

## Temporal Arrows in Space-Time

Temporality (...) has nothing to do with mechanics. It has to do with statistical mechanics, thermodynamics (...).C. Rovelli, in Dieks, 2006, 35

### Abstract

The prevailing current of thought in both physics and philosophy is that relativistic space-time provides no means for the objective measurement of the passage of time. Kurt Gödel, for instance, denied the possibility of an objective lapse of time, both in the Special and the General theory of relativity. From this failure many writers have inferred that a static block universe is the only acceptable conceptual consequence of a four-dimensional world. The aim of this paper is to investigate how arrows of time could be measured objectively in space-time. In order to carry out this investigation it is proposed to consider both local and global arrows of time. In particular the investigation will focus on a) invariant thermodynamic parameters in both the Special and the General theory for local regions of space-time (*passage* of time); b) the evolution of the universe under appropriate boundary conditions for the whole of space-time (*arrow* of time), as envisaged in modern quantum cosmology. The upshot of this investigation is that a number of invariant physical indicators in space-time can be found, which would allow observers to measure the lapse of time and to infer both the existence of an objective passage and an arrow of time.

### Keywords

Arrows of time; entropy; four-dimensional world; invariance; space-time; thermodynamics

## I. Introduction

Philosophical debates about the nature of space-time often centre on questions of its ontology, i.e. on the reality *versus* the unreality of time, on becoming *versus* being. Such debates are usually carried out on the basis of Minkowski's geometric interpretation of space-time, in terms of a light cone structure attached to events, with  $c$  providing the null-like separation between two events 'p' and 'q'. Such debates can then be extended to the General theory of relativity. Gödel, for instance, inferred a Parmenidean block universe from the well-known relative parameters – relative to reference frames or world lines – of the theory, like relative simultaneity and time dilation, because these relative parameters seem to lend themselves readily to a philosophical exploration.<sup>1</sup> Other features of space-time theories, such as invariant properties, which may be important for the issues mentioned, are often neglected. For instance, it is hardly ever considered what effect thermodynamic invariants would have for the measurement of time in a relativistic setting. However, it is appropriate to recall that Einstein was not opposed to a correspondent's suggestion to call his theory *Invariantentheorie* rather than *Relativitätstheorie*, since, as a matter of fact, the theory of relativity is a theory of invariance.<sup>2</sup> This paper pursues two aims: firstly, to consider the possibility of the measurement of the passage of time from the point of view of the invariant relationships, which exist in the theory of relativity. This procedure seems justified since questions of the 'nature' of time (Heraclitean becoming or Parmenidean being) are traditionally *inferred* from the existence of relative simultaneity in the theory of relativity. In the spirit of that theory the appeal to the anisotropy of time is here not understood in a deep metaphysical sense – i.e. whether time exists independently of events, whether as such it flows – but in the sense of clock time and the behaviour of clocks in various physical systems. Furthermore, the idea of the objective passage of time will not be understood here in terms of the foliation of space-time by appropriate hyperplanes but in the sense of invariant features of appropriate clocks. This procedure seems appropriate: even if a cosmic time exists as in some FLRW space-times, it will only refer to the average motion of matter in the universe. For the present purpose it is also important to recall that clocks do not need to be mechanical to measure time. In particular we shall consider thermodynamic clocks and what they say about the measurement of time by two observers moving inertially with respect to each other. However local observers would not be able to determine a global arrow of time from asymmetric experiences in their local space-time area. Kurt Gödel showed that there are solutions of the Einstein field equations, which would lead to closed time-like curves (CTCs), which locally always point in a forward temporal direction but globally return to their starting point. We are led from the STR to the behaviour of thermodynamic clocks in the GTR, where the behaviour of thermodynamic clocks also turns out to be invariant. Secondly, then, Gödel's argument for the unreality of time, based on closed time-like curves (CTCs) leads to a distinction between an asymmetry in local regions of space-time (the passage of time) and the whole of space-time (the arrow of time).<sup>3</sup> For a consideration of the objective, measurable arrow of time, it is necessary to consider the behaviour of the universe under appropriate boundary conditions, as investigated in

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<sup>1</sup> Gödel, 1949, 557-62; Dieks, 2006

<sup>2</sup> Holton, 1996, 132

<sup>3</sup> Cf. Davies, 1974, §2.3

quantum cosmology. The upshot of this paper is that there exists objective indicators of the anisotropy of time, both in the sense of local and global arrows and, consequently, that arguments for the unreality of time – the Parmenidean block universe - are not conclusive.

## II. *From the Special to the General Theory.*

To describe the problem-situation recall that the transition from the Special (STR) to the General theory of relativity (GTR) is marked by a number of changes:

- a) The nature of symmetry principles changes from Lorentz covariance in the Special theory (Lorentz transformations) to general covariance in the General theory (general transformations of the coordinate systems). The covariance group of a space-time theory must retain invariant elements, since in the transition between coordinate systems some structure remains unchanged. The symmetry principles of the theory of relativity are related to invariance, such that the 'factual or physically significant quantities of a theory of space and time are the invariants of its covariance group.'<sup>4</sup> In both the STR and the GTR the invariance associated with the space-time interval,  $ds$ , remains. But the STR has a fixed space-time structure: space-time acts on the inertial systems but no matter acts on space-time itself. Its light cone structure is regarded as its causal structure such that the light cones attached to any event 'p' never tilt. The covariance group of the STR has therefore a larger invariance group than the GTR. The GTR no longer has fixed background structures so that that its covariance group is its invariance group. Nevertheless the invariance associated with the space-time interval  $ds$  remains. It is the aim of this paper to show that this characterization leaves room for a consideration of *other* invariant structures, especially from the theory of relativistic thermodynamics, for the measurement of time.
- b) The inertial reference frames of Einstein's original exposition - the inertial world lines in Minkowski space-time - become arbitrary coordinate systems and curved world lines in the GTR. The inertial reference frames of the STR are equipped with synchronized clocks and rigid rods, which become subject to the effects of time dilation and length contraction, leading to relative simultaneity. The Minkowski metric determines the time-like, null-like and space-like intervals on the world lines in Minkowski space-time. But the rigid measuring rods and synchronized clocks can no longer be used in accelerated systems or gravitational fields. To illustrate, in a Euclidean frame K, which is at rest with respect to a rotating system K', the ratio  $C/D = \pi$  is valid, but in the rotating system the ratio will measure  $C/D > \pi$ . This inequality is due to the length contraction suffered by tangential rods placed along the circumference of K'. In the GTR arbitrary coordinate systems take the place of inertial reference frames in the STR.<sup>5</sup> And the Minkowski metric is replaced by a more general metric, which determines the behaviour of trajectories in curved space-time. Such coordinate systems can be characterized as the 'smooth, invertible assignment of four numbers to events in space-time neighbourhoods.'<sup>6</sup> Despite this abstract characterization, one should not forget that the

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<sup>4</sup> Norton, 1996/1992, 22

<sup>5</sup> Norton, 1993, 836

<sup>6</sup> Norton, 1993, §6.3; Earman, 1974a

space-time models of the GTR are constrained by the Einstein field equations and ultimately meant to make statements about the physical world. Hence even in a curved space-time geometry, the measurement of the passage of time is a viable concern.

How, then, does the measurement of time appear from the point of view of invariant relationships? In the theory of relativity, we encounter Lorentz invariance (STR) and diffeomorphism invariance (GTR), which can be taken in the active sense of general covariance. But as space-time is generally curved, due to the mass-energy distribution in the universe, there is the additional problem, as Gödel pointed out, that the field equations allow closed time-like curves (CTCs) as solutions, which constitute a further challenge to proponents of the 'reality' of time in relativistic space-times. To address this challenge the argument needs to move to the level of quantum cosmology and a consideration of the boundary conditions of the universe.

### III. *Consequences of Lorentz Covariance.*

Invariance results from the operation of a group of transformation rules, which are applied to a system  $A$ , in order to transform it into another system  $B$ . Such a transformation changes certain parameters of  $A$  but leaves other parameters unchanged or invariant. Symmetry principles are of fundamental importance in modern science. They state that certain, specifiable changes can be made to reference systems, without affecting the structure of the physical phenomena. For instance, experimental results are invariant under spatial, temporal and rotational symmetries. Symmetries result from specified group transformations that leave all relevant structure intact.<sup>7</sup> Symmetry is immunity to possible change.<sup>8</sup>

Symmetries are therefore related to invariance. The theories of relativity only deal with space-time symmetries of a global kind (STR) or a local kind (GTR). In the STR the transformation rules are expressed in the Poincaré group; the GTR requires more general transformation groups because the inertial reference frames have been replaced by general coordinate systems.

As we have inertial reference frames or arbitrary coordinate systems, we need transformation rules between them. These transformation rules will transform certain parameters from one system of coordination to another:  $t \rightarrow t'$ ,  $x \rightarrow x'$  but crucially will leave others invariant. The transformed coordinates in the STR affect the time and length coordinates, leading to the well-known phenomena of time dilation, length contraction and relative simultaneity.

But we shall focus here on the invariants of the Lorentz transformations, and ask what they tell us about the measurement of time. The Lorentz rules, which transform coordinates  $x$  and  $t$  of a system at rest,  $S$ , into the coordinates  $x'$  and  $t'$  of a system,  $S'$ , in inertial motion with respect to  $S$ , are global transformations since they are constant throughout Minkowski space-time. The transformation rules, which apply to Minkowski space-time, admit only of two invariants, the space-time interval  $ds$  and the velocity of light,  $c$ . The first question is whether they can be used to measure the passage of time. The space-time interval,  $ds^2$  remains the same in transitions between inertial reference frames and is expressed by the invariant line element

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<sup>7</sup> Van Fraassen, 1989, 243; Martin, 2003

<sup>8</sup> Rosen, 1995, Ch. 1

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 = ds'^2 \quad (1)$$

Equation (1) expresses well the suggestion to call this theory a ‘theory of invariants’. But in a certain sense, the space-time interval,  $ds$ , is ill-suited to determine the objective passage of time since the equation incorporates the changes in spatial ( $x, x'$  etc.) and temporal coordinates ( $t, t'$ ). Travellers in space-time have the problem that they cannot rely on mechanical clocks, because of the time-dilation effects. It is only the computation of  $ds$  and  $ds'$ , which leads to the same result but from different length and time coordinates whose values depend on the velocities of the respective systems. On the other hand, the space-time interval  $ds$  can be used by different observers to determine each other’s clock readings.<sup>9</sup> If the observers consider each other’s clocks, from their respective coordinate systems, which are in relative motion with respect to each other, they will find that they appear to tick at different rates. But it is hard to see why they should conclude from these different perspectival ticking rates that the world is ontologically a four-dimensional static block universe and the passage of time a human illusion. (Yet these inferences are often made.<sup>10</sup>) The only permissible inference from the STR is that the ticking rates of clocks are *not* invariant across different coordinate systems that are in relative inertial motion with respect to each other, (although they tick regularly in their respective frames: invariance of proper times). The further conclusion that time must be a human illusion is no longer a permissible inference, in the sense that it is not deducible from the principles of the STR. From the failure of invariance of ticking rates across different reference frames we cannot derive the thesis that time is an illusion according to the STR, since the question arises whether two inertial observers do not have other objective ways of measuring the passage of time, as we shall attempt to show below.

The velocity of  $c$  is also an invariant of Minkowski space-time but could it serve observers in Minkowski space-time to measure, objectively, the passage of time? This question cannot be answered without taking note of a systematic ambiguity in the status of  $c$  in interpretations of Minkowski space-time. In the standard *geometric* interpretation of Minkowski space-time as a four-dimensional manifold,  $c$  constitutes the limits of the light cones for each observer, as  $c$  is invariant for all observers. The value of  $c$  is the same for each observer irrespective of her/his velocity. Therefore the light cones do not tilt and within the light cones events are time-like connected and clearly constitute an invariant ‘before-after’ order of events. The constant  $c$  is sometimes said to constitute the causal structure of Minkowski space-time, and signals which propagate faster than  $c$  are not only forbidden in the STR but events on such world-lines are space-like connected and provide no invariant causal order. Events on the geometric boundaries of the light cones are said to be null-like connected, and for such observers time stands still, since  $ds^2 = 0$ . The geometric representation of Minkowski space-time is therefore static; events ( $p, q$ ) are connected by world lines, which according to some interpretations are etched into the fabric of space-time. The whole question of whether time may be dynamic gets short shrift, since world lines do not propagate. At best the standard geometric view gives rise to frame-dependent *geometric* time, which by-passes the question whether time can

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<sup>9</sup> Geroch, 1978, 124-8

<sup>10</sup> See Hoyle, 1982; Ehlers, 1997

be measured objectively in Minkowski space-time. On the standard interpretation,  $c$  cannot be used to measure the passage of time.

However, in his original paper Einstein (1905) proposed the use of light signals for the coordination of distant clocks. Einstein's discussion of light pulses, used by different observers to synchronize distant clocks, hints at an alternative reading of  $c$ : it can serve as a basis for a much more dynamic interpretation of space-time. Such alternative approaches were indeed proposed soon after Minkowski's static geometric interpretation.<sup>11</sup> These so-called *axiomatic* approaches are based on the propagation of light signals between space-time events. The fundamental concept is the instant of time rather than the space-time event. And the before-after relation between two instants or elements of time is an asymmetrical relation. These axiomatic approaches are based on optical facts of light propagation, which means that the standard geometric view is regarded as a mere 'formal expression' of the optical facts.<sup>12</sup> The alternative approach is focussed on a light geometry. As such a light geometry is based on the propagation of electromagnetic radiation, it immediately invites thermodynamic considerations, since it touches on questions of irreversibility and asymmetry. The contrast between the two approaches clearly shows that the standard geometric interpretation is restricted to kinematics.

The point of all this is that a given space-time geometry leads to a complete, exhaustive list of detailed numerical predictions of all experiences of a spatial and temporal character. In other words, a space-time geometry represents a complete model for a physical world.<sup>13</sup>

The axiomatic approaches invite us to take thermodynamic aspects into account, in which further invariant relationships come to light, which are of interest to a consideration of the objective measurement of the passage of time in the context of relativistic thermodynamics.

#### IV. Thermodynamics Clocks

In a thermodynamic system, moving with a relative velocity,  $v$ , with respect to another system, considered at rest, several thermodynamic parameters remain invariant.<sup>14</sup>

$$p = p_0; N = N_0; S = S_0(2).$$

The question whether temperature,  $T$ , is also invariant is still controversial and at an experimental stage.<sup>15</sup> If this were the case, then, to adopt Einstein's famous train thought experiment, a cup of coffee drunk by a passenger on a fast-moving train would get neither colder nor hotter; it would have the same temperature as a cup of coffee drunk by a waiting passenger on a platform. If temperature

<sup>11</sup> Robb, 1914; Cunningham, 1915; Carathéodory 1924; Schlick, 1917; Reichenbach, 1924; cf. Weinert 2010a. Note that although the propagation of light signals is the same for all observers in Minkowski space-time,  $c$  would not be a good clock because of a) the phenomenon of aberration (the direction of a light signal depends on the relative angle between the coordinate frames of the observers and the light source); b) the relativistic Doppler effect, which affects both the frequency and wavelength of the signal.

<sup>12</sup> R. Penrose 2010, 94 proposes that the frequency of massive particles could serve as precise quantum clocks:  $\nu = m\left(\frac{c^2}{h}\right)$ ,

which is derived from the two fundamental equations:  $E = mc^2$  and  $E = h\nu$ . This proposal excludes the massless photon as a basis for a quantum clock, although light signals still propagate between events in space-time, but they will be subject to the (relativistic) Doppler Effect.

<sup>13</sup> Geroch, 1978, 161, 121

<sup>14</sup> Einstein, 1907; Planck, 1908

<sup>15</sup> Cubero *et al.*, 2007; Weinert, 2010b

were invariant, a specially constructed thermometer could be used by inertially moving observers at relativistic speeds to objectively measure the passage of time. But relativistic thermodynamics offers other invariant parameters: we shall concentrate here on the pressure,  $p$ , and in particular on the entropy,  $S$ , of thermodynamic systems. Although invariant thermodynamic clocks have been considered before in order to defend alternative interpretations of STR (cf. Schlegel 1975), their implications for temporal arrows in space-time have not been explored. It is important to note that the lapse of time need not be measured by mechanical clocks, whose ticking rates are affected by relativistic speeds and gravitational fields, both resulting in a time dilation effect. It is equally important to note, for the arguments that follow, that physical time is based on physical processes.<sup>16</sup> Such physical processes must display mathematically describable regularities, which are often of a periodic nature, since this periodicity helps to determine finite intervals of time. One problem, which arises in the STR is that clocks tick regularly in inertially moving systems, but their ticking is not invariant across such systems, as is revealed by the difference between proper and coordinate time. But if the regularities are to be invariant across different reference systems, two observers must clearly agree on the length of a temporal interval and the ticking rate of clocks. This is a prerequisite for the objective measurement of time. These features are satisfied by some of the thermodynamic parameters, mentioned above. Consider, for instance, two properties of a gas clock – its pressure and entropy – both of which are invariant according to relativistic thermodynamics. Such a clock could consist of a two-chamber system, connected by a valve, as used in thermodynamics, with gas molecules at  $t = t_0$  being confined at first to, say, chamber A. When the valve is opened the gas molecules will, in accordance with thermodynamic laws, begin to fill chamber B until it reaches a final pressure,  $p_f$ . First, what does the invariance of  $p$  mean for the measurement of time in two inertial reference frames? As the relationship  $p = p_0$  obtains, the pressure (force per unit area) is the same in these two systems, and independent of their respective velocities. In whichever way such clocks are built, the pressure measured will be the same for two observers. Hence the rate of change of pressure,  $\Delta p = \Delta p_0$ , is also the same<sup>17</sup>, with the result that the pressure of gas clocks could be used as primitive clocks for two observers to coordinate activities in relativistically moving systems. When the two observers are at rest, carrying identical gas clocks in their respective systems, they can agree to perform a certain action when the pressure gauge reaches a certain value,  $p$ . What is important from the present point of view is that they can infer an invariant succession of events in Minkowski space-time, since the proper times of the two gas clock will agree. However, this agreement does not constitute a violation of the principle of relativity and does not give rise to a notion of ‘absolute’ simultaneity. Firstly, the gas clocks do not show the ‘correct’ time. They are simply two particular systems, amongst other clocks. Secondly, there are still as many clock times as there are reference frames. Thirdly, the theory of relativity does not allow a particular clock system to be chosen as a ‘preferred’ frame but it does not forbid the choice of a particular clock as the basis for inferences about the nature of time. The proposal simply exploits an invariant feature of relativistic

<sup>16</sup> Rugh and Zinkerhagel, 2009; Ellis 2013

<sup>17</sup> Schlegel, 1968, 137. In response to a referee’s suggestion, it should be noted that in isobaric processes ( $\Delta p = 0$ ) this method cannot be applied. But it should not be inferred that ‘time does not flow’ because other methods to measure passage remain available. Where the pressure and entropy method apply, they can be regarded as physical manifestations of  $ds$ .

thermodynamics, in the same way that  $c$  and  $ds$  are invariant features of Minkowski space-time. Furthermore, the pressure gauge can serve as an invariant clock in the two systems, and hence 'time' in the two gas clocks ticks at the same rate, although they are in inertial motion with respect to each other, and at velocities which are relativistically significant. Whilst the two systems may move at high velocities, the motion of the gas molecules will not be affected by these relativistic speeds of the coordinate systems to which they are attached. And thus these observers will measure the passage of time between their pre-arranged, coordinated actions, in an objective, frame-invariant manner. It may be called *thermal* time.

A similar conclusion can be drawn from the consideration of entropy in relativistic systems. Boltzmann entropy is taken to be valid for all systems.<sup>18</sup> The Lorentz-invariance of entropy in statistical mechanics follows from the equation  $S = k \log N_o$ . As  $N = N_o$  – so that the number of microstates,  $N$ , does not depend on the velocity of the thermodynamic system – we also have  $S = k \log N$ , and hence  $S = S_o$ , where the spatial extension of the system in the x-direction must be kept small. It follows from this equation that, again, when the entropy reaches a certain state in one system, by the spreading of microstates into the available phase space, two observers can observe a frame-invariant succession of events, according to their specific clocks. Note that the spreading of the microstates into the available phase space is a function of time, and this spreading of microstates into phase space occurs at the same rate. The spreading rate can therefore be used to define an entropy clock,  $\Sigma$ . Hence observers in two relativistically moving systems can use the rate of spreading of the microstates, which according to the equation,  $S = S_o$ , must be invariant, as a way of measuring the objective, frame-independent passage of time. Even the fact that one of the two systems, with an entropy clock  $\Sigma'$  attached to it, must be accelerated to reach its relativistic velocity does not change the invariant rate of entropy increment,  $\delta S$ , in the accelerated system. If we consider the velocity increase in the  $\Sigma'$ -clock as  $\delta v$ , then the invariance theorem can be written as  $d/dv(\delta S) = 0$ , and hence the change in velocity has no effect on  $\delta S$ .

We must take it, then, that the  $\Sigma'$ -clock while being accelerated gains the same increments  $\delta S$  which comparable  $\Sigma$  clocks are gaining; if it were otherwise, entropy would not be independent of velocity. In the limiting case of zero velocity increments, we must also have the same entropy increments for the  $\Sigma$  and  $\Sigma'$  clocks, and hence also the same increases in clock readings. We conclude that similar entropy clocks, in relative uniform motion, will run at the same rate.<sup>19</sup>

Once again, then, two observers who use entropy clocks will know that the invariance of  $S$  ensures an objective measurement of the passage of time in their respective systems. These considerations do not, of course, question the validity of the Lorentz transformations for entropy clocks provide no 'preferred' temporal frame and are restricted to the two systems at hand. But they show that, if relativistic thermodynamics is taken into account, new invariant relationships come to light, which can be exploited for the objective measurement of the passage of time in relativistic systems. Hence, the usual claim that only the static block universe is compatible with the STR may be doubted. This claim is based on the failure of invariance in the ticking rate of mechanical clocks, and the relativity of simultaneity. It appears, then, that observers in Minkowski space-time cannot agree on the amount of

<sup>18</sup> Carroll, 2010, 32

<sup>19</sup> Schlegel, 1968, 148; cf. Schlegel, 1977; Davies, 1974, 89f



time which has elapsed between two events, which leads to the usual questioning of the objectivity of time. If, however, invariant relationships, like the ticking rates of entropic clocks can be found, then two observers can measure the passage of time between two events in space-time objectively (in the sense of invariance). Then the image of the Parmenidean block universe, which appeared to be a reasonable philosophical consequence of relative simultaneity, may be questioned.

Note that the inference has a curious consequence: if two observers, who are in inertial motion with respect to each other at relativistic speeds, infer from the time dilation effect that time must be 'unreal', they would have to conclude that time is 'real' when their velocities reduce to non-relativistic values, since in this case their proper times will agree. It may be concluded from this curious effect that inferences to the block universe are unreliable. Hence, two observers would be allowed to infer that the passage of time must be more than an illusion or a subjective experience. According to the gas clock the temporal passage must be objective.

However the doubt about objective time measurements must also be considered at the level of the GTR since the STR turned out to be a limiting case of the GTR. The STR became, in Einstein's eyes, unsatisfactory – it still privileges inertial frames and quasi-Euclidean hyperplanes of simultaneity, whilst ignoring accelerated systems. The effect of acceleration and gravitation on time, which arises in the context of the GTR, must be taken into account. This transition to the GTR does not invalidate the conclusions reached so far since sufficiently small regions of space-time are flat and can be treated according to the methods of the STR. Thus the behaviour of clocks in curved space-time approximates the measurement of the passage of time in the STR, for small regions of space-time.<sup>20</sup>

## *V. Consequences of General Covariance*

Nevertheless there have been a number of arguments purporting to show that the assumption of the objective passage of time is incompatible with the General theory of relativity. These arguments are based on a) the diffeomorphism invariance between space-time manifolds; b) Earman's claim that in the constrained Hamiltonian formulation of the GTR the observables become 'constants of motion' and hence that the GTR offers no basis for the objective passage of time; and c) Gödel's solutions to the Einstein field equations, involving closed time-like curves (CTCs). The question, which arises for all these approaches, is whether they take sufficiently many physical parameters into account to establish their respective conclusion.

### *V. 1. Diffeomorphism Invariance*

In a letter to Ehrenfest (December 26, 1915) Einstein wrote that 'the inertial frame signifies nothing real'.<sup>21</sup> This statement reflects Einstein's awareness that the four numbers (coordinates), assigned to events in space-time neighbourhoods in the GTR, 'have not the least physical significance, but only serve the purpose of numbering the points of the continuum in a definite but arbitrary manner.' In fact these coordinates are so abstract that the numbers ' $x_1, x_2, x_3$ ' do not need to denote space, and ' $x_4$ ' does not need to denote time.<sup>22</sup> Thus Einstein draws from the covariance of the GTR the

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<sup>20</sup> Geroch, 1978, 111

<sup>21</sup> Quoted in Stachel, 1989, 86-7

<sup>22</sup> Einstein, 1920, 94

conclusion that space and time have lost their last ‘vestiges of reality’.<sup>23</sup> The arbitrariness in the choice of coordinates or manifolds, linked by smooth transformations, is seen as an ‘argument against objective temporal passage.’<sup>24</sup>

These general coordinate transformations are referred to as diffeomorphisms. The diffeomorphism group is the covariance group of the GTR. A diffeomorphism is a one-to-one mapping of a differentiable manifold  $M$  onto another differential manifold  $N$ . Then the question of invariance arises as diffeomorphisms (manifold transformations) are applied to manifolds  $M$ ,  $N$ . It may be reasonable to suggest, in line with the consideration above, that what remains invariant are structural elements: If  $\phi$  is a diffeomorphism (or symmetry transformation) between  $M$  and  $N$  -  $\phi: M \rightarrow N$  - then these manifolds have identical manifold structure and are physically indistinguishable.

If a theory describes nature in terms of a spacetime manifold,  $M$ , and tensor fields,  $T^{(i)}$ , defined on the manifold, then if  $\phi: M \rightarrow N$  is a diffeomorphism, the solutions  $(M, T^{(i)})$  and  $(N, \phi^* T^{(i)})$  have identical properties. Any physically meaningful statement about  $(M, T^{(i)})$  will hold with equal validity for  $(N, \phi^* T^{(i)})$ .<sup>25</sup>

We now have to ask what the invariants of the space-time models are. One is the space-time interval  $ds$ . The space-time interval  $ds$ , in the presence of a gravitational field, now takes the general form:

$$ds = g_{ik} dx^i dx^k \quad (3)$$

where the  $g_{ik}$  are functions of the spatial coordinates  $x^1, x^2, x^3$  and the temporal coordinate  $x^4$ . It is now this more general relation, which must remain covariant with respect to ‘arbitrary continuous transformations of the coordinates.’<sup>26</sup>

However, we have already seen that the space-time interval is only useful for the measurement of time in curved space-time in the sense that it allows observers to calculate their respective clock times. This fact in itself should make us hesitant about drawing the conclusion that time is unreal. These observers can calculate the respective ticking of their clocks and the elapsed time they would measure for certain intervals between two events ( $p, q$ ) on their respective world lines; and hence there is no reason for them to conclude that time is unreal. The only permissible inference from the effects of time dilation due to frame velocity and gravitational effects is that the ticking rates of clocks are affected differentially, as seen from reference frames moving inertially with respect to each other. The very fact that from their respective world lines the observers can compute and predict the ticking of clocks on other world lines seems to suggest that time cannot simply be a human illusion.

There is a great temptation, which can also be detected in Gödel’s argument, to draw inferences from the structure of a theory like the GTR to the unreality of time. Thus Earman<sup>27</sup> argues that gauge-invariant observables in the GTR can be understood as ‘constants of motion’ in the constrained Hamiltonian formulation, and hence that there is no change in any genuine physical quantity and hence no time in the worldview of the GTR. This argument has already been subjected to sustained

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<sup>23</sup> Einstein, 1916

<sup>24</sup> Lockwood, 2005, 87

<sup>25</sup> Wald, 1984, 438

<sup>26</sup> Einstein, 1920, 154; Einstein, 1950; cf. Weinert 2007

<sup>27</sup> Earman, 2002

criticism.<sup>28</sup> From the point of view of the present paper such arguments leave out important considerations concerning the measurement of time in gravitational fields. In particular

we can have no empirical reason to believe such a theory if it cannot explain even the possibility of our performing observations and experiments capable of providing evidence to support it. And in the absence of convincing evidence for such a theory we have no good reason to deny the existence of time as a fundamental feature of reality.<sup>29</sup>

On an empirical level, it is the case that gravitation affects the running of clocks. Clocks at the bottom of a mountain run more slowly than clocks at the top. Although clocks undergo time dilation, due to gravity in the GTR (as they suffer time dilation due to velocity in the STR), it does not follow from the diffeomorphism  $\phi: M \rightarrow N$  that there is no objective passage of time. The question of the measurement of time does not affect the diffeomorphism invariance of the space-time manifolds  $M, N$  but much more mundanely how two observers in two systems subject to gravitation (or acceleration) would (or could) measure the passage of time objectively. Let's consider the question from the point of view of two twins who are each travelling in gravitational fields and who wish to measure the passage of time. Would this be possible? In the light of what has been said before the question boils down to the role of thermodynamic variables in gravitational systems, since the twins cannot rely on their mechanical clocks. Does the invariance of pressure and entropy extend to these systems? By the equivalence theorem of the GTR the accelerating systems are equivalent to gravitational systems, and hence the relation  $\frac{d}{dv}(\partial S) = 0$  should apply in this case. And this is indeed the case.<sup>30</sup> Thermodynamics in curved space-time is the same as in flat space-time, whether it be relativistic or classical. Hence we find the same invariants as we found before. What Misner *et al.* call the 'thermodynamic potentials', i.e.

- $n$  = baryon number density
- $T$  = temperature in rest frame
- $\rho$  = density of total mass-energy
- $p$  = isotropic pressure in rest frame
- $S$  = entropy per baryon in rest frame

are invariant in curved space-time.

Despite the use of rest frame to measure the potentials, the potentials are frame-independent functions (scalar fields). At the chosen event  $P_0$ , a given potential (e.g.  $n$ ) has a unique value  $n(P_0)$ ; so  $n$  is a perfectly good frame-independent function.<sup>31</sup>

Hence two observers in curved space-time find themselves in the same situation as those in flat space-time. And by the above arguments they are able to measure the passage of time, objectively, by the use of invariant features of thermodynamic clocks.

However, it may be objected that we are still only talking about the *passage* of time in local neighbourhoods of curved space-time. Recall the distinction made above between an asymmetry in

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<sup>28</sup> Healey, 2002; Healey, 2003; Maudlin, 2002; Rickles, 2008

<sup>29</sup> Healey, 2002, 308

<sup>30</sup> Misner *et al.*, 1973, Ch. 22

<sup>31</sup> Misner *et al.*, 1973, 558

local regions of space-time (the passage of time) and the whole of space-time (the arrow of time). Local observers would not be able to determine a global arrow of time from asymmetric experiences in their environment. Such observers would not be able to determine whether they find themselves on a closed time-like curve, which loops back to its initial state after reaching a maximum expansion. (Figure 2) On such a curve, by Gödel's argument, there would be no clear distinction between past and future, and hence time would still be unreal. Hence we have to move from the consideration of the *passage* of time to a consideration of the *arrow* of time.

## V.2. Gödel's arguments for the unreality of time

Gödel's arguments for the unreality of time from certain solutions in the GTR may be summarized as follows<sup>32</sup>:

- I. Time is real only if there is an objective lapse of time (an infinite layer of Nows succeeding each other).
- II. Time is real only if space-time admits of a global time function.
- III. There are physically possible worlds, filled with rotating dust (R-universes), which do not admit of global time functions (they display closed time-like curves, CTCs).
- IV. In such physically possible R-universes, time must be unreal since they lack a global time function.
- V. Time must also be unreal in *our* universe since it only differs from the R-universe by its contingent distribution of matter and motion, and hence not by lawlike differences.

As Gödel postulates that the reality of time requires a global time function, he does not distinguish the objective passage of time on a section of a closed time-like curve from the arrow of time in the whole universe. A number of authors have pointed out that we retain what Reichenbach called a 'sectional nature of time'.<sup>33</sup> Even in the R-universe observers will experience the objective passage of time on a section of the total closed time-like curve, which amounts to a rejection of Premise II.

Gödel's space-time is time orientable. This means that locally it is possible to distinguish at every point between the future direction and the past direction, even on a closed timelike curve. Then there is a sense in which locally we would experience a change in existing that is consistent with the relation of chronological precedence even on a closed timelike curve.<sup>34</sup>

A space is also orientable in the sense of Earman's 'time direction heresy', which makes temporal orientation an intrinsic feature of space-time.<sup>35</sup> For the purpose of orientability the direction of time is a matter of convention but not, as Earman accepts, for the purpose of orientation.<sup>36</sup> That is, we encounter the same problem as before with respect to diffeomorphism invariance. What we desire to know is not if space-time is temporally orientable but whether the actual universe has a temporal orientation and this is not established by definition. We can fix a time orientation by fiat, irrespective of physical processes, but temporal orientability of space-time does not imply temporal asymmetry.<sup>37</sup>The

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<sup>32</sup> Gödel, 1949; see Calosi, 2009; Dorato, 2002 for careful reconstructions of Gödel's argument. The main text summarizes their findings. This following sections cover some of the material discussed in Weinert, 2013.

<sup>33</sup> Reichenbach, 1956, §15; Calosi, 2009; Dorato, 2002; Dieks, 2006

<sup>34</sup> Calosi, 2009, 127

<sup>35</sup> Earman, 1974, 20

<sup>36</sup> Earman, 1974, 34

<sup>37</sup> Kroes, 1985, 133 ; Castagnino, 2013, 2499

temporal orientation, if any, of the universe requires physical criteria, like, for instance, energy flows or the evolution of space-time from the Big Bang.<sup>38</sup>

Gödel may admit a sectional nature of time and accept thermodynamic clocks to measure its course. The problem arises not on a local but on a cosmic scale. In cosmological terms Gödel assumes the scenario of a re-collapsing universe or closed time (Figure 1a), in which the final condition of the Big Crunch is physically identical or at least very similar to the initial condition of the Big Bang. But it is an open question whether this scenario is correct, even if we exclude from the consideration a linear accelerated expansion of the universe, which will end in a Big Chill. The Big Chill represents the total dissipation of energy in the universe so that the loss of entropic differentials would render all life impossible. The Big Chill, as a final condition, would be very different from the smooth initial conditions of the universe as they obtained in the Big Bang. The scenario of a Big Bang-Big Chill trajectory would clearly indicate a linear arrow of time for the whole universe.

Therefore Gödel's considerations suffer from the same conceptual bias, which we observed in the discussion of the STR. The problem is not the existence of CTCs; it is that the conclusion – the unreality of time in the universe – does not follow from the existence of CTCs. In order to make reasonable statements about the nature of time we need to take into consideration all *physical* features, which affect the anisotropy of time and observers on a CTC. Thus even if CTCs represented physically real worlds, it would not necessarily follow that time is unreal as long as *all* temporal features of the universe have not been taken into account. One feature concerns the role of entropy in such a universe.<sup>39</sup>

In order to evaluate the arrow of time in a re-collapsing and expanding universe, it may be useful to turn to quantum cosmology (and the notion of decoherence), which is an attempt to understand the boundary conditions of the universe and how the observable classical universe may emerge from quantum conditions of the early universe.<sup>40</sup> Although decoherence is only one of many interpretations of quantum mechanics, and its application to cosmology is tentative at this stage, the notion itself is important enough to throw new light on the anisotropy of time.

## VI. Quantum Cosmology

According to modern quantum cosmology the classical world emerges from a quantum state of the universe through a number of mechanisms, which are generally known as decoherence.

Decoherence leads to the disappearance of superpositions and their typical interference patterns on the quantum level. Decoherence in the cosmological context refers to the emergence of decoherent sets of histories from the underlying quantum world. A history is a time-ordered set of properties. Histories are branch-dependent, i.e. conditional on past events. In this approach branch-dependence replaces the familiar notion of quantum measurement, such that decoherent sets of histories result from coarse-graining. A coarse-grained history, from a physical point of view, reveals the link between

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<sup>38</sup> Aiello et al., 2008; Lehmkuhl, 2012; Ellis, 2013

<sup>39</sup> It could be argued, as a referee pointed out, that the existence of invariant parameters is not sufficient to conclude that time is real, either. This is correct but they represent only one of many features, which serve as criteria to infer the anisotropy of time or at least that the arguments for the 'unreality of time' are not conclusive.

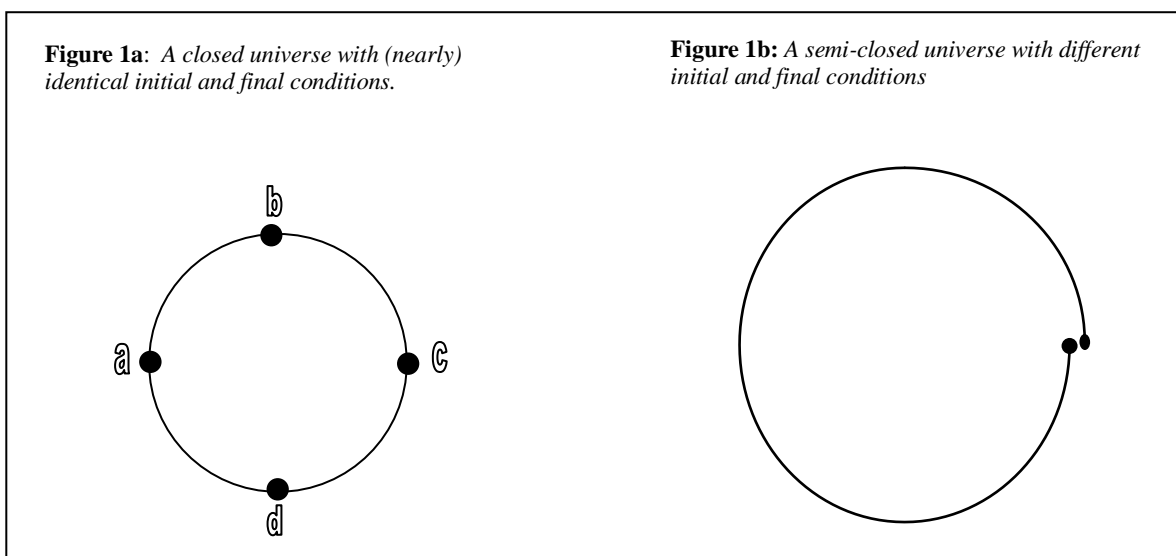
<sup>40</sup> Halliwell et al., 1994; Weinert, 2013, Chapt. IV

decoherence and the existence of indelible records about states of the universe.<sup>41</sup> These decoherent histories, in which quantum superpositions have been suppressed, make space-time (in the classical and relativistic context) a macroscopic phenomenon. According to the decoherence approach, applied to the universe as a whole, which is treated as a closed system,

(s)ets of alternative histories consist of *time sequences* of exhaustive sets of alternatives. A *history* is a particular sequence of alternatives, abbreviated  $[P_\alpha] = (P_{\alpha_1}^1(t_1), P_{\alpha_2}^2(t_2), \dots, P_{\alpha_n}^n(t_n))$ <sup>42</sup>

The classical world is characterized by a certain number of asymmetric processes, even if the classical equations of motion are time-reversal invariant. The asymmetry we observed is usually associated with the Second law of thermodynamics, and low-entropy initial conditions of the universe. But if classical decoherent histories emerge from quantum superpositions through decoherence, then classical trajectories become practically irreversible.

The classical world is generally characterized by entropy increases and many macroscopic phenomena reveal to human observers the passage of time. But how reliable is entropy as an indicator of the *arrow* of time? In terms of entropy, Gödel's assumption that a re-collapsing universe returns to its initial state (or very close to it), means that any current entropic state of the universe would be higher than the initial or the final state. The increase in entropy to the current state could be explained from the point of view of either past or future boundary conditions. But such physically different boundary conditions are usually not considered in CTCs. In such a universe it seems that there would be no clear arrow of time – no global foliation -, which would seem to confirm Gödel's thesis of the unreality of time. However quantum cosmology has added new insights to the question of the arrow of time. Even if we assume that the universe will re-collapse – as in some current cosmological models – it cannot simply be assumed that the universe will return to its initial entropic state. In particular we have to consider two hypotheses, which have been put forward to this effect:



<sup>41</sup> Cf. Gell-Mann and Hartle, 1990, 434; Halliwell, 1994, §3

<sup>42</sup> Gell-Mann and Hartle, 1990, 432, italics in original; cf. Hartle, 2011, Appendix

1) *The role of the Weyl curvature hypothesis.* R. Penrose<sup>43</sup> has proposed that in general relativistic space-times, space-time curvature may experience a gravitational tidal distortion – expressed in the Weyl tensor. The Weyl curvature tensor is approximately zero at the Big Bang but much greater at the Big Crunch. In terms of the topology of time this means that even on a Big Bang – Big Crunch cycle, the final conditions would be very different from the initial conditions and could be represented as an open circle. (Figure 1b) The cosmological conditions would be asymmetric and there would be an arrow of time.<sup>44</sup> Unfortunately, there are several problems with Penrose’s proposal. It necessitates the assumption of asymmetric physics: not only do we have asymmetric boundary conditions, we also get asymmetric laws, like the behaviour of the Weyl curvature tensor, and the Second law of thermodynamics itself.<sup>45</sup> Nor does Penrose’s proposal account for the quantum nature of the initial conditions but rather imposes classical constraints on the initial conditions. The Weyl curvature hypothesis imposes an asymmetric initial condition, rather than deriving the observed asymmetry from a time-symmetric cosmological model.<sup>46</sup> But note that entropy considerations are important in these cosmological scenarios.

2) *The role of entropy in the fate of the future universe.* Hawking’s ‘no-boundary’ proposal at first assumed a time-symmetric evolution of the universe, with boundary conditions imposed at both ends of the evolution. The universe begins in a Big Bang, grows to a maximum expansion, and then re-collapses into a Big Crunch, in real time. (Figure 2) Under such a scenario the arrow of time does not seem to be well defined, especially if we assume that the initial and final conditions are physically similar states. Topologically, this corresponds to a closed universe. At first this model seemed to imply that the thermodynamic arrow of time would have to reverse in the contracting phase of cosmic evolution. If for the time being we adopt the (mistaken) identification of the thermodynamic arrow of time with the increase in disorder (entropy), then a re-collapsing universe seems to imply a flipping of the arrow of time and therefore a decrease in disorder.

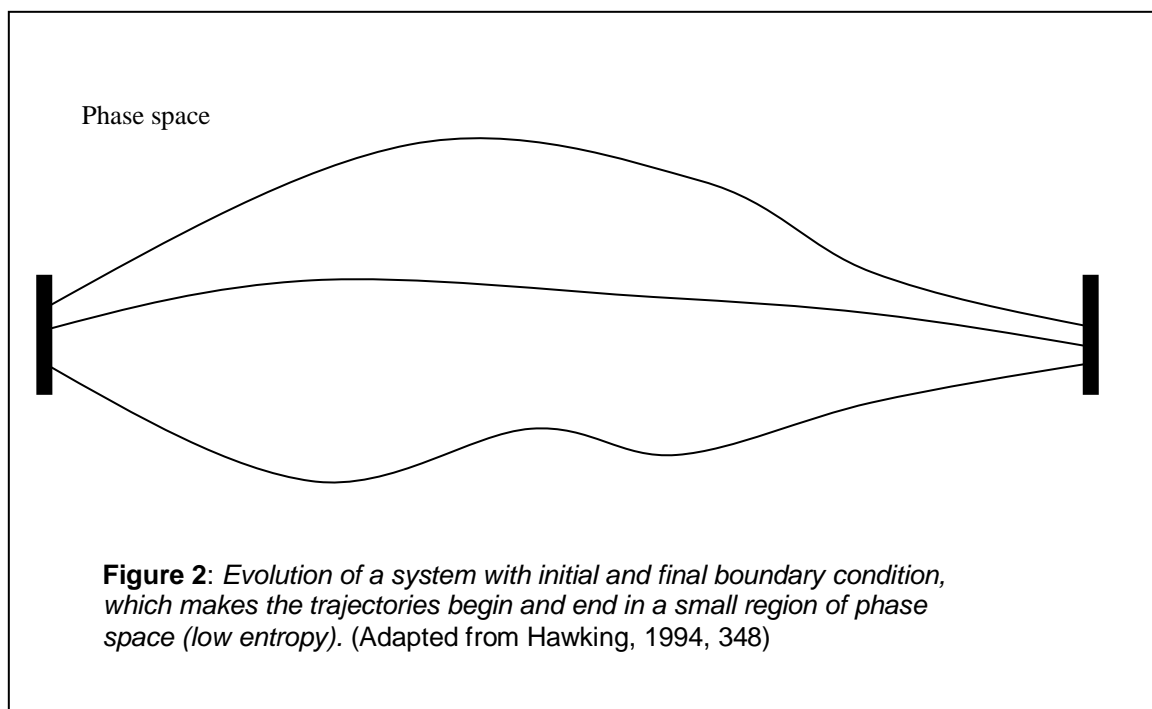
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<sup>43</sup> Penrose, 2005, 765-8

<sup>44</sup> Cf. Gell-Mann and Hartle, 1993, §7

<sup>45</sup> Penrose, 2005, 778; Penrose, 2010, Part I

<sup>46</sup> Halliwell, 1994, 372



At first, I believed that disorder would decrease when the universe re-collapsed. This was because I thought that the universe had to return to a smooth and ordered state when it became small again. This would mean that the contracting phase would be like the time reverse of the expanding phase. People in the contracting phase would live their lives backward: they would die before they were born and get younger as the universe contracted.<sup>47</sup>

A reversal of the thermodynamic arrow of time implies a Gold universe, in which the cosmological arrow drives the thermodynamic arrow. However, there now exist reasonable doubts whether the entropic arrow of time would reverse. First, the expansion of the universe seems to be accelerating, which makes a Gold universe unlikely. Secondly, what actually contracts seems to be the whole of space, not individual arrows in a re-collapsing universe. Even if the universe began to re-contract, galaxies would continue to recede from each other for some considerable time, stars would continue to radiate into the surrounding space and individual systems inside the galaxies would continue their ordinary lives from disequilibrium to equilibrium and things would continue their ordinary careers from birth to death.<sup>48</sup> Inhabitants of a re-collapsing universe would still experience the passage of time. Hawking later corrected his mistake:

The universe does not return to a smooth state in the collapse and the arrow of time continues pointing in the same direction as in the expansion.<sup>49</sup>

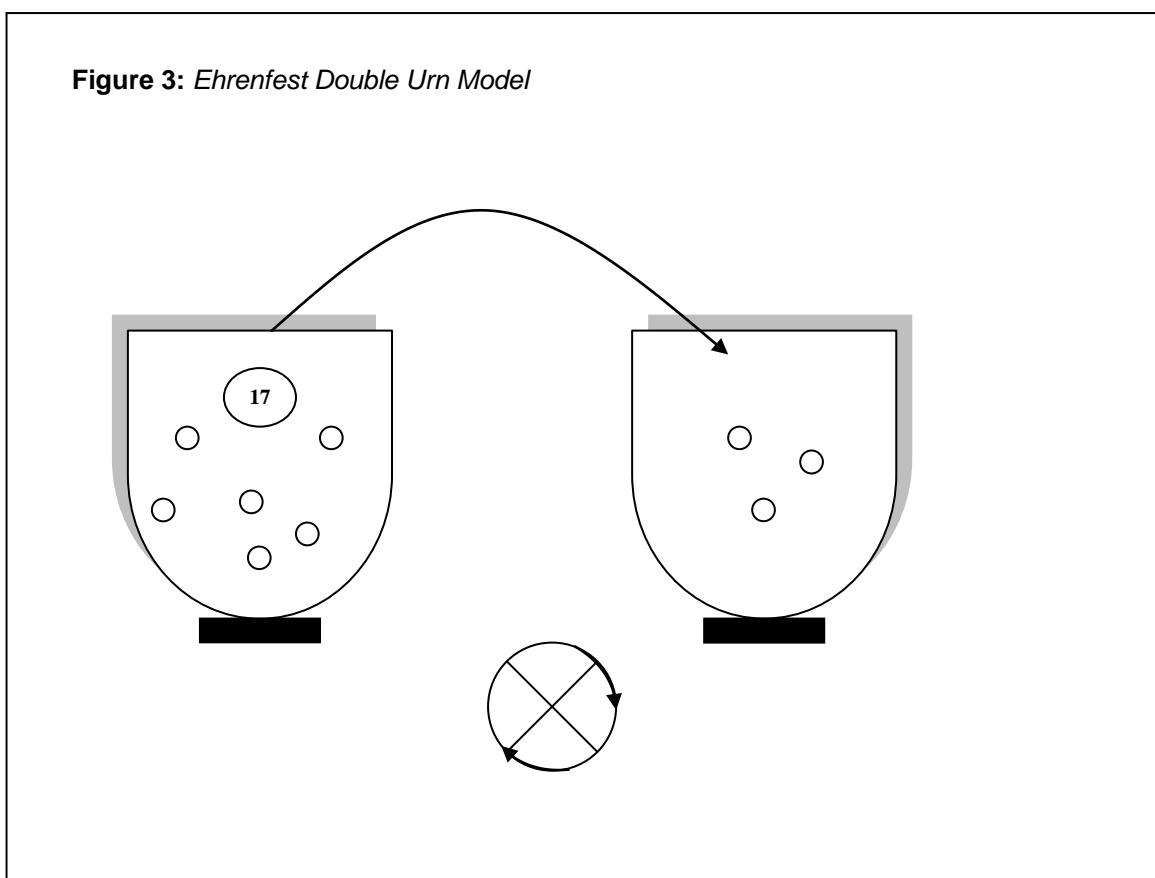
<sup>47</sup> Hawking, 1988, 150

<sup>48</sup> Cf. Davies, 1974; Page, 1985; Penrose, 2005; Carroll, 2010

<sup>49</sup> Hawking and Penrose, 1996, 102-3; cf. Hawking, 1988, 150; Carroll, 2010, 297-8



Hawking's 'no boundary' proposal thus explains the observed asymmetry between the past and the future by an appeal to the difference in the boundary conditions of the universe. The universe starts in smooth conditions in the Big Bang but ends in gravitational collapse in black holes, in real time. Both the Penrose hypothesis and the 'no-boundary' proposal therefore assume an evolution of the universe towards a Big Chill, a state in which even black holes will eventually evaporate and leave nothing but a sea of massless particles, which cannot even form the basis of quantum clocks.<sup>50</sup> Quantum cosmology therefore needs to address the question of the evolution of the universe, given appropriate boundary conditions: whether the initial and the final conditions of the universe are indeed approximately identical, thus leading to the topology of closed time and a 'nice symmetry between the



expanding and contracting phases<sup>51</sup> or whether the two phases end in physically different boundary conditions, leading to the topology of an open circle, and thus an arrow of time. (Figure 1b)

VII. *Cosmological Boundary Conditions.* If boundary conditions have to be taken into account the question arises what happens to the entropy gradients under the assumption a) that the initial and final conditions of the universe are symmetric (two-time boundary universe) and b) that final and initial boundary conditions are asymmetric?

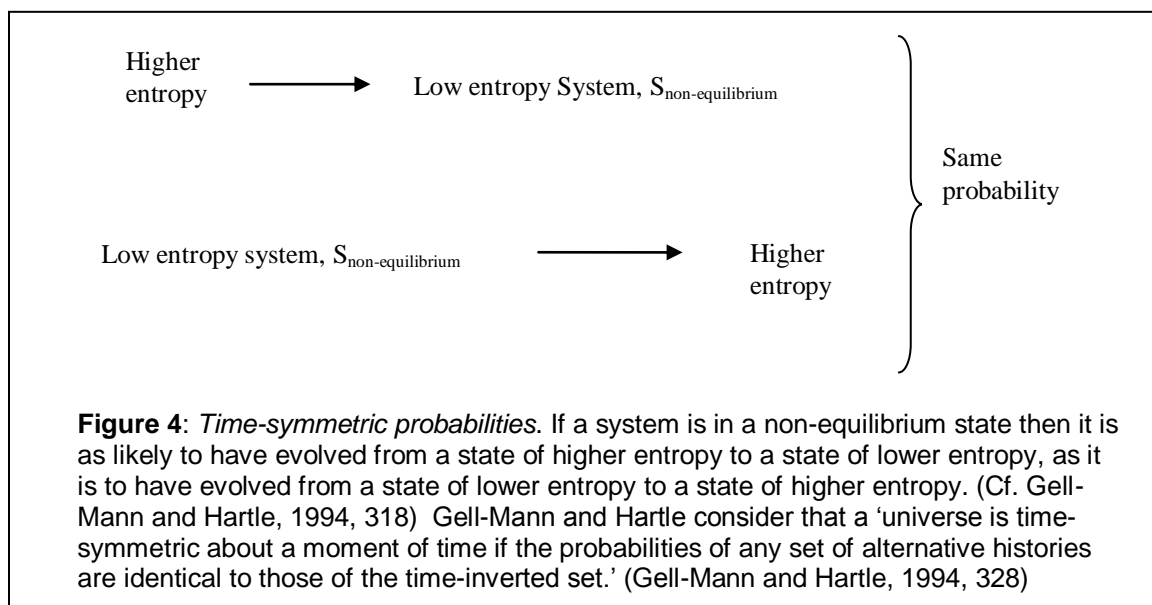
<sup>50</sup> Rugh and Zinkernagel, 2009, 10; cf. Penrose, 2010, 93

<sup>51</sup> Hawking, 1988, 150

A convenient way to model the behaviour of the universe under both conditions is to study the Ehrenfest double urn model.<sup>52</sup> It is a model of two bowls with a numbered set of balls, which are unevenly distributed between them. For instance, there may be 100 balls in the two urns overall, with 75 balls in the left-hand bowl and 25 in the right-hand bowl. (Figure 3)

This system evolves according to a time-symmetric dynamical rule, which is illustrated by the roulette wheel, which randomly selects a number (say 17), which is then moved from the left-hand urn to the right-hand urn. In this situation the initial conditions are unequally distributed, and it is reminiscent of the classic illustration of the mixture of hot and cold water. Such systems will approach equilibrium, as a function of the number of time steps which are allowed. That is, the two liquids will equalize their temperatures and the urns will soon see a 50:50 distribution, if no further conditions are imposed. The entropy of these systems will grow towards their maximum equilibrium value. If we have an asymmetric distribution in the boundary conditions, as the Ehrenfest urn model illustrates, the system evolves ‘normally’ towards equilibrium. The problem is that we assume an asymmetry of boundary conditions from the start, and hence observe the familiar spread of microstates into the available phase space, according to a time-symmetric dynamic rule.

What happens when no boundary conditions are set? If we have a  $t$ -invariant dynamics (without setting any boundary conditions), the relaxation to equilibrium is a time-symmetric process: a current non-equilibrium state is as likely to have evolved from a state of lower as from a state of higher entropy; i.e. probability considerations as such do not justify an arrow of time. (Figure 4)



But what happens if we now consider the evolution of the system under the imposition of boundary (initial and final) conditions (that is assumption a)?<sup>53</sup>

In this scenario it is crucial to consider the relaxation times,  $\tau$  – the time it takes for the system to reach its equilibrium – by comparison to the total lifetime of the system,  $T$ . Let’s start with the normal case when the future condition is far away:  $\tau \ll T$ . In this case, the evolution towards maximum

<sup>52</sup> Gell-Mann and Hartle, 1994, 318-22

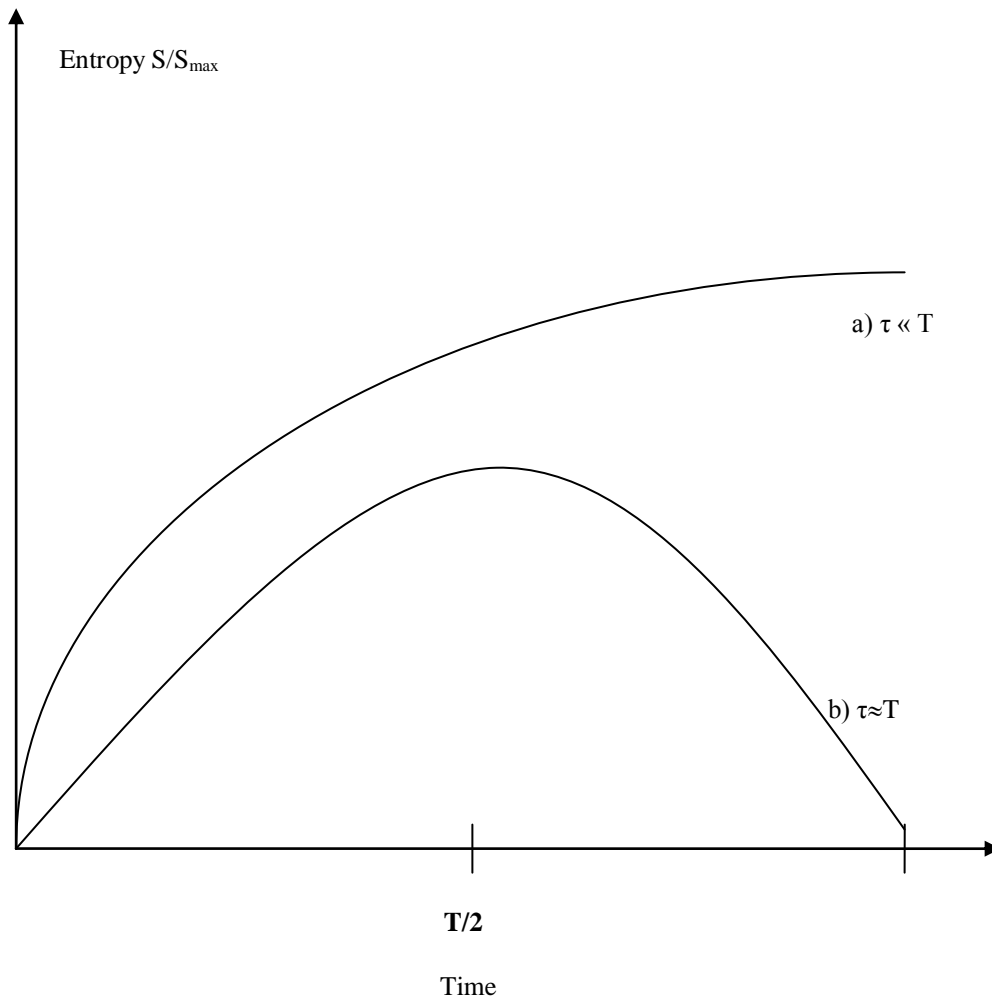
<sup>53</sup> Gell-Mann and Hartle, 1994, §22.4; Schulman, 1997, Ch. 4

equilibrium is typical: the system behaves as if the future boundary condition did not exist. The system behaves 'normally' for all practical purposes, as illustrated in the Ehrenfest urn model. The approach to equilibrium, under the imposition of symmetric boundary conditions, is practically indistinguishable from an approach to equilibrium when no final conditions are imposed. In the words of L. S. Schulman, the future 'is many relaxation times away from the present state.'<sup>54</sup> (Figure 5) In such universes there exists a *de facto* arrow of time.

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<sup>54</sup> Schulman, 1997, 142

**Figure 5:** *Evolution of a system towards equilibrium.* A) When the relaxation time,  $\tau$ , is much smaller than  $T$  (the time difference between initial and final conditions), the system evolves 'normally' towards equilibrium; b) when the relaxation time is closer to the total cosmological time, the final boundary conditions may leave their mark on the evolution of the system. (Schulman 1997, 321)



However, the situation changes when the relaxation time of the system,  $\tau$ , is comparable to the time difference between initial and final conditions. In cosmological terms this means that the final condition is as far away from the present condition as the initial condition and the system's final condition is in a low-entropy state, similar to the initial condition ( $t=0$ ). When this time-symmetric boundary condition is imposed, the evolution of the system deviates significantly from the unconstrained typical approach to equilibrium. The behaviour of the system is time-symmetric around  $T/2$ . (Figure 5) That means that the maximum point of entropy can be explained as arising either from the final condition or from the initial condition. Note that even though the ensemble of systems displays on average a time-symmetric evolution towards equilibrium if time-symmetric boundary conditions are imposed, individual histories may still deviate from this pattern.<sup>55</sup>

The two-time boundary conditions work on the minimal assumption that both initial and final conditions are far from equilibrium. In terms of the topology of time, this minimal assumption leaves open two possibilities. If initial and final conditions are (approximately) identical, we have closed time, since the universe re-collapses to a Big Crunch, which is similar in conditions to a Big Bang. However, if the Big Crunch is not similar in physical conditions to the Big Bang, we have the topology of an open circle, in which the physical conditions at the beginning of the temporal universe do not coincide with its end. However these possibilities now crucially depend on how long the relaxation time,  $\tau$ , is in comparison with the cosmological time,  $T$ . At the present moment we have no empirical data, which would suggest that a final condition exists, which may have an effect on the evolution of the current condition.

In the absence of some compelling theoretical principle mandating time symmetry, the simplest possibility seems to be the usually postulated universe where there is a fundamental distinction between past and future – a universe with a special initial state and a final condition of indifference with respect to state.<sup>56</sup>

According to current cosmology, the final condition is expected to result in the evaporation of black holes in  $10^{100}$  years.<sup>57</sup> If physical time is associated with physical change, then in a re-collapsing universe, in which initial and final conditions are markedly different, as modern cosmology seems to affirm, a global arrow of time emerges in a similar fashion as in the evolution of the universe towards a 'heat death'.<sup>58</sup>

Given these considerations, it is by no means certain that from a physically possible Gödelian R-universe it can be inferred that 'time is unreal' in our universe. Even a time-symmetric model must take into account the boundary conditions of the universe, since otherwise the time-symmetric probabilities impose an atemporal view by default. Gödel assumed that if 'time is real then change is real'.<sup>59</sup> In a re-collapsing universe with the topology of an open circle, the condition 'initial condition  $\neq$

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<sup>55</sup> Gell-Mann and Hartle, 1994, 320; Reichenbach, 1956, §15

<sup>56</sup> Gell-Mann and Hartle, 1994, 335-7; cf. Schlegel, 1968

<sup>57</sup> We exclude from these considerations the attempts in modern cosmology to account for the low-entropy initial conditions of our universe. Multiverse scenarios (bouncing and cyclic universes) are still at a speculative stage.

<sup>58</sup> If we live in a universe with asymmetric boundary conditions, in which entropy even in curved space-time remains invariant, then an entropy gradient must exist, which would allow inhabitants of such a universe to define a global arrow of time. (Schlegel, 1968, 150) This suggestion is reminiscent of the existence of 'cosmic time', which provides a definition of a global foliation of space-time, at least for universes, which satisfy the conditions of homogeneity and isotropy. See Misner *et al.*, 1973, §27.3; cf. Wald 1984, §§5.1, 8.3

<sup>59</sup> Cf. Dorato, 2002, 259

final condition' obtains, and hence there is 'real change'. (Figure 1b) This change can be inferred from the difference in the entropy gradient between initial and final conditions. Gödel argues that the space-time of the R-universe is chronologically vicious since

for every point  $p \in M$  there is a closed timelike curve, i.e. a timelike curve whose tangent vectors at every point are always timelike and future directed which comes back to  $p$ .<sup>60</sup>

If the considerations in this paper are correct, then the mere existence of CTCs, as possible solutions to Einsteins' field equations, are not sufficient to establish the unreality of time. One lesson to derive from quantum cosmology is that mere symmetry considerations are not sufficient to answer questions about the nature of time. Although the boundary conditions of the actual universe are a contingent matter from a theoretical point of view, they cannot be neglected, since time-symmetric probabilities are not sufficient to provide significant information about time in the actual universe. If we accept, following quantum cosmology, that even in a recollapsing universe the entropy gradient will never 'flip', then observers in such a universe will be able to detect a locally 'forward' moving passage of time; they may also be able to detect an arrow of time, if  $\tau \ll T$ , which seems to be the case in our actual universe. Both of these observations will be frame-invariant and may lead to a questioning of the Parmenidean version of the block universe.

#### VIII. Conclusion

In order to determine the 'nature' of time conceptual inferences from the principles of the STR and the GTR may not suffice to support the view of the 'unreality' of time. A focus on thermodynamic systems and the behaviour of the universe under the assumption of appropriate entropic boundary conditions certainly suggest indicators for the objective passage of time. In both the STR and the GTR frame-invariant clocks exist, which would allow observers to agree, objectively, on the lapse of time. And the actual universe seems to be characterized by asymmetric boundary conditions, which determine its actual behaviour. Hence if the relaxation time,  $\tau$ , is much shorter than the lifetime of our universe,  $T$ , which current calculations of the evaporation of black holes seem to suggest ( $10^{100}$  years), then entropic consideration are again of great importance. Given that entropy is frame-independent even in the cosmological context, observers should be able to establish a frame-invariant 'arrow' of time. Overall, then, it is possible to infer temporal arrows in space-time.<sup>61</sup>

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<sup>60</sup> Calosi, 2009, 119

<sup>61</sup> The author would like to thank an anonymous referee for constructive and helpful comments on an earlier draft of this paper.

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