Analysis of scattering of an on-axis zero-order Bessel beam by a chiral sphere

Qu Tan, Wu Zhensen, Mou Yuan, Li Zhengjun

(School of Science, Xidian University, Xi′an 710071, China)

Abstract: By right of its non-diffracting and self-constructing property, Bessel beam has gained growing attentions from scholars since its naissance. Based on the generalized Lorenz-Mie theory (GLMT), the incident on-axis Bessel beam and scattered fields of the chiral sphere were expressed in terms of the spherical vector wave functions (SVWFs). The analytical solution to the scattering of chiral sphere illuminated by an on-axis zero-order Bessel beam was investigated by utilizing the SVWFs and continuous boundary conditions. The on-axis Bessel beam was degenerated to a plane wave incident on a chiral sphere, and the results were found to be in good agreement with the literature available. The effects of beam, particle parameters, and on-axis beam waist center shift z_0 on the scattered intensity were numerically analyzed. The theory present here may play an important role in the fields of the applications of chiral material and chiral-coated target in microwave engineering and shielding targets.

Key words: Bessel beam; chiral media; spherical vector wave functions; light scattering

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在轴零阶贝塞尔波束对手征介质球的散射特性分析

屈 檀,吴振森,牟 媛,李正军

(西安电子科技大学 理学院,陕西 西安 710071)

摘 要: 贝塞尔波束自产生以来,凭借其无衍射、自修复特性已经获得了越来越多学者的关注。基于广义洛伦兹米理论, 将在轴贝塞尔波束与手征介质球相互作用的入射场及散射场展开为球矢量波函数的表达式。结合球矢量波函数的正交完备性及电磁场的连续性边界条件,推导出了在轴零阶贝塞尔波束对手征介质球电磁散射的解析解。数值模拟了散射强度随散射角的分布,将在轴贝塞尔波束退化为平面波照射手征介质球的散射结果与文献比较,吻合得较好。分析了波束及介质球参数、介质球偏离波束中心位置对散射强度的影响。该理论为手征材料和手征涂覆目标在微波工程及目标隐身中的应用提供了很好的理论应用价值。

关键词: 贝塞尔波束; 手征介质; 球矢量波函数; 光散射

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作者简介:屈檀(1987-),女,博士生,主要从事粒子对各种有形波束散射方面的研究。Email:qutandream@gmail.com

导师简介:吴振森(1946-),男,教授,博士生导师,主要从事目标与环境光电特性、随机介质中的电磁/光波传播与散射、粒子散射与粒度分析方面的研究。Email:wuzhs@mail.xidian.edu.cn

0 Introduction

The interaction of electromagnetic waves with chiral media is a topic of current research interest with applications occurring in a variety of areas including chiral microstrip antenna, target shielding, chiral composite material, and so on[1-3]. Many scholars have devoted their endeavor to the scattering of chiral object. Bassiri researched the propagation of plane wave in chiral slab in 1987[4]. Using the method of separation of wave, S. He[5] found that a stratified chiral slab can act as a polarized-conversion transmission filter and as an antireflection filter. Based on Mie theory, Holzwarth gave a crude approximation of the scattering properties of chiral dielectric sphere[6]. Bohren[7-8] successively presented the analytical solution of chiral sphere and cylinder with plane wave utilizing decomposition. The detailed explanation was put forward subsequently by Worasaeate[9].

Because of the electromagnetic fields that radiate from waveguide horns and laser cavities are almost beamlike fields, it is essential for us to explore the scattering of chiral material with laser beams. M. Yokota[10] discussed the near-field scattering properties of chiral sphere illuminated by shaped beams applying the expansion of vector wave functions. Kim and Lee[11] investigated rigorously the scattering of a complex beam by a dielectric sphere. Owing to its non-diffracting and self-reconstructing properties, Bessel beam[12] have found wide applications in remote communication and atmospheric detection, etc.

The purpose of this paper is to study the scattering of a zero-order Bessel beam by a chiral sphere. The Bessel beam can be expressed as the superposition of SVWFs. The electromagnetic fields inside of the sphere are expanded in terms of the two special circular polarized waves. The unknown expansion coefficients for the internal field and the scattered field are determined by the boundary conditions on the surface of the chiral sphere. As numerical examples, the scattered fields of the Bessel beam are calculated and compared with those for an incident plane wave. It is found that the values are in good agreement. Based on the theoretical model, numerical results of the angular distribution of scattered intensity are obtained to show the influence of various beam parameters and chiral sphere parameters on the scattered field.

1 Theoretical formulation

1.1 Expansion of Bessel beam

Consider the Bessel beam located at (x_0,y_0,z_0) in the Cartesian coordinate system with its origin at the center of a chiral sphere as shown in Fig.1.

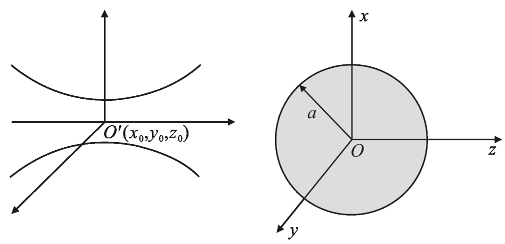

Fig.1 Geometry for light scattering of a Bessel beam by a chiral sphere

The polarized Bessel beam can be expressed in integral form as[13]:

$$\boldsymbol{E}_i(\boldsymbol{r})=E_0\int_0^{2\pi}\boldsymbol{e}_\upsilon(\alpha,\beta)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_0)\mathrm{d}\beta \qquad (1)$$

where E_0 is the peak amplitude; $\boldsymbol{e}_\upsilon(\alpha,\beta)$ indicates the unit polarization vectors; υ represents polarization directions of x and y; $\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})$ denotes a plane wave with wave vector $\boldsymbol{k}=(k,\alpha,\beta)$; parameter α is being called conical angle of Bessel beam and $\boldsymbol{r}_0=x_0\hat{x}+y_0\hat{y}+z_0\hat{z}$. For the x- and y-polarized Bessel beams, $\boldsymbol{e}_\upsilon(\alpha,\beta)$ has the following form, respectively:

$$\boldsymbol{e}_x(\alpha,\beta)=\sin\alpha\cos\beta\boldsymbol{e}_\gamma(\alpha,\beta)+\cos\alpha\cos\beta\boldsymbol{e}_\theta(\alpha,\beta)-\sin\beta\boldsymbol{e}_\phi(\alpha,\beta) \qquad (2)$$

$$\boldsymbol{e}_y(\alpha,\beta)=\sin\alpha\sin\beta\boldsymbol{e}_\gamma(\alpha,\beta)+\cos\alpha\sin\beta\boldsymbol{e}_\theta(\alpha,\beta)-\cos\beta\boldsymbol{e}_\phi(\alpha,\beta) \qquad (3)$$

The polarized plane wave can be represented in terms of SVWFs in spherical system whose origin is at the center of the sphere as[14-15]:

$$\boldsymbol{e}_v(\alpha,\beta)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}D_{mn}[p_{mn}'\boldsymbol{M}_{mn}^{(1)}(\boldsymbol{r},k_0)+q_{mn}'\boldsymbol{N}_{mn}^{(1)}(\boldsymbol{r},k_0)] \quad (4)$$

where

$$D_{mn}=\frac{(2n+1)(n-m)!}{4n(n+1)(n+m)!}$$

$$p_{mn}'=-4i^{n+1}\exp(-\mathrm{i}m\beta)\boldsymbol{e}_v(\alpha,\beta)\times[\pi_{mn}(\cos\alpha)\boldsymbol{e}_\theta(\alpha,\beta)-i\tau_{mn}(\cos\alpha)\boldsymbol{e}_\phi(\alpha,\beta)]$$

$$q_{mn}'=-4i^{n+1}\exp(-\mathrm{i}m\beta)\boldsymbol{e}_v(\alpha,\beta)\times[\tau_{mn}(\cos\alpha)\boldsymbol{e}_\theta(\alpha,\beta)-i\pi_{mn}(\cos\alpha)\boldsymbol{e}_\phi(\alpha,\beta)] \quad (5)$$

In Eq. (5),

$$\pi_{mn}(\cos\alpha)=mP_n^m(\cos\alpha)/\sin\alpha$$

$$\tau_{mn}(\cos\alpha)=\mathrm{d}P_n^m(\cos\alpha)/\mathrm{d}\alpha$$

Substituting Eq. (4) into Eq. (1) and performing integration over β, and utilizing integral localized approximation (ILA)[16], the expansion of the polarized Bessel beam in terms of SVWFs can be derived as:

$$\boldsymbol{E}^i(\boldsymbol{r})=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[p_{mn}^v\boldsymbol{M}_{mn}^{(1)}(\boldsymbol{r},k_0)+q_{mn}^v\boldsymbol{N}_{mn}^{(1)}(\boldsymbol{r},k_0)] \quad (6)$$

where P_{mn}^v and q_{mn}^v are the expansion coefficients, which is so-called beam shape coefficients(BSC):

$$\begin{Bmatrix}P_{mn}^v\\ q_{mn}^v\end{Bmatrix}=-4D_{mn}i^{n+1}\exp(\mathrm{i}kz_0\cos\alpha)\times\left[\cos\alpha\begin{Bmatrix}\pi_{mn}\\ \tau_{mn}\end{Bmatrix}I_+^v+\begin{Bmatrix}\tau_{mn}\\ \pi_{mn}\end{Bmatrix}I_-^v\right]$$

$$I_\pm^x=\pi\exp[\mathrm{i}(1-m)\phi_0]J_{1-m}(\rho_0)\pm\pi\exp[-\mathrm{i}(m+1)\phi_0]J_{-1-m}(\rho_0)$$

$$I_\pm^y=-\pi\mathrm{i}\exp[\mathrm{i}(1-m)\phi_0]J_{1-m}(\rho_0)\pm\pi\mathrm{i}\exp[-\mathrm{i}(m+1)\phi_0]\cdot J_{-1-m}(\rho_0)$$

$$\rho_0=k\sqrt{x_0^2+y_0^2}\sin\alpha$$

$$\phi_0=\arctan(y_0/x_0)+\pi/2 \quad (7)$$

Similarly, the magnetic field of polarized Bessel beam can be expressed as:

$$\boldsymbol{H}^i(\boldsymbol{r})=\frac{k_0}{\mathrm{i}\omega\mu_0}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[q_{mn}^v\boldsymbol{M}_{mn}^{(1)}(\boldsymbol{r},k_0)+p_{mn}^v\boldsymbol{N}_{mn}^{(1)}(\boldsymbol{r},k_0)] \quad (8)$$

1.2 Expansion of internal field of chiral sphere

For an arbitrary chiral sphere, the constitutive relations are[17]:

$$\boldsymbol{D}=\varepsilon\boldsymbol{E}+i\xi_c\boldsymbol{H} \qquad B=-i\xi_c\boldsymbol{E}+\mu\boldsymbol{H} \quad (9)$$

where ξ_c is a parameter describing the chirality properties of the medium.

The special wave fields in the interior of the chiral sphere are right and left circular polarized wave. Therefore, on the basis of the eigen wave number k_R and k_L, the internal electromagnetic field can be expanded in terms of (SVWFs)[18] as:

$$\boldsymbol{E}^I=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[A_{mn}\boldsymbol{M}_{mn}^{(1)}(\boldsymbol{r},k_R)+A_{mn}\boldsymbol{N}_{mn}^{(1)}(\boldsymbol{r},k_R)+B_{mn}\boldsymbol{M}_{mn}^{(1)}(\boldsymbol{r},k_L)-B_{mn}\boldsymbol{N}_{mn}^{(1)}(\boldsymbol{r},k_L)$$

$$\boldsymbol{H}^I=Q\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[A_{mn}\boldsymbol{N}_{mn}^{(1)}(\boldsymbol{r},k_R)+A_{mn}^{(1)}\boldsymbol{M}(\boldsymbol{r},k_R)+B_{mn}\boldsymbol{N}_{mn}^{(1)}(\boldsymbol{r},k_L)-B_{mn}\boldsymbol{M}_{mn}^{(1)}(\boldsymbol{r},k_L) \quad (10)$$

where

$$\left.\begin{matrix}k_R\\ k_L\end{matrix}\right\}=\omega[(\mu\omega)^{1/2}\pm\xi_c] \quad Q=(\mu\varepsilon)^{1/2}/(i\mu) \quad (11)$$

1.3 Expansion of scattered field

Based on the asymptotic expansion and its physical meaning of the first and second Hankel functions, the scattered fields of the chiral sphere illuminated by an zero-order Bessel beam can also be expanded in terms of SVWFs[18]:

$$\boldsymbol{E}^S=\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[A_{mn}^S\boldsymbol{M}_{mn}^{(3)}(r,k_0)+B_{mn}^S\boldsymbol{N}_{mn}^{(3)}(r,k_0)] \quad (12)$$

$$\boldsymbol{H}^S=\frac{k_0}{i\omega\mu_0}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}[A_{mn}^S\boldsymbol{N}_{mn}^{(3)}(r,k_0)+B_{mn}^S\boldsymbol{M}_{mn}^{(3)}(r,k_0)] \quad (13)$$

On the spherical boundary at $r=a$, the tangential components of the electromagnetic fields continue as:

$$\boldsymbol{E}^{I\theta}=\boldsymbol{E}^{i\theta}+\boldsymbol{E}^{s\theta} \quad \boldsymbol{H}^{I\phi}=\boldsymbol{H}^{i\phi}+\boldsymbol{H}^{s\phi} \quad (14)$$

Substituting Eqs. (6), (8), (10), (11), (12) and (13) into Eq. (14), and utilizing the orthogonality of the associated Legendre function, the scattering coefficients can be derived as:

$$A_{mn}^S=\frac{\psi_n(x_0)}{\xi_n(x_0)}\{[\eta_rD_n^{(1)}(x_L)-D_n^{(3)}(x_0)][(p_{mn}^v+\eta_rq_{mn}^v)D_n^{(1)}(x_R)-(q_{mn}^v+\eta_rp_{mn}^v)D_n^{(1)}(x_0)]+[\eta_rD_n^{(1)}(x_L)-D_n^{(3)}(x_0)]\cdot[(p_{mn}^v-\eta_rq_{mn}^v)D_n^{(1)}(x_L)+(q_{mn}^v-\eta_rp_{mn}^v)D_n^{(1)}(x_0)]\}/S_{mn} \quad (15)$$

$$B_{mn}^S=\frac{\psi_n(x_0)}{\xi_n(x_0)}\{[D_n^{(1)}(x_L)-\eta_rD_n^{(3)}(x_0)][(p_{mn}^v+\eta_rq_{mn}^v)D_n^{(1)}(x_R)-(q_{mn}^v+\eta_rp_{mn}^v)D_n^{(1)}(x_0)]-[D_n^{(1)}(x_L)-\eta_rD_n^{(3)}(x_0)]\cdot[(p_{mn}^v-\eta_rq_{mn}^v)D_n^{(1)}(x_L)+(q_{mn}^v-\eta_rp_{mn}^v)D_n^{(1)}(x_0)]\}/S_{mn} \quad (16)$$

where

$$S_{mn}=-[D_n^{(1)}(x_R)-\eta_rD_n^{(3)}(x_0)][\eta_rD_n^{(1)}(x_L)-D_n^{(3)}(x_0)]-[\eta_rD_n^{(1)}(x_R)-D_n^{(3)}(x_0)][D_n^{(1)}(x_L)-\eta_rD_n^{(3)}(x_0)] \quad (17)$$

With these solved coefficients, the field components of the scattered fields can be obtained by corresponding substitutions. Because of great use in radar detection and military purposes, we can calculate the scattered field as:

$$I_S=\lim_{r\to\infty} k^2r^2|E^s|^2/|E^i|^2 \tag{18}$$

where E^i indicates the initial incident electric field.

2 Numerical results

Here, we chose TM wave as the incident beam, which is polarized in $+x$ axis direction, and propagates in $+z$ axis direction.

To check the validity of the aforementioned theory, the results of zero-order Bessel beam degenerated to plane wave incident on a chiral sphere are compared with the numerical results of reference[18] as shown in Fig.2. It is found that the values are in good agreement.

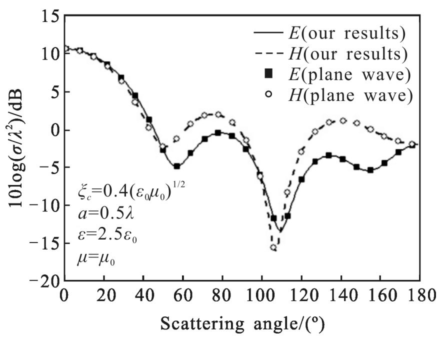

Fig.2 Angular distribution of scattered intensity of Bessel beam degenerated to plane wave compared with that of reference

In Fig.3 the angular distribution of scattered intensity of a chiral sphere illuminated by a zero-order Bessel beam with different conical angles are presented. It can be found that when on-axis Bessel beam incident on chiral sphere, the maximum scattered intensity appears in the direction or neighboring direction of conical angle, which is different from the case of plane wave and Gaussian beam (i.e. α=0°) which exist the maximum scattered intensity at θ=0°. This is determined by the structure characteristics of Bessel beam.

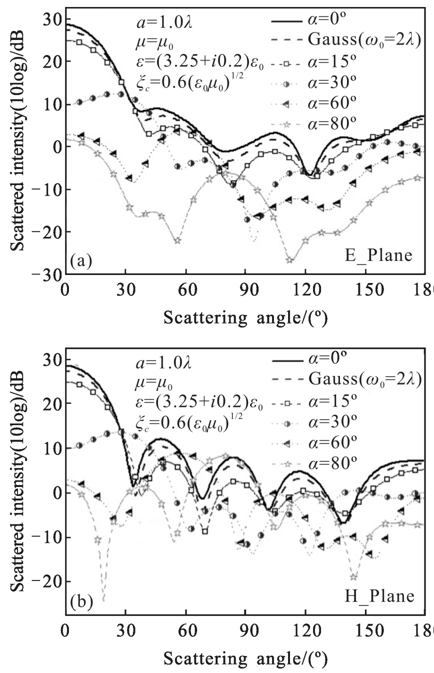

Fig.3 Effects of conical angle on the distribution of scattered intensity

As shown in Fig.4 is the effects of chiral parameter on the angular distribution of scattered intensity. It can be revealed that the backward scattering scattered intensity is more sensitive to the chiral parameter than that of forward. In E-plane and H-plane, with the increase of chiral parameter, the backward scattering scattered intensity increase before decrease.

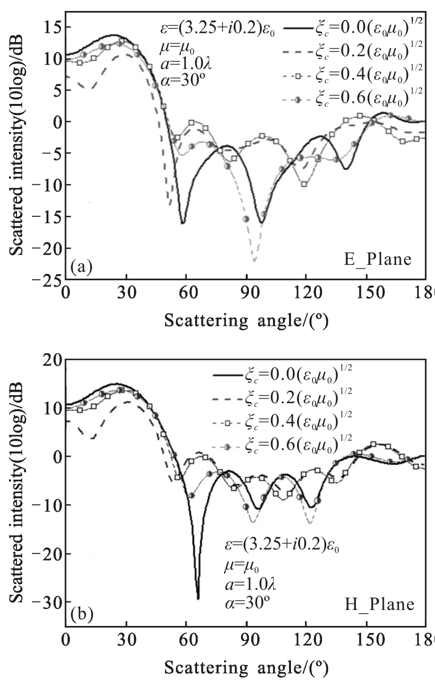

Fig.4 Effects of chiral parameter on the distribution of scattered intensity

The effects of the radius of the chiral sphere on the angular distribution of scattered intensity are calculated in Fig.5. It can be seen from the figure that the value of scattered intensity increases with the increase of the radius. The larger the size, the shaper the oscillation and the shorter the vibration period. However, all maximum scattered intensity appear in the direction or neighboring direction of conical angle $\alpha=30°$ only when $\alpha \geqslant 0.96\lambda$, otherwise there will be no extreme points. Because the central spot size of Bessel beam $\rho=2.405/k\sin\alpha$, (here $\rho \approx 0.77\lambda$) namely, the radius of the sphere must satisfy the critical condition $\alpha/\rho \approx 5/4$ for the existence of extreme point in the direction or neighboring direction of conical angle.

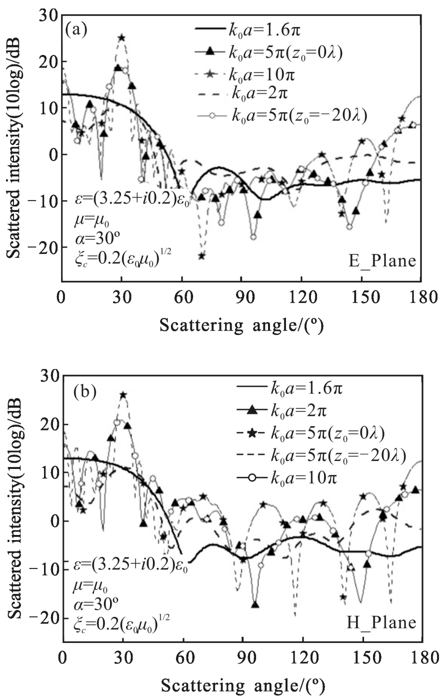

Fig.5 Effects of radius of chiral sphere on the distribution of scattered intensity

Meanwhile, to some extent, the small shift of beam waist center along z axis has little influence on scattered intensity because of the nondiffracting property in the limited distance.

In the above study, only TM wave is considered. In fact, the angular distribution of E-plane for incident TM wave is the same with that of H-plane for incident TE wave. Hence, the figures on the scattered intensity values for TE wave are not given in detail due to length restrictions.

3 Conclusion

In this paper, the scattering of a chiral sphere illuminated by a zero-order Bessel beam is investigated. For a Bessel beam with normal incidence, the angular distributions of scattered field are presented. The influence of conical angle, chiral parameter, size parameter and the bias of the beam center on the distribution of scattered intensity are numerically analyzed. This study provides an analytic exact model for interpretation of Bessel beam scattering for a chiral sphere. It provides an effective calibration for further research on the scattering of stratified chiral sphere, chiral bispheres and configuration of an aggregate of chiral spheres by a Bessel beam.

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