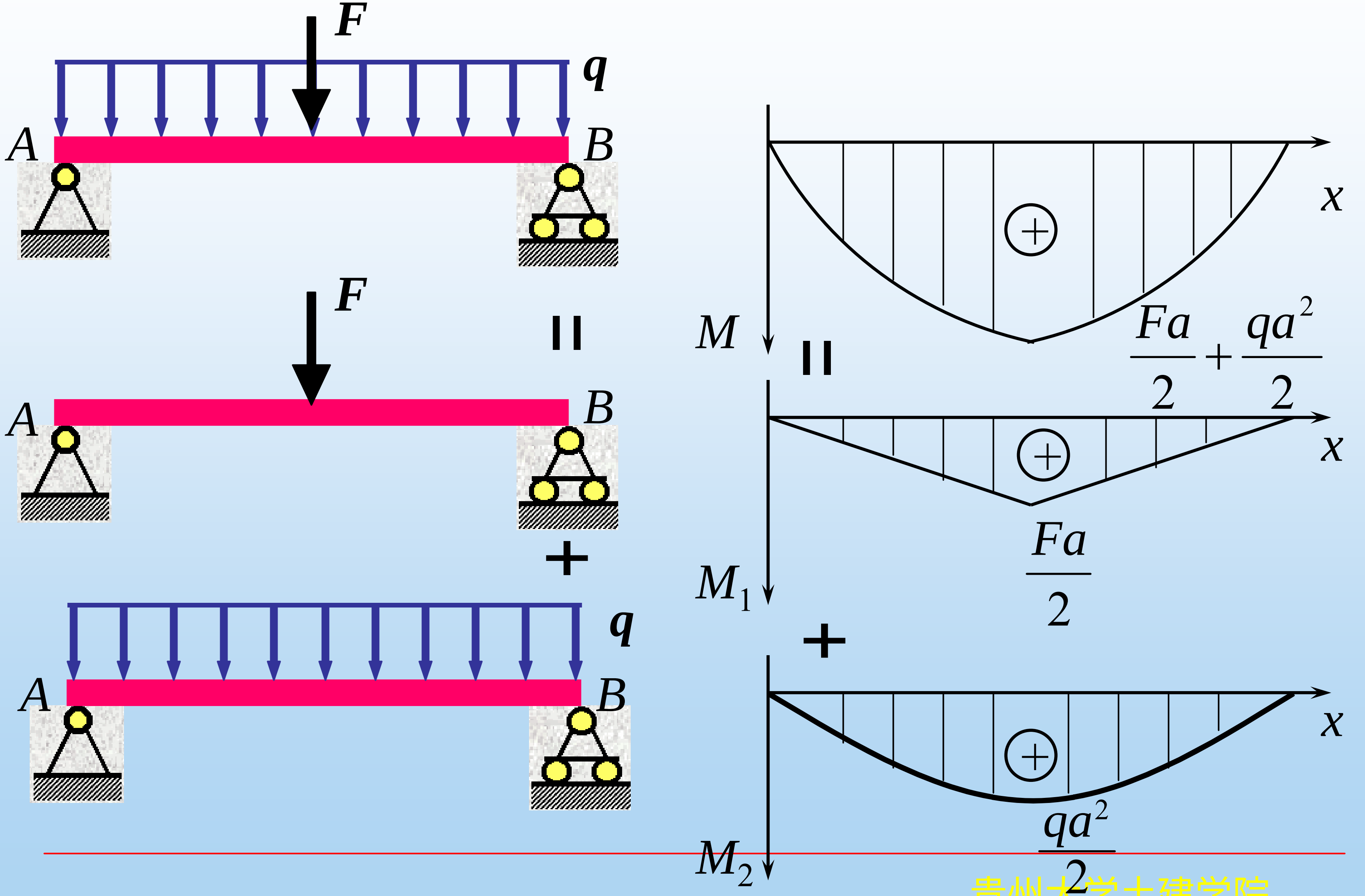
适用条件：所求参数（内力、应力、位移）必然与荷载满足线性关系。即在弹性限度内满足虎克定律。

按叠加原理作弯矩图步骤：

①分别作出各项荷载单独作用下梁的弯矩图；

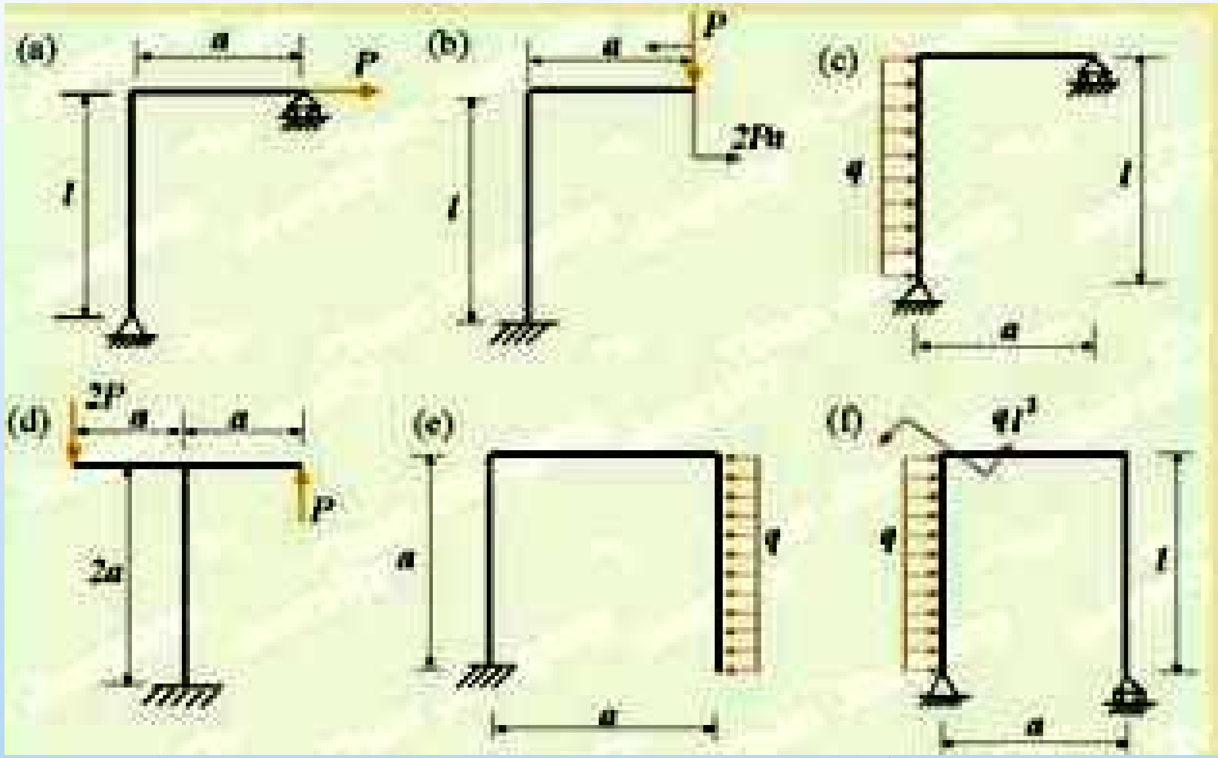
②将其相应的纵坐标叠加即可（注意：不是图形的简单拼凑）。

例4-12 按叠加原理作弯矩图($AB=2a$，力$\boldsymbol{F}$作用在梁AB的中点处）。



§4-3 平面刚架和曲杆的内力图

I、平面刚架 ——由同一平面内不同取向的杆件相互间刚性连接的结构。



面内受力时，平面刚架杆件的内力有：轴力、剪力、弯矩

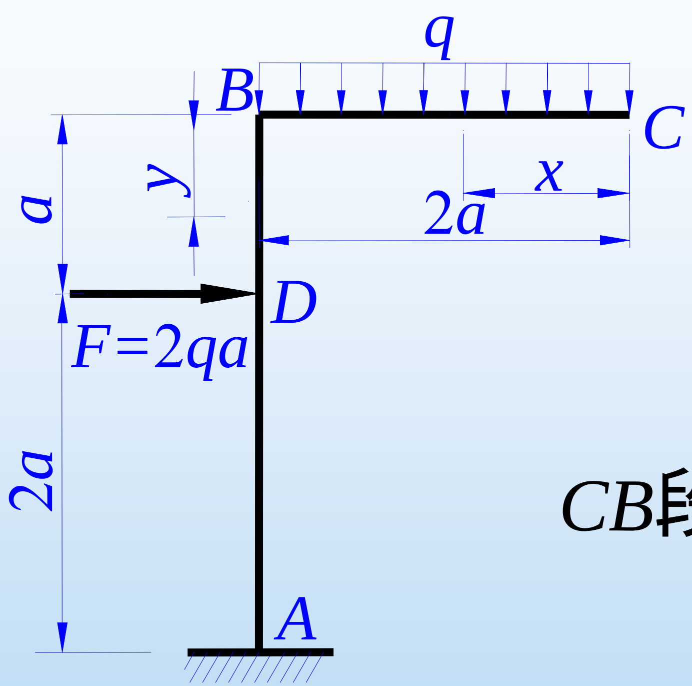
作刚架内力图的约定：

弯矩图：画在各杆的受拉一侧，不注明正、负号；

剪力图及轴力图：可画在刚架轴线的任一侧，但应注明正、负号；

剪力和轴力的正、负规定仍与前面章节一致。

例4-13 试作图示刚架的内力图。



解：从自由端取分离体作为研究对象写各段的内力方程，可不求固定端**A**处的支反力。

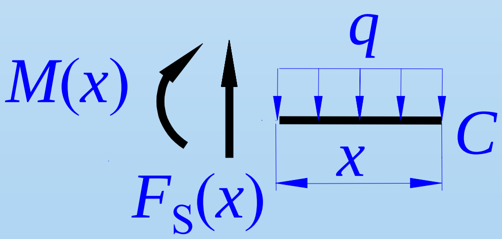
*CB*段：

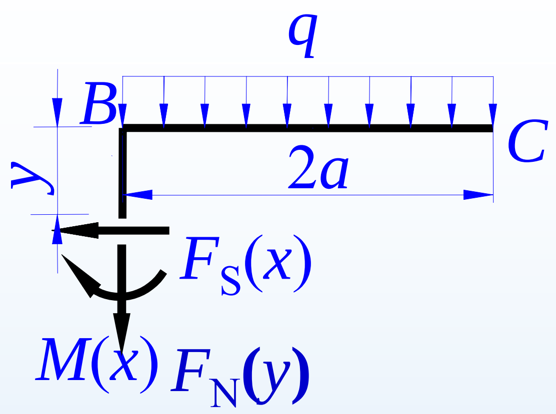
$$F_N(x) = 0$$

$$F_S(x) = qx \qquad (0 \le x \le 2a)$$

$$M(x) = -\frac{qx^2}{2} \, (0 \le x \le 2a)$$

(外侧受拉)





BD段：

$$F_N(y) = -2qa(0 \le y \le a)$$

$$F_S(y) = 0$$

$$M(y) = -2qa^2(0 \le y \le a)$$

(外侧受拉)

DA段：

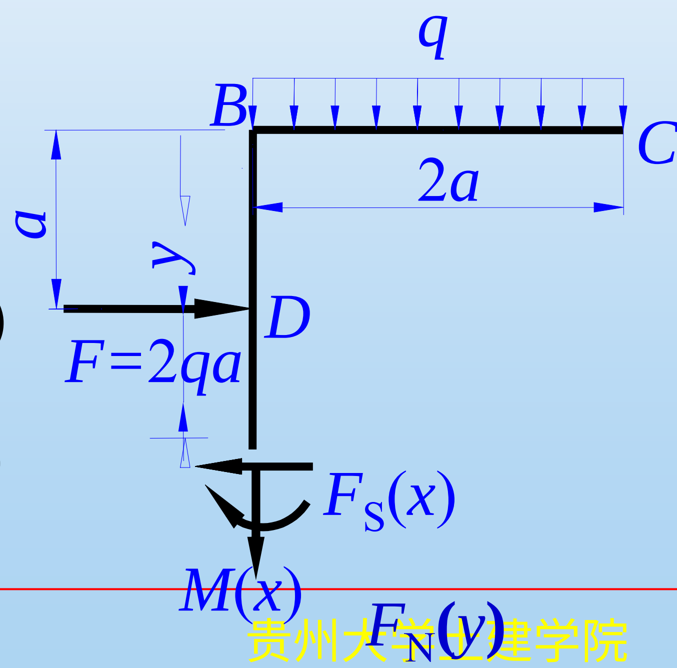
$$F_N(y) = -2qa(a \le y \le 3a)$$

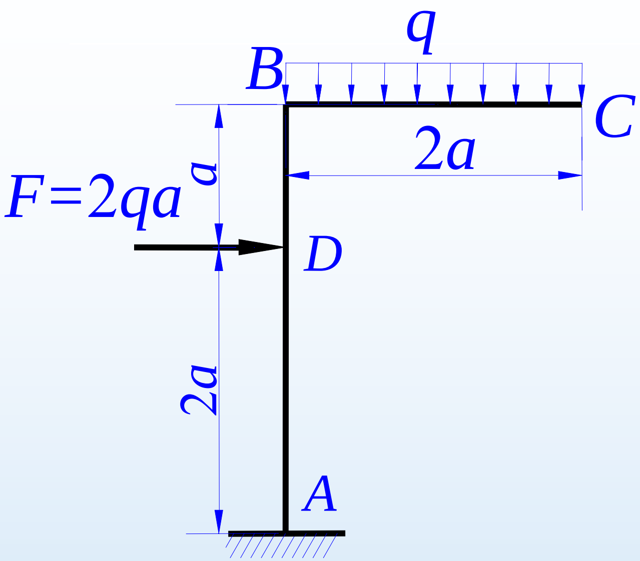
$$F_S(y) = 2qa(a < x < 3a)$$

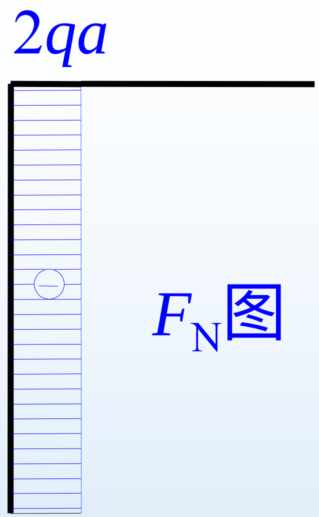
$$M(y) = -2qa^2 - 2qa(y-a)$$

$$= -2qay$$ (外侧受拉)

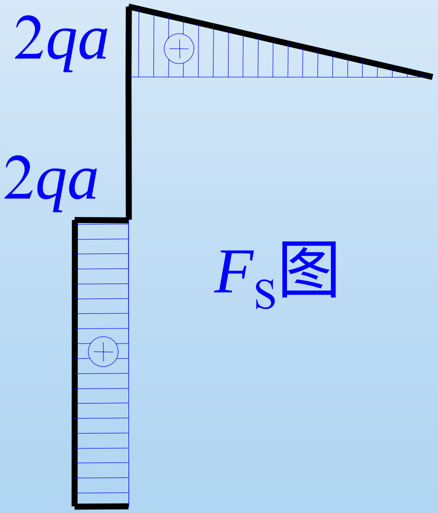
$$(a \le y < 3a)$$

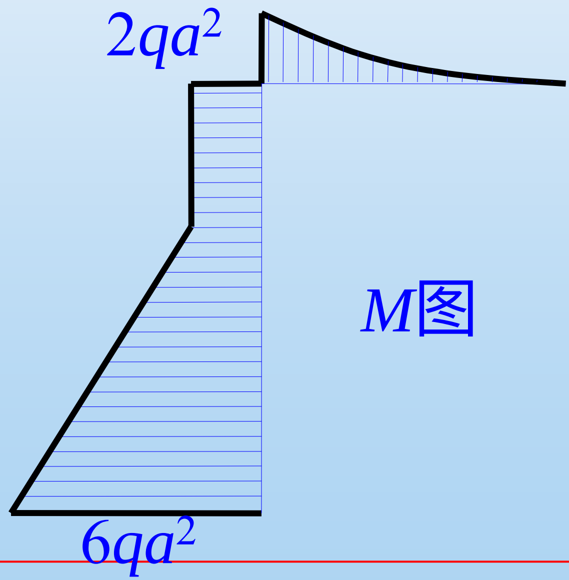


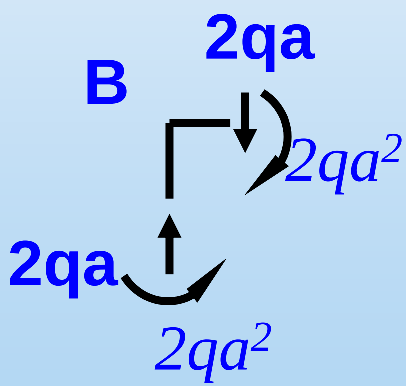




可取刚性结点**B**为分离体，考察该结点是否满足平衡条件来校核内力图的正误。





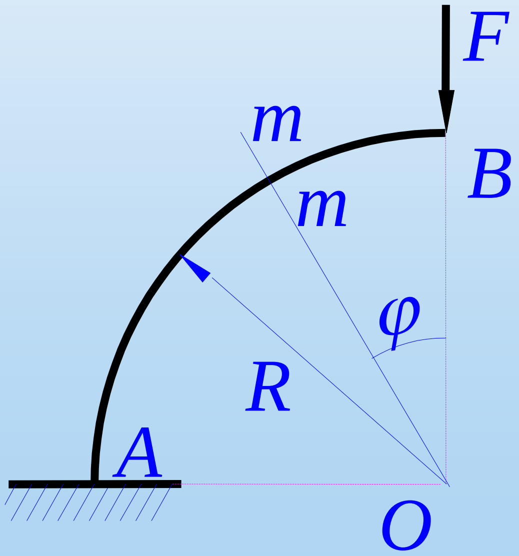


II、平面曲杆

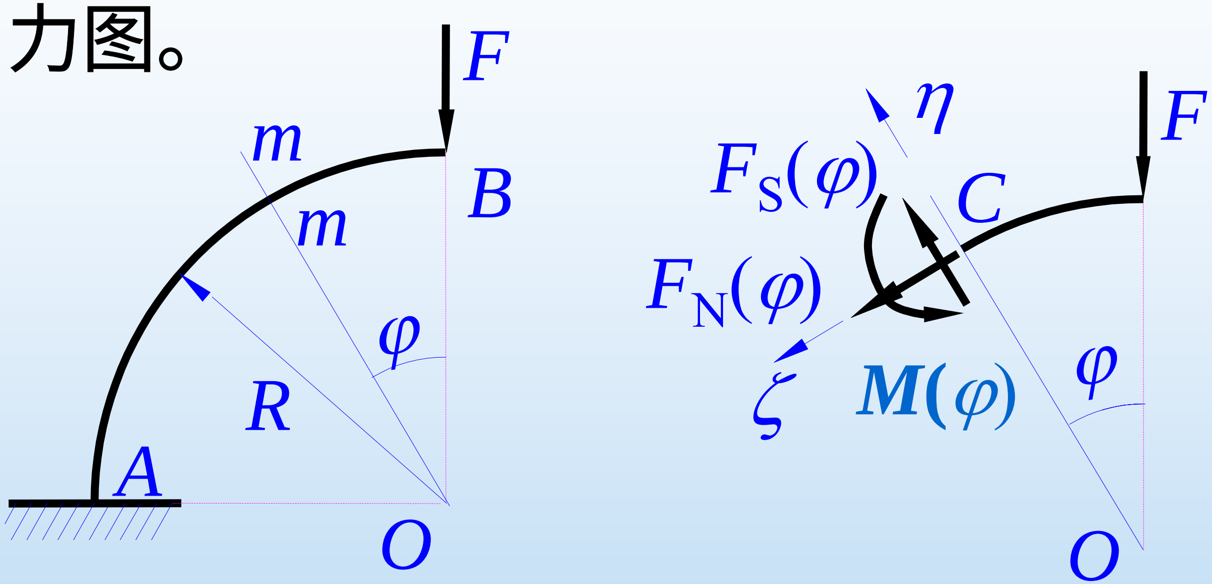
面内受力时的内力——轴力、剪力、弯矩

弯矩的符号约定——使杆的曲率增加（即外侧受拉）为正

作平面曲杆内力图的约定与刚架相同。



例5-14　一端固定的四分之一圆环，半径为R，在自由端B受轴线平面内的集中荷载F作用如图，试作出其内力图。



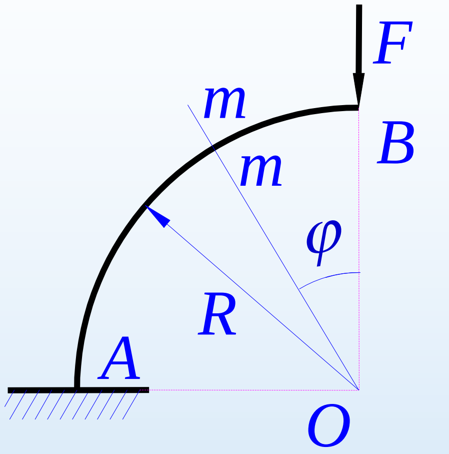
解：取分离体如图写出其任意横截面m-m上的内力方程：

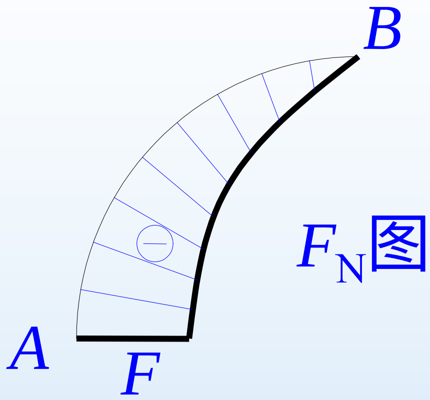
$$F_N(\varphi) = -F\sin\varphi \quad (0 < \varphi < \pi/2)$$

$$F_S(\varphi) = F\cos\varphi \quad (0 < \varphi \le \pi/2)$$

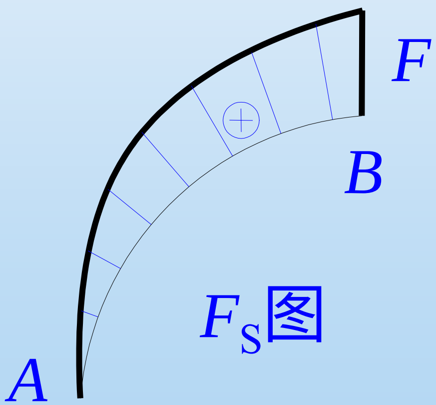
$$M(\varphi) = FR\sin\varphi \quad (0 \le \varphi < \pi/2)$$

根据内力方程绘出内力图，如图所示。

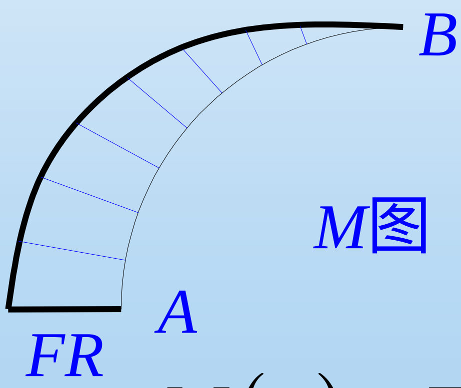




$$F_N(\varphi) = -F\sin\varphi$$

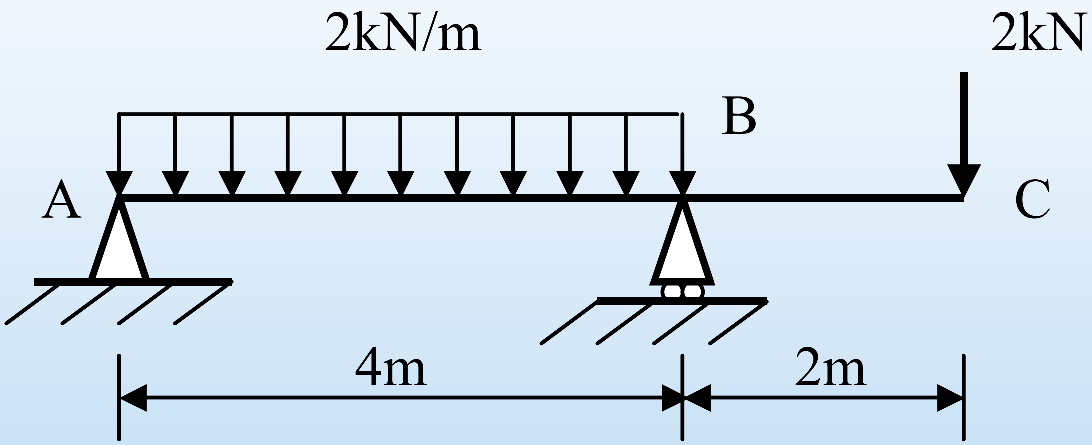


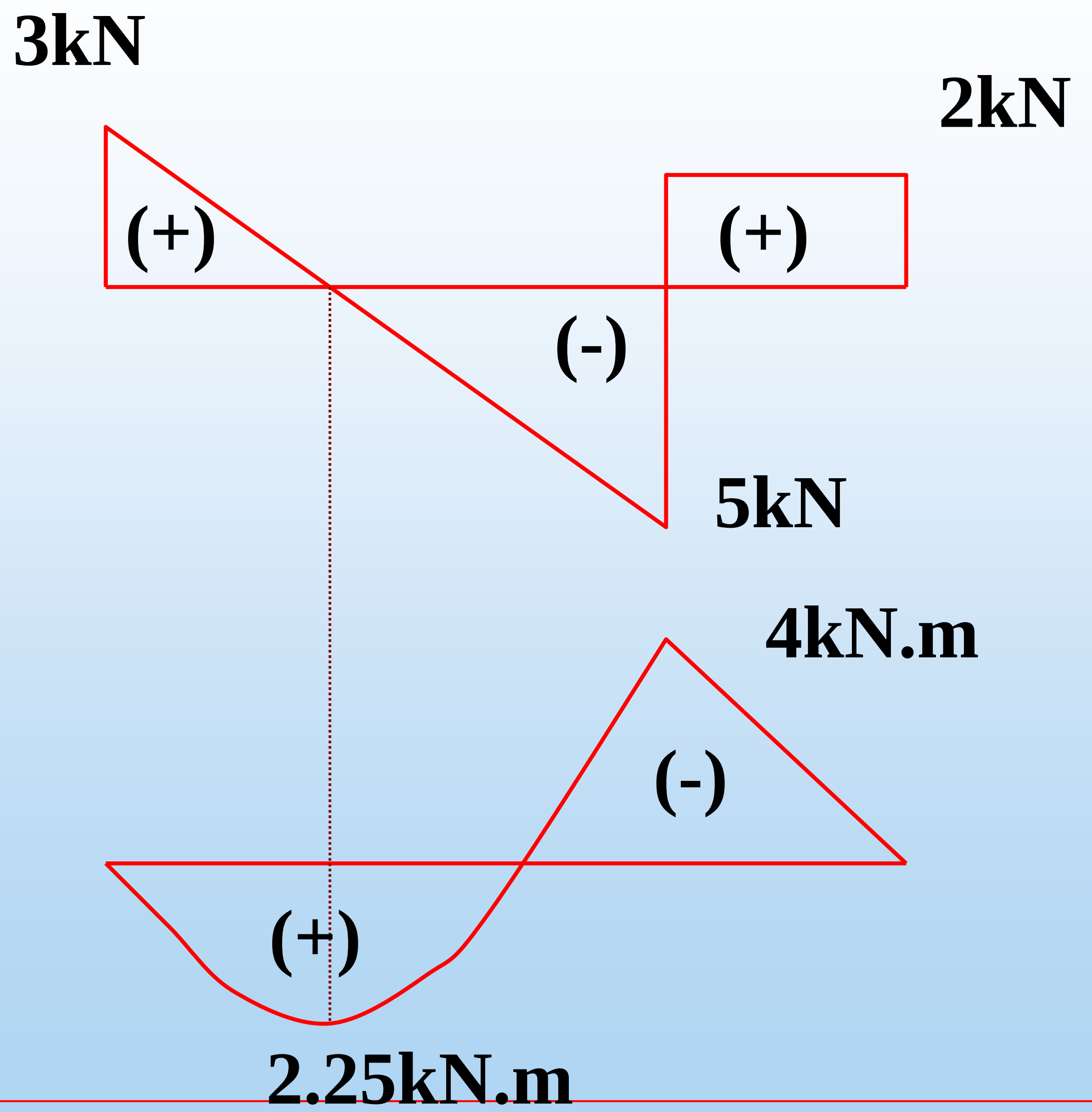
$$F_S(\varphi) = F\cos\varphi$$



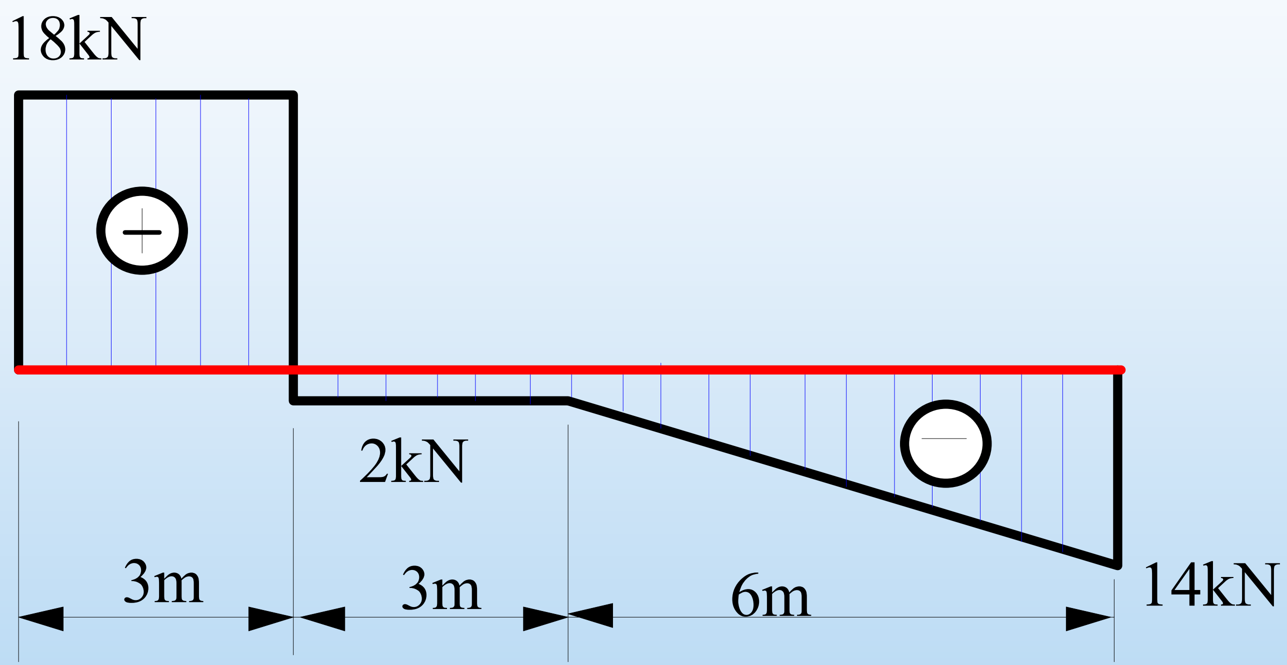
$$M(\varphi) = FR\sin\varphi$$

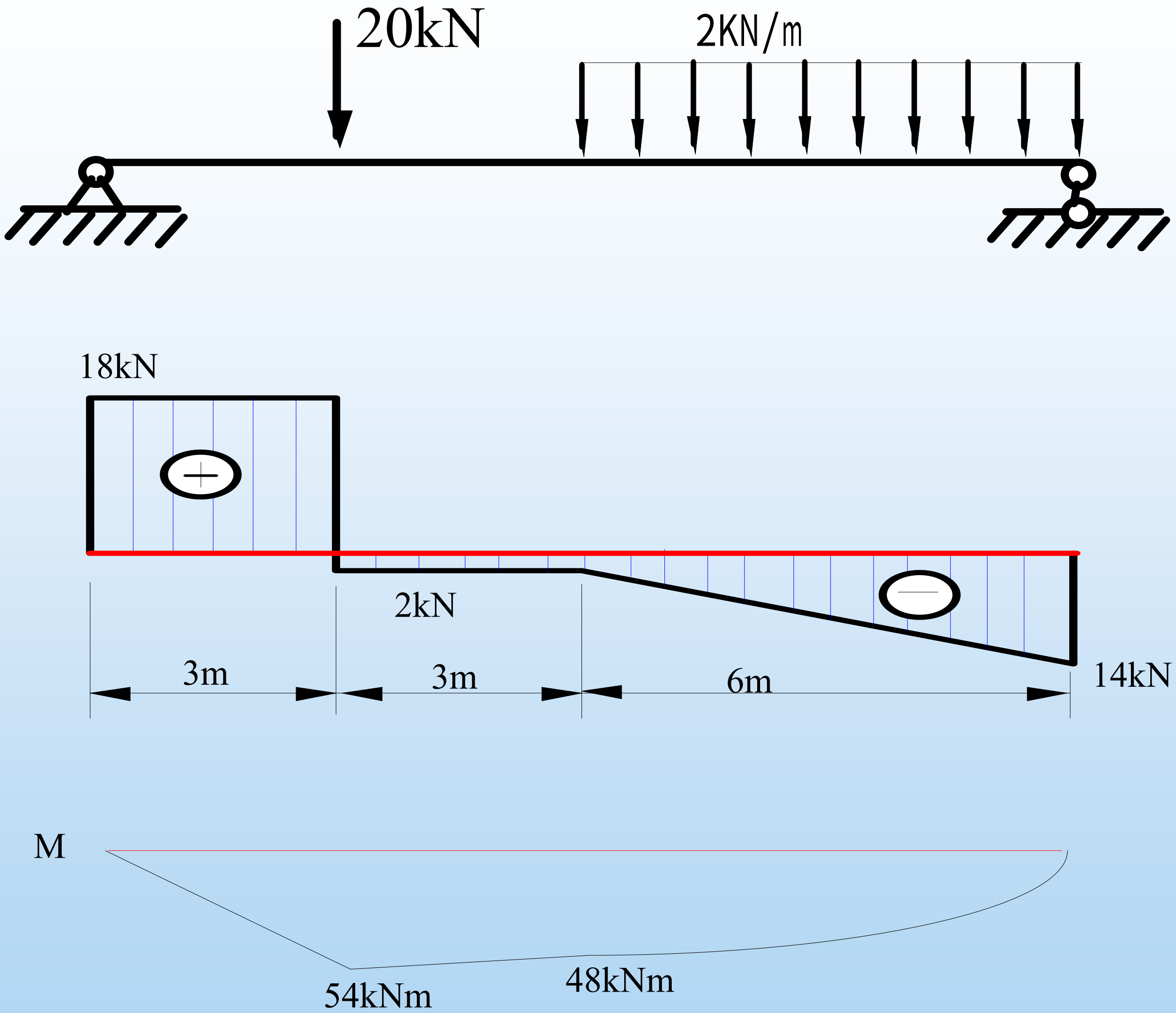
练习题：作图示梁的剪力图和弯矩图



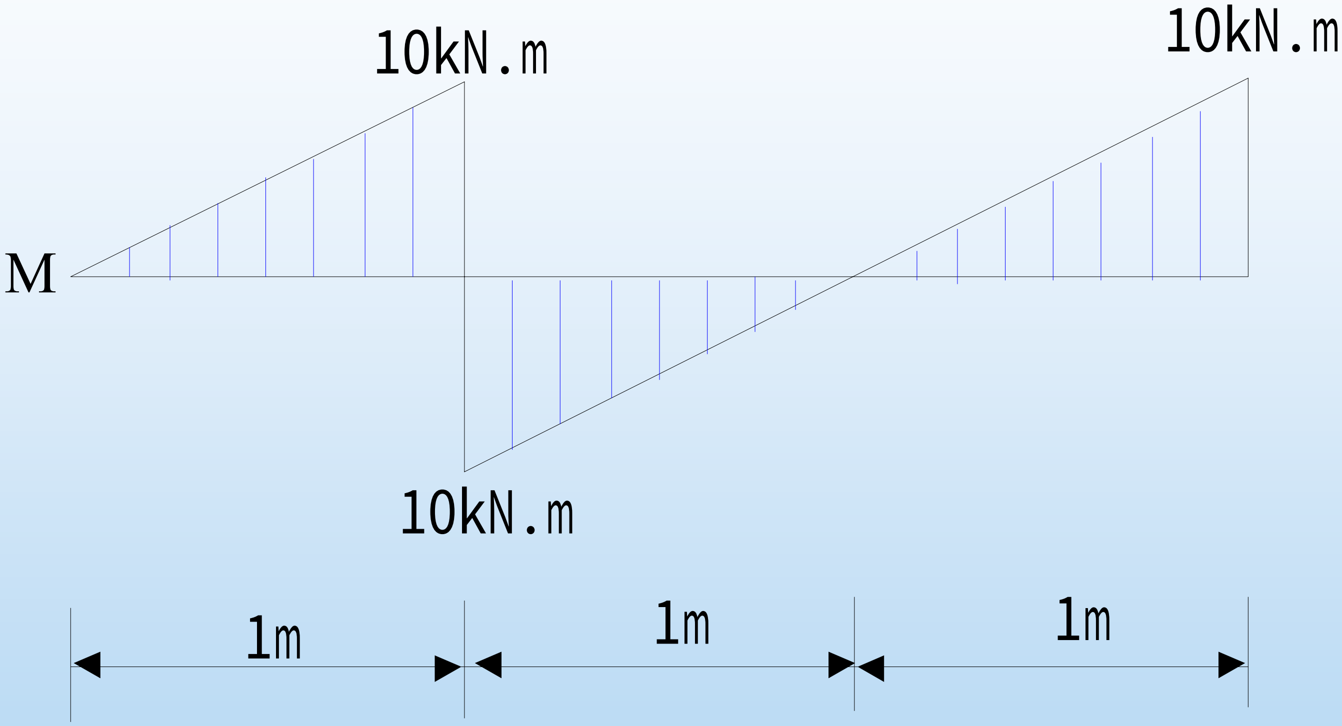

3kN
2kN
(+)
(+)
(-)
5kN
4kN.m
(-)
(+)
2.25kN.m

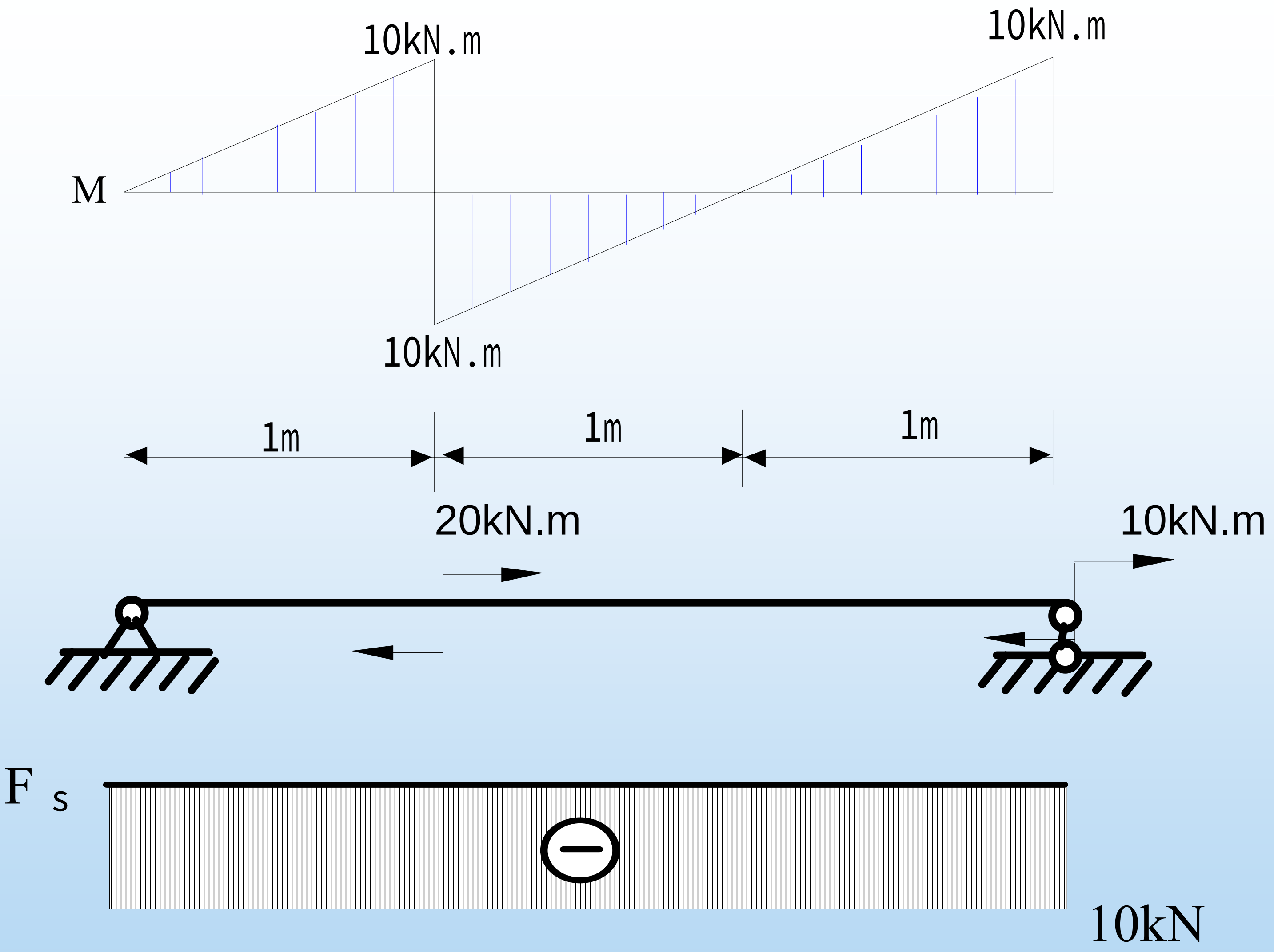
P_{144} 4-6 已知简支梁的剪力图如图所示，试作梁的弯矩图和荷载图。已知梁上没有集中力偶作用。




20kN
2KN/m
18kN
2kN
14kN
3m
3m
6m
M
54kNm
48kNm

P_{145} 4-7 试根据图示简支梁的弯矩图作出两的剪力图与荷载图。




10kN.m
10kN.m
M
10kN.m
1m
1m
1m
20kN.m
10kN.m
F s
10kN

作业

- P_{143} 4-3 a, e, j
- P_{145} 4-8 c, e
- P_{147} 4-15 b

再见！