Harmonic wavelet analysis of TDLAS signals

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Abstract: Wavelength modulation spectroscopy (WMS) technique uses high-frequency modulation signal which is superimposed on the slow scanning signal. The signal is applied to the semiconductor laser drive current, so the laser frequency linear sweep as well as the modulation sweep. The absorption intensity of the lines is also modulated by the same frequency. 2f signal extraction of WMS was achieved by harmonic wavelet, which is the tool for data demodulation. The absorption spectroscopy data was obtained by scanning the beams from a tunable diode laser operating near the 7 185.6 cm^{-1} with the modulation frequency of 180 kHz. The 2f signal of high quality was obtained after harmonic wavelet processing for the data. The 2f signal's identification of its peak center is very clear, and data smoothness, anti-interference ability of the signal is better than method of digital lock-in.

Key words: harmonic wavelet; WMS; TDLAS

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谐波小波在 TDLAS 信号分析中的应用

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摘　要：波长调制谱技术(WMS)在慢扫描信号基础上叠加上高频的调制电流，并加到半导体激光器的驱动电流上，于是激光频率在线性扫描的同时受交流调制。频率调制的激光束在通过吸收气体以后，其吸收线的强度也受到相同频率调制。采用谐波小波方法作为数据解调的手段，对 WMS 技术产生的信号进行处理并实现了 2f 信号的提取。采用这种方法对波数为 7185.6 cm^{-1} 的水蒸气谱线进行测量实验，调制频率达到 180 kHz，对数据进行谐波小波处理后获得了高质量的 2f 信号，该 2f 信号的峰值中心辨识度高，其数据光滑度、信号的抗干扰能力方面均优于数字锁相方法。

关键词：谐波小波；　WMS；　TDLAS

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0 Introduction

Tunable diode laser absorption spectroscopy (TDLAS) has attracted widespread interest for advantages, such as small gas environmental impact, fast response, and high reliability and so on. The TDLAS technologies including direct absorption spectroscopy and wavelength modulation spectroscopy technology[1]. Direct absorption spectroscopy technique measures the change in laser light intensity generated in the measured environment, and detects the various parameters of the gas. Wavelength modulation spectroscopy (WMS) technique uses high-frequency modulation which avoids environment noise, and thus is widely used[2].

WMS technology uses high-frequency modulation signal. The harmonic signal contains measurement information. The conventional method uses the lock-in amplifier to detect the harmonic signals. But the signals detected by this method contain many impurities in the signal. So the result is not very accurate. This paper focuses on signal-processing techniques based on harmonic wavelet analyses of the wavelength modulated spectrum signal. As will be shown, harmonic wavelet has the potential to enable more accurate measurement results.

1 Wavelength modulation spectroscopy

Wavelength modulation spectroscopy technique uses high-frequency modulation signal which is superimposed on the slow scanning signal. The signal is applied to the semiconductor laser drive current, so the laser frequency linear sweep as well as the modulation sweep. The absorption intensity of the lines is also modulated by the same frequency. Passing by the absorption gas, a phase sensitive detection method is used for coherent demodulation, by which information about the absorption curve is obtained. Because the frequency band of the phase-sensitive detector system is typically compressed within a very narrow range of modulation frequencies near, the background noise thereby greatly suppressed. WMS techniques theory will be briefly introduced below[3-5].

The laser beam passes through a uniform gaseous medium. The equation of Beer-Lambert law is simple and straightforward as follow

$$\left(\frac{I}{I_0}\right)=\exp(-\mathbf{K}_v\cdot\mathbf{L}) \qquad (1)$$

Where I_0 and I_t are the incident laser and transmitted intensities; $\mathbf{K}_v$ (cm^{-1}) is the spectral absorption coefficient; $\mathbf{L}$ (cm) is the path length. The spectral absorption coefficient $\mathbf{K}_v$ can be expressed as

$$\mathbf{K}_v=\mathbf{P}\cdot\mathbf{X}_{H_2O}\cdot\mathbf{S}(\mathbf{T})\cdot\phi(\mathbf{v}) \qquad (2)$$

Where $\mathbf{P}$ (atm) is the total pressure; $\mathbf{X}_{H_2O}$ is the mole fraction of H_2O; $\mathbf{S}(\mathbf{T})\cdot(cm^2atm^{-1})$ and $\phi(\mathbf{v})$ (cm) are linestrength and lineshape.

The equations for the instantaneous optical frequency, $\mathbf{v}(t)$, and the output intensity, $I_0(t)$, can be expressed as:

$$\mathbf{v}(t)=\bar{\mathbf{v}}(t)+\alpha\cos(\mathbf{wt}) \qquad (3)$$

$$I_0=\bar{I}_0(t)[1+i_0(t)\cos(\mathbf{wt}+\psi)] \qquad (4)$$

Where $\bar{\mathbf{v}}(t)$ is the average frequency of the laser; $\bar{I}_0(t)$ is the average incident intensity; $\mathbf{w}$ is the modulation frequency; α is the modulation depth, i_0 (t) is the amplitude of light modulation; ψ is the phase shift. Note that, $\bar{\mathbf{v}}$ (t) and $\bar{I}_0$ (t) vary with modulation data. And usually, ψ is considered to π.

The incident light will be some absorbed when it passes through the absorbing gas. The transmission coefficient, τ, can be used to represent the relationship between incident laser and transmitted intensities

$$I_t=I_0(t)\cdot\tau(\bar{\mathbf{v}}+\alpha\cos(\mathbf{wt})) \qquad (5)$$

Since the absorption function τ follows the frequency modulation with no phase delay, it can be expanded as a periodic, even function of ωt in a Fourier cosine series

$$\tau(\bar{\mathbf{v}}+\alpha\cos(\mathbf{wt}))=\sum_{k=0}^{\infty}\mathbf{H}_k(\bar{\mathbf{v}},\alpha)\cdot\cos(\mathbf{kwt}) \qquad (6)$$

The harmonic functions $\mathbf{H}_k$ are described by

$$\mathbf{H}_0(\bar{v},\alpha)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\tau(\bar{v}+\alpha\cos\theta)d\theta \quad (7)$$

$$\mathbf{H}_k(\bar{v},\alpha)=\frac{1}{\pi}\int_{-\pi}^{\pi}\tau(\bar{v}+\alpha\cos\theta)\cos k\theta d\theta \quad (8)$$

Substitution of Eq. (6) into Eq. (5) yields the signal of going into lock-in amplifier. This signal is multiplied by a sinusoidal signal of twice the frequency of a modulated signal, and low-pass filtered the result to get the second harmonic signal. The gain ratio of the signal and photodetector is described by

$$\frac{S_{2f}(\bar{v})}{G\bar{I}_0(\bar{v})}=\frac{1}{2}\left[\mathbf{H}_2+\frac{\bar{i}_0(\bar{v})}{2}(\mathbf{H}_1+\mathbf{H}_3)\right] \quad (9)$$

Since the Eq.(9) fulfills the Beer-Lambert law, τ can be expressed as

$$\tau(\mathbf{v})=\exp(-\mathbf{S}(\mathbf{T})\cdot\mathbf{P}_i\cdot\mathbf{L}\cdot\phi(\mathbf{v})) \quad (10)$$

Substitution of Eq.(10) into Eq.(9) yields the following

$$\mathbf{H}_k(\bar{v},\alpha)=$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\exp[-\mathbf{P}_i\cdot\mathbf{L}\cdot\sum_j(\mathbf{S}_j(\mathbf{T})\cdot\phi_j(\bar{v}+\alpha\cos\theta))]\cos k\theta d\theta \quad (11)$$

When the gas absorption of incident laser intensity is less than 0.1, the transmission coefficient, τ, fulfils the following

$$\tau(\mathbf{v})\approx 1-(\mathbf{S}(\mathbf{T})\cdot\mathbf{P}_i\cdot\mathbf{L}\cdot\phi(\mathbf{v})) \quad (12)$$

Substitution of Eq.(12) into Eq.(11) yields the harmonic equation

$$\mathbf{H}_k(\bar{v},\alpha)=\frac{\mathbf{S}(\mathbf{T})\cdot\mathbf{P}_i\cdot\mathbf{L}}{\pi}\int_{-\pi}^{\pi}\phi(\bar{v}+\alpha\cos\theta)\cos k\theta d\theta \quad (13)$$

This paper focuses on the analyses of the second harmonic signal. First because the signal is an even function, the profile is symmetric, and the peak lies at the center position. Second, the second harmonic signal is strongest in the even harmonics signals. And the signal is abbreviated as WMS-2f.

2 Harmonic wavelet techniques

In recent years, the harmonic wavelet have attracted widespread concern in many areas such as dynamic model of nonlinear partial differential equations, biomedical signal processing, pattern recognition, especially in the field of noise and vibration[6]. A lot of researchers have successfully applied harmonic wavelet. The wavelet is a function which meets the agreed conditions. If a wavelet fulfils absolute spectrum characteristics of the "box-shaped", the wavelet will be ideal. Based on this idea, Newland constructed the wavelet from the frequency domain Having the real even function $\mathbf{h}_e(t)$ and real odd function $\mathbf{h}_o(t)$, their Fourier transforms are expressed as[7-9]:

$$\hat{\mathbf{h}}_e(\omega)=\begin{cases}1/4\pi & \omega\in[-4\pi,-2\pi]\cup[2\pi,4\pi]\\ 0 & \text{others}\end{cases} \quad (14)$$

$$\hat{\mathbf{h}}_o(\omega)=\begin{cases}i/4\pi & \omega\in[-4\pi,-2\pi]\\ -i/4\pi & \omega\in[2\pi,4\pi]\\ 0 & \text{others}\end{cases} \quad (15)$$

The combination of Eq.(14) and Eq.(15) is expressed as

$$\hat{\mathbf{h}}(\omega)=\hat{\mathbf{h}}_e(\omega)+i\hat{\mathbf{h}}_o(\omega)=\begin{cases}1/2\pi & \omega\in[2\pi,4\pi]\\ 0 & \text{others}\end{cases} \quad (16)$$

Making the inverse Fourier transform with Eq.(14) and Eq. (15), their time-domain expression is expressed as

$$\mathbf{h}_e(t)=\int_{-\infty}^{\infty}\hat{\mathbf{h}}_e(\omega)\exp(i\omega t)d\omega=(\cos 4\pi t-\cos 2\pi t)/2\pi t \quad (17)$$

$$\mathbf{h}_o(t)=\int_{-\infty}^{\infty}\hat{\mathbf{h}}_o(\omega)\exp(i\omega t)d\omega=(\sin 4\pi t-\sin 2\pi t)/2\pi t \quad (18)$$

The combination of Eq.(17) and Eq.(18) is called harmonic wavelet function

$$\mathbf{h}(t)=\mathbf{h}_e(t)+i\mathbf{h}_o(t)=[\exp(i4\pi t)-\exp(i2\pi t)]/i2\pi t \quad (19)$$

Substitution of $(2^j t-\mathbf{k})$ into t yields the following

$$\mathbf{h}(2^j t-\mathbf{k})=[\exp(i4\pi t(2^j t-\mathbf{k}))-\exp(i2\pi(2^j t-\mathbf{k}))]/i2\pi(2^j t-\mathbf{k}) \quad (20)$$

Where $j,\mathbf{k}\in\mathbf{Z}$. The shape of the wavelet has not changed, but it is compressed about 2^j times in the scale direction, and its position has a shift about $\mathbf{k}$ units.

Setting the dilation and translation of the $\mathbf{h}(t)$ as the family of functions $\mathbf{w}(t)$

$$\omega(t)=\mathbf{h}(2^j t-\mathbf{k}) \quad (j,\mathbf{k}\in\mathbf{Z}) \quad (21)$$

The Fourier transform of **w**(t) is expressed as

$$\hat{\mathbf{w}}(\omega)=\int_{-\infty}^{\infty}\mathbf{w}(t)e^{-i\omega t}dt=\int_{-\infty}^{\infty}\mathbf{h}(2^{j}t-\mathbf{k})e^{-i\omega t}dt \qquad (22)$$

Assuming $\xi=2^{j}t-\mathbf{k}$ and $dt=d\xi/2^{j}$, it can yield the following

$$\hat{\mathbf{w}}(\omega)=\frac{e^{-i\omega t}\hat{\mathbf{h}}(\omega/2^{j})}{2^{j}} \qquad (23)$$

Equation (23) shows the band of harmonic wavelet increases 2^{j} times and the amplitude reduces 2^{j} times as the wavelet layer j increases. Based on wavelet theory, the harmonic wavelet **h** (t) and Eq.(20) constitute the orthonormal basis of $\mathbf{L}^2$ (**R**) space. Signal can be orthogonal decomposed to the independent space and can be decom posed its frequency components to corresponding bands by setting the harmonic wavelet as the basis function family. So, the harmonic wavelet constitudes the orthonormal basis of $\mathbf{L}^2$ (**R**) space and any real signal **x** (t) can be expressed into linear combination of harmonic wavelet

$$\mathbf{x}(t)=\sum_{j=-\infty}^{+\infty}\sum_{k=-\infty}^{+\infty}\mathbf{a}_{j,k}\mathbf{h}(2^{j}t-\mathbf{k}) \qquad (24)$$

Equation (24) shows the harm onic wavelet expansion of the signal. Since the harmonic wavelet is complex wavelet, so the expansion coefficients $\mathbf{a}_{j,k}$ is also complex, which can be expressed as

$$\mathbf{a}_{j,k}=\langle \mathbf{x}(t),\mathbf{h}(2^{j}t-\mathbf{k})\rangle \quad (j,\mathbf{k}\in\mathbf{Z}) \qquad (25)$$

3 Experimental methods

This experiment measured water vapor spectra by using a DFB laser. The wave number of measured spectrum is 7 185.6 cm^{-1}, the ambient temperature is 273 K, the length of the gas chamber is 50 cm, the pressure is 10^5 Pa, and the modulation frequency of the laser is 180 kHz. Experimental schematic diagram is shown in Fig.1. The computer generates a modulated signal, which is transmitted by data acquisition card. After the modulation signal goes into the laser control, DFB laser is modulated by the corresponding current. The laser passes through the fiber which connects the collima tor lens. Then, the laser passes through the gas chamber and the gas absorbs the laser. The transmitted laser is focused by the collimator lens, and which is passed through fiber. The photodiode transport the laser to electrical signals. The data acquisition card catches the signal and then brings it to the computer. The computer processes the signal by harmonic wavelet techniques and the 2f signal is obtained.

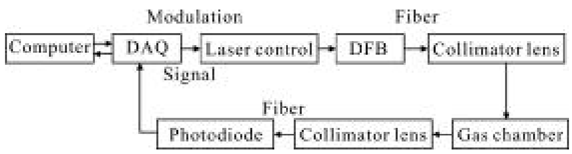

Fig.1 Experimental schematic

4 Harmonic wavelet analysis of W M S signal

Figure 2 shows representative data for the case where laser modulation was applied. The intensity signal consists of ramp signal with modulation signal. The laser was absorbed by the gas chamber, so the data in the middle has an obvious reduction in strength. This signal contains a harmonic signal, and we can detect its harmonic signal to obtain the desired gas parameters. This paper focuses on abstracting harmonic signal by harmonic wavelet techniques, and the techniques will be discussed as follows[10].

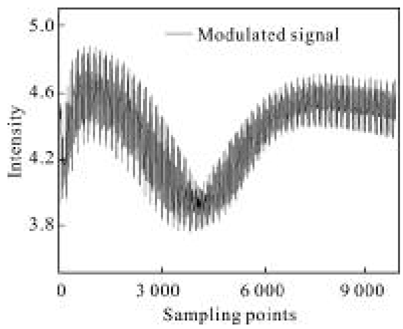

Fig.2 W MS signal

Wave let coefficients are the same as Fourier coefficient, which has the same conjugate law as follows

$$\mathbf{a}_{N-r}=\mathbf{a}_{r} \quad r=1,2,\cdots,\mathbf{N}/2 \qquad (26)$$

Similar to the Fourier coefficients, except for $\mathbf{a}_0$ and $\mathbf{a}_{N/2}$, the rest are plural. $\mathbf{a}_r$ is in accordance with the different frequency bands. The jth layer' wavelet has 2^j wavelet coefficients $\mathbf{a}_{2^j+k}$ ($\mathbf{k}$=0,1,…,2^j-1), which exists a corresponding relationship with the Fourier coefficients, the relationship between the two of formula can be expressed as

$$\mathbf{F}_k=\frac{1}{2^j}\sum_{m=0}^{2^j-1}\mathbf{a}_{2^j+m}\exp\left(\frac{-i2\pi\mathbf{km}}{2^j}\right)(\mathbf{k}=0,1,\ldots,2^j-1) \quad (27)$$

Wavelet coefficients are complex sequence, which include messages both time-domain and frequency-domain. A certain period of $\mathbf{a}_r$ is corresponding to certain layer of wavelet decomposition. The relationship between decomposition layer and the band is that the band decrease by 1/2 from high frequency to low frequency when the decomposition layer has decreased. It can get the half-top frequency of the signal when j is max.

Sampling results obtained by using the harmonic wavelet method is shown in Fig.3. The signal is better to keep the original appearance of the second harmonic signal. The middle of the profile is the main peak of the second harmonic signal. Flanks are on both sides of the main peak. Based on this signal, the 2f signal was obtained by harmonic wavelet techniques, which is shown in Fig.4.

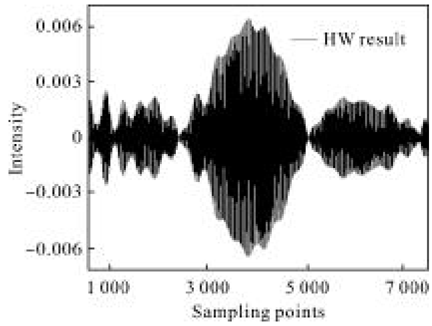

Fig.3 Signal of two times frequency

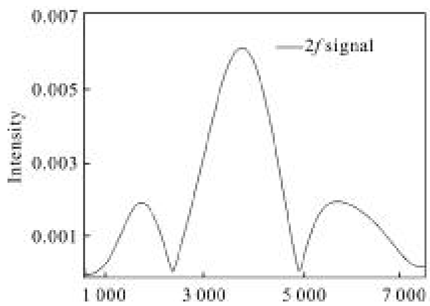

Fig.4 2f signal

In order to compare the pros and cons of the 2f signal, the 2f/1f signal is used to be explained. The 2f/1f signal is mainly used for temperature measurement as the general approach uses digital phase-locked to process the signal. In this paper, the 2f/1f signal of harmonic wavelet method with digital phase-locked signal to define the pros and cons of the two methods are compared. First, the data obtained by the two methods is subjected to the drawing shown in Fig.5. You can see the differences of two signals. Next, the amplifications of two signal's details show several differences. First, as the peak of digital phase-locked signal is rough, the signal of harmonic wavelet is smooth, and the peaks are different. Second, the peak centers are different. The above two points are shown in Fig.6.

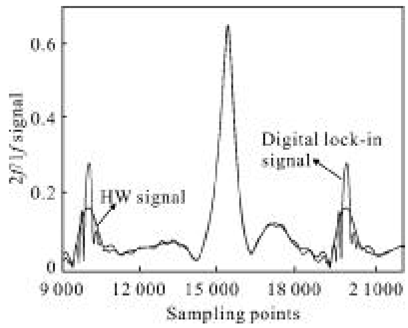

Fig.5 Two kinds of 2f/1f signals

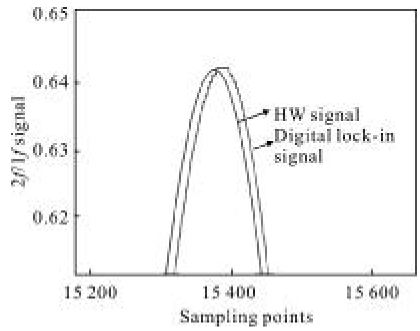

Fig.6 Comparisons with peaks

Third, as can be seen under the waveform segments, the harmonic wavelet signal is smoother than the digital-locked signal; there is an interference signal, as shown in Fig.7. Fourth, in the initial portion of the signal, the harmonic wavelet signal is lower amplitude, relatively smooth; digital lock signal amplitude is large, smoothness almost; the harmonic wavelet signals has less interference, as Fig.8 shows.

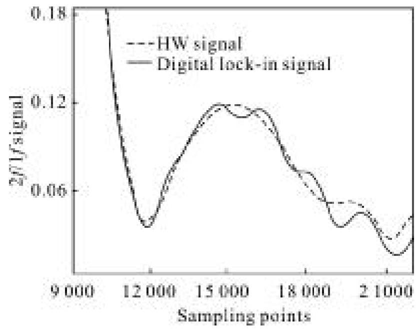

Fig.7 Comparison with wings of both signals

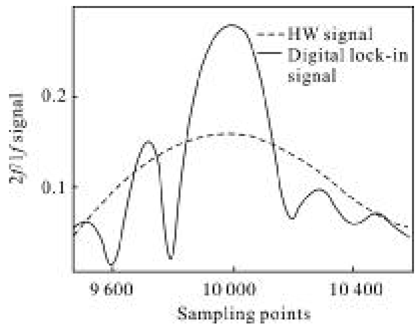

Fig.8 Comparison with initial portions of both signals

5 Conclusions

Harmonic wavelet has complete "box-shaped" spectrum characteristics, which is able to generate the spectral windows of the particular frequency range. Using the harmonic wavelet techniques as well as the spectral characteristics of the "box-shaped", 2f signal of the data is successfully extracted by the harmonic wavelet decomposition of the modulation spectrum. Compared the 2f signal extracted by harmonic wavelet with the signal obtained by the digital phase-locked manner in the water vapor spectrum measurement test, the 2f signal has advantages such as less interference, good smoothness, easy to find the peak of the waveform, etc. In conclusion, the usage of harmonic wavelet to analysis the TDLAS modulation signal can improve the quality of the 2f signal, which provides an ideal data base for wavelength modulation spectroscopy techniques.

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