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Effect of non-Kolmogorov turbulence on fluctuations in angle of arrival of starlight

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Abstract: Based on a power spectrum of non-Kolmogorov turbulence developed by A S Gurvich et al, the variance of angle-of-arrival (AOA) fluctuations was derived. The concise closed-form expression was obtained and used to analyze the joint influence of Kolmogorov tropospheric turbulence and non-Kolmogorov stratospheric one on the fluctuations in the angle of arrival (AOA) of starlight. It is shown that the AOA fluctuations of starlight were mainly determined by Kolmogorov tropospheric turbulence. And the non-Kolmogorov stratospheric turbulence was responsible for 5-14 percent of the total of AOA fluctuations for different receiver apertures in weak fluctuations regime. In addition, the AOA fluctuations induced by non-Kolmogorov turbulence depended on the receiver aperture and the outer scale and the intensity of non-Kolmogorov turbulence.

Key words: atmospheric optics; non-Kolmogorov turbulence; Kolmogorov model; angle-of-arrival fluctuations

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非柯尔莫哥洛夫湍流对星光到达角起伏影响研究

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摘 要: 基于A. S. Gurvich等人所提出的非柯尔莫哥洛夫湍流功率谱密度模型，推导了弱起伏条件下的到达角起伏方差,得到了一个解析的结果;然后,利用该结果分析了对流层柯尔莫哥洛夫湍流和平流层非柯尔莫哥洛夫湍流对星光到达角起伏的联合影响。结果表明：星光到达角起伏主要是由对流层柯尔莫哥洛夫湍流决定;对于不同的接收孔径,到达角起伏 5%~14%是由平流层非柯尔莫哥洛夫湍流引起的。此外,非柯尔莫哥洛夫湍流对到达角起伏还取决于接收孔径、湍流外尺度及非柯尔莫哥洛夫湍流起伏强度。

关键词: 大气光学； 非柯尔莫哥洛夫湍流； 柯尔莫哥洛夫模型； 到达角起伏

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0 Introduction

In recent years, laser has been used to extend radio-frequency (RF) atmospheric communication to the optical-frequency band. Free space laser optics communication (FSO) has some potential advantages than conventional RF communications, including high data rate, the low probability of detection, low weight and volume, etc. Modulation of the laser signal by optical turbulence is a major factor for communication links that traverse atmospheric paths. Optical turbulence is an important atmospheric phenomenon that adversely affects optical wave propagation. It is brought about by fluctuations in the atmosphere's refractive index due to inhomogeneities in temperature and pressure caused by solar heating and wind. When laser light, which is simply one form of optical wave, propagates through the atmosphere, optical turbulence distorts the optical path and further induces fluctuations in its amplitude and phase[1-7]. These effects can severely affect the performance of FSO systems[8-11].

For a long time, the Kolmogorov model for atmospheric turbulence has been extensively accepted and applied widely in the researches of light wave propagation in the atmosphere[12-15] and further estimate the performance of FSO systems[16]. Although Kolmogorov model has been confirmed by results of numerous experiments, both theoretical [17] and experimental works[18-19] concerning non-Kolmogorov turbulence over the past 10 -15 years have shown that Kolmogorov model is not the only possible one in the atmosphere. This also has prompted the investigations on the effect of non-Kolmogorov turbulence on optical wave propagation[20-23].

A S Gurvich and M S Belen' kii, based on experimental data from in situ measurements in the stratosphere and on a theory of saturated internal gravity waves, have developed a model for the power spectrum of non-Kolmogorov turbulence and researched the effects of non-Kolmogorov stratospheric turbulence on the scintillation and the coherence of starlight, as well as on the degradation of star image[24]. Later M. S. Belen'kii investigated the influence of non-Kolmogorov stratospheric turbulence on star image motion again[25].

In this paper, considering a power spectrum of non-Kolmogorov turbulence introduced in ref[24], the variance of AOA fluctuations has been derived. The concise closed-form expression is obtained and used to analyze the effect of non-Kolmogorov stratospheric turbulence on the AOA fluctuations of starlight.

1 Joint power spectrum for refractive-index fluctuations

To analyze the joint influence of non-Kolmogorov stratospheric turbulence and Kolmogorov tropospheric one on the fluctuations in the AOA of starlight, the three-dimensional power spectrum model $\Phi_n(\kappa,z)$ for the refractive-index fluctuations developed by A S Gurvich et al is used[26], which has this form:

$$\Phi_n(\kappa,z)=\Phi_n^1(\kappa,z)+\Phi_n^2(\kappa,z) \tag{1}$$

where $\Phi_n^1(\kappa,z)$ characterizes the refractive - index fluctuations for Kolmogorov turbulence in the troposphere, while $\Phi_n^2(\kappa,z)$ describes the inhomogeneities of refractive index for non-Kolmogorov turbulence in the stratosphere. $\Phi_n^1(\kappa,z)$ has the form:

$$\Phi_n^1(\kappa,z)=0.033\mathbf{C}_n^2(z)\left[\kappa^2+\frac{\kappa_0^2}{\mathbf{a}^2}\right]^{-11/3} \tag{2}$$

where $\mathbf{C}_n^2(z)$ is the refractive-index structure parameter of Kolmogorov turbulence with units $\mathbf{m}^{-2/3}$ and κ is the magnitude of spatial frequency vector, rad/m. It is noted that, considering that the outer scale of atmospheric turbulence $\mathbf{L}_0(z)$ is an important relevant parameter for high angular resolution imaging [26], we introduced it in Eq.(2) by adding a term $\kappa_0^2/\mathbf{a}^2$ based on the work of H Trinquet et al in 2008. Here $\kappa=2\pi/\mathbf{L}_0(z)$ and $1/\mathbf{a}^2=6.2\times10^{-3}$.

$\Phi_n^2(\kappa,z)$ has the form:

$$\Phi_n^2(\kappa,z)=\tilde{\mathbf{C}}_n^2(z)\kappa^{-5}\left[\frac{\kappa^2}{\kappa_{0N}^2+\kappa^2}\right]^{-5/2} \tag{3}$$

where $\tilde{\mathbf{C}}_n^2(z)$ is the index-of-refraction structure parameter of non-Kolmogorov turbulence with units $\mathbf{m}^{-2}$ and

$\kappa_{0N}=2\pi/L_{0N}$, with L_{0N} being the outer scale of non-Kolmogorov stratospheric turbulence. $\tilde{C}_n^2(z)$ uses the following representation:

$$\tilde{C}_n^2(z)=\tilde{C}_{n0}^2\left[\frac{2(z-H_{Tr})}{H_0}\right], z\geqslant H_{St}$$

$$\tilde{C}_n^2(z)=0, z<H_{St} \tag{4}$$

where $\tilde{C}_{n0}^2(z)$ represents the structure characteristic of non-Kolmogorov turbulence at the altitude of tropopause, H_0 is the atmospheric scale height $H_0=6\,000$ m and H_{St} is the starting altitude of non-Kolmogorov turbulence and is equal to 10 km.

2 Angle-of-arrival fluctuations of starlight

As a star is observed on the ground, the starlight may be considered as a plane wave. Following the same procedure discussed in Ref[1], substituting Eq.(1) into the conventional formula for the AOA fluctuations variance of plane wave (obtained using the Rytov approximation) yields:

$$\sigma_{pl}^2(z)=\sigma_{pl(Tr)}^2(z)+\sigma_{pl(St)}^2(z) \tag{5}$$

where $\sigma_{pl(Tr)}^2(z)$ and $\sigma_{pl(St)}^2(z)$ are the components of the variance of AOA fluctuations induced by Kolmogorov and non-Kolmogorov turbulence, respectively. They have the following representations:

$$\sigma_{pl(Tr)}^2(z)=0.033\pi^2\int_0^{H_{St}} C_n^2(z)\left(\frac{\kappa_0}{a}\right)^{1/3}\times U\left(2;\frac{7}{6};\frac{0.058\,4D^2\kappa_0^2}{a^2}\right)dz \tag{6}$$

$$\sigma_{pl(St)}^2(z)=\frac{1}{2}\pi^2\tilde{C}_{n0}^2 H_0\kappa_{0N}^{-1}U\left(2;\frac{1}{2};0.058\,4D^2\kappa_{0N}^2\right) \tag{7}$$

where $U(a;c;x)$ is the confluent hypergeometric function of the second kind. Since the above results are obtained using the geometrical approximation, the receiver aperture D satisfies the condition, $D>>\sqrt{L/k}$, where $\sqrt{L/k}$ is the Fresnel zone(L is the length of the optical path and k is optical wave number). Moreover, the valid range of the power spectrum of the refractive-index fluctuations imposes the constraints $L_{0N}>>D$ and $L_0>>D$ on them again.

From Eq. (5) it can be concluded that the fluctuations in the AOA of starlight in the plane of a ground-based telescope are determined by both the troposphere and the stratosphere. As a result, an expression for the variance of the AOA fluctuations of starlight contains two terms, each depending on different characteristics of the atmosphere. The tropospheric component of the AOA fluctuations of starlight $\sigma_{pl(Tr)}^2(z)$ is determined by the optical turbulence intensity of troposphere C_n^2, the outer scale of the tropospheric turbulence L_0, and the telescope diameter D, whereas the stratospheric component $\sigma_{pl(St)}^2(z)$ depends on the optical turbulence intensity of stratosphere $\tilde{C}_{n0}^2$, the outer scale of the stratospheric turbulence L_{0N} on the altitude of tropopause, and the telescope diameter D. In addition, it is noted that the stratospheric component is a concise closed-form expression.

3 Simulation and discussions

To estimate quantitatively the contribution of non-Kolmogorov stratospheric turbulence to AOA fluctuations of starlight, the v ariance of AOA fluctuations $\sigma_{pl}^2(z)$, the tropospheric component $\sigma_{pl(Tr)}^2(z)$, and stratospheric one $\sigma_{pl(Sr)}^2(z)$ at the zenith are computed using Eq. (5) -(7), respectively. In addition, the variance of AOA fluctuations $\sigma_{pl(C)}^2(z)$ based on the conventional Kolmogorov theory is also computed for comparison. Table 1 summarizes the rms $\sigma_{pl(C)}^2(z)$, $\sigma_{pl(Tr)}^2(z)$, $\sigma_{pl(St)}^2(z)$, $\sigma_{pl}^2(z)$ and $\sigma_{pl(St)}^2(z)/\sigma_{pl}^2(z)$(the ratio of stratospheric component to the total AOA fluctuations) for different receiver apertures.

For the $C_n^2(z)$ profile in the troposphere, the most widely used Hufnagel-Valley model is chosen as:

$$C_n^2(z)=0.005\,94\left(\frac{v}{27}\right)^2(10^{-5}z)^{10}\exp\left(\frac{-z}{1\,000}\right)+2.7\times10^{-16}\exp\left(\frac{-z}{1\,500}\right)+C_n^2(0)\exp\left(\frac{-z}{1\,000}\right) \tag{8}$$

with the wind speed $v=21$ m/s and $C_n^2(0)=1.7\times10^{-14}\ m^{-2/3}$. For the $L_0(z)$ profile in the troposphere, following Ref[29], the function in the boundary layer is chosen, $L_0(z)=3.21\times$

$\mathbf{z}^{-0.11}$, while the Coulman-Vernin model is use $\mathbf{L}_0(\mathbf{z})=4/\{1+[(\mathbf{z}-8\,500)/2\,500]^2\}$ in the free atmosphere.

For the parameters of stratospheric turbulence, based on ref [27], $\mathbf{C}_{n0}^2=4.5\times10^{-19}\ \mathrm{m}^{-2}$ and $\mathbf{L}_{0N}=100$ m are taken.

As it is shown in Tab.1, the joint model theoretical predictions for the AOA fluctuations are smaller than the conventional ones. Furthermore, by comparing the Kolmogorov component with the total AOA fluctuations, the conclusion can be drawn that the AOA fluctuations are determined primarily by the lower layer Kolmogorov turbulence near the receiver, namely, the conventional point of view is tenable for the joint model theory. At last, the tropospheric component, the stratospheric component, and the total AOA fluctuations decrease with the increase of the receiver aperture owing to averaging of the phase fluctuations by the receiver aperture, and the stratospheric non-Kolmogorov turbulence accounts for 5-14 percent of the total fluctuations in the AOA of starlight for different receiver apertures.

Tab.1 Ratio of stratospheric component of RMS AOA fluctuations to total RMS for different receiver aperture

$\mathbf{D}$/m	$\sigma_{pl(\mathbf{c})}$/μrad	$\sigma_{pl(Tr)}$/μrad	$\sigma_{pl(St)}$/μrad	σ_{pl}/μrad	$\sigma_{pl(St)}/\sigma_{pl}$/%
0.1	13.687	9.647	0.531	9.661	5.49
0.2	10.446	7.364	0.530	7.383	7.17
0.4	7.327	5.166	0.528	5.193	10.16
0.6	5.621	3.964	0.525	4.000	13.14

To find out specially the effect of the receiver aperture on rms $\sigma_{pl(Tr)}^2(\mathbf{z})$, $\sigma_{pl(St)}^2(\mathbf{z})$, $\sigma_{pl}^2(\mathbf{z})$, their variations with $\mathbf{D}$ are plotted in Fig.1. From this figure it is apparent that the AOA fluctuations induced by non-Kolmogorov stratospheric turbulence lightly (almost appear a horizontal line) fall off linearly with the receiver aperture, while the AOA fluctuations induced by Kolmogorov tropospheric turbulence firstly fall off more quickly and then more slowly with the receiver aperture. This leads to the similar variations of the total RMS variance with the receiver aperture to the tropospheric component.

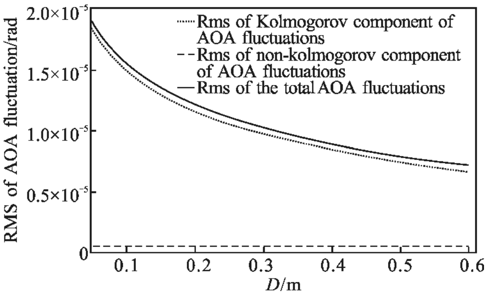

Fig.1 Kolmogorov component non-Kolmogorov one, and total RMS AOA fluctuations for different receiver apertures

To investigate the effect of tropospheric and stratospheric turbulent strength on the AOA fluctuations, the variations of RMS AOA fluctuations with Kolmogorov tropospheric turbulent strength KTTS $\mathbf{C}_n^2(0)$ on the ground for different non-Kolmogorov stratospheric turbulent strength NSTS $\tilde{\mathbf{C}}_{n0}^2$ at the altitude of tropopause are plotted in Fig.2, taking $\mathbf{D}=0.2$ m. As it is shown in Fig.2, the AOA fluctuations increase with KTTS and the variations of NSTS results in the smaller increase of the AOA fluctuation due to the smaller contribution of stratospheric turbulence to the total AOA fluctuations.

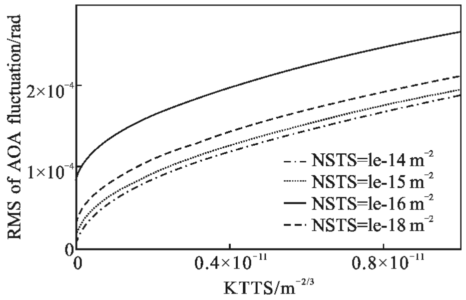

Fig.2 Variations of total RMS AOA fluctuations with KTTS for different NSTS

To study the effect of the outer scale on the AOA fluctuations of starlight, the variations of RMS AOA fluctuations with outer scales are plotted in Fig.3. As it is shown that the RMS AOA fluctuations increase with increase in the outer scale, i.e., it is physically correct. In fact, the AOA, like the beam wander, is caused mostly by large-scale turbulence cells; therefore, when the outer scale assumes high values, the optical wave meets a large number of large-scale turbulence cells along its propagation length, and these cells lead to a higher AOA value with respect to the case of a low outer scale value.

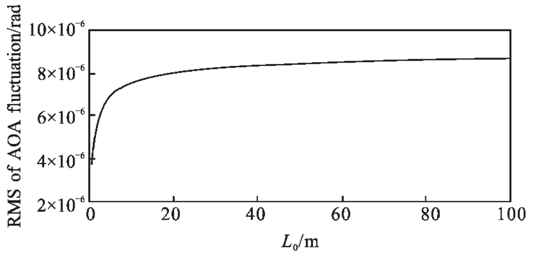

Fig.3 Variations of total RMS AOA fluctuations with outer scales L_0

4 Conclusion

In conclusion, considering a power spectrum of non-Kolmogorov turbulence introduced by A S Gurvich et al, the variance of AOA fluctuations is derived, and then the contribution of non-Kolmogorov stratospheric turbulence to the fluctuations in the AOA of starlight has been estimated. It is shown that the AOA fluctuations are primarily determined by Kolmogorov tropospheric turbulence nearest the receiver and non-Kolmogorov strapospheric turbulence is only responsible for the 5-14 percent of the total fluctuations. The AOA fluctuations induced by non-Kolmogorov turbulence depend on the receiver aperture D, the outer scale L_{0N} and the intensity of non-Kolmogorov turbulence. The numeric simulation has shown that the AOA fluctuations induced by non-Kolmogorov turbulence decrease linearly with the receiver aperture.

To differentiate the contribution of stratospheric non-Kolmogorov turbulence and tropospheric Kolmogorov one to the total AOA fluctuations, it is necessary that an experimental investigation should be performed based on astronomical observation at the ground accompanied by monitoring simultaneously the turbulence at the altitude of tropopause and at the ground.

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