# Modeling and quantitative analysis of infrared polarization characteristics 

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#### Abstract

Aiming at the theory requirement of the application of infrared polarization information in target detection and identification，the polarization effect produced in the light transmission process of infrared reflectance，thermal radiation，infrared scattering etc．was analyzed and deduced in detail in this paper based on the theory of infrared radiation，and three kinds of polarization effects were modeled and analyzed quantitatively in detail，wherein，the polarization of infrared reflection on object surface was calculated by using Fresnel reflection law，thermal radiation of the object was calculated using Fresnel refraction mechanism combined with the object surface model，and the polarization of infrared scattering on the object surface was calculated by using the Rayleigh and Mie scattering theory respectively．The above conclusion from the study in this paper provides sufficient theoretical foundation for the realization of target detection and recognition by better utilization of infrared polarization information．


Key words：infrared polarization modeling；infrared reflection；thermal radiation； infrared scattering；modeling
CLC number：TN219 Document code：A DOI：10．3788／IRLA201645．0604001

# 红外偏振特性建模与实验验证 

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#### Abstract

摘 要：针对红外偏振信息应用于目标探测识别的理论需求，基于红外辐射基本理论，详细分析和推导了红外反射，热辐射，红外散射等光波传输过程产生的偏振效应，并对这三种偏振效应进行了详细的建模与实验验证，其中利用菲涅耳反射定律对物体表面红外反射的偏振性进行了分析，并得出确定的反射偏振度解析式；结合物体表面模型利用菲涅耳折射机制对物体的热辐射进行了计算，并利用文献中的数据重现了密立根关于金属铂和金属银的偏振性实验，充分验证了红外辐射偏振性的存在；分别利用瑞利散射和米氏散射理论对物体表面红外散射的偏振性进行了分析和计算，计算结果充分体现了散射产生偏振的规律性。上述研究结论为更好地利用红外偏振信息实现目标探测识别提供了充分的理论依据。


关键词：红外偏振建模；红外反射；热辐射；红外散射

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## 0 Introduction

Infrared polarization imaging is a new infrared detection technology developed in recent ten years． Different from the traditional detection method of infrared imaging，infrared polarization imaging can not only obtain infrared radiation intensity information of targets，but also obtain the infrared radiation polarization information．So the polarization of the target scene is the physical basis of infrared polarization imaging detection．In order to make better use of the polarization information for infrared target detection and recognition，mechanism of infrared polarization was analyzed in this paper，from the principle of infrared radiation，the polarization effect generated from infrared radiation and object interaction （such as reflection，refraction，scattering and emission） was studied in focus，providing the theoretical basis for applying polarization information in target detection and identification．

## 1 Analysis of polarization produced by infrared reflection

Reflection and refraction will take place when the waves reach the interface between two different medium．Fresnel formula can be used to describe the reflection and refraction phenomena quantitatively， which are introduced by French physicist Fresnel in 1823，as shown in formula（1）－（4）．Relative to the incident surface of the light，the electric field vector of the light can be decomposed into two orthogonal components，as shown in Fig． $1{ }^{[1]}$ ，one parallel with the incident face，the other perpendicular to the incident plane，and the two polarization components are independent of each other，functioning with the interface based on their propagation characteristics．

Four expressions of Fresnel formula are the reflection amplitude ratio and transmission amplitude ratio of the two orthogonal components of electric field vector．


Fig． 1 Reflection and refraction of incident light at the interface

$$
\begin{gather*}
r_{\perp}=\frac{E_{\perp}^{r}}{E_{\perp}^{i}}=-\frac{\sin \left(\theta_{1}-\theta_{2}\right)}{\sin \left(\theta_{1}+\theta_{2}\right)}=\frac{n_{1} \cos \theta_{1}-n_{2} \cos \theta_{2}}{n_{1} \cos \theta_{1}+n_{2} \cos \theta_{2}}  \tag{1}\\
r_{p}=\frac{E_{p}^{r}}{E_{p}^{i}}=\frac{\tan \left(\theta_{1}-\theta_{2}\right)}{\tan \left(\theta_{1}+\theta_{2}\right)}=\frac{n_{1} \cos \theta_{1}-n_{2} \cos \theta_{2}}{n_{1} \cos \theta_{1}+n_{2} \cos \theta_{2}}  \tag{2}\\
t_{\perp}=\frac{E_{\perp}^{t}}{E_{\perp}^{i}}=\frac{2 \cos \theta_{1} \sin \theta_{2}}{\sin \left(\theta_{1}+\theta_{2}\right)}=\frac{2 n_{1} \cos \theta_{1}}{n_{1} \cos \theta_{1}+n_{2} \cos \theta_{2}}  \tag{3}\\
t_{p}=\frac{E_{p}^{t}}{E_{p}^{i}}=\frac{2 \cos \theta_{1} \sin \theta_{2}}{\sin \left(\theta_{1}+\theta_{2}\right) \cos \left(\theta_{1}-\theta_{2}\right)}=\frac{2 n_{1} \cos \theta_{1}}{n_{1} \cos \theta_{1}+n_{2} \cos \theta_{2}} \tag{4}
\end{gather*}
$$

Where，$n_{1}$ is the refraction rate of incident medium，$n_{2}$ is the refraction rate of transmission medium，$\theta_{1}$ is the incident angle，$\theta_{2}$ is the refraction angle，and $\theta_{1}{ }^{\prime}$ is the reflection angle．The reflection radiation at the interface follows Snell＇s Law $\left(n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}\right)$ ．If the refraction rate on both sides of the interfaces $n_{1}$ and $n_{2}$ ，incident angle $\theta_{1}$ are known，the refraction angle $\theta_{2}$ can be calculated by Snail＇s law，and then the amplitude reflection ratio $r_{\perp}$ and $r_{p}$ can be calculated by the above equation．

The reflection at the interface follows Fresnel＇s law．When the incident wave arrives at the interface of two media，if not considering the energy loss of absorption，scattering and other forms，the energy of the incident light will only redistribute between the reflected light and refracted light，and the total energy remains constant．The energy distribution mode can be determined using the Fresnel formula ${ }^{[2]}$ ．The reflection rate of the horizontal and vertical components $R_{\perp}$ and $R_{p}$ can be obtained by squared amplitude reflectance， as shown in expression（5）（6），and the total reflection rate $R$ can be obtained by（7）．

$$
\begin{gather*}
R_{\perp}=\frac{I_{\perp}^{r}}{I_{\perp}}=\frac{\left|E_{\perp}^{r}\right|^{2}}{\left|E_{\perp}^{i}\right|^{2}}=\left|\frac{n_{1} \cos \theta_{1}-n_{2} \cos \theta_{2}}{n_{1} \cos \theta_{1}+n_{2} \cos \theta_{2}}\right|^{2}  \tag{5}\\
R_{p}=\frac{I_{p}^{r}}{I_{p}}=\frac{\left|E_{p}^{r}\right|^{2}}{\left|E_{p}^{i}\right|^{2}}=\left|\frac{n_{1} \cos \theta_{1}-n_{2} \cos \theta_{2}}{n_{1} \cos \theta_{1}+n_{2} \cos \theta_{2}}\right|^{2}  \tag{6}\\
R=R_{p} \cos ^{2} \alpha+R_{\perp} \sin ^{2} \alpha \tag{7}
\end{gather*}
$$

Where $\alpha$ equals to $\arccos \frac{E_{p}^{i}}{E^{i}}$ ．The polarization degree of reflection components is defined as：

$$
\begin{align*}
\mathrm{DOP}_{R}= & \left|\frac{R_{p}-R_{\perp}}{R_{p}+R_{\perp}}\right|= \\
& \left|\frac{\left|n_{2} \cos \theta_{1}-n_{1} \cos \theta_{2}\right|^{2}-\left|n_{1} \cos \theta_{1}-n_{2} \cos \theta_{2}\right|^{2}}{\left|n_{2} \cos \theta_{1}-n_{1} \cos \theta_{2}\right|^{2}+\left|n_{1} \cos \theta_{1}-n_{2} \cos \theta_{2}\right|^{2}}\right|^{2} \tag{8}
\end{align*}
$$

## 2 Analysis of the state of polarization generated by thermal radiation

Study on polarization effect of thermal radiation began with Arago，in 1824 his qualitative experiments showed that both the solid and liquid，whether the surface is smooth，their thermal radiation are partially polarized．In 1895，Millikan conducted massive qualitatively experimental study on thermal radiation polarization effect of all kinds of metal and insulator． He found that heat radiation of relatively smooth metal surface has high degree of polarization，and the thermal radiation polarization of transparent and opaque non metal material is very weak．In order to conduct a quantitative study on metal，Millikan studied fluorescence of uranium glass，and experiments also showed that the fluorescence is indeed similar to thermal radiation．

By summarizing previous research on the thermal radiation polarization，Sandus ${ }^{[3]}$ proposed Fresnel refraction mechanism of smooth surface thermal radiation polarization．By the transmission amplitude ratio between the vertical component and the parallel component in Fresnel formula，namely Formula（3） （4），it is not difficult to get the transmissivity of vertical and parallel components，as follows：

$$
\begin{equation*}
R_{\perp}=\frac{\sin 2 \theta_{1} \sin 2 \theta_{2}}{\sin ^{2}\left(\theta_{1}+\theta_{2}\right)} \tag{9}
\end{equation*}
$$

$$
\begin{gather*}
T_{p}=\frac{\sin 2 \theta_{1} \sin 2 \theta_{2}}{\sin ^{2}\left(\theta_{1}+\theta_{2}\right) \cos ^{2}\left(\theta_{1}-\theta_{2}\right)}  \tag{10}\\
T=T_{p} \cos ^{2} \alpha+T_{\perp} \sin ^{2} \alpha \tag{11}
\end{gather*}
$$

By the definition of degree of polarization and Formula（9）（10），the degree of polarization of transmission component is：

$$
\begin{equation*}
\mathrm{DOP}_{T}=\left|\frac{T_{p}-T_{\perp}}{T_{p}+T_{\perp}}\right|=\frac{1-\cos ^{2}\left(\theta_{1}-\theta_{2}\right)}{1+\cos ^{2}\left(\theta_{1}-\theta_{2}\right)} \tag{12}
\end{equation*}
$$

From the energy conservation law we can get that transmissivity and reflectivity have the following relationship：

$$
\begin{equation*}
T=1-R \tag{13}
\end{equation*}
$$

According to Kirchhoff＇s law，for opaque objects，energy transmitted to the internal of object is absorbed by the object，under condition of thermal equilibrium，the emission rate is proportional to the absorption rate $\varepsilon$ ，whose relationship with the reflectivity of $R$ is as follows

$$
\begin{equation*}
\varepsilon=1-R \tag{14}
\end{equation*}
$$

Applying Kirchhoff＇s law and Fresnel formula， radiation rate with the polarization direction perpendicular and parallel to the radiation surface is calculated as follows：

$$
\begin{gather*}
\varepsilon_{\perp}=T_{\perp}=\frac{\sin 2 \theta_{1} \sin 2 \theta_{2}}{\sin ^{2}\left(\theta_{1}+\theta_{2}\right)}  \tag{15}\\
\varepsilon_{p}=T_{p}=\frac{\sin 2 \theta_{1} \sin 2 \theta_{2}}{\sin ^{2}\left(\theta_{1}+\theta_{2}\right) \cos ^{2}\left(\theta_{1}-\theta_{2}\right)} \tag{16}
\end{gather*}
$$

Total radiation rate is the average of the parallel and perpendicular components：

$$
\begin{equation*}
\varepsilon=\frac{1}{2}\left(\varepsilon_{p}+\varepsilon_{\perp}\right) \tag{17}
\end{equation*}
$$

The degree of radiation polarization is defined as follows：

$$
\begin{equation*}
\mathrm{DOP}_{\varepsilon}=\left|\frac{\varepsilon_{p}-\varepsilon_{\perp}}{\varepsilon_{p}+\varepsilon_{\perp}}\right|=\frac{1-\cos ^{2}\left(\theta_{1}-\theta_{2}\right)}{1+\cos ^{2}\left(\theta_{1}-\theta_{2}\right)} \tag{18}
\end{equation*}
$$

Where $\theta_{2}$ is he at radiation angle，$\theta_{1}$ is calculated according to the refractive index of two media．

Based on the viewpoint that refraction produces radiation，we have used Formula（18）to reproduce the experiments of Millikan，which included the experiment of polarization of uranium glass fluorescent radiation and the experiments of the thermal radiation polarization of solid platinum and liquid silver．We
have carried out observation experiment，and then the calculation results are compared with the experimental results，as shown in Fig．2．The experimental results and the calculation results are consistent in the range of allowable error．This consistency means that at least for fluorescence radiation of uranium glass， polarization can only come from refraction on the surface，that is to say all thermal radiation will undergo the process of refraction．


Fig． 2 Comparison between the experimental results and the calculation results of uranium glass fluorescent polarization

Subsequently，Millikan carried out experiments on metal in order to determine the thermal radiation polarization of solid platinum and liquid silver under different thermal radiation angle，as shown in Fig．3．

$$
\begin{aligned}
& 0.9 \\
& \hline
\end{aligned}
$$



Fig． 3 Comparison between the experimental results and the calculation results of polarization of metal platinum and metal silver

It can be seen that experimental results of radiation polarization degree are basically consistent with calculation results，and so the conclusion Millikan got from uranium glass experiment，that thermal radiation on objects surface come from refraction，was also confirmed in the metal experiments．

Millikan＇s experiments showed that all object emitting light was from a depth of the object under the surface，including the surface emitting light would go through refraction．The depth is defined as the depth of penetration．In 1926，Worthing studied the thermal radiation polarization of incandescent tungsten wire，tantalum wire and molybdenum wire，and he found that the maximum degree of polarization appears in the grazing observation（incident angle is $90^{\circ}$ ），which was $93 \%$ ，and further confirmed the above discussion．

Based on the experimental results and theoretical summary of Millikan，thermal radiation on object surface are emitted through the refraction at the interface，refraction undoubtedly brought the polarization of thermal radiation，and so it can be concluded that the polarization effect of thermal radiation exists universally．For relatively rough surface，when the thermal radiation dip angle is large， the polarization is quite strong，which is because the rough surface in large dip observation angle would appear very smooth．For opaque objects，according to the principle of energy conservation，sum of reflectivity rate and absorption rate equals 1. According to the Fresnel formula，the reflecting degree of object to parallel component and vertical component of light is different，and so the extent of absorption to them are different，which means that absorption will also change the polarization properties of light ${ }^{[4]}$ ．From Kirchhoff＇s law，objects good at absorbing are also good at emitting，the emitting of object on the two components of light is different，and therefore，the thermal radiation of the object will also have a certain polarization．

## 3 Analysis of the state of polarization generated by infrared scattering

Light scattering is caused by uneven medium， this heterogeneity may be local fluctuations of density caused by thermal motion of material molecules，may also be suspended with other small particles ${ }^{[5]}$ ．For example，the scattering caused by the gas molecules of atmosphere，including nitrogen and oxygen molecules，is called molecules scattering；various types of suspended particles in the atmosphere will also cause scattering．The scattering strength has close relation with scattered size，density and radiation wavelength，polarization of scattering is still related with the polarization state of incident light．

The scattering law of particles is different with different particle size．Usually a size factor $\chi$ is applied to describe that：

$$
\begin{equation*}
\chi=\frac{2 \pi r}{\lambda} \tag{19}
\end{equation*}
$$

Where $r$ is the radius of scattering particles，$\lambda$ is the wavelength of incident light．When $\chi$ is small（usually refers to $\chi<0.3-1.0$ ），scattering follows the law of Rayleigh scattering，which is called Rayleigh scattering； when $x$ exceeds the range of Rayleigh scattering （usually refers to $\chi=1.0-2.0$ ），it enters the scope of Mie scattering，when scattering particles are large，it belongs to large particle scattering range of geometrical optics ${ }^{[6]}$ ．

## 3．1 Analysis of polarization effect of Rayleigh scattering

The Rayleigh scattering theory is used to describe particles especially molecular（the radius meets $2 \pi r / \lambda=1$ ）． Other assumptions of Rayleigh scattering include isotropic non ionizing particles，refraction rate close to 1 ， and the frequency of incident radiation is different from the inherent frequency of particles．Because diatomic nitrogen molecules and oxygen molecules are the main ingredients of atmosphere，the isotropy assumption needs anisotropic molecular modification
coefficient．Rayleigh scattering follows equation：

$$
\begin{equation*}
\frac{I}{I^{\prime}}=K^{4} \frac{V^{2}}{\lambda^{4} r^{2}}\left(1+\cos ^{2} \theta\right) \tag{20}
\end{equation*}
$$

Where，$I$ is scattering radiation intensity，$I^{\prime}$ is incident radiation intensity，$K$ is the medium character，$V$ is the mass of particles，$r$ is the distance from particles to the observation point，$\lambda$ is the wavelength of radiation，$\theta$ is scattering angle．

Rayleigh scattering happens to the moment introducing dipole into incident radiation synchronously．The frequency of scattering radiation is equal to the frequency of the incident radiation． Therefore Rayleigh scattering is elastic ${ }^{[7]}$ ．Chandrasekhar calculated the polarization properties of atmospheric molecule after scattering and absorption according to Rayleigh scattering principles．

According to the calculation of Chandrasekhar， the phase matrix of Rayleigh scattering is ${ }^{[8]}$ ：

$$
M=\left[\begin{array}{cccc}
\frac{3}{4}\left(1+\cos ^{2} \theta\right) & \frac{3}{4} \sin ^{2} \theta & 0 & 0  \tag{21}\\
-\frac{3}{4} \sin ^{2} \theta & \frac{3}{4}\left(1+\cos ^{2} \theta\right) & 0 & 0 \\
0 & 0 & \frac{3}{2} \cos \theta & 0 \\
0 & 0 & 0 & \frac{3}{2} \cos \theta
\end{array}\right]
$$

It is clear that，the incident light is horizontal linearly polarized（with normalized Stokes vector $S_{\text {in }}=$ $\left.\frac{1}{\sqrt{2}}\left[\begin{array}{llll}1 & 1 & 0 & 0\end{array}\right]^{\mathrm{T}}\right)$ ，Rayleigh scattering light is horizontal linearly polarized light．Similarly，when the incident light is vertical polarized light，Rayleigh scattering light is still perpendicular linearly polarized light．

For the linearly polarized light of other polarization direction，take the $45^{\circ}$ linearly polarized light for example，Rayleigh light scattering（with Stokes vector $\left.S_{\text {out }}=\left[\begin{array}{llll}3 & (1+\cos 2 \theta) / 4 & -3 \sin ^{2} \theta / 4 & 3 \cos \theta / 2\end{array} 0\right]^{T}\right)$ is still linearly polarized light，with only the polarization angle changed．

When the incident light is natural light，the degree of polarization for Rayleigh scattering light is $P=\frac{\sin ^{2} \theta}{1+\cos ^{2} \theta}$ ．As shown in Fig． 4 is the change curve
of Rayleigh scattering polarization degree with scattering angle．At $\theta=\pi / 2$ ，the scattering light is linearly polarized light，with the polarization degree of $100 \%$ ；at $\theta=0$ and $\theta=\pi$ ，scattering light has no polarization；at other scattering angles，scattering light is partially polarized light．At the same time，at Rayleigh scattering area，polarization of the scattered light is basically independent of scattering particle size．


Fig． 4 Change curve of Rayleigh scattering polarization degree with scattering angle

## 3．2 Analysis of the polarization effect of Mie scattering

Mie scattering is applied to particle of comparative size with the incident wavelength．The theory assumes the isolated spherical particle as scattering medium．Although most particles are not spherical，but a large number of non spherical particles with disorder directions can be approximated spherical particle distribution except when determining backscattering and polarization properties．As with the Rayleigh scattering，Mie scattering describes excitation of incident radiation to particles，and re radiation of energy in a spherical solid angle of $4 \pi$ ．Cross regional Mie scattering of unit volume follows the distribution as：

$$
\begin{equation*}
\beta_{\mathrm{sca}}=\int_{0}^{\infty} C_{\mathrm{sca}}(r) n(r) \mathrm{d} r \tag{22}
\end{equation*}
$$

Where $\beta_{\text {sca }}$ is cross－region scattering of unit volume，is cross－region scattering，$C_{\mathrm{sca}}(r)$ is the distribution function of particle size ${ }^{[9]}$ ．

For particles of arbitrary shape and composition， the electric field component of scattering light at
distance of $R$ can be described as

$$
\left\{\begin{array}{c}
E_{p}^{o}  \tag{23}\\
E_{s}^{o}
\end{array}\right\}=\frac{\exp (-i k R+i k z)}{i k R}\left[\begin{array}{ll}
S_{1}(\theta, \varphi) & S_{2}(\theta, \varphi) \\
S_{3}(\theta, \varphi) & S_{4}(\theta, \varphi)
\end{array}\right]\left\{\begin{array}{c}
E_{p}^{i} \\
E_{s}^{i}
\end{array}\right\}
$$

Where，$E_{p}^{i}$ and $E_{s}^{i}$ are respectively the parallel component and vertical component of the incident electric field on the scattering plane，$E_{p}^{o}$ and $E_{s}^{o}$ are respectively parallel component and vertical component of scattering electric field，$R$ is the distance between scattering light and the particle center，$z$ is the distance from incident light to the particle center，$k=$ $2 \pi / \lambda, \theta$ is scattering angle，and $\varphi$ is azimuth angle．In practice，the solving process of scattering matrix $S$ is difficult，but for homogeneous isotropic circular particles，the scattering matrix $S$ can be simplified to：

$$
S=\left[\begin{array}{cc}
S_{1}(\theta) & 0  \tag{24}\\
0 & S_{2}(\theta)
\end{array}\right]
$$

Where $S_{1}(\theta)$ and $S_{2}(\theta)$ are scattering amplitude functions， with expression of ${ }^{[10]}$

$$
\left\{\begin{array}{l}
S_{1}(\theta)=\sum_{n=1}^{\infty} \frac{2 n+1}{n(n+1)}\left(\pi_{n} a_{n}+\tau_{n} b_{n}\right)  \tag{25}\\
S_{2}(\theta)=\sum_{n=1}^{\infty} \frac{2 n+1}{n(n+1)}\left(\tau_{n} a_{n}+\pi_{n} b_{n}\right)
\end{array}\right.
$$

Where $a_{n}$ and $b_{n}$ are Mie functions，defined as

$$
\left\{\begin{array}{l}
a_{n}=\frac{\psi_{n}(\chi) \psi_{n}{ }^{\prime}(m \chi)-m \psi_{n}{ }^{\prime}(\chi) \psi_{n}(m \chi)}{\zeta_{n}(\chi) \psi_{n}^{\prime}(m \chi)-m \zeta_{n}{ }^{\prime}(\chi) \psi_{n}(m \chi)}  \tag{26}\\
b_{n}=\frac{m \psi_{n}(\chi) \psi_{n}{ }^{\prime}(m \chi)-\psi_{n}{ }^{\prime}(\chi) \psi_{n}(m \chi)}{m \zeta_{n}(\chi) \psi_{n}{ }^{\prime}(m \chi)-\zeta_{n}{ }^{\prime}(\chi) \psi_{n}(m \chi)}
\end{array}\right.
$$

Where $\psi_{n}(\chi)$ and $\zeta_{n}(\chi)$ are Riccati－Bessel functions， defined as

$$
\left\{\begin{array}{l}
\psi_{n}(\chi)=(\pi \chi / 2)^{1 / 2} J_{n+1 / 2}(\chi)  \tag{27}\\
\zeta_{n}(\chi)=(\pi \chi / 2)^{1 / 2} H_{n+1 / 2}^{(2)}(\chi)
\end{array}\right.
$$

Where，$J_{n+1 / 2}(\chi)$ is the first kind of Bessel function of half odd order，$H_{n+1 / 2}^{(2)}(\chi)$ is the second kind of Hankel function．

In the formula，$m$ is the refraction index，size factor $\chi=2 \pi r / \lambda, r$ is the particle radius，angle coefficient $\pi_{n}$ and $\tau_{n}$ are defined as：

$$
\left\{\begin{array}{l}
\pi_{n}=\frac{\mathrm{d} P_{n}(\cos \theta)}{\mathrm{d}(\cos \theta)}  \tag{28}\\
\tau_{n}=\frac{\mathrm{d} P_{n}^{(1)}(\cos \theta)}{\mathrm{d} \theta}
\end{array}\right.
$$

Where，$P_{n}(\cos \theta)$ is the first kind of Legendre function，$P_{n}^{(1)}(\cos \theta)$ is the first kind of Associated Legendre function of first－order $n$－th degree．

Using the above formula，when the ratio of scattering particle radius and the wavelength $\mathrm{r} / \lambda$ ， scattering angle $\theta$ ，and the refraction rate $m$ are known，the scattering amplitude function $S_{1}(\theta)$ and $S_{2}(\theta)$ can be calculated，and then obtain the scattering intensity function value $i_{1}(\theta)=\left|S_{1}(\theta)\right|^{2}, i_{2}(\theta)=\left|S_{2}(\theta)\right|^{2}$ ．

When the incident light is linearly polarized，let its electric field vibration direction be the $X$ axis direction，and parallel component $E_{p}^{i}$ and vertical component $E_{s}^{i}$ of the incident light are respectively ${ }^{[11]}$ ：

$$
\left\{\begin{array}{l}
E_{p}^{i}=E^{i} \mathrm{e}^{-i k} \cos \varphi  \tag{29}\\
E_{s}^{i}=E^{i} \mathrm{e}^{-i k} \sin \varphi
\end{array}\right.
$$

Combining expressions（23）（24）（29），parallel component $E_{p}^{o}$ and vertical component $E_{s}^{o}$ of scattered light are respectively：

$$
\left\{\begin{array}{l}
E_{p}^{o}=\frac{E^{i} \mathrm{e}^{-i k R} S_{1}(\theta)}{i k R}  \tag{30}\\
E_{s}^{o}=\frac{E^{i} \mathrm{e}^{-i k R} S_{2}(\theta)}{i k R}
\end{array}\right.
$$

Using the relationship equation $I=E \cdot E^{*}$ between the light intensity and the electric field strength，the parallel component and vertical component of the scattering intensity are respectively：

$$
\left\{\begin{array}{l}
I_{p}=\frac{I^{i}}{k^{2} R^{2}} i_{1}(\theta) \cos ^{2} \varphi  \tag{31}\\
I_{s}=\frac{I}{k^{2} R^{2}} i_{2}(\theta) \sin ^{2} \varphi
\end{array}\right.
$$

The total scattering intensity is：

$$
\begin{equation*}
I=\frac{I^{i}}{k^{2} R^{2}}\left[i_{1}(\theta) \cos ^{2} \varphi+i_{2}(\theta) \sin ^{2} \varphi\right] \tag{32}
\end{equation*}
$$

When the incident light is natural light，the parallel component，the vertical component of scattering intensity and the total scattering intensity are respectively：

$$
\left\{\begin{array}{l}
I_{p}=\frac{I^{i}}{2 k^{2} R^{2}} i_{1}(\theta)  \tag{33}\\
I_{s}=\frac{I^{i}}{2 k^{2} R^{2}} i_{2}(\theta) \\
I=\frac{I}{2 k^{2} R^{2}}\left[i_{1}(\theta)+i_{2}(\theta)\right]
\end{array}\right.
$$

It is obvious that，for incident natural light， scattering light turns to partially polarized light，and only the forward scattering and backscattering light are still natural light．

## 4 Conclusions

According to the basic principle of infrared radiation，various polarization effects are analyzed and deduced in detail in this paper when infrared radiation interacts with matter，and the following three important conclusions are obtained：（1）infrared reflectance， thermal radiation，infrared scattering will produce polarization，infrared polarization effects are ubiquitous；（2）the polarization of infrared reflection at object surface can be calculated by Fresnel reflection law，and polarization is closely related to attributes of objects（such as material，texture，roughness，structure， shape and so on），polarization measurement can be used for interpretation of target attribute；（3）the heat radiation of objects has a process of refraction through the interface，polarization generated can be calculated by Fresnel refraction mechanism combining with corresponding surface model；（4）the linearly polarized light after Rayleigh scattering is still linearly polarized light，non polarized light changes into partially polarized light（with the exception of individual scattering angle）；linearly polarized and non polarized light will be turned into partially polarized light by Mie scattering（with the exception of individual scattering angle）．The above conclusion of this paper provides sufficient theory basis for the realization of target detection and recognition based on infrared polarization information by making better use of the target infrared polarization information．

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[^0]:    收稿日期：2015－10－05；修订日期：2015－11－14
    基金项目：国家自然科学基金（61302145）
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