Influence of Fabry-Perot cavity on frequency discrimination curve in Pound-Drever-Hall method

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Abstract: A laser frequency stabilization technique based on Pound-Drever-Hall (PDH) method was introduced and analyzed in detail. The influences of the Fabry-Perot(F-P) mirror's reflectivity and loss of F-P cavity were analyzed by numerical simulation. The theoretical linear dynamic range (LDR) and sensitivity were 9.6 MHz and 0.2 MHz^{-1} without loss respectively if the reflectivity was 0.2. Meanwhile, the LDR was less than 14.5 MHz and the sensitivity preceded 0.05 MHz^{-1} in a situation considering that the cavity loss was about 1%. The results demonstrate that both the two factors cause significant influence on the LDR and sensitivity of the system.

Key words: PDH method; F-P cavity; reflectivity; cavity loss
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PDH 技术中 F-P 腔特性对鉴频曲线的影响

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摘　要: 详细介绍和分析了基于Pound-Drever-Hall (PDH) 方法的激光器稳频技术。仿真分析了法布里-珀罗(F-P)腔镜面反射率及腔内损耗对鉴频曲线的影响,当反射率为 0.2、不考虑腔内损耗时,鉴频曲线线性动态范围和灵敏度分别为 9.6 MHz 和 0.2 MHz^{-1}。与此同时,当腔内损耗为 1%时,线性动态范围和灵敏度变化为 14.5 MHz 和 0.05 MHz^{-1}。结果证明了镜面反射率及腔内损耗都会对鉴频曲线产生显著影响。

关键词: PDH 技术; F-P 腔; 反射率; 腔内损耗

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0 Introduction

Highly frequency-stabilized laser is of considerable interest in numerous fields such as ultra-cold atom [1], optical fiber sensing and communications[2], atmospheric quality monitoring[3]. Therefore, it is meaningful to the science and technology development and utilitarian for the engineering application. In the research of high accuracy resonator optic gyro, the frequency-stabilized laser is needed because the light intensity error signal produced by multiple-beam interference is sensitive to the phase. Pound-Drever-Hall frequency stabilization method[4] (PDH method) was invented by Drever and Hall in 80′s last century, for inspiring of Pound′s microwave frequency stabilization technique[5]. This method has ultrahigh detecting sensitivity and reaches the limit of shot noise, as well as avoiding low frequency range which containslarge laser noise. The method relies on a reference frequency provided by F-P cavity[6] and then uses the sideband spectrum produced by phase modulation to achieve optical heterodyne detection[7]. Therefore, the cavity parameters directly affect the frequency stabilization result.

Dissimilar to the ideal etalon, for the limit of manufacturing technique, high mirror flatness and reflectivity could not be achieved in practical situation, which leads to dispersion loss[8]. These factors influence the linear dynamic range(LDR) and sensitivity, generate the non-linear error and reduce the efficiency of the frequency stabilization system.

In this paper, the relationship between three parameters which are closely relative to the performance of frequency stabilization is analyzedin detail according to PDH method. The frequency discrimination curve is simulated, and the alteration of LDR and sensitivity caused by reflectivity and loss are calculated.

1 Principle

In laser spectrum technique, the optical heterodyne detecting is realized by sideband spectrum produced by utility of laser phase modulation of PDH method. It can satisfy phase condition of optical heterodyne detecting and weaken the influence of system noise of detecting and amplifying. It also possesses high sensitivity and signal to noise ratio that the other methods could hardly reach.

The system of laser frequency stabilization is shown in Fig.1.

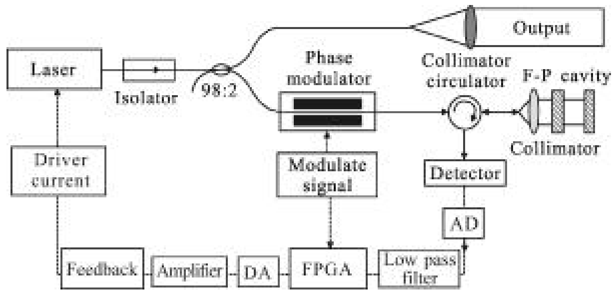

Fig.1 Schematic diagram of frequency stabilization system

The laser electric field modulated by sine wave is described as:

$$\mathbf{E}=\mathbf{E}_0 e^{i(2\pi ft+\beta\sin(2\pi vt))} \tag{1}$$

where f, β, **v** are laser frequency, modulation depth and modulation frequency, respectively. The Bessel function expanded expression of Eq.(1) is:

$$\mathbf{E}=\mathbf{E}_0[\mathbf{J}_0(\beta)+2i\mathbf{J}_1(\beta)\sin(2\pi\mathbf{v}t)]e^{i2\pi ft}+\mathbf{o}(\sin(e(2\pi\mathbf{v}t)))\approx \mathbf{E}_0[\mathbf{J}_0(\beta)e^{i2\pi ft}+\mathbf{J}_1(\beta)e^{i2\pi(f+v)t}-\mathbf{J}_1(\beta)e^{i2\pi(v-f)t}] \tag{2}$$

There are three strong frequencies after electric field modulation: (1) The laser intrinsic frequency f; (2) The sideband frequency f+**v** and f-**v** located in each part of f; (3) The sideband power is proportional to $\mathbf{J}_0(\beta)\mathbf{J}_1(\beta)$, so the proper β choice can restrain carrier and improve sideband power. The frequency discrimination curve is the best when $\beta\approx1.08$[9].

The free spectral range of confocal F-P cavity is $\Delta_{FSR}=\mathbf{c}/4\mathbf{nL}$, **c**, **n**, **L** are light speed in the vacuum, refractive index and cavity length, respectively. Depending on the multi-beam interference theory, the reflective coefficient of glossy F-P cavity is:

$$\mathbf{F}(\upsilon)=\frac{\mathbf{R}\cdot\mathbf{C}_\mathbf{R}\left[(1-\mathbf{C}_\mathbf{d})^2 e^{i\frac{2\pi\upsilon}{\Delta_{FSR}}}-1\right]}{1-\mathbf{R}\cdot(1-\mathbf{C}_\mathbf{d})^2 e^{i\frac{2\pi\upsilon}{\Delta_{FSR}}}} \tag{3}$$

where **R**, $\mathbf{C}_\mathbf{R}$, $\mathbf{C}_\mathbf{d}$ are the reflectivity of F-P etalon, incident loss of F-P material and dispersion loss, respectively, and shown in Fig.2.

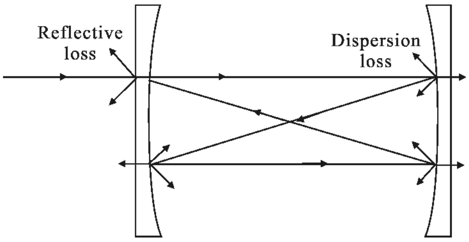

Fig.2 Sketch map of cavity loss, including reflection, dispersion and transmission loss

Considering the response of electric field modulated by F-P cavity. Based on linear system theory, the modulated electric field is the product of reflective coefficient of F-P cavity and corresponding frequency:

$$\mathbf{E}_{out}=\mathbf{E}_0[\mathbf{J}_0(\beta)\mathbf{F}(f)e^{i2\pi ft}+\mathbf{J}_1(\beta)\mathbf{F}(f+\mathbf{v})e^{i2\pi(f+\mathbf{v})t}-\mathbf{J}_1(\beta)\mathbf{F}(f-\mathbf{v})e^{i2\pi(f-\mathbf{v})t}] \quad (4)$$

The corresponding light intensity:

$$\mathbf{I}_{out}=\mathbf{E}_{out}\cdot\mathbf{E}_{out}^*=\mathbf{E}_0^2\mathbf{J}_0^2(\beta)|\mathbf{F}(f)|^2+\mathbf{E}_0^2\mathbf{J}_0^2(\beta)\cdot\{|\mathbf{F}(f+\mathbf{v})|^2+|\mathbf{F}(f-\mathbf{v})|^2\}+2\mathbf{E}_0^2\mathbf{J}_0(\beta)\mathbf{J}_1(\beta)\cdot\{\mathrm{Re}[\mathbf{F}(f)\mathbf{F}^*(f+\mathbf{v})-\mathbf{F}^*(f)\mathbf{F}(f-\mathbf{v})]\times\cos(2\pi\mathbf{v}t)+\mathrm{Im}[\mathbf{F}(f)\mathbf{F}^*(f+\mathbf{v})-\mathbf{F}^*(f)\mathbf{F}(f-\mathbf{v})]\times\sin(2\pi\mathbf{v}t)\}+\mathbf{o}(\mathbf{I}(2\mathbf{v})) \quad (5)$$

The first and second term are direct current, which represents the intensity produced by the carrier and sideband independently. The third and fourth term are the beat frequencies signal produced by mutual interaction between the sideband and carrier. In PDH method, the sine term is usually chosen to be the frequency discrimination signal.

$$\mathbf{e}=2\mathbf{E}_0^2\mathbf{J}_0(\beta)\mathbf{J}_1(\beta)\mathrm{Im}[\mathbf{F}(f)\mathbf{F}^*(f+\mathbf{v})-\mathbf{F}^*(f)\mathbf{F}(f-\mathbf{v})] \quad (6)$$

The demodulation curve based on lock-in technique is shown in Fig.3.

It is clear in the figure that there is a region with the character of linear and high slope near the resonant frequency. The better scale factor and sensitivity are obtained by using this region as frequency discrimination curve. So the part is defined as linear dynamic range (LDR) of PDH curve, it is also the working area of frequency stabilization system. The frequency range means ability of system feedback control, if frequency deviation is larger than frequency range, the non-linear phenomenon such as losing lock would appear, cause system unstable and produce errors.

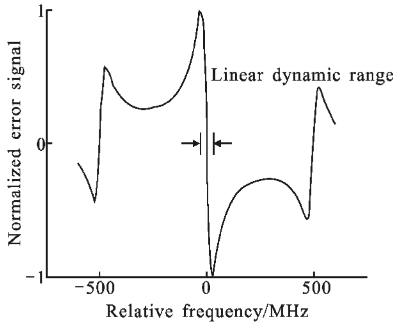

Fig.3 Frequency discrimination curve

For obtaining LDR, the first-order derivative of frequency discrimination curve is calculated and then the maximum and minimum value is obtained. the difference between corresponding frequency is LDR. The slope of LDR is defined as sensitivity of frequency discrimination signal, which represents the rate of error signal change with shifting of frequency.

2 Simulation analysis

2.1 Reflectivity of inner mirror

The center frequency of semi-conductor laser is 1 550 nm, free spectrum range of the cavity is 1.5 GHz, and the modulation frequency is large enough (f = 500 MHz). Only the mirror's reflectivity is taken into consideration. The reflectivity is set as 0.95, 0.97, 0.99 and the result is shown in Fig.4.

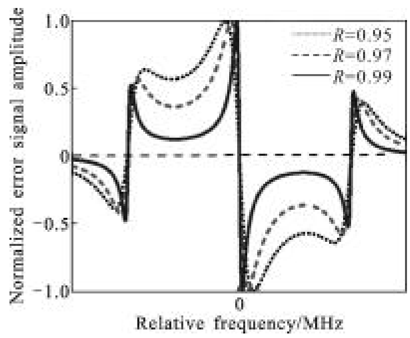

Fig.4 Relationship between reflectivity and frequency discrimination curve

From Fig.5 it can be seen that the cavity finesse ($F=\pi\sqrt{R}/(1-R)$) is enlarged with the increase of R when the reflectivity is close to 1, accordingly, the full width at half maximum (FWHM, $v_{FWHM}=v_{fsr}/F$), as well as the LDR which is proportional to the FWHM, is reduced.

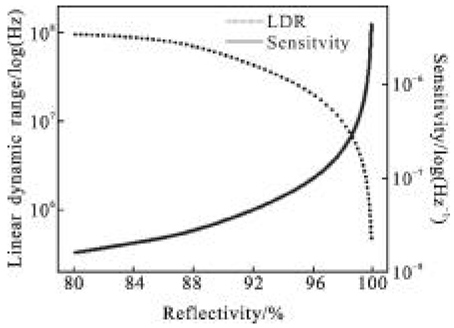

Fig.5 Influence of reflectivity on LDR and sensitivity

The trend of LDR and sensitivity is opposite with the increase of reflectivity. In another word, the LDR is reduced as sensitivity is increased. When R=0.98, LDR and sensitivity are 9.6 MHz, 0.2 MHz^{-1}, respectively.

2.2 Incident loss C_R

The incident loss exists because the route of light entering F-P cavity is air-medium-air. Part of the light is reflected before entering the cavity, which means that the luminous flux is diminished. When the light propagates in the cavity, the dispersion phenomenon also causes light energy loss. Both of them could affect the output signal.

From Fig.6 it can be seen that the incident loss C_R does not contribute to the LDR but only affects the amplitude of the light that takes part in resonance. The slope of the frequency discrimination curve

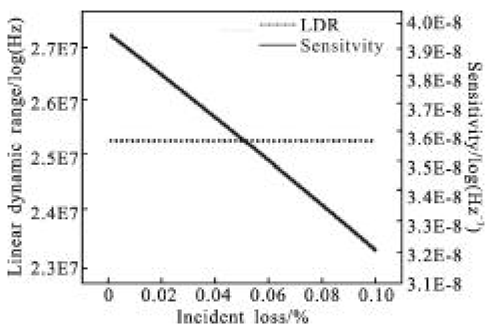

Fig.6 Effect of incident loss on linear dynamic range and sensitivity

within LDR regionis decreased, in another word, the sensitivity is reduced. Therefore an anti-reflective film coating is needed for the incident mirror to reduce the total insertion loss.

2.3 Dispersion loss C_d

The dispersion loss is set from 0.1% to 10% in the simulation: the FWHM is enlarged with the increase of the F-P cavity mirror's dispersion loss, as shown in Fig.7. Therefore, the LDR is raised and the sensitivity is diminished.

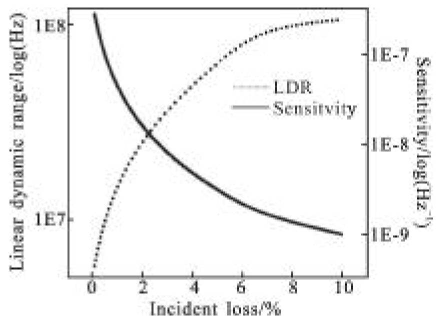

Fig.7 Effect of dispersion losson LDR and sensitivity

Figure 8 shows that the cavity finesse and the sensitivity are reduced when the dispersion loss in resonant cavity increases. The calculation results are listed in Tab.1.

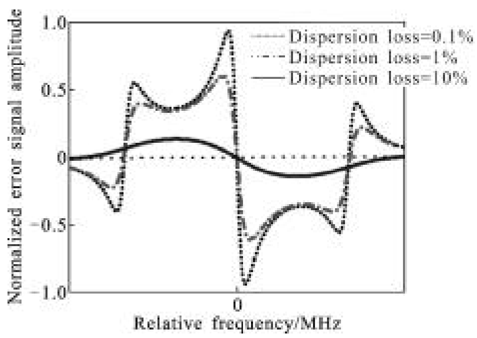

Fig.8 Influence of dispersion loss on the frequency discrimination curve

Tab.1 Calculation results of LDR and sensitivity when dispersion loss changes

Dispersion loss	LDR/MHz	Sensitivity/MHz^{-1}
0.1%	5.78	0.288
1%	14.54	0.05
10%	106	9.98×10^{-4}

3 Conclusion

In this paper, the theory of PDH frequency stabilization method is introduced, including principle of PDH method, reduction of frequency discrimination curve with lossy resonator and calculation scheme of LDR and sensitivity. The influence of reflectivity and cavity loss is simulated. The simulation consequence demonstrates that the linear dynamic range is 9.6 MHz and sensitivity is 0.2 MHz^{-1} with reflectivity 0.2 in ideal situation. When the dispersion loss is below 1%, the LDR is less than 14.5 MHz and sensitivity is exceeded 0.05 MHz^{-1}. It improves sensitivity of PDH system when the F-P cavity with high-reflectivity and low loss is adopted. This work provides useful guidance for optimizing the parameters of F-P cavity used in laser frequency stabilization based on PDH method.

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