§ 6.5 相对论电动力学

1.学习目标

了解四维电荷守恒、达朗伯方程

了解迈氏方程的协变形式

2.复习和思考

电荷守恒、达朗伯方程、迈氏方程

四维速度、特殊的洛仑兹变换

上节简介

$$x_1'^2 + x_2'^2 + x_3'^2 - c^2t'^2 = x_1^2 + x_2^2 + x_3^2 - c^2t^2 = \text{不变量}$$

$$a = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$\left.\begin{aligned} A_1' &= \gamma(A_1 + i\beta A_4) \\ A_2' &= A_2 \\ A_3' &= A_3 \\ A_4' &= \gamma(A_4 - i\beta A_1) \end{aligned}\right\}$$

$$u_\mu = \frac{\mathrm{d}x_\mu}{\mathrm{d}\tau}$$

定义为四维速度矢量

$$u_\mu = (\gamma v_i, \gamma ic) = \gamma(v_i, ic)$$

3. 四维波矢量

$$\sum : e^{i\phi} \qquad \phi = \vec{k}\cdot\vec{x} - \omega t$$

$$\sum{}' : e^{i\phi'} \qquad \phi' = \vec{k}'\cdot\vec{x}' = \omega' t'$$

假设:$\sum$、$\sum'$在$t = t' = 0$时重合，此时$\phi = \phi' = 0$

可以证明：$\phi = \phi' = const$

$$\therefore \vec{k} \cdot \vec{x} - \omega t = \vec{k}' \cdot \vec{x}' - \omega' t' = const$$

$$由 x_4 = ict \Rightarrow \begin{cases} k_4 \cdot x_4 = -\omega t \\ k_4 = \dfrac{i\omega}{c} \end{cases} \Rightarrow k_\mu = (\vec{k}, i\frac{\omega}{c})$$

$$\therefore k_\mu' x_\mu' = k_\mu x_\mu = const$$

在特殊的洛仑兹变换下，$k'_{\mu} = \alpha_{\mu\gamma} k_{\gamma}$

$$\begin{cases} k_1' = \gamma(k_1 - \frac{v}{c^2}\omega) \\ k_2' = k_2 \\ k_3' = k_3 \\ \omega' = \gamma(\omega - v k_1) \end{cases} \Leftarrow \begin{cases} k_1' = \gamma(k_1 + i\beta k_4) \\ k_2' = k_2 \\ k_3' = k_3 \\ k_4' = \gamma(k_4 - i\beta k_1) \end{cases}$$

若：$k_1 = \frac{\omega}{c}\cos\theta, k_1' = \frac{\omega'}{c}\cos\theta'$

($\vec{k}$与x成θ，$\vec{k}'$与x'成θ')

$$k_1' = \gamma(k_1 - \frac{v}{c^2}\omega) \qquad \omega' = \gamma(\omega - v k_1)$$

若： $k_1 = \dfrac{\omega}{c}\cos\theta, k_1' = \dfrac{\omega'}{c}\cos\theta'$

$$\omega' = \omega\gamma(1 - \frac{v}{c}\cos\theta)$$

$$\frac{\omega'}{\mathrm{c}}\cos\theta' = \gamma(\frac{\omega}{\mathrm{c}}\cos\theta - \frac{\mathrm{v}}{\mathrm{c}^2}\omega)$$

$$\frac{1}{\mathrm{c}}\omega\gamma(1 - \frac{\mathrm{v}}{\mathrm{c}}\cos\theta)\cos\theta' = \gamma(\frac{\omega}{\mathrm{c}}\cos\theta - \frac{\mathrm{v}}{\mathrm{c}^2}\omega)$$

$$\omega' = \omega\gamma(1 - \frac{v}{c}\cos\theta)$$

相对论的多普勒效应

$$\cos'\theta = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}$$

$$tg\theta' = \frac{\sin\theta}{r(\cos\theta - \frac{v}{c})} \Rightarrow$$ 光行差公式

一. 四维电流与连续性方程的协变性：

1. 连续性方程的四维形式：

连续性方程为 $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

$$\therefore \quad \frac{\partial J_1}{\partial x_1} + \frac{\partial J_2}{\partial x_2} + \frac{\partial J_3}{\partial x_3} + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} = \frac{ic}{ic}\frac{\partial \rho}{\partial t} = \frac{1}{ic}\frac{\partial ic\rho}{\partial t}$$

$$= \frac{\partial ic\rho}{\partial ict} = \frac{\partial ic\rho}{\partial x_4}$$

令： $J_4 = ic\rho$ ，则有：

$$\frac{\partial J_1}{\partial x_1} + \frac{\partial J_2}{\partial x_2} + \frac{\partial J_3}{\partial x_3} + \frac{\partial J_4}{\partial x_4} = 0$$

$$\boxed{\frac{\partial J_\mu}{\partial x_\mu} = 0}$$

$$J_\mu = (\vec{J}, J_4) = (J_1, J_2, J_3, ic\rho)$$

式中 $\dfrac{\partial J_\mu}{\partial x_\mu}$ 表示四维散度，因而 J_μ 为四维矢量。

其中 $\vec{J}$ 为 J_μ 的空间分量，J_4 为时间分量。

2. 四维电流密度 J_μ ：

电荷守恒定律指出，系统总电荷在所有情况下守恒，故在一切惯性系中为不变量，即为一个洛仑兹标量。

设与带电系统相对静止的坐标系 K' 中，体系总电荷为 Q_0，

电荷密度为 ρ_0，体积元为$\mathrm{d}V$，则有：$Q_0 = \int \rho_0 \mathrm{d}V_0$

在另一惯性系 K 中观察，则带电系统以 $\vec{v}$ 运动，K 中电

荷密度为 ρ ，体积元为$\mathrm{d}V$ ，则带电系统的总电荷为：

$$Q = \int \rho \mathrm{d}V$$

电荷守恒，有$Q_0 = Q$ 即 $\int \rho_0 \mathrm{d}V_0 = \int \rho \mathrm{d}V$

$$\mathrm{d}V = \sqrt{1 - \frac{v^2}{c^2}} \mathrm{d}V_0$$

代入 $\int \rho_0 \mathrm{d}V_0 = \int \rho \mathrm{d}V$ 得到：

$$\int \rho_0 \mathrm{d}V_0 = \int \rho \sqrt{1 - \frac{v^2}{c^2}} \mathrm{d}V_0$$

有：$\rho_0 = \rho\sqrt{1-\frac{v^2}{c^2}}$ 或 $\rho = \frac{\rho_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\rho_0$

$$\because \vec{J} = \rho\vec{v}$$

$$\therefore J_1 = \rho v_1 = \frac{\rho_0 v_1}{\sqrt{1-\frac{v^2}{c^2}}} = \rho_0\gamma v_1 = \rho_0 u_1$$

$$J_i = \rho_0 u_i$$

$$J_4 = ic\rho = \frac{ic\rho_0}{\sqrt{1-\frac{v^2}{c^2}}} = \rho_0 \gamma ic = \rho_0 u_4$$

$$\therefore \quad \boxed{J_\mu = \rho_0 u_\mu}$$

由此可知J_μ为四维矢量（$\because u_\mu$为四维矢量，而ρ_0为标量）

由 J_μ 前三分量 $\vec{J} = \rho_0 \vec{u} = \rho_0 \gamma \vec{v} = \gamma \rho_0 \vec{v}$

$= \rho \vec{v}$ 含电流密度的物理意义。

则称J_μ为四维电流矢量。其第四分量$J_4 \propto \rho_0$。

含电荷密度因子，四维电流 J_μ 把电流密度和电荷密度同一起来。

二. 四维势矢量及势方程的协变性：

1. 四维形式的达朗伯方程：

洛仑兹规范：

$$\nabla \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0$$

势达朗伯方程：

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{j}$$

$$\nabla^2 \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{1}{\varepsilon_0}\rho$$

势达朗伯方程： $$\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{j}$$

$$\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{1}{\varepsilon_0}\rho$$

达朗伯算符： $$\Box=\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}=\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\mu}$$

于是： $$\Box\vec{A}=-\mu_0\vec{j}$$

$$\Box\varphi=-\frac{1}{\varepsilon_0}\rho=-\mu_0c^2\rho\quad(c^2=\frac{1}{\mu_0\varepsilon_0})$$

由于 $\vec{j}$ 和 ρ 统一在 J_μ 中，那么 $\vec{A}$ 和 φ 也应统一

$\Box\varphi = -\mu_0 c^2\rho$ 同乘以 $\frac{i}{c}$，有： $\frac{i}{c}\Box\varphi = -\mu_0 ic\rho$

$$\therefore \quad \Box\frac{i}{c}\varphi = -\mu_0(ic\rho) = -\mu_0 J_4$$

令： $A_4 = \frac{i}{c}\varphi$ ， $A_i = \vec{A}\ (i = 1,2,3)$

$\Box A_\mu = -\mu_0 J_\mu$ —— 四维形式的达朗伯方程

分量式为：$\Box A_i = -\mu_0 J_i \quad (i = 1,2,3)$

$\Box A_4 = -\mu_0 J_4$

$\because \quad \Box = \dfrac{\partial}{\partial x_\mu}\dfrac{\partial}{\partial x_\mu}$ 为矢量$\dfrac{\partial}{\partial x_\mu}$的标积为标量，而$J_\mu$

为四维矢量。$\therefore A_\mu$ 也为四维矢量，是一个相对论物理量

$$\Box A_\mu = -\mu_0 J_\mu$$ 具有洛仑兹协变性

2. 四维势矢量： A_μ

洛仑兹条件：$\nabla \cdot \vec{A} + \dfrac{1}{c^2}\dfrac{\partial \varphi}{\partial t} = 0$

$$\frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} + \frac{\partial A_3}{\partial x_3} + \frac{\partial A_4}{\partial x_4} = 0 \quad \left(\frac{1}{c^2}\frac{\partial \varphi}{\partial t} = \frac{\partial(\frac{i}{c}\varphi)}{\partial(ict)} = \frac{\partial A_4}{\partial x_4}\right)$$

$$\boxed{\frac{\partial A_\mu}{\partial x_\mu} = 0}$$

即 $$\frac{\partial A_\mu}{\partial x_\mu} = 0$$

式中 A_μ 定义为四维势矢量，它满足特殊洛仑兹变换：

$$A_\mu^{'} = a_{\mu\gamma} A_\gamma$$

$$\therefore \quad A_1^{'} = \gamma(A_1 + i\beta A_4) = \gamma(A_1 - \frac{v}{c^2}\varphi)$$

$$A_2^{'} = A_2$$

$$A_3^{'} = A_3$$

$$A_4^{'} = \gamma(A_4 + i\beta A_1) = \gamma(A_4 - i\frac{v}{c}A_1)$$

⇓

$$\varphi^{'} = \gamma(\varphi - vA_1)$$

$$\frac{\partial J_{\mu}}{\partial x_{\mu}} = 0$$

$$J_{\mu} = (\vec{J}, J_4) = (J_1, J_2, J_3, ic\rho)$$

$$\frac{\partial A_{\mu}}{\partial x_{\mu}} = 0$$

$$A_4 = \frac{i}{c}\varphi \quad , \quad A_i = \vec{A}\,(i = 1,2,3)$$

$$\Box A_{\mu} = -\mu_0 J_{\mu}$$

三. 电磁场张量：

1. 电磁场张量定义：

对于 φ 和 A 来讲，

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} \quad ①$$

$$\vec{B} = \nabla\times\vec{A} \quad ②$$

$$\because \quad A_4 = \frac{i}{c}\varphi \, , \, x_4 = ict$$

①：

$$\therefore \vec{E} = ic\nabla A_4 - \frac{\partial\vec{A}}{\partial ict}\cdot ic = ic(\nabla A_4 - \frac{\partial\vec{A}}{\partial x_4})$$

或

$$-\frac{i}{c}\vec{E} = \nabla A_4 - \frac{\partial\vec{A}}{\partial x_4}$$

其分量式为（自己展开）？

$$-\frac{i}{c}(E_1\vec{e}_1 + E_2\vec{e}_2 + E_3\vec{e}_3)$$

$$=\frac{\partial A_4}{\partial x_1}\vec{e}_1 + \frac{\partial A_4}{\partial x_2}\vec{e}_2 + \frac{\partial A_4}{\partial x_3}\vec{e}_3 - \frac{\partial A_1}{\partial x_4}\vec{e}_1 - \frac{\partial A_2}{\partial x_4}\vec{e}_2 - \frac{\partial A_3}{\partial x_4}\vec{e}_3$$

$$=(\frac{\partial A_4}{\partial x_1} - \frac{\partial A_1}{\partial x_4})\vec{e}_1 + (\frac{\partial A_4}{\partial x_2} - \frac{\partial A_2}{\partial x_4})\vec{e}_2 + (\frac{\partial A_4}{\partial x_3} - \frac{\partial A_3}{\partial x_4})\vec{e}_3$$

其分量式为：

$$-\frac{i}{c}(E_1\vec{e}_1 + E_2\vec{e}_2 + E_3\vec{e}_3)$$

$$= \frac{\partial A_4}{\partial x_1}\vec{e}_1 + \frac{\partial A_4}{\partial x_2}\vec{e}_2 + \frac{\partial A_4}{\partial x_3}\vec{e}_3 - \frac{\partial A_1}{\partial x_4}\vec{e}_1 - \frac{\partial A_2}{\partial x_4}\vec{e}_2 - \frac{\partial A_3}{\partial x_4}\vec{e}_3$$

$$= (\frac{\partial A_4}{\partial x_1} - \frac{\partial A_1}{\partial x_4})\vec{e}_1 + (\frac{\partial A_4}{\partial x_2} - \frac{\partial A_2}{\partial x_4})\vec{e}_2 + (\frac{\partial A_4}{\partial x_3} - \frac{\partial A_3}{\partial x_4})\vec{e}_3$$

② 式： $\vec{B} = \nabla \times \vec{A}$ 其分量式为：

$$B_1\vec{e}_1 + B_2\vec{e}_2 + B_3\vec{e}_3$$

$$= (\frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3})\vec{e}_1 + (\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1})\vec{e}_2 + (\frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2})\vec{e}_3$$

规律？

$$F_{\mu\gamma} = \frac{\partial A_\gamma}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\gamma} \qquad (\mu, \gamma = 1, 2, 3, 4)$$

由组合形式可以看出： $F_{\mu\gamma} = -F_{\gamma\mu}$

$F_{\mu\gamma}$ 称为电磁场张量，它为一个二阶反对称张量。

2. 电磁场张量 $F_{\mu\gamma}$ 的矩阵形式：

写出电磁场张量各分量： $F_{11}=F_{22}=F_{33}=F_{44}=0$

$$F_{11}=\frac{\partial A_1}{\partial x_1}-\frac{\partial A_1}{\partial x_1}=0 \qquad F_{14}=-F_{41}=\frac{\partial A_4}{\partial x_1}-\frac{\partial A_1}{\partial x_4}$$

$$F_{12}=\frac{\partial A_2}{\partial x_1}-\frac{\partial A_1}{\partial x_2}=-F_{21} \qquad F_{13}=-F_{31}=\frac{\partial A_3}{\partial x_1}-\frac{\partial A_1}{\partial x_3}$$

$$F_{23}=-F_{32}=\frac{\partial A_3}{\partial x_2}-\frac{\partial A_2}{\partial x_3} \qquad F_{24}=-F_{42}=\frac{\partial A_4}{\partial x_2}-\frac{\partial A_2}{\partial x_4}$$

$$F_{34}=-F_{43}=\frac{\partial A_4}{\partial x_3}-\frac{\partial A_3}{\partial x_4}$$

$$-\frac{i}{c}(E_1\vec{e}_1 + E_2\vec{e}_2 + E_3\vec{e}_3)$$

$$= (\frac{\partial A_4}{\partial x_1} - \frac{\partial A_1}{\partial x_4})\vec{e}_1 + (\frac{\partial A_4}{\partial x_2} - \frac{\partial A_2}{\partial x_4})\vec{e}_2 + (\frac{\partial A_4}{\partial x_3} - \frac{\partial A_3}{\partial x_4})\vec{e}_3$$

$$B_1\vec{e}_1 + B_2\vec{e}_2 + B_3\vec{e}_3$$

$$= (\frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3})\vec{e}_1 + (\frac{\partial A_1}{\partial x_3} - \frac{\partial A_3}{\partial x_1})\vec{e}_2 + (\frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2})\vec{e}_3$$

代入①、②式的分量式，有

$$-\frac{i}{c}(E_1\vec{e}_1 + E_2\vec{e}_2 + E_3\vec{e}_3) = F_{14}\vec{e}_1 + F_{24}\vec{e}_2 + F_{34}\vec{e}_3$$

$$B_1\vec{e}_1 + B_2\vec{e}_2 + B_3\vec{e}_3 = F_{23}\vec{e}_1 + F_{31}\vec{e}_2 + F_{12}\vec{e}_3$$

代入①、②式的分量式，有

$$-\frac{i}{c}(E_1\vec{e}_1 + E_2\vec{e}_2 + E_3\vec{e}_3) = F_{14}\vec{e}_1 + F_{24}\vec{e}_2 + F_{34}\vec{e}_3$$

$$B_1\vec{e}_1 + B_2\vec{e}_2 + B_3\vec{e}_3 = F_{23}\vec{e}_1 + F_{31}\vec{e}_2 + F_{12}\vec{e}_3$$

比较两边系数，有：

$$F_{14} = -\frac{i}{c}E_1, F_{24} = -\frac{i}{c}E_2, F_{34} = -\frac{i}{c}E_3$$

$$F_{23} = B_1, F_{31} = B_2, F_{12} = B_3$$

加上 $F_{11} = F_{22} = F_{33} = F_{44} = 0$ 可以将电磁张量写成矩阵：

比较两边系数，有：

$$F_{14}=-\frac{i}{c}E_1, F_{24}=-\frac{i}{c}E_2, F_{34}=-\frac{i}{c}E_3$$

$$F_{23}=B_1, F_{31}=B_2, F_{12}=B_3$$

$$F_{\mu\gamma}=\begin{pmatrix} 0 & B_3 & -B_2 & -\frac{i}{c}E_1 \\ -B_3 & 0 & B_1 & -\frac{i}{c}E_2 \\ B_2 & -B_1 & 0 & -\frac{i}{c}E_3 \\ \frac{i}{c}E_1 & \frac{i}{c}E_2 & \frac{i}{c}E_3 & 0 \end{pmatrix}$$

四. 麦克斯韦方程组的协变形式：

1. 麦克斯韦方程组的四维形式：

麦克斯韦方程：

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j} \Rightarrow \nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{j}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \nabla \cdot \vec{E} = \mu_0 c^2 \rho$$

$$\nabla \cdot \vec{B} = 0 \quad \Rightarrow \nabla \cdot \vec{B} = 0 \quad (c^2 = \frac{1}{\mu_0 \varepsilon_0})$$

一、三式可以合写成一个四维方程，二、四式可以合写成一个四维方程。

（1）一、三式可以合写成一个四维方程：

写出第一式和第二式的直角坐标系中分量式：

$$(\frac{\partial B_3}{\partial x_2}-\frac{\partial B_2}{\partial x_3})\vec{e}_1+(\frac{\partial B_1}{\partial x_3}-\frac{\partial B_3}{\partial x_1})\vec{e}_2+(\frac{\partial B_2}{\partial x_1}-\frac{\partial B_1}{\partial x_2})\vec{e}_3$$

$$-\frac{1}{c^2}(\frac{\partial E_1}{\partial t}\vec{e}_1+\frac{\partial E_2}{\partial t}\vec{e}_2+\frac{\partial E_3}{\partial t}\vec{e}_3)=\mu_0(j_1\vec{e}_1+j_2\vec{e}_2+j_3\vec{e}_3)$$

$$\frac{\partial E_1}{\partial x_1}+\frac{\partial E_2}{\partial x_2}+\frac{\partial E_3}{\partial x_3}=\mu_0 c^2\rho \qquad (c^2=\frac{1}{\mu_0\varepsilon_0})$$

通过比较两边系数，上边二个方程可以写成四个方程

（调整了顺序）

$$0+\frac{\partial B_3}{\partial x_2}-\frac{\partial B_2}{\partial x_3}-\frac{1}{c^2}\frac{\partial E_1}{\partial t}=\mu_0 j_1$$

$$-\frac{\partial B_3}{\partial x_1}+0+\frac{\partial B_1}{\partial x_3}-\frac{1}{c^2}\frac{\partial E_2}{\partial t}=\mu_0 j_2$$

$$\frac{\partial B_2}{\partial x_1}-\frac{\partial B_1}{\partial x_2}+0-\frac{1}{c^2}\frac{\partial E_3}{\partial t}=\mu_0 j_3$$

$$\frac{\partial E_1}{\partial x_1}+\frac{\partial E_2}{\partial x_2}+\frac{\partial E_3}{\partial x_3}+0=\mu_0 c^2 \rho$$

有电磁场张量 $F_{\mu\gamma}$的矩阵及 $j_4 = ic\rho$ $x_4 = ict$

可将上边方程写成：

$$0+\frac{\partial B_3}{\partial x_2}-\frac{\partial B_2}{\partial x_3}-\frac{1}{c^2}\frac{\partial E_1}{\partial t}=\mu_0 j_1$$

$$-\frac{\partial B_3}{\partial x_1}+0+\frac{\partial B_1}{\partial x_3}-\frac{1}{c^2}\frac{\partial E_2}{\partial t}=\mu_0 j_2$$

$$\frac{\partial B_2}{\partial x_1}-\frac{\partial B_1}{\partial x_2}+0-\frac{1}{c^2}\frac{\partial E_3}{\partial t}=\mu_0 j_3$$

$$\frac{\partial E_1}{\partial x_1}+\frac{\partial E_2}{\partial x_2}+\frac{\partial E_3}{\partial x_3}+0=\mu_0 c^2 \rho$$

有电磁场张量 $F_{\mu\gamma}$的矩阵及 $j_4 = ic\rho$ $x_4 = ict$

可将上边方程写成：

$$0+\frac{\partial \boldsymbol{F}_{12}}{\partial \boldsymbol{x}_2}+\frac{\partial \boldsymbol{F}_{13}}{\partial \boldsymbol{x}_3}+\frac{\partial \boldsymbol{F}_{14}}{\partial \boldsymbol{x}_4}=\mu_0 \boldsymbol{j}_1$$

$$\frac{\partial F_{21}}{\partial x_1}+0+\frac{\partial F_{23}}{\partial x_3}+\frac{\partial F_{24}}{\partial x_4}=\mu_0 j_2$$

$$\frac{\partial \boldsymbol{F}_{31}}{\partial \boldsymbol{x}_1}+\frac{\partial \boldsymbol{F}_{32}}{\partial \boldsymbol{x}_2}+0+\frac{\partial \boldsymbol{F}_{34}}{\partial \boldsymbol{x}_4}=\mu_0 \boldsymbol{j}_3$$

$$\frac{\partial F_{41}}{\partial x_1}+\frac{\partial F_{42}}{\partial x_2}+\frac{\partial F_{43}}{\partial x_3}+0=\mu_0 j_4$$

于是可以写成 $$\frac{\partial F_{\mu\gamma}}{\partial x_\gamma}=\mu_0 J_\mu \qquad (\mu,\gamma=1,2,3,4)$$

这样通过$F_{\mu\gamma}$把麦克斯韦方程一、三合写成一个四维方程。

练习：展开下列两式

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \cdot \vec{B} = 0$$

（2）二、四式可以合写成一个四维方程：

写 二、四式直角坐标系分量式：

仿一、三式分量式，比较系数，可得四个方程：

$$0+\frac{\partial E_3}{\partial x_2}-\frac{\partial E_2}{\partial x_3}-\frac{\partial B_1}{\partial t}=0$$

$$-\frac{\partial E_3}{\partial x_1}+0+\frac{\partial E_1}{\partial x_3}-\frac{\partial B_2}{\partial t}=0$$

$$\frac{\partial E_3}{\partial x_1}-\frac{\partial E_1}{\partial x_2}+0-\frac{\partial B_3}{\partial t}=0$$

$$\frac{\partial B_1}{\partial x_1}+\frac{\partial B_2}{\partial x_2}+\frac{\partial B_3}{\partial x_3}+0=0$$

有电磁场张量 $F_{\mu\gamma}$ 的矩阵形式及 $x_4 = ict$

可将上边四个方程写成：

序号

$$
\left.
\begin{array}{ll}
1 & 0+\dfrac{\partial \boldsymbol{F}_{43}}{\partial \boldsymbol{x}_2}+\dfrac{\partial \boldsymbol{F}_{24}}{\partial \boldsymbol{x}_3}+\dfrac{\partial \boldsymbol{F}_{32}}{\partial \boldsymbol{x}_4}=0 \\
2 & \dfrac{\partial F_{42}}{\partial x_1}+0+\dfrac{\partial F_{41}}{\partial x_3}+\dfrac{\partial F_{13}}{\partial x_4}=0 \\
3 & \dfrac{\partial \boldsymbol{F}_{42}}{\partial \boldsymbol{x}_1}+\dfrac{\partial \boldsymbol{F}_{14}}{\partial \boldsymbol{x}_2}+0+\dfrac{\partial \boldsymbol{F}_{21}}{\partial \boldsymbol{x}_4}=0 \\
4 & \dfrac{\partial F_{23}}{\partial x_1}+\dfrac{\partial F_{14}}{\partial x_2}+\dfrac{\partial F_{12}}{\partial x_3}+0=0
\end{array}
\right\} \quad \text{(A)}
$$

于是可以写成：

$$\frac{\partial F_{\mu\gamma}}{\partial x_{\lambda}}+\frac{\partial F_{\gamma\lambda}}{\partial x_{\mu}}+\frac{\partial F_{\lambda\mu}}{\partial x_{\gamma}}=0 \qquad (\mu,\gamma,\lambda=1,2,3,4)$$

2. 麦氏方程的协变性：

$$\left.\begin{aligned}&\frac{\partial F_{\mu\gamma}}{\partial x_{\gamma}}=\mu_0 J_{\mu}\\&\frac{\partial F_{\mu\gamma}}{\partial x_{\lambda}}+\frac{\partial F_{\gamma\lambda}}{\partial x_{\mu}}+\frac{\partial F_{\lambda\mu}}{\partial x_{\gamma}}=0\end{aligned}\right\} \qquad (\mu,\gamma,\lambda=1,2,3,4)$$

为麦氏方程四维形式中个两为四维量，在进行坐标变换时，均按同一规律变换，因而适用于一切惯性系，故具有四维形式的协变性。

期末考试说明

1、主要内容：预备知识、**1-6**章

2、复习范围：书本、笔记、作业

3、题目形式：简答、证明、计算（期中）

4、注意问题：矢量/条件/计算过程

5、分数安排：**20+80**