doi: 10.17221/293/2015-AGRICECON

Economic efficiency of the AOQL single sampling plans for the inspection by variables

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Abstract: The paper refers to the AOQL (Average Outgoing Quality Limit) single sampling plans when the remainder of the rejected lots is inspected. These rectifying AOQL plans for inspection by variables were created by the author of this paper and published in the Statistical Papers. These new plans were compared with the corresponding Dodge-Romig AOQL plans for inspection by attributes from the economic point of view. Numerical investigations confirm that under the same protection of consumer, the AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig AOQL attribute sampling plans. The dependence of the saving of the inspection cost on the input parameters of acceptance sampling (the average outgoing quality limit, the lot size and the process average proportion defective) is analysed in the paper. Moreover, a criterion for deciding if the inspection by variables should be considered instead of the inspection by attributes is suggested in the paper.

Keywords: acceptance sampling, average outgoing quality limit, economical aspects

In a book written by Dodge and Romig, the acceptance sampling plans (*n, c*) are considered which minimize the mean number of items inspected per lot of the process average quality, assuming that the remainder of the rejected lots is inspected

$$I_s = N - (N - n) \cdot L(\overline{p}; n, c) \qquad (1)$$

under the condition

$$\max_{0<p<1} AOQ(p) = p_L \qquad (2)$$

(AOQL single sampling plans), where N is the number of items in the lot (the given parameter), $\overline{p}$ is the process average fraction defective (the given parameter), p_L is the average outgoing quality limit (the given parameter, denoted AOQL), n is the number of items in the sample (the search parameter, $n < N$, c) is the acceptance number (the search parameter).

The inspection procedure is as follows: The lot is rejected when the number of defective items in the sample is greater than *c.*

The function L is the operating characteristic, $L(p)$ is the probability of accepting a submitted lot with the fraction defective p. The function AOQ is the average outgoing quality, $AOQ(p)$ is the mean fraction defective after inspection when the fraction defective before inspection was p.

The condition (2) protects the consumer against the acceptance of a bad lot, the average outgoing quality is less or equal to p_L (the chosen value) for each fraction defective p before inspection.

The AOQL plans for inspection by attributes are extensively tabulated – see Dodge and Romig (1998).

The Dodge-Romig AOQL plans can be used under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes – e. g. Hald 1981).

The corresponding AOQL plans for the inspection by variables (all items from the sample and from the remainder of rejected lots are inspected by variables) have been introduced in Klufa (1997) – the basic notions of the variables sampling plans are addressed in Jennett and Welch (1939). The exact calculation of these plans, when the non-central t distribution is used for the operating characteristic L is considerably difficult. This problem was solved in Klufa (2008), the exact solution is in Kaspříková (2012) – LTPDvar is an add-on package to the R software (see R Development Core Team 2011). Similar problems are solved in Chen and Chou (2001), Kaspříková and Klufa (2015), Wilrich (2012), Ho et al. (2012), Yen et al. (2014), Klufa (2015), Aslam et al. (2015), Wang and Lo (2015), Balamurali et al. (2014).

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The dependence economic efficiency of AOQL plans for the inspection by variables on input parameters of acceptance sampling is analysed in the presented paper. A criterion for deciding if the inspection by variables should be considered instead of the inspection by attributes is suggested in this paper.

MATERIAL AND METHODS

The problem to find AOQL plans for inspection by variables has been solved under the following assumptions:

Measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic X, there is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter s is estimated from the sample standard deviation s.

The inspection procedure is as follows:

(1) Draw a random sample of n items and compute

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \,,\; s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \tag{3}$$

(2) Compute $\frac{U-\bar{x}}{s}$ for an upper specification limit, or $\frac{\bar{x}-L}{s}$ for a lower specification limit.

(3) Accept the lot if

$$\frac{U-\bar{x}}{s} \ge k \,,\text{ or } \frac{\bar{x}-L}{s} \ge k \tag{4}$$

We have determined the sample size n and the critical value k. As well as Dodge and Romig, we shall look for the acceptance plan (n, k) minimizing the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of the rejected lots are inspected by variables

$$I_m = N - \left(N - n\right)\cdot L\left(\bar{p}; n, k\right) \tag{5}$$

under the condition (2) (the AOQL single sampling plans for inspection by variables). The condition (2) is the same one as used for the protection of the consumer Dodge and Romig.

The AOQL plans for the inspection by variables (all items from the sample are inspected by variables, the remainder of the rejected lots is inspected by variables) were created by the author of this paper – Klůfa (1997). The exact calculation of the AOQL plans for inspection by variables when the non-central t distribution is used for the operating characteristic $L(p, n, k)$ is considerably difficult. This problem was solved in Klůfa (2008), the exact solution is in Kaspříková (2012). Now we shall study the economic aspects of these plans.

RESULTS AND DISCUSSION

For the comparison of the AOQL single sampling plans for the inspection by variables with the corresponding Dodge-Romig AOQL plans for inspection by attributes from economical point of view, we introduce parameter E defined by the relation (see (1) and (5))

$$E = \frac{I_m}{I_s} 100 \tag{6}$$

Let us denote

$$c_m = c_m^* / c_s^* \tag{7}$$

where c_s^* is the cost of inspection of one item by attributes, c_m^* is the cost of inspection of one item by variables. For the comparison of these plans, the parameter c_m (the ratio of cost of inspection of one item by variables to cost of inspection of this item by attributes) must be estimated in each real situation. Usually c_m is greater than 1 (for $c_m \le 1$ the AOQL plans for the inspection by variables are evidently the most economical). Let us denote

$$\varepsilon = Ec_m = 100(I_m c_m^*)/(I_s c_s^*) \tag{8}$$

where $I_m c_m^*$ is the mean cost of the inspection by variables and $I_m c_s^*$ is the mean cost of the inspection by attributes. Therefore, if c_m is statistically estimated and $\varepsilon < 100$ then the AOQL plans for the inspection by variables are more economical than the corresponding Dodge-Romig AOQL plans for the inspection by attributes.

The difference

$$s = 100 - \varepsilon \tag{9}$$

then represents the percentage of *savings in inspection cost* when the sampling plan for the inspection by variables is used instead of the corresponding plan for the inspection by attributes.

If

$s > 0$

doi: 10.17221/293/2015-AGRICECON

then the AOQL plans for the inspection by variables are more economical than the corresponding Dodge-Romig AOQL plans for the inspection by attributes, if

$s < 0$

then Dodge-Romig AOQL attribute sampling plans are more economical.

Example 1. Let $N = 500$, $p_L = 0.500$, $\bar{p} = 0.002$ and $c_m = 1.5$ (the cost of inspection of one item by variables is higher by 50% than the cost of inspection of one item by attributes). We shall look for the AOQL plan for the inspection by variables. Furthermore, we shall compare this plan and the corresponding Dodge-Romig AOQL plan for the inspection by attributes from the economic point of view.

For the given parameters $N = 500$, $p_L = 0.500$, $\bar{p} = 0.002$, we shall compute the AOQL plan for the inspection by variables Klufa (2008)

$n = 38$, $k = 2.2967$

and $E = 44$. The corresponding AOQL plan for the inspection by attributes, we find in Dodge and Romig (1998). For the given parameters $N = 500$, $p_L = 0.500$, $\bar{p} = 0.002$ we have

$n = 65$, $c = 0$

For $c_m = 1.5$ the economic parameter s is

$s = 100 - 66 = 34$

From this result, it follows that under the same protection of the consumer, the AOQL plan for the inspection by variables (38, 2.2967) is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (65, 0). Since $s = 34$, there can be expected approximately **34% saving of the inspection cost** (Table 1).

The percentage of savings in the inspection cost when the sampling plan for the inspection by variables is used instead of the corresponding plan for the inspection by attributes S depends on the acceptance sampling characteristics p_L, N, $\bar{p}$ and c_m, i.e. s is a function of four variables

$$s = s(p_L, N, \bar{p}, c_m) \qquad (10)$$

Values of this function for some parameters p_L, N, $\bar{p}$ and c_m are in Table 1.

From Table 1 and from the results of the numerical investigations, it follows that under the same protection of the consumer, the AOQL plans for the inspection by variables are in many situations *more economical* (saving of the inspection cost is 70% in any cases) than the corresponding Dodge-Romig attribute sampling plans.

Table 1. Values of the function s for $p_L = 0.005$, $c_m = 1.5$

$\bar{p} \setminus N$	100	500	1 000	4 000	10 000	50 000	100 000
0.00025	48	54	57	72	69	76	75
0.00050	40	48	54	64	63	69	69
0.00075	34	45	52	60	61	66	69
0.00100	30	42	51	57	63	66	72
0.00125	27	39	51	54	60	64	67
0.00150	22	37	49	54	57	63	64
0.00175	19	36	49	52	55	64	64
0.00200	16	34	48	52	55	66	66
0.00225	13	31	39	48	54	60	61
0.00250	12	30	36	45	51	58	60
0.00275	9	28	33	43	49	57	60
0.00300	7	27	30	40	48	58	61
0.00325	4	24	28	37	45	52	55
0.00350	3	22	25	36	42	51	54
0.00375	1	21	22	33	39	49	52
0.00400	–2	19	19	30	36	48	52
0.00425	–3	12	16	25	30	40	43
0.00450	–5	9	13	21	25	36	39
0.00475	–6	7	10	16	21	30	31
0.00500	–8	4	7	12	15	21	21

Source: Own construction

doi: 10.17221/293/2015-AGRICECON

Dependence of the percentage of savings s on the lot size N:

Let p_L, $\overline{p}$, c_m be the given parameters. For the given parameters p_L, $\overline{p}$, c_m, the function s in (10) is a function of one variable N, which has an increasing trend in N (it is confirmed by numerical investigations – also Table 1). Therefore, *when lot size N increases, then saving of the inspection cost increases* (using the AOQL plan for the inspection by variables instead of the corresponding plan for the inspection by attributes).

Dependence of the percentage of savings s on $\overline{p}$:

Now we shall study the dependence of the economic efficiency of the AOQL plans for the inspection by variables measured by the parameter s on the process average fraction defective $\overline{p}$. Let p_L, N, c_m, be given parameters. For the given p_L, N, c_m, the function s in (10) is a function of one variable $\overline{p}$, which has mostly a decreasing trend in $\overline{p}$ (it is confirmed by numerical investigations – also Table 1). Therefore, *when the process average fraction defective* $\overline{p}$ *increases, then saving of the inspection cost decreases* (using the AOQL plan for the inspection by variables instead of the corresponding plan for the inspection by attributes).

Dependence of the percentage of savings s on c_m:

Finally, we shall study the dependence of the percentage of savings s on the fraction of the cost of the inspection of one item by variables to the cost of the inspection of one item by attributes c_m. Let p_L, N, $\overline{p}$ be the given parameters. Function (10) for the given p_L, N, $\overline{p}$ is a function of one variable c_m. Since E in (6) for given p_L, N, $\overline{p}$ is a constant function of c_m (E does not depend on c_m), this function is a linear function of c_m (see (9)). Due to $E > 0$ this function is decreasing (Figure 1). It means that *when the fraction of the cost of inspection of one item by variables to the cost of inspection of one item by attributes* c_m *increases, then saving of the inspection cost* s *decreases.*

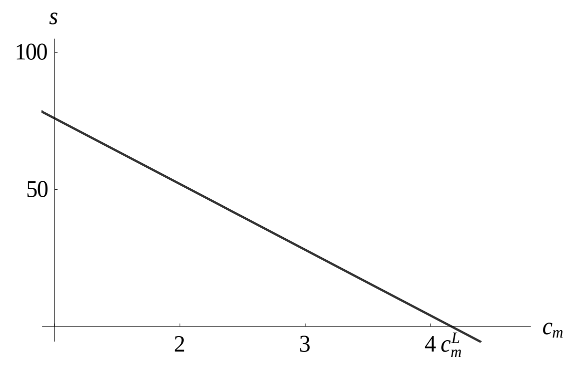

Figure 1. Graph of the function $s = s(c_m)$ for $p_L = 0.005$, $N = 4000$ $\overline{p} = 0.0005$

Now we shall decide according to c_m, if inspection by variables should be considered in place of the inspection by attributes. Let p_L, N, $\overline{p}$ be the given parameters. Let us define

$$c_m^L \tag{11}$$

as the value of c_m for which $s = 0$.

According to the definition c_m^L, is such value of c_m for which the mean inspection cost per lot of process average quality for the inspection by variables is equal to the mean inspection cost per lot of process average quality for the inspection by attributes (Figure 1). From equation $s = 0$ (see (9)) we have

$$c_m^L = 100/E \tag{12}$$

If $c_m = c_m^L$, then $s = 0$. If

$$c_m < c_m^L \tag{13}$$

then $s > 0$, i.e. the AOQL plans for the inspection by variables are more economical than the corresponding Dodge-Romig AOQL attribute sampling plans. On the other hand, if

$$c_m > c_m^L \tag{14}$$

then $s < 0$, i.e. the inspection by attributes is better.

Example 2. Let $N = 4000$, $p_L = 0.005$, $\overline{p} = 0.0005$. We shall determine c_m^L (a limit value of parameter c_m).

For the given parameters N, p_L, $\overline{p}$, we shall compute (Klůfa 2008) the parameter $E = 24$ Therefore (see (12)) $c_m^L = 4.2$, i.e. the AOQL plan for inspection by variables is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan when the ratio of the cost of the inspection of one item by variables to the cost of the inspection of this item by attributes $c_m < 4.2$ (Figure 1).

If the value of c_m parameter is not known in some situation in practice, then c_m^L (a limit value of c_m parameter) may be calculated to provide some guidance in deciding if the inspection by variables is worth considering. If c_m^L is high, then using the inspection by variables may be efficient (and one should try to estimate c_m to make some more precise evaluation), on the other hand, if c_m^L is near 1, then the inspection by variables cannot be supposed to bring any significant advantage over the inspection by attributes. Calculation of c_m^L value is implemented in LTPDvar package (Kaspříková 2012).

doi: 10.17221/293/2015-AGRICECON

The limit value c_m^L in (12) is a function of three variables N, p_L, $\bar{p}$, i.e.

$$c_m^L = c_m^L(p_L, N, \bar{p}) \qquad (15)$$

Values of this function for some parameters N, p_L, $\bar{p}$ are in Table 2.

Dependence of the limit value c_m^L on the lot size N:
Let p_L and $\bar{p}$ be the given parameters. For the given parameters p_L and $\bar{p}$, the function c_m^L in (15) is a function of one variable N, which has an increasing trend in N (it is confirmed by numerical investigations – also Table 2). Therefore, *when lot size N increases, then limit value c_m^L increases* (using the AOQL plan for the inspection by variables instead of the corresponding plan for the inspection by attributes is efficient).

Dependence of the limit value c_m^L on $\bar{p}$:
Let p_L and N be the given parameters. For the given parameters p_L and N, the function c_m^L in (15) is a function of one variable $\bar{p}$, which has a decreasing trend in $\bar{p}$ (it is confirmed by numerical investigations – also Table 2). Therefore, *when the process average fraction defective $\bar{p}$ increases, then limit value c_m^L decreases.*

CONCLUSIONS

The AOQL single sampling plans for the inspection by variables are (under the same protection of the consumer) in many cases *more economical* than the corresponding Dodge-Romig AOQL plans for the inspection by attributes. Economic efficiency of these plans depends (for chosen value of the average outgoing quality limit AOQL) on the input acceptance sampling characteristics (the lot size, the process average fraction defective, the fraction of the cost of the inspection of one item by variables to the cost of inspection of one item by attributes). From the results of this paper, it follows that
(1) the saving of the inspection cost increases when the lot size N increases,
(2) the saving of the inspection cost decreases when the process average fraction defective $\bar{p}$ increases,
(3) the saving of the inspection cost decreases when the fraction of the cost of inspection of one item by variables to the cost of inspection of one item by attributes c_m increases.

The limit value of the parameter c_m (denoted c_m^L) was suggested in this paper as a criterion for deciding if the inspection by variables should be considered

Table 2. Values of the function c_m^L for p_L = 0.001

$\bar{p} \setminus N$	100	500	1 000	4 000	10 000	50 000	100 000
0.00005	4.2	6.3	7.1	7.7	9.1	12.5	11.1
0.00010	3.7	5.0	5.6	6.7	8.3	10.0	10.0
0.00015	3.4	4.3	5.0	6.3	9.1	9.1	11.1
0.00020	3.2	4.0	4.5	5.9	8.3	9.1	12.5
0.00025	3.0	3.7	4.2	5.9	6.7	8.3	9.1
0.00030	2.9	3.4	3.8	5.6	6.3	7.7	9.1
0.00035	2.8	3.2	3.6	5.3	5.9	7.7	9.1
0.00040	2.6	3.0	3.4	5.0	5.6	7.1	9.1
0.00045	2.6	2.9	3.2	4.5	5.3	6.7	7.7
0.00050	2.5	2.8	3.0	4.3	5.0	6.3	7.1
0.00055	2.4	2.6	2.9	4.0	4.8	6.3	6.7
0.00060	2.4	2.5	2.8	3.7	4.3	5.9	6.3
0.00065	2.3	2.4	2.6	3.4	4.2	5.3	5.9
0.00070	2.3	2.3	2.6	3.2	4.0	5.0	5.6
0.00075	2.2	2.2	2.4	3.0	3.7	4.5	5.3
0.00080	2.2	2.2	2.3	2.9	3.4	4.2	4.8
0.00085	2.1	2.1	2.3	2.7	3.0	3.7	4.0
0.00090	2.1	2.0	2.2	2.5	2.8	3.3	3.6
0.00095	2.0	2.0	2.1	2.4	2.6	2.9	3.0
0.00100	2.0	1.9	2.0	2.2	2.3	2.5	2.6

Source: Own construction

doi: 10.17221/293/2015-AGRICECON

instead of the inspection by attributes (when $c_m < c_m^L$, then the AOQL plans for the inspection by variables are more economical than the corresponding Dodge-Romig AOQL attribute sampling plans). Values of the parameter c_m^L depend (for chosen value of the average outgoing quality limit AOQL) on the lot size and the process average fraction defective. From the results of this paper, it follows that the limit value c_m^L increases when the lot size N is increasing and decreases when the process average fraction defective $\bar{p}$ increases (some values of c_m^L are in Table 2).

Acknowledgment

This paper was processed with the contribution of the long term institutional support of research activities by the Faculty of Informatics and Statistics, University of Economics, Prague.

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Received: 14th October 2015
Accepted: 11th December 2015

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