Priority and Particle Physics: Ontic Structural Realism as a Fundamentality Thesis

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1 Introduction

Talk of whether there exists a fundamental basis to the world pervades contemporary metaphysics. Increasingly prominent in the metaphysics literature are arguments as to whether the world resolves itself into a set of fundamental objects or whether, on the contrary, each object is composed of more fundamental objects *ad infinitum*.[1] But there is a school of thought that takes it that, whatever its outcome, this whole debate gets off on the wrong foot, since this school denies that fundamentality ought to be construed in terms of *objects* at all. According to its proponents, even if there should exist a set of 'fundamental objects' – in the sense of a set of entities privileged with respect to other members *of the category of objects*, such as the fundamental particles or fields described in our best physics – it is not the case that such objects would qualify as truly fundamental. Rather, in their view, the most basic metaphysical level of the world is constituted solely by the *structures* that our best physics theories describe.[2]

The thesis just outlined is that of *radical ontic structuralism*. Indeed, ontic structuralism in general is often explicitly characterized in fundamentality terms, and in particular as a thesis concerning *ontological priority*. In his survey article for the *Stanford Encyclopedia of Philosophy*, for example, James Ladyman characterizes ontic structuralism (often denoted 'OSR') on its 'broadest construal' as 'any form of structural realism based on an ontological or metaphysical thesis that inflates the ontological priority of structure and relations.'[3] It is thus a novel priority claim concerning the status that structures enjoy relative to objects that we may take to be most definitive of structuralist metaphysics.[4]

[1] See eg. Cameron [2008]; Schaffer [2003].

[2] French and Ladyman [2003], p. 27.

[3] Ladyman [2007]. Likewise, Esfeld and Lam state that 'the issue of the relationship between objects and relations within OSR has mainly been addressed in the literature in terms of ontological primacy' (Esfeld and Lam [2010], p. 145).

[4] In what follows, by 'objects' I will mean the bearers of determinate physical properties – the paradigmatic examples in the particle physics context being of course particles and fields. However,

Ladyman himself is a structuralist of the radical stripe. But since structuralists hold that mainstream metaphysics is highly 'object oriented', they will hold that a position that fell short of imparting a superior status to structure, but simply raised it to the status of objects, would on this construal qualify as just as legitimate a form of ontic structuralism as one that held the stronger 'superiority' view. There are thus two discernible structuralist positions that may be defined with regard to priority. There is, on the one hand, the *radical* position in which structures enjoy an unreciprocated, one-way priority over objects – a situation which they claim sanctions the *elimination* of objects *qua* elements of fundamental, hence properly philosophical, ontology.[5] There is, on the other hand, the so-called *moderate* position in which the two categories are taken to be 'ontologically on a par' with one another, so that – to the extent that it makes sense to speak of 'priority' at all – these priority relations are reciprocated.[6] Thus while the radical view takes it that 'relational structure is more ontologically fundamental than objects', according to the moderate view objects and structures are 'on the same ontological footing, being given "at once" in the sense that they are mutually ontologically dependent on each other.'[7] The radical structuralist therefore holds that the fundamental basis of the world consists exclusively of structures, while the moderate structuralist sees an indispensible role for objects in that basis as well.

It is the claims regarding an elevated priority status for structures that is most distinctive of OSR. But how exactly the priority alluded is to be understood has not received a great deal of attention in the structuralist literature – something that is rather anomalous given its evident centrality to the doctrine as a whole.[8] Moreover, on those occasions in which efforts have been made to spell out priority, they are often ambiguous, with different and inequivalent characterizations being used interchangeably. Hawley for example has thereby objected that 'the ways in which structures are somehow prior to objects' – and thus what it means for structure to be ontologically fundamental and for objects by contrast to fail to be 'self-subsistent' – have not been clearly articulated.[9] If structuralism is to present itself as a viable alternative to the predominant 'object oriented' metaphysics and its attendant fundamentality assumptions, precisifying priority ought therefore to be put at the top of its agenda.

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exactly how structuralists ought to define structure in general has proved to be a much more controversial matter. In what follows, I will appeal only to two specific examples that are held up as paradigms of structures in physics – namely entanglement relations and group-theoretic structures – and will not offer any general definition. To be clear, however, these structures are to be understood as interpreted, and not merely in terms of what is preserved under isomorphisms.

[5] See e.g. Ladyman [1998], French and Ladyman [2003a]. For statements on how the secondary status of objects prompts their elimination, see French [2010]; Ladyman and Ross [2007], Chapter 3.

[6] See e.g. Esfeld and Lam [2008]; Esfeld [2004]. I appreciate that it is awkward to speak of reciprocated or 'symmetric' priority when the word 'priority' connotes asymmetry. The use of this term may be put down to a widespread bias against symmetric dependence relations – a bias I reject – but I think it best to use the extant terminology nonetheless.

[7] Esfeld and Lam [2008]; see also their [2010], p. 146.

[8] A recent and notable exception to this is French [2010], which lays out some of the options but remains uncommitted on how priority is to be understood; nor is Fine's approach considered, though I will argue that it is a natural candidate. Linnebo [2008] also discusses priority, though in the context of mathematical structuralism.

[9] Hawley [2008].

The purpose of this paper is to do exactly that. I will begin by considering how the notion of priority may best be articulated, given that it is to play a definitive role within OSR. With that in place, I will then examine whether any of structuralism's priority claims can be regarded as supported. Since structuralism's most revisionary claims regard what is fundamental, I will focus on two case studies that concern fundamental particles (each of which receives considerable attention in the structuralist literature): first, the case of entangled quantum particles, and second, the group-theoretic conception of elementary particles. In each case, I will consider whether either of structuralism's radical and moderate versions represents a feasible interpreration of these cases, and from there, whether either represents a viable alternative to the standard 'object oriented' vision of what is ontologically fundamental. I therefore begin by thinking about how exactly to conceive of priority in the context set by ontic structualism.

2 The Right Priority Relation for Structuralism: Supervenience or Dependence?

Since priority lies at the heart of ontic structuralism, the priority relation most appropriate to it will ideally be reflective of structuralist metaphysics as a whole. Structuralist philosophy of physics is motivated by no one single fact or thesis but rather by a whole constellation of ideas, and as such it incorporates a variety of metaphysical concerns.[10] In spite of the breadth of these considerations, however, two broad metaphysical themes may be traced throughout the discussions surrounding them. There is, of course, a principal claim concerning the *ontological priority* relations that exist between objects and structures, and it is the issue of how exactly structuralists should understand this claim that we will try to come to grips with. But to do that, we must note that there is a second core theme to structuralism – namely, that objects are to be *reconceptualized* in terms that make ineliminable reference to the dynamical structures in which those objects participate.[11] While I will not attempt here to spell out explicitly what this 'reconceptualization' amounts to, I *will* contemplate how we ought to understand priority, given that both of these claims constitute major themes in ontic structuralism.

So how should structuralists understand priority? No doubt the most familiar way in which contemporary philosophers express ideas about ontological privilege is by invoking *supervenience* relations, and as one might expect sometimes structuralists themselves likewise frame things in exactly these terms. Ladyman and Ross, for example, write that

> OSR is the view that the world has an objective modal structure that is ontologically fundamental, in the sense of *not supervening* on the intrinsic

[10] See Ladyman [2007] for discussion.

[11] French and Ladyman, for example, write that they 'are not "anti-ontology" in the sense of urging a move away from electrons, elementary particles etc. [...] rather, [they] urge the reconceptualization of electrons, elementary particles and so forth in structural instead of individualistic terms.' (French and Ladyman [2003], p. 37). See also Bokulich [2010], p. xiv.

properties of a set of individuals.[12]

So it is clear that sometimes structuralists vouch for supervenience as the right way to cash out their fundamentality claims. On the other hand, and more often perhaps, structuralists allude to relations of *dependence* to articulate what they mean by priority. French, for example,

> take[s] it that a core feature of OSR is the claim that putative 'objects' are *dependent* in some manner upon the relevant relations (and hence these putative objects can be reconceptualized as mere nodes in the relevant structure).[13]

(Note another allusion to 'reconceptualization' here.) So given that both of these relations are evidently in play in the literature, there is the question of which, if either, is the better relation with which to express priority. And what we should definitely be clear on, in any case, is that the two cannot simply be used interchangeably: not only are supervenience and dependence conceptually quite distinct – the former being most naturally viewed as a sufficient condition for the existence of the non-fundamental and the latter a (type of) necessary condition – one may moreover argue that the two fail to be co-extensive with one another.[14]

So let us contemplate for a moment how the two relations compare.[15] I have already mentioned that supervenience is by now very familiar, and it is also deemed to be amply clear. But the idea that supervenience can be used to cash out priority has of course also had its critics, and these often focus on the idea that supervenience is not at all *explanatory* of any relationship between the sub- and supervenient relata; it is often regarded as at best an indication that it is worth *looking* for an explanation of the evident connection between them, while not at all explanatory of it.[16] Now, that need not be a criticism in itself, of course – one can in fact imagine situations in which it might be positively advantageous, such as when it is believed that there is no such explanation to be had.[17] But insofar as it posits a relation between things without giving us any indication of why it holds, it may be difficult to stomach the idea that supervenience gives us the last word on fundamentality.

On the other hand, it is fair to say that the predicament with dependence has been almost the diametric opposite: while dependence is taken to have some deep connection with explanation, it has frequently not been seen as sufficiently clear. As evidence for this, one could cite the fact that Lewis – in one of his many papers attempting to define

[12] Ladyman and Ross [2007], p. 130; my italics.

[13] French [2010], p. 104; my italics.

[14] A full discussion of the distinctness of the two relations may be found in Yoshimi [2007].

[15] To avoid potential confusion, I should point out that although it is common to lay down *a priori* restrictions on the logical form of either relation, I am not going to do so here, and in particular I am not going to demand that dependence must be asymmetric (Fine likewise permits symmetric dependence; see Fine [1995a], Section III). Thus as far as I am concerned, either relation could be used to express both the radical and the moderate position.

[16] See Kim [1993], especially pages 146, 148 and 156, and works cited therein.

[17] Emergentists of various stripes believe that there are such cases.

intrinsicality – commented that 'if we had a clear enough understanding of "soley in virtue of" [a cognate of "dependent upon"], we would need no further definition of "intrinsic"'.[18] But it seems to be the case that philosophers have grown more sanguine about dependence in recent years, and that it is increasingly regarded as something that we do have a good enough working grasp of. (This is indeed seemingly evidenced by the number of new dependence-based analyses of intrinsicality that are now on the market.)[19] Whatever it is that accounts for this change, if there *are* good reasons to be more sanguine, then it appears that dependence – unlike supervenience – gets 'a tick in both boxes' insofar as it is regarded as both sufficiently clear and appropriately explanatory, and we might want to endorse it on these grounds.

Whatever virtues either relation might have in the abstract, however, the principled question we should be asking in this context is which is the right relation *for structuralism*; and a little more thought will deliver that it is almost certainly *dependence* that is the more apt of the two. Recall that I pressed at the start of this section that structuralism is more than just a priority claim: it is also an invocation for us to *reconceptualize* object-based ontology in some way that integrates structures. It is in that sense revisionary with respect to the *nature* of objects as well as to their supposed priority, and both of these are part of the structuralist deal. But if we don't want the priority claim and the proposed reconceptualization to constitute independent theses, *then supervenience is not what we are looking for* as a means of expressing priority. Half the *point* of invoking supervenience is, after all, to liberate priority attributions from specific claims regarding the nature of the relata – which is why people can claim that categories so wildly distinct as the modal, the moral and the mental all supervene on the natural and have all these claims amount to basically the same thing.[20] But if, on the other hand, we want the two key metaphysical aspects of structuralism to come as a package – and hence for the priority claims to be in some sense 'tied to the nature of the dependent entity', a nature which structuralism itself will elucidate – then there is an obvious candidate for conceptualizing priority in structuralism. It is, after all, the idea that priority attributions ought to issue from the nature of the dependent entity that is distinctive of Fine's analysis of ontological dependence.

What I think we ought to do at this point, then, is put the talk of supervenience behind us and take a closer look at what exactly is involved in the account of dependence just alluded to, since such an account has the resources to bind together structuralism's two core ontological themes. With that in place, I will address how structuralists might explicitly invoke Fine's account to promote into real *demonstrations* what it seems fair to say have, in the absence of a clear and unambiguous understanding of what is meant by 'priority', largely just been plausibility arguments for the priority of structure over objects (and, where appropriate, vice versa).[21] So with that, let me now expand a little

[18] Lewis [2001], p. 384. See also Lewis [1983], p. 29.

[19] On the defence, see Jenkins [forthcoming]. On intrinsicality, see Rosen [2010], Witmer *et al.* [2005].

[20] That is, supervenience is seen as 'entirely topic neutral' (Kim [1993], p. 132).

[21] For example, and with regard to the latter point, the extant major argument for the moderate over the radical position hinges on the idea that 'relations require relata, that is, things which stand in the relations' (see e.g. Esfeld [2004], Esfeld and Lam [2009]). However, the nature of this requirement is

more on ontological dependence and, in particular, Fine's account of it.

3 Introducing Ontological Dependence

If we go with our first intuitions on the matter, we will want to say that an entity x is ontologically dependent upon an entity y just if *x exists only if y exists*. It additionally seems clear that this should hold with metaphysical necessity (for if there were a world in which x could after all exist without y we would presumably want to retract that x was dependent upon it). To a first approximation, then, we may say that an entity x is ontologically dependent upon an entity y just if, necessarily, x exists only if y does, where the force of the necessity is metaphysical.[22]

Intuitive though it may seem at a first pass, the starting point for Fine's analysis is that such a simple and purely modal construal of ontological dependence in fact turns out to be hopeless; it is simply not fine-grained enough to exclude some patently spurious cases.[23] As already indicated, Fine proposes in its place an analysis in which

> The necessity of the conditional 'x exists only if y does' should be appropriately tied to the nature of the dependent item x,[24]

from which it follows that such conditionals are 'not necessary simpliciter' but are additionally 'tied' in some sense to the nature of their relata. Now, the 'natures' involved in this 'tying' are to be understood in terms that are at least close to what has traditionally been associated with *essence* – for Fine's analysis is indeed explicitly essentialist. This raises an immediate concern, however, for we probably do have to grant to French that 'essentialism has not typically been viewed all that favourably in the context of modern physics.'[25] Hence we face the worry that, by invoking Fine's theory in discussions of structuralism in physics, we are attempting to shoe-horn a very contemporary ontology into a hallowed framework that was not designed to accommodate it. It is perhaps partly for this reason that structuralist discussions of priority have tended to focus on the notion of *identity* without any mention of essence – the former being deemed to connote something altogether more innocuous than the latter.[26]

However, *I* would argue that keeping the discussion confined to identity considerations to the exclusion of this thing called 'essence' is neither realistic nor necessary.

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not spelled out, and seems perfectly consistent with relations necessitating the existence of relata but as an *ontologically secondary* phenomenon. It is after all presumably this circumstance that French and Ladyman have in mind when they invoke Cassirer's conception of objects as the 'points of intersection' of relations (French and Ladyman [2003a].)

[22] See Fine [1995a] for discussion of and references to modal accounts.

[23] See however Wildman [unpublished].

[24] Fine [1995a], p. 272.

[25] French [2010], p. 106. This is in part because essentialist doctrines in the philosophy of science have tended to be tied up in the language of dispositions, and hence of cause and effect – concepts that are problematic to say the least in the context of fundamental physics.

[26] Thus while French's discussion of dependence in structuralism cites the notion of 'essential dependence', he has little more to say than that 'the essence of an object is closely bound up with its identity', and thus effectively reverts to discussing identity dependence (French [2010], p. 101).

Talk of ontological dependence is after all metaphysical talk, and thus the fact that *physics* never speaks explicitly of essence need not constitute any reason to shun it. Furthermore, and as Lowe has argued, talk of the 'identity dependence' that structuralists seem more comfortable with apparently still requires us to ultimately invoke something close to essence if we are to put constraints (as we must) on what sort of properties should feature in the analysis of the identity of an entity.[27] But more positively, we should, I think, be open to the possibility that essence too may be relatively innocuous in the sorts of structuralist contexts that we will be working within here. It may turn out, in fact, that what Fine has in mind by 'essence' is just the sort of thing that we're used to dealing with *all the time* in particle physics. But before I can assert any of that, of course, I need to clarify what commitment to essence involves, and to do so I will defer to a recent and useful survey article on contemporary approaches to ontological dependence. Here, Correia writes that

> The conception of essence Fine has in mind is a traditional conception according to which what is essential to an object pertains to *what the object is*, or *defines the object* (at least in part).[28]

This we will take as our starting point. Now, perhaps the two conceptions of essence just alluded to are intended in a somewhat technical sense in which they come out as synonymous, but it doesn't seem so, and at a first pass they do seem to invoke different things – both of which we intuitively want to be brought into the discussion. If we understand the first characterization in the statement as meaning 'pertaining to what the object is, *as opposed to what it is not*', we seem to have invoked matters of *individuation* and hence of properties that might confer distinctness or individuality – something that philosophers of physics are routinely happy to discuss. But since definitions need not be individuating, this needn't be the same thing as *defining* the object.[29] Indeed, in particle physics we usually take defining an object to be a matter of listing off the determinate, fundamental, state-independent properties common to all members of the particle's *kind*, all of which are indistinguishable with respect to these properties. These properties *physicists* usually call 'intrinsic' properties, but – as Ladyman and Ross remind us – they correspond much more naturally to what *philosophers* would call 'essential' properties (partly because such properties are always, among other things, permanent and observer-independent).[30]

So given that preliminary reassurance, and in keeping with Correia's rough-and-ready criterion, we can say, to a first approximation, that the properties we should take to feature in a particle's essence are

- The fundamental, determinate, state-independent properties that serve to define its kind; and

[27] Lowe [1998], p. 148.

[28] Correia [2008], p. 1018; italics in original. Note that it is better to speak of 'entities' than 'objects' here; more on this below.

[29] See Fine *op. cit.*, p. 275. Whether a definition is individuating or not will depend on its relationship to a pre-agreed principle of individuation. Compare, say, the definition of a set in terms of its members, and the definition of an electron in terms of its having mass m, charge e and spin s.

[30] Ladyman and Ross [2007], p. 134.

- (Some of) the properties involved in conferring distinctness from other members of its kind.

I will leave the latter very vague, as I will not go into much depth about how essence and individuation fit together; all I am going to do is invoke a couple of theorems of Fine's as and when we need them. But hopefully I have done enough to dispel at least some of the initial misgivings about what essence might involve in the fundamental physics context, and with that, let me introduce Fine's system.

4 Fine's System

The first thing to be clear on is that Fine's analysis of ontological dependence is intended as completely general in scope, incorporating dependencies between (among other things) objects, properties, numbers, sets, persons, and states of things at a time. Thus it is unfortunate – and especially so in the current context – that Fine often uses the term 'object' to refer to the relata of dependence relations when the more generic 'entity' would be a better term. So while to preserve ease of reading I will not replace 'object' with 'entity' in the ensuing quotes from Fine, the reader should be clear that Fine's theory is to apply to (possible) entities of *any* category and is not restricted to the category of objects alone; in the structuralist case studies, however, references to objects should be understood as references to entities *in the category of objects* and thus to entities distinct from those in category of structures.[31]

What we must acquaint ourselves with next is Fine's primitive operator '$\Box_x$', which generates the prefix 'it is true in virtue of the identity of x that':

$$\Box_x =_{def} \text{it is true in virtue of the identity of } x \text{ that}$$

The idea behind this operator is that when it operates on a predication ϕ of an object x, it generates the proposition that ϕ is an essential property of x:

$$\Box_x \phi(x) =_{def} \phi \text{ is an essential property of } x.$$

As I have already pointed out, Fine takes essential attributions to be more discriminating than necessary attributions, and as a consequence he

> accept[s] that if an object essentially has a certain property then it is necessary that it has that property (or has the property if it exists); but [he] reject[s] the converse,[32]

and it is clear that we can schematize the part he accepts as follows:

$$\Box_x \phi(x) \rightarrow \Box(Ex \rightarrow \phi(x)), \tag{1}$$

[31] I thank Kit Fine for clarifying this point.

[32] Fine [1994], p. 4.

which says that if an entity is essentially ϕ, then its necessary that it is ϕ if it exists. Here, following Fine and the standard literature on dependence I use this 'E' to denote 'the existence predicate'.[33] As Correia notes for us again, this schema is basically uncontested in the debate and here I will call it the 'basic schema'. Now, given that the necessary truths that we obtain in this way are 'not necessary simpliciter' but 'flow from the nature of the objects in question', we may say that the necessity attached to the consequent here is itself reflective of the nature of x.[34] It follows that ϕ's being an essential property of x plus the basic schema implies

$$\Box_x \phi(x) \rightarrow \Box_x(Ex \rightarrow \phi(x)), \tag{2}$$

which states that it is essential to an entity that if it exists, it is ϕ.[35] I've also said that, for Fine, 'ontological dependence should be tied to the nature of the dependent entity', which we can now express as

$$\Box_x(Ex \rightarrow Ey), \tag{3}$$

which is Fine's analysis of the statement that *x ontologically depends upon y*. So what I will try to do in what follows is derive statements of this form, with objects and structures in the appropriate positions, from what I have called the basic schema.

In order to do that, two more things will be required. First of all, when it is the natures of *two* entities that are involved in some dependence, we need a suitable generalization of the basic schema, and this is presumably

$$\Box_{x,y} \psi(x, y) \rightarrow \Box_{x,y}((Ex \& Ey) \rightarrow \psi(x, y)), \tag{4}$$

which says that if ψ holds of x and y in virtue of their essences, then if they exist, ψ is true of them. For the same reasons as before, we may infer from this, plus the fact that ψ holds essentially of them, that

$$\Box_{x,y}((Ex \& Ey) \rightarrow \psi(x, y)) \tag{5}$$

is true, which says that it follows from the natures of x and y that if they exist, then ψ is true of them. There is also the corresponding statement of ontological dependence of a pair x and y on some z:

$$\Box_{x,y}((Ex \& Ey) \rightarrow Ez). \tag{6}$$

As well as these generalizations to two or more entities, we are going to need the notion of the *consequential essence*, and, according to Fine, the most intuitive way to grasp this notion is via the concept of the *constitutive* essence (though we should note that ultimately the constitutive essence may be dispensed with to leave a purely consequence-based account).[36] As Fine puts matters:

[33] See for example Correia [2008], p. 1017. To be clear, Fine does not regard '$\exists y(y = x)$' to adequately express 'x exists'; rather than endorsing a *quantificational* conception of existence, he argues for a *predicational* conception instead. See Fine [2009] for an exposition of his view.

[34] Fine [1995b] pp. 7–8.

[35] Though I will not show it here, this is a theorem of Fine's system and may be demonstrated through the principles laid down in Fine [1995b]. I would like to express my gratitude to Fabrice Correia for providing me with the proof.

[36] See Fine [1995a], Section II.

> A property belongs to the *constitutive* essence of an object if it is not had in virtue of being a logical consequence of some more basic essential properties; and a property might be said to belong to the *consequential* essence of an object if it is a logical consequence of properties that belong to the constitutive essence. [...] Thus a property of containing Socrates as a member will presumably be part of the constitutive essence of singleton Socrates, whereas the property of containing some member or other will presumably only be part of its consequential essence.[37]

Intuitive as that may be, we have unfortunately still got work to do in defining the consequential essence – for, as Fine concedes, as it stands the proposed definition of consequence will be useless (and this relates to the idea that the essential is more fine-grained than necessary). This is because the property of being the same as, or distinct from, any object (or better, 'entity') y will be a logical consequence of *any* proposition about *any* given object, and so will form part of the consequential essence of anything. It will follow that everything depends on everything else – clearly an awkward result. What is needed is 'an independent way of distinguishing between those objects that enter into the consequential essence as a result of logical closure and those that enter in 'their own right', i.e. by way of the constitutive essence,' and to this end Fine proposes that we impose a test on the logical consequences of any essential attribution.[38] The test for x to be said to depend upon y is that y cannot be 'generalized out' of the consequentialist essence of x, or, in other words, that x will depend upon y 'just in case some proposition $P(y)$ belongs to the essence without its generalization belonging to the essence.'[39]

While the motivation for this test is clear enough, it is also apparent that if we are to gain any purchase on which truths are universalizable and which not, we must first of all specify what it is that we take the appropriate domain of quantification to consist of. Fine states that 'the quantifier can and, indeed, should be taken to range over every possible object' in order to preserve the Barcan principle (which again we take as the statement that the quantifier ought to range over every possible *entity*), and it is clear that if Fine's thesis is to be of any use to structuralism, then the domain had better include any of the structures that may be deemed relevant in the metaphysics of present or future fundamental physics (such as laws, groups, metric structures and so on).[40] While *that* is clear, what is far less so is that we can specify all of these structures that must feature in the domain in a clear and well-defined way. There may

[37] *Ibid.*, p. 276.

[38] *Ibid.*, p. 278.

[39] *Ibid.*, p. 278. So, for example, 'although it is part of the consequentialist essence of Socrates that 2 = 2, it is also part of his consequentialist essence that every object whatever is self-identical,' and so the number 2 does not after all feature in the singleton's consequential essence (at least not on this basis). On the other hand, while it is *also* a logical consequence of the nature of, say, Socrates' singleton that there is *something* that it contains, it does not logically follow that, for any object whatsoever, *that* object is contained. In accordance with Fine's criterion, then, we recover the idea that the property of containing something does belong to the consequential essence of Socrates' singleton set – and hence to its essence *simpliciter*.

[40] *Ibid.*, p. 277.

therefore be a worry that we will not be in a position to state that a given proposition $P(y)$ is satisfiable for every y unless we can find a way to specify in advance every y that should feature in the domain. Nonetheless, it is obvious that the task of securing *positive* attributions of dependence will not require us to trawl through every element of an infinite domain, for if we can find just one entity that we indisputably *do* want to be included, but that does not satisfy the proposition, then we will be home and hosed. So let us try to get along for the moment without worrying too much about how to specify the domain beyond the fact that it contains all the entities – including all the objects and structures – that structuralism will want to make use of.

That is basically all that will be needed of Fine's machinery, so let me now show how we can get it to work in structuralism. In what remains, I will examine two prominent case studies in structuralism, beginning with the case of entangled quantum particles and then the case of the group-theoretic conception of elementary particles, and in each case use Fine's analysis to rigorously recover the core structuralist claim that objects ontologically depend on structures. But I will also address the issue of whether this dependence is reciprocated, and hence try to make some progress toward adjudicating between structuralism's radical and moderate variants.

5 The Dependence of Objects on Structure 1: Entangled Quantum Particles

As already mentioned, where structuralists have gone into any kind of detail about the nature of the priority they have in mind it has tended to concern *identity*; structuralism is indeed often explicitly presented as the thesis that objects lack 'primitive identity'.[41] Perhaps the most discussed and clearly presented statements of this thesis revolve around the seminal work by Saunders on Leibniz's principles and their application to physics.[42] While this work was judged, by many at least, to undermine the metaphysical underdetermination claims that had previous motivated the eliminativist thesis associated with radical structuralism, the same work was redeployed to sustain the latter's core priority thesis: while the claim that the structure of quantum relations ruled out the individuality of fermions was dropped, in its place emerged the claim that those relations secured individuality only at the price of rendering them ontologically secondary (and as such, it is claimed, still metaphysically eliminable).[43] Ladyman and Ross, for example, note that in their opinion 'while Saunders' view vindicates an ontology of individuals in the context of QM, it is a thoroughly structuralist one in so far as individuals are nothing over and above the nexus of relations in which they stand'.[44] They furthermore take this 'structuralist' account of particle identity to form a core plank of the foundation for their ontic structuralism as a whole, a position that

[41] A review of this approach to structuralism may be found in Section 4 of Ladyman [2007].

[42] Saunders [2003].

[43] See e.g. French [1998] and Ladyman [1998] for statements of the underdetermination argument. It is my understanding that, at present, Ladyman accepts that Saunders' argument vindicates an ontology of individuals, while French still holds that the dilemma persists.

[44] Ladyman and Ross [2007], p. 138.

consists of 'a conjunction of eliminativism about self-subsistent individuals, the view that relational structure is ontologically fundamental, and structural realism.'[45] As already mentioned, however, Hawley has complained that what exactly these claims amount to is not clear as it stands. Esfeld and Lam have also noted that it is not at all obvious how the observations regarding identity that Saunders brings to the table 'could ground an ontological priority of relations over relata.'[46] Let me now attempt to repair this situation by explicitly setting the priority claims that are taken to follow from Saunders' discussion into a Finean framework and seeing whether they do indeed follow.

In order to do so, I must first briefly recap the main thrust of Saunders' argument. The issue at hand is identity, and Saunders takes as his starting point the analysis of identity in a modern logical context. In that context, he argues that the *principle of identity of indiscernibles* or 'PII' – the statement that if two objects possess all the same properties and stand in all the same relations to all the same things, then it follows that they are identical – can be regarded as well-motivated. Indeed, Saunders argues that the PII may be identifed with (what he calls) the 'Hilbert-Bernays principle', and since he takes the latter to provide an explicit *definition* of identity it follows that the PII may equally well be regarded, in this context, as *true by definition* as well.[47]

The relevance of this to the specific issue of identity in quantum mechanics may be put as follows. It is a postulate of quantum mechanics that the states of interacting indistinguishable (i.e. 'same kind') quantum systems must be subject to *permutation invariance* – that is, they must be determinately either symmetric or antisymmetric with respect to the exchange of particle labels, producing either bosons and fermions respectively.[48] This invariance under exchange has the consequence that there is nothing in terms of the essential properties that define their kind, nor in terms of the dynamical relationships that they bear to one another, that can be used to distinguish between the particles in a given state in such a way that we may determinately refer to any one of them to the exclusion of any of the others. This predicament gave rise to the 'Received View' that quantum particles violated the PII and thus could not be regarded as individuals.[49] Saunders' insight, however, was that the idea that entangled quantum particles are thereby indiscernible rests on a view of discernability that is unduly restrictive. So long as there exists an irreflexive relation between the objects at hand (a relation which, in the context of permutation invariance, will necessarily be symmetric), the particles can be regarded as distinct consistently with the PII: it will *not* be the case that the objects stand in all the same relations to all the same things due to the irreflexivity of the symmetric relation. And the relevant states in the case of fermions – which we regard as the fundamental constituents of matter – are of the

[45] *Ibid.*, p. 145.

[46] Esfeld and Lam [2010], p. 148.

[47] See Saunders *op. cit.*, Section 1. I note that here I am merely presenting Saunders' argument as I find it: my principal purpose is to show how priority attributions may be rigorously built upon these foundations, not to criticize those foundations themselves.

[48] Paraparticle states are also permitted, but do not seem to be instantiated. I thank a referee for reminding me of this.

[49] See, e.g., French and Krause [2006].

form

$$\frac{1}{\sqrt{2}}(\psi_x(\uparrow)\psi_y(\downarrow) - \psi_x(\downarrow)\psi_y(\uparrow)), \tag{7}$$

which may be interpreted as meaning that two particles stand in the symmetric but irreflexive relation of having equal but opposite spin. Objects such as these, which may be secured as distinct only by appeal to the presence of a symmetric but irreflexive relation, are said to be *weakly discernible*.[50]

That Saunders' argument yields this much has not been without controversy but is nonetheless widely accepted. What is by contrast much less clear is how exactly his argument may be used to underwrite the claim that fermions are thereby somehow ontologically secondary. By inserting this claim into Fine's framework, however, we may justify the claim as follows.

We have agreed that quantum mechanics supplies an irreflexive relation between entangled fermions, and Saunders has shown that objects satisfying such a relation are distinct. Thus we may write

$$E(R : R^{irref}(x, y)) \rightarrow x \neq y, \tag{8}$$

where I use '$E(R : R^{irref}(x, y))$' to express 'there exists some irreflexive relation holding between x and y'.[51] What we need now is one of Fine's theorems connecting essence and identity that were mentioned before – namely,

$$x \neq y \rightarrow \Box_{x,y}(x \neq y). \tag{9}$$

(To get a sense of what this means, it may be helpful to contrast it with

$$x = y \rightarrow \Box_x x = y \tag{10}$$

and note that, as Fine says, 'whereas a true identity $x = y$ depends upon the nature of the one object x, a true non-identity depends upon the nature of both objects' – which intuitively seems right.[52]) So while Saunders' analysis delivers that entangled fermions are distinct, Fine's analysis then tells us that it is *essential* to them to be distinct, and we have

$$\Box_{x,y} x \neq y. \tag{11}$$

What else can we deduce to be essential to these objects? Recall that, on the assumption that we are dealing with particles that are at best weakly discernible, it follows from the PII that if the objects are distinct then there must be an irreflexive relation between them. To put it schematically, we may write

$$PII(x, y) \rightarrow (x \neq y \rightarrow E(R : R^{irref}(x, y))), \tag{12}$$

[50] Saunders' analysis was subsequently extended beyond fermions to bosons (see Muller and Saunders [2008]), but this has proved more controversial.

[51] French and Krause ([2006], p. 9) suggest there is a 'worry' that by apppealing to irreflexive relations we beg the question; see Hawley [2009], Section 3.2 for discussion of related points. However, and to repeat, I am here simply presenting Saunders' argument as I find it.

[52] Fine [1995b], pp. 255-6.

where '$PII(x, y)$' symbolizes that x and y obey the PII (that is, that x and y are identical if they have all the same properties and stand in all the same relations to all the same things). Now, as emphasized above, according to Saunders the PII can be regarded as a *definition* of identity, so that '$PII(x, y)$' may be regarded as *true by definition* as well. We may thus move from (12) to

$$x \neq y \rightarrow E(R : R^{irref}(x, y)). \tag{13}$$

Now, we know from (9) that the property of being distinct that features in the antecedent of this expression is essential to x and y. Whatever follows logically from this property will therefore pertain to the consequential essence of both objects, *so long as* the implied proposition cannot be universalized – for that, to recap, is the test we must apply to see if the implied proposition belongs to the consequential essence. What is then left to do now is to test whether the corresponding universalized statement can be derived from the non-identity of x and y. Now, while we have remained quiet on the full content of the domain, we know that structuralists will hold that physical relations must feature in it. But from the fact that our two particles are distinct and obey the PII, we of course cannot deduce that *every* physical relation that the two particles enter into is irreflexive:

$$(PII(x, y)\&x \neq y) \nRightarrow All(R : R^{irref}(x, y)); \tag{14}$$

indeed, 'being of the same species' is presumably one physically significant but reflexive relation that holds between the (by assumption indistinguishable) x and y. We may therefore deduce that

$$\Box_{x,y} E(R : R^{irref}(x, y)), \tag{15}$$

and hence confirm that it is essential to x and y that there exists some irreflexive relation that they stand in. This, therefore, represents a further essential property of the pair.

Now let us go back to (4), which is what I called the 'basic schema' extended to two objects:

$$\Box_{x,y}\psi(x, y) \rightarrow \Box_{x,y}((Ex\&Ey) \rightarrow \psi(x, y)).$$

Since we have established that it follows from the natures of x and y that there exists some irreflexive relation for them to stand in, we may substitute in and write

$$\Box_{x,y} E(R : R^{irref}(x, y)) \rightarrow \Box_{x,y}((Ex\&Ey) \rightarrow E(R : R^{irref}(x, y))), \tag{16}$$

or more simply (cf. the move from (4) to (5) above),

$$\Box_{x,y}((Ex\&Ey) \rightarrow E(R : R^{irref}(x, y))). \tag{17}$$

But this is just the statement of the ontological dependence of the particles upon irreflexive relations, in accordance with Fine's definition (6).[53] Since structures are supposed to be 'nexuses of relations' (vague though that notion no doubt is), we seem to

[53] Since no particular irreflexive relation is being singled out here, the dependence is a *generic* dependence (to quote some standard terminology: see Correia [2008]).

have arrived at just what the structuralists want us to buy into: namely, the *ontological dependence of quantum objects on structures*.

The steps that have just been gone through seem to come close to the sought-for *demonstration* of the claim that quantum objects depend on structures (and hence are not 'self-subsistent'), demonstrated using Fine's principles. It thus appears that this flagship statement of structuralism may indeed be sustained. In order to fulfil our objectives, however, we need to go further and ask whether the *radical* structuralist claim that 'relational structure is more ontologically fundamental than objects' thereby goes through (or at least does so in the case of fermions), or whether it is the *moderate* position that should be adopted here. Now, it is clear that this question is not settled by what has so far been shown, for the issue of whether there is any *reciprocated dependence of relations on objects* must be investigated before any one position can be chosen. Whether this reciprocated dependence holds or not will be a function of how we choose to conceive of relations – in Fine's picture, on what we think their *nature* is – and traditionally of course there have been two ways to do this: we can either conceive of them *extensionally*, or we can conceive of them *intensionally*. But then it becomes clear that whichever we adopt, the radical structuralist in particular potentially has a problem.

The reason for this is that if we choose to conceive of relations extensionally, then given the identity criteria for relations in extension – namely, that two relations are distinct iff there is an ordered tuple in the extension of one that is not in the extension of the other – then by deploying exactly the same sort of reasoning as that just gone through we will be able to deduce that the relations are likewise ontologically dependent on objects.[54] By adopting an extensional account of relations, then, the moderate position would be vindicated.[55] Now, all that radical structuralists will take that to imply is that this route must be rejected – indeed, rejected as nothing other than a pillar in the whole 'object oriented' approach to metaphysics that they explicitly denounce. And that it *is* an intensional understanding that structuralists like Ladyman have in mind is in fact sometimes gestured at. In a couple of places, for example, Ladyman writes

> We eschew an extensional understanding of relations [...] According to Zahar, the continuity in science is in the intension, not the extension, of its concepts [...][56]

Exactly what this is intended to mean does not seem to be fully developed anywhere in the literature.[57] But of course the big problem in the vicinity of any consideration of this sort is that the whole reason that Quine, for example, rejected intensional entities was that he deemed it very unclear what their identity criteria were supposed to be.[58]

[54]Note that once again we would obtain a generic dependence in this case.

[55]I note too that Fine's analysis permits cyclical dependence, which would be the case here: see Fine [1995a], Section III.

[56]Ladyman and Ross, p. 128; also Ladyman [1998], p. 418.

[57]A referee has suggested that it may be connected with the idea that structures possess 'primitive modality'.

[58]See, for example, Quine [1975].

Now, this is of course not to say that such criteria cannot be provided in principle. The point is just that such criteria *need* to be provided, and defended, by structuralists of the radical stripe if their priority claim is to go through. For although many of those who defend intensional entities do so not because they think that intensional entities necessarily *have* perspicuous and reductive identity conditions, but because they *reject the idea that they must* if they are to be philosophically legitimate (and often on some sort of *tu quoque*-type grounds), this is not an option that is available to the radical structuralist (or at least not obviously).[59] Why, after all, should it be deemed obviously acceptable that *relations* can lack reductive identity conditions and thus possess primitive identity if the structuralist judges it to be so objectionable in the case of objects?[60]

The conclusion at this point can therefore only be stated in conditional form. If an extensional account of relations is adopted, then it seems that the moderate position wins out as the right metaphysics of fermions. If, on the other hand, an intensional account should be adopted, then in lieu of some identity criteria for relations-in-intension and, in particular, a policy on whether those identities are functions of objects or not, we simply *have no idea* whether or not the dependence is reciprocated – hence nor which structuralist stance is best recommended to us. In other words, without a positive statement on the identity criteria for relations, it seems there is nowhere for this debate to go.

That structuralists have had so little to say about the matter of the identity conditions of relations is on reflection a little surprising, given the centrality of both relations and identity considerations in structuralist metaphysics. And of course, since Fine's analysis *assumes* an understanding of the nature of the relata involved in dependence attributions, it can be of no help to structuralism in resolving this dispute. What has nonetheless been positively established by this point is that the flagship claim of structuralism – that objects are dependent on structures – may be shown to go through on Fine's conception of dependence. There is thus at least something for us to take home from this study of quantum mechanics and identity from a Finean point of view.

6 The Priority of Structure 2: The Group-Theoretic Conception of Elementary Particles

While the case just discussed is perhaps the most vaunted of all of the priority arguments in structuralist philosophy of physics, I want to turn now to another aspect

[59] See for example Loux [2002], pp. 57–8.

[60] I am not denying that it may be consistent for a radical structuralist to be a quidditist. But since identity considerations do so much work in structuralism, and given that proposals for analyses of property identities exist (as an example in the intensional case, see Hale [2012]), I do think that structuralists have to say *something* by way of explaining why it is that primitive identities are acceptable in the case of properties and relations, if so objectionable in the case of objects. In any case, *some* explicit line must be taken regarding relation identity conditions if radical structuralists are to use Saunders' argument as support for their view, but as yet this seems to be lacking.

of modern physics held up as a poster-child for structuralism – namely, the *group-theoretic conception* of fundamental particles. The relevant structure here is the *symmetry* structure of physical laws, and thus the issue at hand is whether such symmetries may be claimed to be more fundamental than even the so-called fundamental particles.[61] The modern attitude to symmetries arguably emerged in the context of special relativity, in which Einstein deduced the laws governing free relativistic systems by assuming invariance under the Poincaré transformations; since then, assumptions about symmetry structure have also been used to derive the laws that govern the fundamental interactions. While this deductive relationship between symmetries and laws is of critical importance, another reason why symmetries are paramount in particle physics is that knowledge of the symmetries associated with a law can also allow us to derive the *particles* whose behaviour is described by that law, and it is this that has largely precipitated the group-theoretic claims that are prominent in the structuralist literature.

Our ability to perform this derivation of elementary particles is grounded in the fact that the states of physical systems obeying laws of a given symmetry will fall into what are known as the *irreducible representations*, or 'irreps', of the group associated with that symmetry. These irreps we may think of as sets of states that are mapped into one another by the action of the transformations that together comprise the group, so that an irrep in this sense constitutes a vector space.[62] An important property of irreps is that states from different irreps cannot be mapped into one another by the group transformations, and the significance of this in the case of, in particular, the Poincaré group is that the differences between states drawn from different such representations may not be effaced by a mere change in perspective. It thus makes sense in this context to regard the states from different irreps of this group to be states of *different physical systems*. As such, it was proposed that *different irreducible representations* of the Poincaré group correspond to *different species of relativistic particle*, and it was in this way that the connection between symmetries and particle species was born.

To assess the viability of this proposal concerning the intimate connection between particle types and symmetry groups, what was needed was a classification of the irreducible representations of the Poincaré group.[63] This task was undertaken by Wigner and – glossing over some of the subtleties that led to certain representations being discarded – his analysis demonstrated that the irreducible representations, and hence relativistic particles, should either have

- some determinate *mass* $\in \mathbb{R} > 0$ and *spin* $\in \mathbb{Z}/2$, or
- some determinate *mass* $= 0$ and *helicity* $\in \mathbb{Z}$.

[61] To say that a law possesses a symmetry is to say that there is a set of transformations under which the form of that law remains invariant, and such a set of transformations may be shown to form a group in the mathematical sense.

[62] More than this is in fact required of an *irreducible* representation – in particular that it contains no 'smaller' representations. But this will do for our purposes just now. Note too that in physics it is common to refer to these spaces themselves as representations, though strictly speaking they are the spaces that 'carry' the representations.

[63] Wigner [1939].

But it turns out that these are precisely the properties that the fundamental particles do in fact have. The first class describes the electrons, the quarks, the massive bosons – pretty much everything, in fact, except the photon and gluons, which are in turn described by the second. *All* of the elementary particles we know of so far conform perfectly to this scheme. This spectacular success of the classification of free relativistic particles in terms of the representations of the Poincaré group caused the general strategy to be emulated outwith the context of free inertial motion and in the study of the fundamental interactions, and it has been this study of the symmetry groups of dynamical laws that has facilitated the successful prediction of whole new families of fundamental particles. The first three (up, down and strange) quarks, for example, were identified with the three states in the fundamental irrep of the SU(3) flavour group; eleven of the twelve gauge bosons were also predicted through an analysis of the representations of the local SU(3) and SU(2)⊗U(1) gauge symmetry groups that are the lynchpin of the Standard Model (the photon was already known).[64]

This newfound ability to not just *describe* the particles that we regard as fundamental, but also to *predict* their existence and properties, clearly represents an extraordinary development in our understanding of matter. The strategy outlined above has in general been so fruitful, in fact, that one can find prominent physicists saying things such as

> ever since the fundamental paper of Wigner on the irreducible representations of the Poincaré group, it has been a (perhaps implicit) definition in physics that an elementary particle 'is' an irreducible representation of the group, G, of symmetries of nature.[65]

Such an adage may be found all over the particle physics literature (in one form or another). It therefore seems that through these manifest successes, physicists have grown to conceive of an elementary particle as something inherently group-theoretic. It is of course this conception of fundamental particles that contemporary physicists now seem to have adopted that structuralists believe should be imported into fundamental metaphysics. And the implications of this idea for fundamental metaphysics are indeed potentially enormous. For one thing, given that fundamental entities are often taken to be those 'whose existence and features have no further explanation', it is no longer clear that there even *are* any fundamental particles by this definition since the properties that particles have, and the way in which they are knitted together, appear to be explicable via considerations of group structure (at least to a significant extent).[66] It is this apparent consequence that fuels the structuralist claim that even 'fundamental particles' are not truly fundamental, and that what *should* be regarded as properly fundamental is the symmetry structure, or group structure, that explains their basic features. The particle physicist and avowed ontic structuralist Kantorovich, for example, has claimed that these sorts of considerations demonstrate that the representations of groups, and hence fundamental particles, have 'a lower ontological status'

[64]It is interesting to consider why it is that in some situations we take the full representation to represent a particle and in others only a basis state, but I won't dwell on that issue here.

[65]Ne'eman and Sternberg [1991], p. 327; quoted in Roberts [2011].

[66]deRosset [2010], p. 74.

than the symmetries themselves.[67] Likewise, Lyre takes the above considerations to 'support structural realism' on the grounds that

> a group theoretic definition of an object takes the group structure as primarily given, group representations are then constructed from this structure and have a mere derivative status.[68]

These claims regarding the 'lower' or 'derivative' status of irreducible representations seem to be gesturing toward attributions of priority, but so far these claims remain largely unanalyzed.[69] In order to better articulate them, the focus on *definition* in the quote above naturally invites us to seek an approach to ontological priority based on it; but that, of course, is exactly what we have already been considering. Let me therefore now show how Fine's conception of dependence may be used to sharpen up this move from the reconceptualization of objects in group-theoretic terms to novel claims about priority, focussing for concreteness on the case of the Poincaré group and its representations.

If what it is to be an elementary particle is *defined* in terms of its being an irreducible representation of the Poincaré group, then – in accordance with our discussion in Section 3 above – that forms part of its essence. Thus, where x is a relativistic particle, we have

$$\Box_x IR_{PG}(x), \tag{18}$$

where '$IR_{PG}(x)$' means 'x is an irreducible representation of the Poincaré group'. And it is clear that from that essential property one may deduce the existence of the Poincaré group, for it is the transformations *of* this group that define the representation. (To recap, an irreducible representation of a group is defined in terms of a set of states that is closed under the action of the group transformations.) We may represent this as

$$IR_{PG}(x) \Rightarrow E(G : G = PG), \tag{19}$$

where '$E(G : G = PG)$' stands for 'there exists a group which is the Poincaré group' (or more simply, that the Poincaré group exists).[70] What we must do now is check that the deduced statement cannot be universalized and thus that it passes the test alluded to above. But it is immediately clear that it *cannot*. It is plainly not the case that anything other than the Poincaré group is the Poincaré group; that accolade, obviously, belongs only to that particular group itself. We may therefore confirm that it is indeed part of the consequential essence of a relativistic particle that the Poincaré group exists:

$$\Box_x E(G : G = PG). \tag{20}$$

[67] Kantorovich [2009], pp.79 - 80.

[68] Lyre [2004], Section 3.2. This quote may be found repeated all over the survey literature: see, e.g., Ladyman and Ross [2007], p. 147; Ladyman [2007].

[69] While Kantorovich does attempt to articulate and defend the notion of priority that he has in mind (see e.g. [2009], p. 664), I believe that this argument begs the question, though I cannot go into that here.

[70] Just to repeat, the ontic structuralist will insist that the domain of quantification in dependence attributions must contain structures, including group structures.

From this, we may then derive the ontological dependence of elementary particles on this group, just as we did before; substituting into the basic schema we obtain

$$\Box_x E(G : G = PG) \rightarrow \Box_x(Ex \rightarrow E(G : G = PG)), \tag{21}$$

which together with (20) gives us

$$\Box_x(Ex \rightarrow E(G : G = PG)). \tag{22}$$

We thus obtain exactly what the structuralists want – namely, the dependence of relativistic particles on the Poincaré group, and hence on the group structure of relativistic laws.[71]

Once again, therefore, we see that the central structuralist claim – that objects depend upon and hence are not prior to structures – may be straightforwardly established through Fine's analysis. But as before, we are not yet done: insofar as we want to ascertain whether a *superior* status may be accorded to group structure, and hence decide whether it is the radical or the moderate position that represents the right philosophy for this revolution in physics, we need to address the converse relationship between the groups used in physics and their representations. This, however, is a more difficult question to answer, because although structuralists (and physicists) have had plenty to say about particles *qua* irreducible representations, it seems that less attention has been paid to the *ontological interpretation* of group structure. Of course, it is perfectly straightforward to say how a given group is defined *mathematically* – we can go and look that up in a book – but structuralists take the statement that symmetry structure is ontologically fundamental to be a statement about what should be regarded as fundamental *to physical ontology*.[72] The question of what qualifies some, but only some, mathematical structures to enjoy the status of aspects of physical reality is therefore one that structuralists about physics cannot avoid facing up to; given that Fine's analysis ties dependence to the nature of the dependent entitity, until we are clearer on this issue there is little progress to be made on the question of whether the group structures that structuralists promote themselves depend on objects. But there are a few things that I think we *can* say, however.

The first thing to mention is that not all groups need be on a par with one another when it comes to matters of physical interpretation. The Poincaré group, for example, consists of a set of operations that each have a clear physical meaning, since we know very well what it means to boost, or rotate, or translate a physical system such as an observer through space and time (and can indeed in principle observe that such a transformation has taken place). This feature does not, however, seem to be a general feature of the groups that we use in physics: the groups mentioned above that encode facts about interactions, for example, do not in general enjoy this luxury.[73] Indeed, there appears to be *no* physical interpretation to be had in the case of the local gauge

[71] Note that this time it is a 'rigid' dependence: the nature of these particles *qua* representations of the Poincaré group demands not just that some group exists, but that a *specific* group does.

[72] See e.g. French and Ladyman [2003b].

[73] See e.g. Wigner [1968], p. 810.

transformations that underpin the Standard Model, since such transformations may be shown to correspond to mere changes in representation only.[74]

How, then, are we to do it? Something that structuralists such as French and Ladyman have cited as a means of distinguishing the structures they wish to reify as part of their metaphysical base from merely *mathematical* structures, and thus rebut the accusation of 'Platonism', is to characterize them as *causal*.[75] Now, while at that time French and Ladyman 'acknowledged that causal relations constitute a fundamental feature of the structure of the world,' this is something that Ladyman at least now seems to have retracted.[76] In any case, causality is notoriously problematic – especially when dealing with quantum systems – and it would be nice not to have to appeal to it. Furthermore, deciding how exactly to cash out group structures as 'causal' in anything like the sense in which we regard objects as such is likely to prove difficult – not least in the absence of a clear physical interpretation of the group in the first place.[77]

But there is a related and less metaphysically loaded consideration in the neighbourhood. Surely a very minimal, necessary condition on the physical significance of some structures over others is that they have *empirical consequences*. Indeed, this is something that French and Ladyman themselves acknowledge; they ask, for example,

> What makes a structure 'physical'? Well, crudely, that it can be related – via partial isomorphisms in our framework – to physical 'phenomena'. This is how physical content enters.[78]

Now, if symmetry structures are to be 'related to phyical phenomena', they must of course be relatable to *measurement*. But empirical testing is all about the detection of determinate properties – something that is made especially explicit in the basic formalism of quantum mechanics, in which a measurement is represented as the obtaining of a real eigenvalue. But this makes it clear that some reference to a group's representations must enter into any characterization of the group if it is to be considered as a part of empirical reality, for it is the *irreducible representations* of the symmetry groups that carry determinate values, not the symmetry groups themselves.[79] The irreps of the Poincaré group, for example, possess determinate mass and spin; the Poincaré group itself clearly does not. (It clearly doesn't make sense to ascribe mass and spin to a set of transformations between observers.) Likewise, it is the *states in* the irreps of the SU(3) flavour group that possess the determinate properties of isospin

[74] See e.g. Lyre [2004], p. 650. Here I mean 'representation' in the generic sense of mathematical representation, not in the sense of (reducible or irreducible) group-theoretic representations.

[75] *Op. cit.*, p. 75; see also French [2010], Section 4.

[76] *Op. cit.*; Ladyman and Ross [2007].

[77] In any case, since those who do take causation seriously often offer the fundamental determinate properties as the ultimate locus of causal agency, the conclusion of the argument regarding the empirically testable nature of structures that I am about to put forward will apply even if we choose to cash out the physical nature of structures in causal terms.

[78] *Op. cit.*

[79] I stress that here I am not saying that something must itself *possess* determinate physical properties to count as part of empirical reality, only that it must be *suitably related* to them. I would like to thank a referee for pressing this point.

and hypercharge that define the first three quarks; the SU(3) group does not. Putting everything together, then, it seems that reference to representations must be included in the definition of group structure *qua* denizen of physical reality, since it is only these that can furnish the required connection with phenomena. And that, as will by now be clear, will generate a *reciprocal dependence of group structures on objects* once we turn the handle on Fine's machinery, so that it appears to be decisively the *moderate* stance that is vindicated in this case.[80]

All in all, while one could certainly claim that, *qua* mathematical abstractions, there is no essential dependence of groups on vector spaces or their irreducible representations, as denizens of physical reality the matter looks very different. And unless the radical structuralist can find another way of characterizing the physical significance, including the testability, of the groups used in particle physics that does not involve any reference to the representations, we cannot assert that the representations have *merely* 'derivative status'. Rather, the irreducible representations and the symmetries of nature should each be said to be ontologically dependent on the other; given the 'reconceptualization' of fundamental particles in terms of group-theoretic representations, that in turn means that fundamental particles and group structures are likewise on an ontological par.

7 Concluding Remarks

In the course of this paper I firstly argued that ontic structuralism can and should make use of Fine's notion of dependence to articulate its core priority claims. I then put Fine's system to work to show that, in both the entanglement and group-theoretic cases, the ontological dependence of objects on structure can be rigorously sustained. In the case of entanglement structures, while the dependence of objects on structure could be secured without any trouble, we got hamstrung in trying to either establish or deny the existence of reciprocated dependence relations. In the context of group-theoretic structures and their associated objects, by contrast, we were able, through a plausible interpretation of the physical nature of group structures in empirical terms, to mount good arguments that the dependence should postively be taken to be reciprocated in this case, and thus that the moderate position wins out here.

As a first conclusion, we may note that in both cases we encountered much more difficulty in assessing whether there is any dependence of structures on objects than we did in assessing the converse. Given that Fine's analysis ties dependence claims to the nature of the dependent entity, this suggests that there is a lack of clarity not in our understanding of priority but rather in our understanding of *the nature of structures*. And since it is precisely these entities that structuralists entreat us to regard as constituting the very foundation of physical ontology, the fact that we struggled to ontologically articulate these entities carries a serious message for structuralists. As

[80] We should note for completeness that unless there are special reasons for any *one* representation to be realized, we should expect this dependence to be *generic.* (There may be such special reasons in the case of the adjoint representation of the gauge symmetry groups featuring in the Standard Model, for example, since these correspond to the gauge bosons.)

a second conclusion, it seems that we are in a position to determinately declare that an unqualified acceptance of the radical position is untenable, since our second case study showed that particles do indeed have to be regarded as on a par with at least one extremely important class of structures. That of course entails in turn that any 'eliminative' structuralism in which objects are purged from the fundamental basis is likewise untenable as a general thesis about physics, since one cannot eliminate the objects without thereby eliminating the structures – something which would clearly be disastrous from the structuralist point of view. Radical structuralists thus cannot maintain the two theses most closely associated with them when it comes to particle physics: they cannot both maintain that objects must be re-conceptualized in terms of structures *and* that they be eliminated, *qua* metaphysically secondary entities, in favour of the associated structures.[81]

The net result of this discussion, then, is that the more radical claims made by ontic structuralists must be regarded as unjustified. But what we are left with nonetheless is a picture of the fundamental basis that is very different from that which is presented to us by purely 'object oriented' metaphysics. It is a picture in which we regard elementary particles as no more fundamental than (at least some of) the dynamical structures of contemporary physics, and in which a rich nexus of metaphysical dependencies weaves the various entities in the fundamental basis together. Thus while I think we all must agree that the more radical versions must be left behind, it remains the case that ontic structuralism has a highly revisionist, and hopefully now more rigorously supported, proposal to make to contemporary fundamental metaphysics.

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[81] Note, however, that in places French writes as though the group-theoretic representations are *themselves* to be regarded as part of the structure: see, e.g. French [2012]. But then the claim that structures are more fundamental than objects is simply obviated, since there *are* no objects distinct from the structures. And such a response, in this context at least, perhaps also has a ring of 'moving the goal posts'.

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