Galilei invariance and the *welcher Weg* problem

Contribution to the Foundations of Physics Conference, Munich,

29–31 July 2013

R N Sen*

Department of Mathematics
Ben-Gurion University of the Negev
84105 Beer Sheva, Israel
E-mail: rsen@cs.bgu.ac.il

15 July 2013

Abstract

If it is possible to detect the path of an atom in a double-slit atom interferometer, then interference will not be observed even if path detection causes no change in the atom's momentum. This fact was explained by Scully et al and Dürr et al by using a principle of complementarity that could not be formulated mathematically, together with a rather peculiar notion of entanglement. We provide an explanation based on considerations of Galilei invariance, without recourse to any notion of complementarity or entanglement. However, we do have to assume that, under conditions of the experiment, composite systems such as atoms and molecules may be regarded as single particles. This assumption is the basis of modern atom and molecule interferometry, but it lacks justification from the first principles of quantum mechanics. In our world-view the *welcher Weg* problem merely highlights this fact. The empirical validity of this assumption seems to reveal an aspect of quantum mechanics which can justly be termed anti-reductionist, and we conclude that use of unnecessarily strong hypotheses to explain observed phenomena can hide important physical – and occasionally philosophical – problems.

*Retired

1 Prologue

One of the reviewers of my abstracts wrote: "It is unclear what the question is, therefore it is unclear what the answer is, and it is unclear what the point of all of that is." I have added this prologue and an epilogue in the hope that my audience will arrive at different conclusions.

There were several lacunae in the axiomatization of quantum mechanics proposed by von Neumann in his book *Mathematische Grundlagen der Quantenmechanik* [12]. One was that its relativity group was undefined (time did not enter into the construction of the Hilbert space), and mass remained an ad hoc parameter. These gaps were filled by Bargmann's 1954 paper on ray representations [1]. Another flaw, first pointed out by Wigner [13] in 1963 (I am referring only to published papers) was that collapse of the state vector was merely postulated, without any discussion of interactions that could be responsible for it. An interaction operator had to be self-adjoint, but (particularly in many-particle systems) there were far too many self adjoint operators.[1] In relativistic quantum field theory, interactions between fields are limited by two requirements: (i) locality (which means that only a single space-time argument should appear in the interaction density, as in the operator $ie\,\bar{\psi}(x)\gamma_\mu\psi(x)A_\mu(x)$ of QED), and (ii) relativistic invariance; as Fermi showed in his beta-decay paper (1933), these principles were quite restrictive. In nonrelativistic quantum theory the locality restriction does not apply – indeed, most many-particle observables can be said to be inherently nonlocal – and the principle of Galilei invariance may be of limited use when a particle interacts with a (large) measuring apparatus. These facts suggest that limitations on allowed interactions would have been a subject of lively discussion in nonrelativistic quantum mechanics, at least after the publication of Bargmann's 1954 paper; this discussion is yet to begin.

Although my talk has nothing to do with measurement theory itself, the world-view on which it is based is, at least in part, a reaction to chapter VI of von Neumann's book. It was in this chapter (on 'the measurement process') that von Neumann introduced the observer's *conscious ego* into the measurement process, effectively declaring that the quantum measurement problem was a philosophical rather than a scientific problem.[2] He justified it by invoking the 'principle of psycho-physical parallelism', overriding one of the basic tenets of mathematical research, namely that results should be established under the weakest possible conditions. The conscious ego

[1] See, for example, [9], chapter 9.

[2] I am using the terms *scientific* and *philosophical* as they were understood by Bertrand Russell in his *History of Western Philosophy*; he described philosophy as the no-man's land between definite knowledge, i.e., science, on the one hand, and dogma, on the other.

hypothesis was a spectacular rejection of this tenet. In my opinion, the precedent it set for physics was profoundly negative, and its ramifications are being felt to this day.

Under the imprimateur of von Neumann, attention was diverted from real problems of physics. In the case of measurement theory, the real problems lay in the measurement interaction, as suggested by Wigner in 1963 and substantiated brilliantly by Sewell [11] in 2005.[3] In the *welcher Weg* problem, loss of interference was explained by Scully, Englert and Walther (hereafter SEW) [8] and Dürr, Nonn and Rempe (hereafter DNR) [2] using a 'principle of complementarity' that floated above any mathematical formulation. It seems to me that this 'principle' also diverts attention from a very important physical problem,[4] which is: how does quantum mechanics, in which de Broglie's wave-particle duality is subsumed under the superposition principle, explain the fact that wave-particle duality is not limited to elementary systems such as electrons (Davisson and Germer, 1927) but, under certain conditions, applies equally to composite systems such as atoms and molecules (Estermann and Stern, 1930; helium atoms)? The same question must be answered to justify the application of Galilei invariance considerations to composite systems such as atoms and molecules.

Philosophically, the problem may be said to reveal the existence of an essential anti-reductionist component in nonrelativistic quantum mechanics.

2 The *welcher Weg* problem

The welcher Weg problem appears to have been suggested by the *Feynman Lectures on Physics.* In Chapter 1 of Part III of these Lectures [5], Feynman prepares his undergraduate audience for the shock of quantum mechanics by discussing two double-slit gedankenexperiments with electrons. The first experiment is the standard one; an interference pattern is gradually built up as electrons strike the detection screen. In the second experiment, 'which way' an electron takes is determined by shining a light beam on it at the slits. The pattern that builds up now no longer shows interference. The loss of interference is explained by the (position-momentum) uncertainty principle. Feynman goes on to affirm that:

> "If an apparatus is capable of determining which hole the electron goes through, it *cannot* be so delicate that it does not disturb

[3] A textbook-level discussion of Sewell's theory may be found in [9]; Bohr would have approved, but Wigner would probably have been aghast at Sewell's solution.

[4] To be fair to the authors of [8, 2], I must add that it has seldom been regarded as a problem, let alone an important one.

> the pattern in an essential way. No one has ever found (or even thought of) a way around the uncertainty principle."

However, in 1991 Scully, Englert and Walther (hereafter SEW) seemed to controvert Feynman's assertion. They began as follows:

> "We say that two observables are complementary if precise knowledge of one of them implies that all possible outcomes of measuring the other are equally probable."

The term *observable* itself was left undefined. They then stated their 'Principle of Complementarity':

> "For each degree of freedom the dynamical variables are a pair of complementary observables"

and added (apparently as a self-evident truth) that

> "As is true for all physical principles, the actual mechanisms that enforce complementarity vary from one experimental situation to another."

They then claimed that

> "...we have found a way around this position-momentum uncertainty obstacle...That is, we have found a way...to obtain which-path or particle-like information without scattering or otherwise introducing large uncontrolled phase factors into the interfering beams."

Finally, they asserted that, in their Gedankenexperiment (in which interference is not seen),

> "The principle of complementarity is manifest although the position-momentum uncertainty relation plays no role."

These assertions seem very problematical to me. SEW's principle of complementarity would seem to leave no room for different 'enforcers' unless their notions of 'degrees of freedom' and 'dynamical variables' are quite different from those of standard quantum mechanics. 'Path' and 'fringe visibility', the complementary observables used by SEW and DNR, are not representable by self-adjoint operators on Hilbert space.[5] SEW were proposing to adjoin, to standard quantum mechanics, a principle which cannot be formulated mathematically – like von Neumann's conscious ego hypothesis. However, such a drastic departure from standard quantum mechanics is not needed to explain the loss of interference either in the SEW gedankenexperiment or in the one actually carried out by DNR. Standard quantum mechanics – by which I mean von Neumann's Hilbert space formulation plus Galilei invariance – is perfectly adequate, if the applicability of wave-particle duality to composite systems can be justified theoretically.

In the next three sections, we shall sketch the considerations of SEW, the experiment of DNR, and the Galilei invariance explanation of the observations of DNR.

3 The gedankenexperiment of Scully, Englert and Walther

The experimental scheme of SEW is a double-slit interferometer modified by the placement of a resonant cavity just before each slit, shown schematically in Fig. 1. The incoming particles are rubidium atoms in the long-lived Rydberg state $63p_{3/2}$ (denote it $|a\rangle$). This state is higher, by ~ 21 GHz, than the states $61d_{3/2}$ and $61d_{5/2}$ (denote either of these by $|b\rangle$). The two cavities, each tuned to resonate at 21 GHz and prepared in the zero-photon state, together constitute the which-way detector. The two paths are marked 1 and 2 in the figure. Formulae (1) and (2) are taken from the section on *Gedanken experiments illustrating complementarity* in [8].

If the resonant cavities are not present, the state vector of the atom, after it emerges from the double slit, will be will be given by Eq. (4) of [8], namely (we adhere to their notation):

$$\Psi(\boldsymbol{r}) = \frac{1}{\sqrt{2}}[\psi_1(\boldsymbol{r}) + \psi_2(\boldsymbol{r})]|i\rangle. \tag{1}$$

In (1), $\boldsymbol{r}$ is the coordinate of the centre of mass and $|i\rangle$ the internal state of the atom. The subscripts 1 and 2 refer to paths 1 and 2 (Fig. 1). The atom is prepared in the state $|a\rangle$ and makes the transition $|a\rangle \to |b\rangle$ in the

[5]Dürr and Rempe have proposed operator representations for these quantities [3]. In my opinion their assignments are arbitrary. If $A_\alpha = q+\alpha p$ and $B_\beta = q+\beta p$, then $[A_\alpha, B_\beta] = iI$ for $\beta - \alpha = 1$. There are uncountably many such pairs of operators (A_α, B_β), but they do not have useful interpretations. It is not easy to interpret self-adjoint operators, or to find a self-adjoint operator for a given physical quantity (see [9], chapter 9). Wigner spent decades looking for position operators, without arriving at definitive answers.

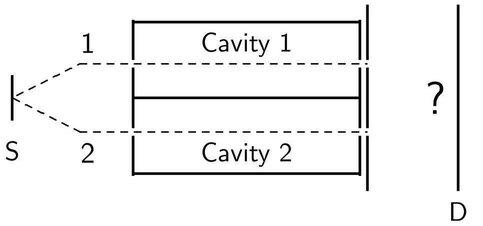

Figure 1: Scheme of the SEW *gedankenexperiment*

which-way detector when the latter is present. When the atom has emitted a photon in one of the cavities, (1) changes to (Eq. (6) in [8])

$$\Psi(\boldsymbol{r}) = \frac{1}{\sqrt{2}}[\psi_1(\boldsymbol{r})|1_1 0_2\rangle + \psi_2(\boldsymbol{r})|0_1 1_2\rangle]|b\rangle, \tag{2}$$

where $|0_1 1_2\rangle$ is the state of the which-way detector with no photon in cavity 1 and one photon in cavity 2, and similarly for $|1_1 0_2\rangle$. The authors write that: "Please note that unlike (4) this $\Psi(\boldsymbol{r})$ is not a product of two factors, one referring to the atomic and the other to photonic degrees of freedom. The system and the detector have become entangled by their interaction." The orthogonality condition $\langle 0_1 1_2|1_1 0_2\rangle = 0$ will now cause the interference term in this $|\Psi(\boldsymbol{r})|^2$ to vanish.

The validity of the representation (2) of $\Psi(\boldsymbol{r})$ may be disputed, but I shall not enter into this dispute because (2) is *not* used by DNR to interpret the results of the experiment they actually performed.

4 The experiment of Dürr, Nonn and Rempe

In 1998, Dürr, Nonn and Rempe performed the actual experiment [2]. Their experiment was somewhat different from the one suggested by SEW, but their findings agreed with the latter's expectations; the mere possibility of detection of the path taken by an atom resulted in loss of interference, and the effect could not be explained by the position-momentum uncertainty relation. The title that DNR gave to their paper was "Origin of quantum-mechanical complementarity probed by a 'which-way' experiment in an atom interferometer".

The ground state of the ^{85}Rb atom splits into two hyperfine states with total angular momenta $F = 2$ and 3. A microwave field of $\sim$3 GHz induces Rabi oscillations between the two. A standing light wave will split a beam of

^{85}Rb atoms into two (Kapitza-Dirac effect), and will also – if its frequency is right – shift the phase of the $F = 2$ component of the refracted beam by π. By a skilful use of these mechanisms, DNR produced two parallel atomic beams, one in the state $F = 2$ and the other in the state $F = 3$, thus encoding which-way information into the internal state of the atom.

According to DNR, states of the atom are vectors in a Hilbert space

$$\mathfrak{H} = \mathfrak{H}_{\mathsf{cm}} \otimes \mathfrak{H}_{\mathsf{inn}}, \tag{3}$$

whare $\psi \in \mathfrak{H}_{\mathsf{cm}}$ is a state of the centre of mass, and $f_k \in \mathfrak{H}_{\mathsf{inn}}$ an inner state of the atom. The state vector of the initial beam is $\Psi^{\mathsf{I}} = \psi^{\mathsf{I}} \otimes f_2$; that of the (split) final beam is

$$\Psi^{\mathsf{F}} = \frac{1}{\sqrt{2}} \left(\psi^{\mathsf{A}} \otimes f_2 + \psi^{\mathsf{B}} \otimes f_3 \right). \tag{4}$$

In (4), the superscripts A and B denote final-state beams which interfere in the far-field region. However, the interference term in $(\Psi^{\mathsf{F}}, \Psi^{\mathsf{F}})$ vanishes, because $(f_2, f_3) = 0$. DNR assert that "precisely the same entanglement that was required to store the which-way information is now responsible for the loss of interference".

Formula (4) is a 'reduced' description of the final state of the atom. We shall offer an alternative description based on Galilei invariance, one which carries a little more information, and as a result makes a testable prediction that is not made by (4).

5 Using Galilei invariance

A nonrelativistic particle of mass m is described by a ray representation of the Galilei group G, or equivalently by a true representation of (its one-parameter) extension $\tilde{G}_m$.[6] The centre of the Lie algebra of $\tilde{G}_m$ is spanned the operators

$$\begin{aligned} M &= mI \\ \boldsymbol{S}^2 &= \left(\boldsymbol{J} - \frac{1}{m} \boldsymbol{K} \times \boldsymbol{P} \right)^2 \\ U &= H - \frac{\boldsymbol{P}^2}{2m}. \end{aligned} \tag{5}$$

[6]The basic results on ray representations of the Galilei group were already contained in Bargmann's paper [1]. A more detailed exposition was given by Lévy-Leblond in [7]. An adequate account will be found in [9], chapter 7.

Here I is the identity operator and $H, \boldsymbol{P}, \boldsymbol{J}$ and $\boldsymbol{K}$ are the generators of time translations, space translations, rotations and boosts respectively. An irreducible unitary ray representation $\Delta_{m,\sigma,u}$ of G is labelled by the mass m, spin σ and internal energy u. The zero of internal energy is arbitrary. The Hilbert space of this representation is

$$\mathfrak{H} = L^2(\boldsymbol{p}, \mathrm{d}\boldsymbol{p}) \otimes \mathfrak{H}_\sigma \otimes \mathfrak{H}_u \tag{6}$$

where $\mathfrak{H}_\sigma$ is $2\sigma + 1$-dimensional and carries the irreducible representation D_σ of the three-dimensional rotation group. $\mathfrak{H}_u$ is one-dimensional, and consists of scalar multiples of a character $\chi(u)$. Let $\{\psi_k\}, k = 1, 2, \ldots$ be an orthonormal basis for $L^2(\boldsymbol{p}, \mathrm{d}\boldsymbol{p})$. The vectors $\psi_k \otimes \varphi_{\sigma,\sigma_z} \otimes \chi(u)$ form a base for $\mathfrak{H}$. We shall denote this state by $\psi_{\sigma;u}(\boldsymbol{p})$, absorbing the subscript k in the symbol ψ and omitting the label σ_z.

In this description, incoming atoms in the DNR experiment are prepared in a state $\psi^{\mathsf{I}}_{2,u(2)}(\boldsymbol{p})$. If the which-way detector is not present, the final state would be

$$\Psi^{\mathsf{F}} = \psi^{\mathsf{A}}_{2,u(2)}(\boldsymbol{p}) + \psi^{\mathsf{B}}_{2,u(2)}(\boldsymbol{p}). \tag{7}$$

Both terms on the right belong to the representation $\Delta_{m,2,u(2)}$; the beam-splitter affects only the centre-of-mass motion. If the which-way detector is present, the second term on the right of (7) will be $\psi^{\mathsf{B}}_{3,u(3)}$, which belongs to a representation $\Delta_{m,3,u(s)}$. This representation is *not* equivalent to the $\Delta_{m,2,u(2)}$. The which-way detector *has changed the representation of* G in the path B. Therefore (7) will have to be replaced by the direct sum

$$\Psi^{\mathsf{F}}_{\mathsf{ww}} = \psi^{\mathsf{A}}_{2,u(2)}(\boldsymbol{p}) \oplus \psi^{\mathsf{B}}_{3,u(3)}(\boldsymbol{p}). \tag{8}$$

As a result, the substates in the two paths will be

$$\begin{aligned} \Psi^{\mathsf{A}} &= \psi^{\mathsf{A}}_{2,u(2)}(\boldsymbol{p}) \oplus 0 \\ \Psi^{\mathsf{B}} &= 0 \oplus \psi^{\mathsf{B}}_{3,u(3)}(\boldsymbol{p}) \end{aligned} \tag{9}$$

and the inner product $(\Psi^{\mathsf{A}}, \Psi^{\mathsf{B}})$ will *always* vanish, irrespective of how much momentum is transferred in the path detection process; the position-momentum uncertainty relation is absent from the theatre of war, but *so are the concepts of entanglement and complementarity.*

If now a quantum eraser is used to delete the which-way information in (8), the final state should be

$$\Psi^{\mathsf{F}}_{\mathsf{qe}} = \psi^{\mathsf{A}}_{f,u(f)}(\boldsymbol{p}) \oplus \psi^{\mathsf{B}}_{f,u(f)}(\boldsymbol{p}), \tag{10}$$

where $f = 2$ or $f = 3$. This state belongs to the direct sum of *two copies of the same representation* of $\tilde{G}_m$. According to (9), the orthogonality relation $(\Psi^{\mathsf{A}}, \Psi^{\mathsf{B}}) = 0$ will continue to hold, i.e., *there should still be no interference*; there appears to be no a priori reason to expect that erasure of which-way information will collapse the direct sum in (8) into a single representation, as in (7).

This prediction, which should be experimentally testable, is at variance with the explanation offered by SEW and DNR. Additionally, if in the SEW gedankenexperiment the two cavities are prepared in coherent states, *no interference should be observed, although which-path information will be impossible to obtain.*

6 Epilogue

The explanation offered in the last section is based on the implicit assumption that *a bound state of n particles with total mass m can, under certain conditions, be described as a single particle corresponding to an irreducible unitary representation of* $\tilde{G}_m$. This assumption is amply verified by experiment – the whole subject of atom and molecule interferometry is based upon it – but has not yet been substantiated from the basic principles of quantum mechanics. In other words, the conditions under which it holds have not been elucidated. One may note that under these conditions the distinction between 'elementary' and 'composite' loses much of its relevance in nonrelativistic quantum mechanics.

The problem of elucidation of these conditions is highly nontrivial. One difficulty is that the key symmetry group of an atom in its ground state is the rotation group around the nucleus, whereas that of a free particle is the full Galilei group.[7] An atom in an excited state will generally be unstable, and will not therefore be describable by a representation of the full Galilei group; it *may* be describable by a representation of the Galilei *semigroup*, in which time translations are not invertible. (See [10], or [9], chapter 12.) If it is, theorists and experimentalists will have to answer, independently, the question whether wave-particle duality applies to unstable particles in the nonrelativistic domain.

[7]Broken and unbroken symmetries can sometimes be accommodated together. One possibility, using Hilbert bundles, is sketched in [9], chapter 12, and the references cited there.

Since 1999, the Vienna group has performed several beautiful interference experiments with fullerene molecules. In one of them the temperature of the source could be varied, and it was found that the fringes became weaker and weaker as the temperature of the source was increased; the molecules lost some energy by emission of photons on the way to the interferometer. Interpreting the progressive loss of interference as the *quantum-classical transition*, Hackermüller et al wrote [6] in 2004 that:

> "In summary, we have presented conclusive empirical and numerical evidence for observation of the quantum-to-classical transition of a material object caused by its own emission of thermal radiation."

This conclusion may be premature. Interference fringes will be observed only as long as the molecule interacts with the interferometer *as if* it were a single quantum-mechanical particle. If interference is not observed, one may only infer that the molecule is *not* interacting with the interferometer as if it were a single quantum-mechanical particle. This condition may be necessary for a classical description to be valid, but the discussion in the first part of this Epilogue suggests that its sufficiency remains to be established.

So, to answer the referee, the point of all of that is that if one makes unnecessarily strong hypotheses to explain observed phenomena, one is likely to miss important physical problems, and sometimes philosophical ones as well (as mentioned at the end of the Prologue).

References

[1] Bargmann, V (1954). On unitary ray representations of continuous groups, *Ann Math* **59**, 1–46.

[2] Dürr, S, Nonn, T and Rempe, G (1998). Origin of quantum-mechanical complementarity probed by a 'which-way' experiment in an atom interferometer, *Nature*, **395**, 33–37.

[3] Dürr, S and Rempe, G (2000). Can wave-particle duality be based on the uncertainty relation? *Am J Phys* **68**, 1021–1024.

[4] Englert, B-E (1996). Fringe visibility and which-way information: an inequality, *Phys Rev Lett* **77**, 2154–2157.

[5] Feynman, R P, Leighton, R B and Sands, M (2006). *The Feynman Lectures on Physics, Definitive Edition.* Vol III, Quantum Mechanics, San Francisco: Pearson Addison Wesley.

[6] Hackermüller, L, Hornberger, K, Brezger, B, Zeilinger, A and Arndt, M (2004). Decoherence of matter waves by thermal emission of radiation, *Nature* **427**, 711–714.

[7] Lévy-Leblond, J-M (1963). Galilei group and nonrelativistic quantum mechanics, *J Math Phys* **4**, 776–788.

[8] Scully, M O, Englert, B-G and Walther, H (1991). Quantum optical tests of complementarity, *Nature* **351**, 111–116.

[9] Sen, R N (2010). *Causality, Measurement Theory and the Differentiable Structure of Space-Time*, Cambridge: Cambridge University Press.

[10] Sen, R N and Sewell, G L (2002). Fiber bundles in quantum physics, *J Math Phys* **43**, 1323–1339.

[11] Sewell, G L (2005). On the mathematical structure of the quantum measurement problem, *Rep Math Phys* **56**, 271–290.

[12] von Neumann, J (1932). *Mathematische Grundlagen der Quantenmechanik* (Berlin: Julius Springer). English translation (revised by the author): *Mathematical Foundations of Quantum Mechanics*, 1955 (Princeton: Princeton University Press).

[13] Wigner, E P (1963). The problem of measurement, *Amer J Phys* **31**, 6-15.