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EXTENDED CONCEPTUAL NEIGHBORHOODS

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ABSTRACT

Based on the smooth transition model for conceptual neighborhood graphs among binary topological relations, an extended model, *the movement model*, is proposed. A neighborhood relation between two topological relations describes one of the possible movements of one object with respect to the second one. The example of the construction of the graph for line-zone relations is provided. Such extended graphs may be used for conception of analyzers and human/machine dialogue engines for visual query languages for geographical information systems. A prospective discussion gives some insights of their applications in this context.

1 INTRODUCTION

Querying Geographical Information Systems (GIS) is a difficult task. Low level query languages (Arc/Info scripts (Morehouse, 1985), Space (Apic Systmes, 1996), various SQL geographical flavors) are difficult to use and are reserved for specialists. GUI-based interfaces are usually limited to a predefined set of spatial queries and to the possibility to zoom/unzoom and to pan the resulting map. While well adapted to repetitive and pre-defined usage, such interfaces are not suited for advanced users who are concerned by activities such as datamining on spatial or thematic georeferenced data. Natural language may be thought as a possible solution but it rapidly appears that it is rather difficult for the user to to simply specify a spatial configuration of objects (Szmurlo et al., 1998, Egenhofer et al., 1994, Mark et al., 1995). Furthermore, natural language implies many difficulties in analysis (references, anaphora, writer's cultural background, etc.). Finally, the most natural way to specify a set of spatial constraints (i.e.: a query) appears to be a graphical representation of the configuration: a sketch, a schema. Such query interfaces belong to the family of Query-By-Example (QBE) languages where an example of what is searched is "shown" to the machine. While intuitive for the users, the main problem is the interpretation of what the user actually is looking for.

Querying a geographical database requires the specification of two kinds of constraints: the spatial constraints and the thematic constraints. Spatial constraints specify what are the spatial relationships between the objects: relative positions (A is north to B), topological relations (A is in B, A crosses B, etc.), metrical relations, etc.. Thematic constraints specify the geographical type of the object (e.g.: road, town) and constraints for alphanumeric data (*population* of the town *less than 10.000 inhabitants*, for example). In the past decade, there have been proposed several projects for querying spatial databases with visual QBE-like interfaces.

Lee and Chin proposed a constrained drawing tool (*à la* MacDraw) for building the spatial configuration (Lee and Chin, 1995). This interface was "object driven" in the sense that the user had total freedom to draw the objects; it was the machine which was in charge for analyzing the drawing and extracting the spatial relationships. The drawback of this interface was the way for specifying thematic constraints which was performed with a more or less SQL-like language, difficult for non specialists. Calcinelli *et al.* developed a prototype, *Cigalles*, of an iconic language where any spatial object or associated thematic data was represented by an icon (Calcinelli and Mainguenaud, 1994). Cigalles is "relation driven" rather than "object driven" which means that before specifying a query, the user has to know which spatial relations he or she wants to express. Egenhofer's "Spatial-Query-by-Sketch" (Egenhofer, 1996) seems to be the most intuitive interface as it is object driven and the user sketches the spatial configuration with an electronic pen. This interface however requires an important object recognition/interpretation work before the analysis of the configurations actually begins. Finally, we also are currently working on a visual language for GIS querying which is a derivate from Lee's project: thematic constraints are entered as natural language expressions while spatial constrains are drawn with a constrained MacDraw-like tool. This project, the *Geographical Anteserver*, was described in (Szmurlo et al., 1998, Szmurlo and Gaio, 1998).

In object driven interfaces the most difficult part is the analysis of the schema in order to derivate the spatial relations that are implied by the relative positions of the objects, and finally the interpretation of the schema into a set of spatial concepts. Our analyzer is based on Egenhofer's *et al.* work on modelization of topological relations (the 9-intersection model (Egenhofer et al., 1994)), on modelization of conceptual neighborhoods among topological relations (Egenhofer and Mark, 1995) and their interpretation (Mark and Egenhofer, 1994, Mark et al., 1995).

Conceptual neighborhoods based on the smooth transition model (STM) are suited for rough partitioning of sets of topological relations into sets expressing some spatial concepts. As it is based on single movement of sub-parts of objects, it is not flexible enough, however, to express partial or total movements. Our contribution in this paper mainly concerns the the definition of an extension of the conceptual neighborhood graph based on the STM by using a different definition of neighborhoods. Our model is called the "movement model" as it takes into account partial but also entire displacement of an object with respect to another.

This paper is organized as follows. In section 2 we present some basic definitions for the 9-intersection model for topological relations and the graph of conceptual neighborhoods based on the STM for line-zone topological relations. In section 3 we develop the general model and present its application to line-zone relations. Finally, section 4 prospectively discusses the application of this model to spatial concept modelization and usage for human/machine dialogue.

2 BASIC DEFINITIONS

This section briefly presents the definitions used for the construction of topological relations that exist between linear and zonal objects, and the method for building the graph of conceptual neighborhoods based on the smooth transitions model as proposed by Egenhofer *et al.* in (Egenhofer et al., 1994) and (Egenhofer and Mark, 1995).

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2.1 The 9-intersection model

Definition 1 (Part, Proper part)
An object A partitions its embedding universe into three "parts": A's exterior (A^-), A's boundary (∂A) and A's interior (A^o).

∂A and A^o are called the "proper parts" of A as their union equals A.

A's parts are mutually exclusive as the intersection of any part of A with an other part of A is empty.

A topological relation R_{AB} between two objects A and B is defined by the nine possible intersections of A's parts with parts of B. These intersections can be represented in matrix form as shown in equation 1.

$$R_{AB} \equiv M_{AB} = \begin{bmatrix} A^o \cap B^o & A^o \cap \partial B & A^o \cap B^- \\ \partial A \cap B^o & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^o & A^- \cap \partial B & A^- \cap B^- \end{bmatrix} \quad (1)$$

The notation $M[P_B, P_A]$ represents the cell corresponding to the intersection $P_A \cap P_B$, where P_A and P_B are parts of A and B, respectively. Among several topological invariants (Munkres, 1966), that is properties that are invariant upon topological transformations, we consider the values empty ($\emptyset$) and not empty ($\neg\emptyset$) for each intersection. This results in $2^9 = 512$ possible candidate relations, most of them being impossible for topological reasons. For simple zones and lines, Egenhofer *et al.* did identify 19 line-zone relations, 8 zone-zone relations, and 33 line-line relations. Some line-zone relations are shown on figure 1 along with their matrices (lines as columns, zones as rows).

2.2 Conceptual neighborhoods

An intuitive examination of figure 1 shows similarities among subsets of relations. For example, R_1 is "closer" to R_9 than to R_2: in order to reach R_9 from R_1 one would only need to move one line's boundary from Z^- on ∂Z while to reach R_2 from R_1 it would be necessary to move the whole line thought zone's boundary. Identically, R_7 is "closer" to R_{11} than to R_1. By defining a closeness measure, it might then be possible to build graph where an edge between R_i and R_j expresses the fact that R_i is close to R_j. Egenhofer and Mark (Egenhofer and Mark, 1995) proposed two modelizations for building such graphs. In this paper we are only concerned by the so called "smooth transitions" model.

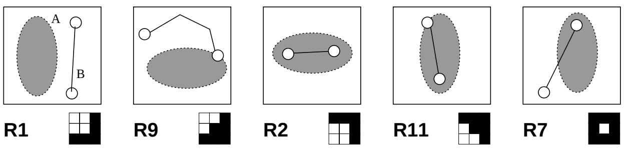

Figure 1: Some line-zone relations. The matrices are represented in graphical form: a black point represents an non-empty intersection while a white point represents an empty intersection.

First, we recall some definitions that will also be necessary in the extended model.

Definition 2 (Extent)
The "extent" of a part P_A of A in the object B is the set of parts of B that have a non-empty intersection with P_A. The extent of P_A in B will be noted $E_B(P_A)$.

This definition implies that P_A is included or equal to the union of the elements from $E_B(P_A)$[1]. For example, for relation R_{11} we have: $E_Z(\partial L) = \{Z^o, \partial Z\}$ and $E_Z(L^o) = \{Z^o\}$.

[1] Note that in their original paper, Egenhofer *et al.* defined the *extent* as the cardinality of $E_B(P_A)$.

Definition 3 (Adjacency)
The "adjacency" of a part P_A of an object A, written as Adj (P_A), is the set of parts of A that are directly reachable from P_A.

For a zone Z we have:

$$\text{Adj}\,(Z^o) = \{\partial Z\} \quad (2)$$
$$\text{Adj}\,(\partial Z) = \{Z^o, Z^-\} \quad (3)$$
$$\text{Adj}\,(Z^-) = \{\partial Z\} \quad (4)$$

The construction of neighbors of the relation R_i between a line L and a zone Z consists in "moving"[2] in turn a part of each proper part P_L from the part $P_Z, P_Z \in E_Z(P_L)$ of Z with which P_L intersects on each element of Adj (P_Z). "Moving" is practically performed by setting to $\emptyset$ or to $\neg\emptyset$ cells in the matrix corresponding to R_i. Such operations may produce non-valid matrices; as the objects are connected, it is necessary to introduce consistency constraints that will correct the matrix:

$$L^o \cap Z^o = \neg\emptyset \wedge L^o \cap Z^- = \neg\emptyset \quad \Rightarrow \quad L^o \cap \partial Z = \neg\emptyset \quad (5)$$
$$\partial L \cap Z^o = \neg\emptyset \quad \Rightarrow \quad L^o \cap Z^o = \neg\emptyset \quad (6)$$
$$\partial L \cap Z^- = \neg\emptyset \quad \Rightarrow \quad L^o \cap Z^- = \neg\emptyset \quad (7)$$

The full algorithm is provided in (Egenhofer and Mark, 1995).

Figure 2 depicts the graph of conceptual neighborhoods obtained by this model.

2.3 Discussion

An interesting point to note is that the graph is oriented as not all the transitions are symmetric (for example $R_{11} \rightarrow R_7$ but $R_7 \not\rightarrow R_{11}$). The transition $R_{11} \rightarrow R_7$ may seem to violate the principle of the model as a part of L^o moves directly from Z^o to Z^-. This transition is obtained by moving ∂L from ∂Z on Z^-. Thus, L^o must "follow" (due to consistency constraint 7). On the contrary, $R_7 \rightarrow R_{11}$ is not obtained because moving either L^o or ∂L from Z^- does never require L^o to fully enter Z^o.

The intuitive geometrical interpretation of $R_7 \rightarrow R_{11}$ is however reasonable. Imagine pulling on the line's boundary that lays in Z^o, then, at some time, the configuration R_7 will be changed into R_{11} as shown on figure 3. This interpretation corresponds to a translation of the whole line.

If we accept that lines may be moved entirely (translation, rotation, deformation, etc.) and not only partially, then many new transitions may be obtained as for example the one depicted figure 4. Note that a similar approach was also used by Egenhofer and Al-Taha for zone-zone relations (Egenhofer and Al-Taha, 1992).

3 EXTENDED CONCEPTUAL NEIGHBORHOODS BASED ON A MOVEMENT MODEL

This section presents an extended model for building an enriched graph of conceptual neighborhoods of topological relations. As an example, we will use line-zone relations. The model is based on movement of the line with respect to the zone. By the generic term "movement" we mean translation, rotation, expansion, reduction, deformation, etc. of the line. This implies that we will consider independent movement of each proper parts of the line either partially or totally as well as simultaneous movement of all proper parts.

This section is divided in four parts. We first will give an intuitive idea of the model based on an example and state some notations. Then we will provide a formal definition and its application to line-zone topological relations. Finally, we will prune the graph from non-atomic transitions and present the final graph for line-zone relations.

[2] It can also be seen as a deformation of the object.

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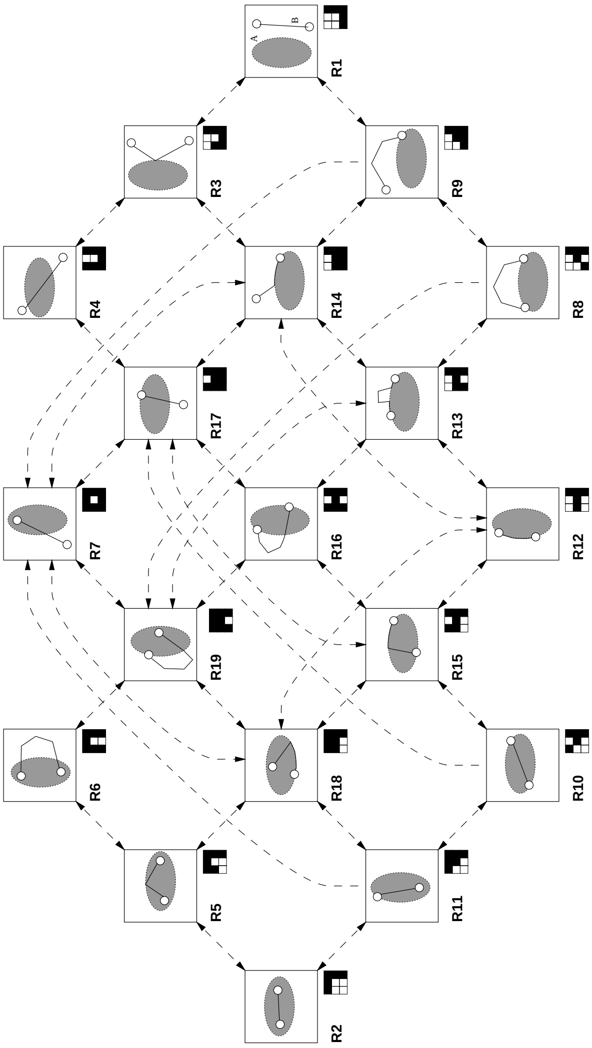

Figure 2: Conceptual neighborhoods based on the smooth transitions model.

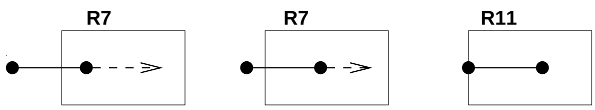

Figure 3: Translation of an entire linear object.

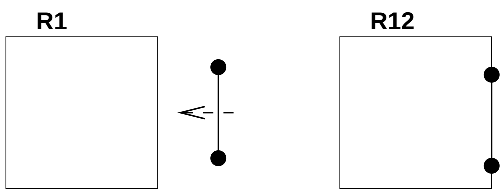

Figure 4: New possible transition: $R_1 \rightarrow R_{12}$.

3.1 Intuitive idea of the model

This section provides an intuitive idea of the model based on the example of relation R_6 (see figure 2). For this configuration we have $E_Z(\partial L) = \{Z^o\}$ and $E_Z(L^o) = \{Z^o, \partial Z, Z^-\}$. We first consider possible movements of ∂L, then movements of L^o and finally simultaneous movements of ∂L and of L^o.

Movement of ∂L only
As $E_Z(\partial L) = \{Z^o\}$, ∂L will be moved either partially or totally on elements of $\text{Adj}\,(Z^o) = \{\partial Z\}$, that is ∂Z. The movement's source is the union of elements from $E_Z(\partial L)$, that is for R_6, Z^o. If ∂L moves partially, the target relation is required to respect $\partial L \cap Z^o = \neg\emptyset$, $\partial L \cap \partial Z = \neg\emptyset$, and $\partial L \subset Z^o \cup \partial Z$. On the other hand, if ∂L moves totally, the relation must respect $\partial L \cap Z^o = \emptyset$ and $\partial L \cap \partial Z = \neg\emptyset$, with $\partial L \subset \partial Z$.

As a short hand notation for:

- Z^o is the source of the movement of ∂L,
- $\partial L \cap Z^o = \neg\emptyset$, $\partial L \cap \partial Z = \neg\emptyset$, and $\partial L \subseteq Z^o \cup \partial Z$, or in other words that ∂L's destination is the union $Z^o \cup \partial Z$,
- $\partial L \cap Z^o = \emptyset$, $\partial L \cap \partial Z = \neg\emptyset$, and $\partial L \subseteq \partial Z$, or in other words that ∂L's destination is ∂L,

we will write: $\{^o\} \xrightarrow{\partial L} \{\partial, {}^o\partial\}$. The source of the movement (^{o}) corresponds to the union of elements of ∂L's extent (currently Z^o only); each element of $\{\partial, {}^o\partial\}$ is a possible destination for ∂L.

The computation of the target relations is performed as follows. For the destination ${}^o\partial$: $M_6[\partial L, \partial Z] \leftarrow \neg\emptyset$ and $M_6[\partial L, Z^o] \leftarrow \neg\emptyset$; for the destination ∂: $M_6[\partial L, Z^o] \leftarrow \emptyset$ and $M_6[\partial L, \partial Z] \leftarrow \neg\emptyset$. By performing these operations, we obtain R_{19} and R_{16} as neighbors of R_6. Note that the destinations ∂ and ${}^o\partial$ are short hand notations for unions of elements of the extents of ∂L in the zone for the target relations, R_{16} and R_{19} respectively.

Movement of L^o only
As $E_Z(L^o) = \{Z^o, \partial Z, Z^-\}$, the source of the movement of L^o will be written as ${}^o\partial^-$. Let's call $L^o_{Z^o}$ the sub-part of L^o that intersects with Z^o, $L^o_{\partial Z}$ the sub-part that intersects with ∂Z, and $L^o_{Z^-}$ the sub-part that intersects with Z^-. Moving L^o means that we will move in turn $L^o_{Z^o}$, $L^o_{\partial Z}$ and $L^o_{Z^-}$ while the two other sup-parts remain still, then we will move simultaneously two of the sub parts while the third remains still, and finally, we will move simultaneously all three. As $L^o_{Z^o}$ intersects with Z^o, the possible destinations of $L^o_{Z^o}$ in a total or partial movement are in $\{\partial, {}^o\partial\}$. Identically, destinations of $L^o_{\partial Z}$ and $L^o_{Z^-}$ are respectively in $\{^o, ^-, {}^o\partial, \partial^-, {}^o\partial^-\}$ and $\{\partial, {}^-\partial\}$.

When $L^o_{Z^o}$ moves alone, we will obtain two possible destinations for L^o, each of these corresponds to the two possible destinations of $L^o_{Z^o}$ while the two other sub-parts remain still. Thus we obtain the following destination set: $\{\partial\partial^-, {}^o\partial\partial^-\}$. The notation $\partial\partial$ expresses the fact that $L^o \cap \partial Z = \neg\emptyset$, and $L^o \cap \partial Z = \neg\emptyset$, and $L^o \subset \partial Z$ which is equivalent to say that $L^o \cap \partial Z = \neg\emptyset$ and $L^o \subset \partial Z$. In other words $\partial\partial$ is equivalent to ∂. The same holds for ^{oo} and $^{--}$ which are respectively equivalent to ^{o} and $^-$. With these simplifications rules, we can write that the possible destinations for L^o when $L^o_{Z^o}$ moves alone are in the set $\{\partial^-, {}^o\partial^-\}$.

Similarly, we will obtain the following respective sets of destinations when $L^o_{\partial Z}$ and $L^o_{Z^-}$ are moving alone: $\{^{o-}, {}^o\partial^-\}$ and $\{{}^o\partial, {}^o\partial^-\}$.

The next step would consists in moving two sub-parts while the third one would remain still (for example move $L^o_{Z^o}$ and $L^o_{\partial Z}$ simultaneously, while $L^o_{Z^-}$ remains still), then we would move all three sub-parts. These operations are rather painful when performed by hand; the courageous reader may verify that after simplifications we will obtain:

$$\{{}^o\partial^-\} \xrightarrow{L^o} \{\partial^-, {}^{o-}, {}^o\partial, {}^o\partial^-, \partial\}$$

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As L^o's source is ${}^o\partial^-$, the movement toward the destination ${}^o\partial^-$ will result in the relation R_6 itself: this transition is the identity transition. The destination ${}^{-o}$ implies that the target relation must satisfy: $L^o \cap Z^o = \neg\emptyset$, $L^o \cap Z^- = \neg\emptyset$, and $L^o \cap \partial Z = \emptyset$. The last constraint is unsatisfiable as lines are connected objects. Likewise the smooth transition model, it is necessary to introduce consistency constraints. In this case, constraint 5 (see above) would be used. The destination then becomes ${}^o\partial^-$ which again corresponds to the identity transition. Finally, we end up with three non-trivial destinations: $\{\partial^-, {}^o\partial, \partial\}$.

For destination ∂^- the following operations are to be done: $M[L^o, Z^o] \leftarrow \emptyset$, $M[L^o, \partial Z] \leftarrow \neg\emptyset$, and $M[L^o, Z^-] \leftarrow \neg\emptyset$. The resulting matrix corresponds to a non-existing relation. If we apply the consistency constraints from the smooth transition model, constraint 6 would be used. This constraints moves the interior of the line according to the new position of the boundary. In the present case, however, we are moving the interior of the line: it is the line's boundary which should be moved according to interior's new position and not the opposite. We therefore need to setup a new set of consistency constraints based on the movement of L^o rather than on the movement of ∂L. For this particular transition, the constraints will be: "if the boundary of the line moves *completely* from Z^o on ∂Z and if there was an intersection between ∂L and Z^o, then ∂L also moves completely from Z^o on ∂Z". An equivalent constraints will be defined for movements from Z^- to ∂Z. By applying this constraint we obtain R_{13} as R_6's neighbor. Destination ${}^o\partial$ directly gives R_5 as target, and destination ∂ gives R_{12} after application of the constraint presented above.

Simultaneous movement of L^o and ∂L

The set of possible destinations for L^o and ∂L in a simultaneous movement is the Cartesian product of individual destinations of L^o and ∂L that were computed above. Thus, we obtain the following set of couples where the first element is the destination of L^o and the second of ∂L: $\{(\partial -, {}^o\partial), ({}^o\partial, {}^o\partial), (\partial, {}^o\partial), (\partial^-, \partial)({}^o\partial, \partial)(\partial, \partial)\}$.

By applying these destinations, we respectively obtain the following neighbors for R_6: R_{19}, R_{18}, R_{18}, R_{13}, R_{15}, and R_{12}. R_{18} is produced twice; R_{19}, R_{13} and R_{12} have already been produced in the two previous stages. Finally only two new neighbors are found: R_{15} and R_{18}. As previously consistency constraints must be applied. When a matrix of a non-existing relation is produced, we first apply interior's consistency constraints, then boundary's consistency constraints. in order to obtain two target relations.

Conclusion

In this section we constructed the neighbors for the relation R_6. By moving ∂L only we obtained R_{16} and R_{19}; by moving L^o only we obtained R_{13}, R_5, and R_{12}; by moving simultaneously L^o and ∂L we obtain R_{15} and R_{18}. Note that R_{19} and R_5 have already been obtained in the smooth transition model . Unlike this model however, different set of constraints are used depending on which part of the object is being moved.

The next section presents the general definition of the movement model and its application to all line-zone relations.

3.2 The movement model for conceptual neighborhoods

In this section we first propose a general description of the movement model for building the conceptual neighborhood graph among topological relations. In a second subsection we will apply this model to line-zone relations and present the graph. Finally, we will provide some insights for geometrical interpretation of the transitions.

The movement model

This section presents the general definition of the movement model. The movement of an object is defined as a movement of any number of its proper parts. In the first paragraph of this section we will define the movement of one proper part. Then we will define the composition of those simple movements in order to make all the object moving.

In order to have a general definition, we extend the definition of topological relations themselves. An object A partitions the universe into a given number p_A of mutually exclusive parts $P_A^1, P_A^2, \ldots, P_A^{p_A}$ which union equals the universe. In the 9-intersection model, the number of parts was equal to 3. Let Π_A be the set of proper parts of A, (their union is A). Let B be a second object which makes a partition of the universe into p_B parts. The topological relation between A and B is defined by a $p_A \times p_B$ boolean matrix M which cell $M[P_B^i, P_A^j]$ represents the emptiness of the intersection $P_A^i \cap P_B^j$. The foreseen application of this extension in our analyzer will be explained later.

We assume furthermore that A remains still and that B moves.

Movement of a single proper part of B:

Let P be one of B's proper parts, let $Q_1, \ldots, Q_n, n \leq p_A$ be some elements of $E_A(P)$, and let $Q = Q_1 \cup \ldots \cup Q_n$. The *extended adjacency* of order n of Q, written as $\mathcal{A}^n(Q)$, is the set of possible destinations of P in a movement which source is Q. $\mathcal{A}^n(Q)$ is defined as follows.

1. If $n = 1$ ($P \subseteq Q = Q_1$):

$$\mathcal{A}^1(Q) = \mathcal{P}^*(\text{ Adj } (Q) \cup \{Q\}) - \{Q\}, \quad (8)$$

where $\mathcal{P}^*(E)$ is the set of parts of a set E, from which the empty set had been removed.

As an example, lets consider the relation R_6 and $P = \partial L$. In this case $Q = Z^o$ (as $E_Z(\partial L) = \{Z^o\}$), Adj $(Q) \cup \{Q\} = \{\partial Z, Z^o\}$, and finally $\mathcal{A}^1(Z^o) = \{\partial Z, Z^o \cup \partial Z\}$ (which can be written as $\{\partial, {}^o\partial\}$ if we use the shorthand notation defined in section 3.1).

2. If $n = 2$ ($P \subseteq Q = Q_1 \cup Q_2$):

$$\begin{aligned} \mathcal{A}^2(Q) &= \{Q_1' \cup Q_2 | Q_1' \in \mathcal{A}^1(Q_1)\} \cup & (9)\\ & \{Q_1 \cup Q_2' | Q_2' \in \mathcal{A}^1(Q_2)\} \cup & (10)\\ & \{Q_1' \cup Q_2' | Q_i' \in \mathcal{A}^1(Q_i)\} & (11) \end{aligned}$$

Equation 9 corresponds to a movement of the sub-part of P that lays in Q_1, equation 10 corresponds to a movement of the sub-part of P that lays in Q_2, and equation 11 corresponds to a simultaneous movement of both sub-parts.

As an example, lets consider relation R_{18} and $P = L^o$. In this case we have: $Q_1 = Z^o$; $Q_2 = \partial Z$, $\mathcal{A}^1(Q_1) = \{{}^o\partial, \partial\}$, $\mathcal{A}^1(Q_2) = \{{}^o, {}^-, {}^o\partial, {}^{o-}, \partial^-, {}^o\partial^-\}$. Thus, we obtain:

$$\begin{aligned} \mathcal{A}^2(Q) &= \{{}^o\partial\partial, \partial\partial\} \cup \\ & \{{}^{oo}, {}^{o-}, {}^{oo}\partial, {}^{oo-}, {}^o\partial^-, {}^{oo}\partial^-\} \cup \\ & \{{}^o\partial^o, {}^o\partial^-, {}^o\partial^o\partial, {}^o\partial^{o-}, {}^o\partial\partial^-, {}^o\partial^o\partial^-, \\ & \partial^o, \partial^-, \partial^o\partial, \partial^{o-}, \partial\partial^-, \partial^o\partial^-\} \\ &= \{{}^o\partial, \partial, {}^o, {}^{o-}, {}^o\partial^-, \partial^-\} \end{aligned}$$

3. In the general case, when $P \subseteq Q = Q_1 \cup \ldots \cup Q_l, l \leq n$, $\mathcal{A}^l(Q)$ is defined by:

$$\mathcal{A}^l(Q) = \{Q_1' \cup \ldots \cup Q_l' \mid Q_i' \in \mathcal{A}^1(Q) \cup \{Q_i\}, 2 \leq i \leq l\} \quad (12)$$

It is clear from these definitions that partial and total movement of P is considered. A partial movement, defined by a movement of one sub-part of P, corresponds to the smooth transition model as defined in (Egenhofer and Mark, 1995). The transitions obtained by this model will therefore also be obtained in the movement model.

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Movement of the object B:

Once the movement of a single proper part has been defined, moving the object partially or totally simply is the matter of moving in turn all the elements of $\mathcal{P}^*(\Pi_B)$.

From the programmatic point of view, computation of all the transitions for a given relation R is performed as follows:

FOREACH E of $\mathcal{P}^*(\Pi_B)$ **DO**
  FOREACH P_i proper part of B element of E **DO**
    Compute the union Q of parts of A such that $P_i \subseteq Q$
    FOREACH destination D in $\mathcal{A}^{\#E(Q)}(Q)$ **DO**
      $N_R \leftarrow M_R$, where M_R is the matrix for the relation R
      Modify N_R according to the movement $Q \rightarrow D$
    DONE
  IF N_R is not a valid matrix
    apply consistency constraints to N_R
  DONE
DONE

"Modifying N_R according to the movement $Q \rightarrow D$" consists in setting $\emptyset$ and $\neg\emptyset$ in cells of the matrix N_R. For example, if $Q = \{^o\}$, $\mathcal{A}(Q) = \{^o\partial, \partial\}$, and $E = \partial L$ (section 3.1, movement of ∂L only), we will perform the following operations:

$$N_{R_6}[\partial L, Z^o] \leftarrow \emptyset,\ N_{R_6}[\partial L, Z^o] \leftarrow \neg\emptyset,\ N_{R_6}[\partial L, \partial Z] \leftarrow \neg\emptyset$$

for the first movement, and

$$N_{R_6}[\partial L, Z^o] \leftarrow \emptyset,\ N_{R_6}[\partial L, \partial Z] \leftarrow \neg\emptyset$$

for the second one.

Consistency constraints depend of the objects we are working on.

Extended conceptual neighborhoods for line-zone relations

This section presents the application of the movement model to line-zone relations (defined by the 9-intersection model) in order to build the extended graph of conceptual neighborhoods. In the construction the line L is moved while the zone Z is still.

In the 9-intersection model, L has two proper parts: L^o and ∂L. For modeling the movement of L, we need to perform in turn the movement of L^o while ∂L is still, the movement of ∂L while L^o is still, and finally the simultaneous movement of L^o and ∂L.

As we are working with simple lines (lines having 2 boundaries), we need the number of cells such that $N_R[\partial L, i] = \neg\emptyset, i \in \{Z^o, \partial Z, Z^-\}$ to be strictly less than 3. This constraint must be verified in all cases. If it is not verified, the transition must be rejected.

When ∂L is moved alone, we use consistency constraints that were used in the smooth transition model (equations 5, 6, and 7). These three constraints act on the newly produced matrix N_R. Constraint 5 does not act on ∂L; constraints 6 and 7 modify the position of L^o according to the new position of ∂L.

When L^o is moved alone it is required to correct the position of ∂L according to the new position of L^o. The following consistency constraints are defined (where "$=_{M_R}$" tests equality in the matrix M_R while "$=$" tests equality in the newly constructed matrix N_R):

$$\begin{array}{l} \partial L \cap Z^o =_{M_R} \neg\emptyset \wedge L^o \cap Z^o = \emptyset \Rightarrow \\ \partial L \cap Z^o = \emptyset \wedge \partial L \cap \partial Z = \neg\emptyset \end{array} \quad (13)$$

This constraint specifies that "if L^o moves *completely* from Z^o on ∂Z and if there was an intersection between ∂L and Z^o, then ∂L also moves completely from Z^o on ∂Z". Note that $\partial L \cap Z^o =_{M_R} \neg\emptyset$ implies that $L^o \cap Z^o =_{M_R} \neg\emptyset$ (as all proper parts of the line are connected) and that $L^o \cap Z^o = \emptyset$ specifies that all the line's interior has left Z^o (the destination ∂Z is implied as $\operatorname{Adj}(Z^o) = \{\partial Z\}$).

The constraint:

$$\begin{array}{l} \partial L \cap Z^- =_{M_R} \neg\emptyset \wedge L^o \cap Z^- = \emptyset \Rightarrow \\ \partial L \cap Z^- = \emptyset \wedge \partial L \cap \partial Z = \neg\emptyset \end{array} \quad (14)$$

is equivalent to the constraint 13 for Z^-.

Finally, we add the constraint 5 which insures that the interior of the line is connected.

When L^o and ∂L are moved simultaneously, both sets of constraints should be applied in turn on matrices that correspond to non-existing relations. The experience shows however, that after the boundary corrective constraints have been applied, no new transition is obtained if interior corrective constraints are used.

As we are gaining *many* new transitions, a graphical representation is not readable. The graph is thus represented in tabular form in table 1 where target relations between parenthesis represent targets obtained by the smooth transitions model. As we will see in the next section, some of these transitions are not "atomic" and must be rejected. The targets of non-atomic transitions are written in bold.

Pruning non-atomic transitions

For many transitions in table 1 it is rather easy to find a geometrical interpretation of the line's movement (see for example figure 4). This is however not the case for all transitions, as for example $R_3 \rightarrow R_{15}$. It actually appears that this transition cannot be performed atomically. This section describes the method for detecting end eliminating these non-atomic transitions. The discussion is based on the example of line-zone relations but can be adapted to any objects.

R_{15} is obtained from R_3 if L^o's sub-part that lays on ∂Z (let's call it $L^o_{\partial Z}$) moves in Z^o while L^o sub-part that lays in Z^- ($L^o_{Z^-}$) moves entirely on ∂Z. Due to the constraint defined in equation 14 the boundary of the lines is also required to move on ∂Z. An intuitive reason for the difficulty of interpretation of this transition is that it requires "less time" to leave ∂Z than to leave Z^-. In other words, when $L^o_{\partial Z}$ enters Z^o, $L^o_{Z^-}$ cannot have left *entirely* Z^-, as depicted on figure 5. The transition $R_3 \rightarrow R_{15}$ requires an intermediate step: it is not *atomic*.

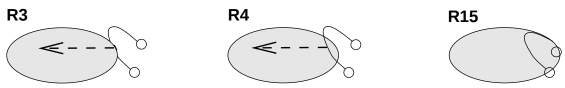

Figure 5: A possible interpretation for transition $R_3 \rightarrow R_{15}$.

A more formal explanation requires to take into account the dimensions of the parts that are sources of the movement. The definitions below apply in the case when a line is moving with respect to a zone; they can however be easily adapted to other objects.

Definition 4 (instantaneous, continuous movement)

A movement is said to be "instantaneous" if its source is ∂Z and its destination is either Z^o or Z^-.

A movement is said to be "continuous" if its source either is Z^o or Z^- and its destination is ∂Z.

Definition 5 (Atomic line's movement)

The movement of a line is said to be "atomic" if the movements of all the proper parts or sub-parts of the proper parts implied by the general movement are of same nature (all instantaneous or continuous).

Let's take some examples from table 1:

- $R_6 \rightarrow R_{12}$ is obtained by: $\{^o\partial^-\} \xrightarrow{L^o} \{\partial\}$.
  As neither ^{o} or $^-$ appear in the destination, the sub-parts of L^o that were lying in Z^o and in Z^- did move on ∂Z. We therefore had two sub-movements: $Z^o \rightarrow \partial Z$ and $Z^- \rightarrow \partial Z$ which according to definition 4 are both continuous. The whole movement is therefore atomic (def. 5).

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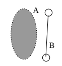
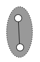
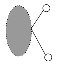
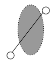
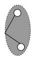
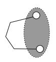
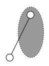
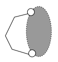
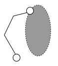
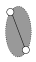
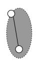
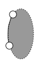
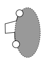
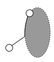
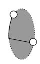
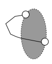
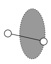
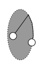
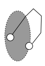
Relation	Neighbors
 R_1	(R_9), R_8
	(R_3), R_{12}
	R_{13}, R_{14}
 R_2	(R_{11}), R_{10}
	(R_5), R_{12}
	R_{15}, R_{18}
 R_3	(R_{14}), R_{13}
	(R_1, R_4), R_{12}, $\mathbf{R_{15}}$
	$\mathbf{R_8}$, $\mathbf{R_9}$, $\mathbf{R_{16}}$, $\mathbf{R_{17}}$
 R_4	(R_{17}), R_{16}
	(R_3), R_{12}, R_{15}
	R_{13}, R_{14}
 R_5	(R_{18}), R_{15}
	(R_2, R_6), R_{12}, $\mathbf{R_{13}}$
	$\mathbf{R_{10}}$, $\mathbf{R_{11}}$, $\mathbf{R_{16}}$, $\mathbf{R_{19}}$
 R_6	(R_{19}), R_{16}
	(R_5), R_{12}, R_{13}
	R_{15}, R_{18}
 R_7	(R_{17}, R_{19}), R_{16}
	(R_{14}, R_{18}), R_{12}
	R_{13}, R_{15}
 R_8	(R_9, R_{19}), R_1, R_6, R_7
	(R_{13}), R_{12}
	$\mathbf{R_5}$, $\mathbf{R_{18}}$, $\mathbf{R_3}$, $\mathbf{R_{14}}$
 R_9	(R_1, R_7, R_8), $\mathbf{R_{19}}$
	(R_{14}), R_{12}
	$\mathbf{R_{18}}$, R_{13}, $\mathbf{R_3}$
 R_{10}	(R_{11}, R_{17}), R_2, R_4, R_7
	(R_{15}), R_{12}
	$\mathbf{R_3}$, $\mathbf{R_{14}}$, $\mathbf{R_5}$, $\mathbf{R_{18}}$
 R_{11}	(R_2, R_7, R_{10}), $\mathbf{R_{17}}$
	(R_{18}), R_{12}
	$\mathbf{R_{14}}$, $\mathbf{R_5}$, R_{15}
 R_{12}	(R_{18}, R_{14}), R_3, R_5, R_7
	(R_{13}, R_{15}), R_{10}, R_8, R_{16}
	R_2, R_{11}, R_1, R_9, R_6, R_4, R_{19}, R_{17}
 R_{13}	(R_{19}, R_{14}), R_6, R_3, R_7
	(R_{12}, R_8, R_{16}), $\mathbf{R_{15}}$
	R_1, R_9, $\mathbf{R_5}$, $\mathbf{R_{18}}$, R_4, R_{17}
 R_{14}	(R_3, R_{13}, R_7), $\mathbf{R_{19}}$
	(R_{12}, R_9, R_{17}), $\mathbf{R_{15}}$
	$\mathbf{R_8}$, R_1, $\mathbf{R_{18}}$, $\mathbf{R_{16}}$, R_4
 R_{15}	(R_{17}, R_{18}), R_5, R_4, R_7
	(R_{10}, R_{12}, R_{16}), $\mathbf{R_{13}}$
	R_2, R_{11}, $\mathbf{R_3}$, $\mathbf{R_{14}}$, R_6, R_{19}
 R_{16}	(R_{17}, R_{19}), R_6, R_4, R_7
	(R_{13}, R_{15}), R_{12}
	$\mathbf{R_3}$, $\mathbf{R_{14}}$, $\mathbf{R_5}$, $\mathbf{R_{18}}$
 R_{17}	(R_{16}, R_4, R_7), $\mathbf{R_{19}}$
	(R_{14}, R_{15}), R_{12}
	R_{13}, $\mathbf{R_3}$, $\mathbf{R_{18}}$
 R_{18}	(R_5, R_7, R_{15}), $\mathbf{R_{17}}$
	(R_{11}, R_{12}, R_{19}), $\mathbf{R_{13}}$
	R_2, $\mathbf{R_{10}}$, $\mathbf{R_{14}}$, R_6, $\mathbf{R_{16}}$
 R_{19}	(R_6, R_7, R_{16}), $\mathbf{R_{17}}$
	(R_{13}, R_{18}), R_{12}
	$\mathbf{R_{14}}$, $\mathbf{R_5}$, R_{15}

Table 1: Neighbors obtained by the movement model. The left column is the source relation, the right columns contain the names of the target relations. The first line are the targets obtained by moving line's boundary, the second line are targets obtained by moving line's interior, and the third line are additional targets obtained by simultaneous movements of the interior and of the boundary. Relations between parenthesis are those obtained by the STM. Relations written in bold font are to be rejected as they are not atomic (see text).

- $R_6 \rightarrow R_{18}$ is obtained by: $\{^o\} \xrightarrow{\partial L} \{^o\partial\}$.
  As ∂ appears in the destination and not in the source, the movement to take into account is the partial movement of ∂L from Z^o on ∂Z. By definition 4, this movement is continuous, and as it is the only movement in the transition, the transition is atomic.

- $R_3 \rightarrow R_{15}$ is obtained by: $\{\partial^-\} \xrightarrow{L^o} \{^o\partial\}$.
  Because of the definition of the adjacency on the zone object, this movement requires that the sub-part of L^o that lays on ∂Z moves in Z^o and that the sub-part that lays in Z^- moves on ∂Z. The first sub-movement is instantaneous, while the second one is continuous. The transition $R_3 \rightarrow R_{15}$ therefore is non-atomic and must be rejected.

Practically, the test for deciding whether a transition is atomic or not will be performed as follows. First assume one proper part P of the line is moving from the source $s_1 \ldots s_n$ on the destination $d_1 \ldots d_m$, where s_i and d_j are zone's parts ($\{s_1 \ldots s_n\} \xrightarrow{P} \{d_1 \ldots d_m\}$). First, determine what sub-movements are required for the realization of the whole movement of P. Then for each of these sub-movements, determine their nature (continuous or instantaneous). If all the sub-movements are of identical nature, accept the transition. If the two proper parts of the line are moving, then accept the transition if the sub-movements required by both whole movements of both proper parts are of identical nature.

It is important to notice that this test must be applied on proper parts that are actually moving and only on these. In other words, it must be applied before any consistency constraint was used for correcting the new matrix N_R. Otherwise, transitions like $R_{11} \rightarrow R_7$ would be rejected as L^o "jumps" over ∂Z.

Once non-atomic transitions have been pruned, we obtain the extended graph of conceptual neighborhoods shown on figure 6. For readability, only the new transitions obtained by the movement model are depicted.

Conclusion
This section described a general method for building conceptual neighborhood graphs among topological relations. As an example, we did build the graph for line-zone relations. If other objects are considered, only consistency constraints and the pruning algorithm have to be adapted.

The graph we obtain for line-zone relations is oriented. As the movement model is an extension of the smooth transition model, we inherit all the asymmetric transitions ($R_{11} \rightarrow R_7$, for example). Moreover, few new asymmetric transitions are produced by the new model ($R_8 \rightarrow \{R_7, R_6\}$ and $R_{10} \rightarrow \{R_7, R_4\}$). As for the STM, counterparts of asymmetric transitions are easily interpreted in terms of movement. The graph can therefore be considered as non-oriented.

4 DISCUSSION AND PROSPECTIVE WORK

This section shortly presents two applications of the conceptual neighborhood graphs based on the movement model presented above. First, it is however necessary to state the context of our work.

As stated in the introduction, we are working on a GIS query interface based on a graphical representation of spatial configurations (Szmurlo et al., 1998, Szmurlo and Gaio, 1998). This interface is hybrid, in the sense that spatial constraints are represented as a sketch while the types of the objects and the thematic constraints are expressed as expressions in natural language. Distance relations are partly expressed as graphics (to specify which objects are in relation) and partly in natural language (to specify the actual distance). In order to avoid the problem of image preprocessing and object recognition, our drawing tool is a constrained

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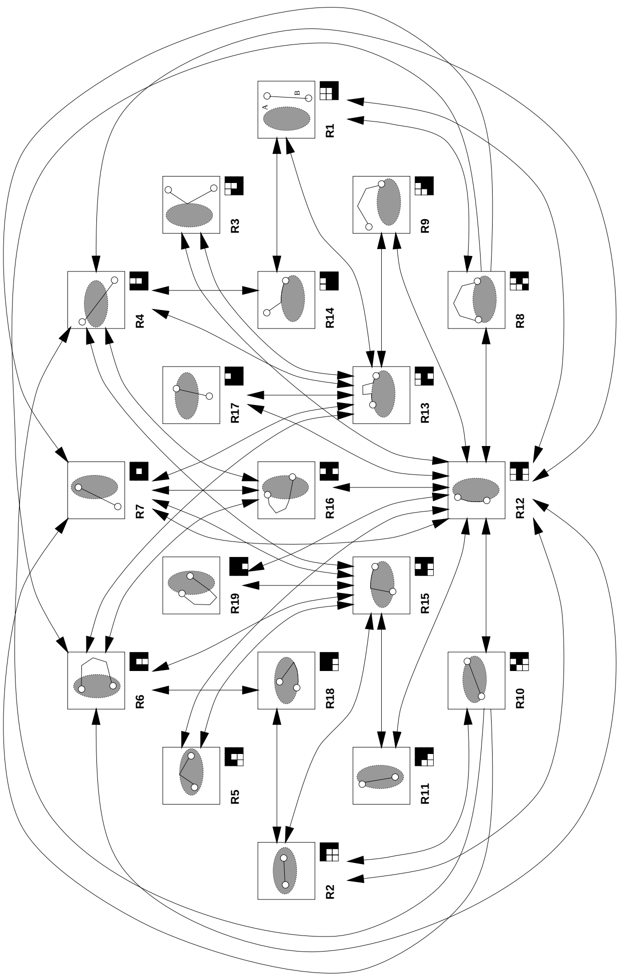

Figure 6: Conceptual neighborhoods based on the movement model. Transitions obtained by the smooth transitions model are not shown, for readability.

vector drawing application *à la* MacDraw which allows the user to only draw points, segments, bi-segments, and octagons. These four graphical elements allow the user to express all but two line-line relations defined by Egenhofer *et. al*.

The user is fully integrated in the processing loop at both stages of query analysis and acceptance of the response. First the user builds the query which is sent to the analyzer. The analyzer attempts to verbalize with short expressions the concepts the user introduced by the sketch and the labels. Spatial concepts will be modeled (at least partially) with an extended set of topological relations as briefly explained below. If the user accepts system's analysis, a query is constructed and sent to the GIS. If a part of the analysis is refused by the user, the system must make a proposition of a close concept, or a spatial configuration if verbalization is not possible, that loosely corresponds to the sketch. Neighborhoods of the configurations the user sketched will be used in order to find these "close concepts". The second foreseen usage is to relax spatial constraints if either there is no objects corresponding to the query, or if the response does not satisfy the user. Only the first point is discussed below.

Several authors proposed to express spatial concepts with topological relations or sets of these, while taking into account whole objects regardless of their semantics (Gting, 1988, Clementini et al., 1993). This may be called a *global* approach. While applicable in some limited cases (zone-zone relations (Egenhofer et al., 1991)), we believe that such mapping should be *local*. In other words, relations between only parts of the objects should be taken

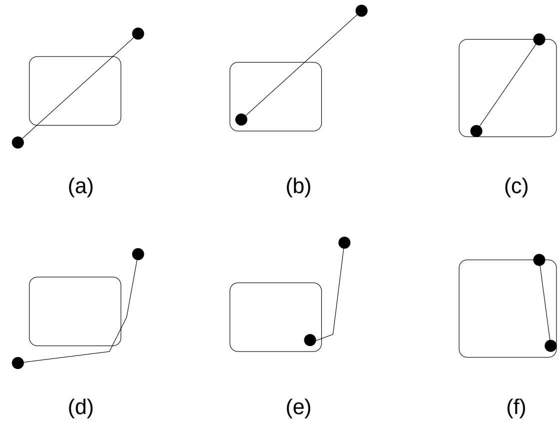

Figure 7: Top: a set of topological relations that may be interpreted as "crossing". Bottom: identical topological relations that cannot be interpreted as "crossing".

into account. Let's consider sketches *(a)*, *(b)* and *(c)* from figure 7. All three correspond to different topological relations but all may express the "crossing" concept applied to different types of objects: a river *crossing* a town (rarely a town contains an entire river), a road (that begins in the town) *crossing* a town (and that goes away), and an avenue *crossing* a town[3]. Conversely, even if sketches *(d)*, *(e)* and *(f)* represent the same topological relations as sketches *(a)*, *(b)* and *(c)*, respectively, they do not correspond to a "*crossing*" but rather to "circumvent" (*(d)*) and "go along" (*(e)* and *(f)*) configurations. The information that captures the "crossing" concept is that, at some moment, the line is near the zone's boundary, then it goes trough the central part of the zone, and finally, it is again close the zone's boundary[4], *regardless* of what happen to line's boundary and the rest of line's interior.

To be able to model concepts this way, and in order to have a generic model for spatial relations, we define a new set of topological regions for defining objects. For a zone we may need: the *central* interior Z^{o_c}, the *peripherical* interior Z^{o_p}, the intermediate interior Z^{o_i} located between Z^{o_c} and Z^{o_p}, the *peripherical* exterior Z^{-_p}, and the *external* exterior Z^{-_e}. These parts are mutually exclusive. By considering the 6×3 boolean matrix that defines all the possible intersection between the six new zone's parts and the three "traditional" parts of the line, we define a new set of topological relations between a zone and a line. The new topological parts are shown figure 8. Such an extension results in *many* new relations. However, as we intend to model concepts locally, only those parts of the matrices that enter in the definition of the concept are to be considered.

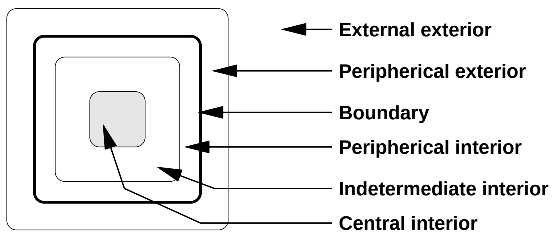

Figure 8: Parts defined by the new definition of the zone object..

In this context, the graph based on the movement model may be used for two purposes. Assume that the sketch is interpreted into a

[3] This interpretation cannot be applied on sketch *(b)* as an avenue is only defined within the limits of a town. Note here the importance of the geographical semantics given to the objects.

[4] Actually, some directional constraint should also be added in this attempt to model "crossing".

given concept C and that this interpretation is refused by the user. The the analyzer would perform movements of the parts that enter in the definition of C in order to obtain new configurations. This is straightforward as the graph's model is defined independently of the number of parts implied by the embedding of the object in the universe. Some of these new configurations may correspond to different concepts C', other may still correspond to C itself, and finally, some new configurations may correspond to no particular concept at all. In the first case, new propositions based on C' will be done to the user. Once the user accepts the propositions of the analyzer, the system must generate the query that will be issued to the GIS. Furthermore, it must take into account the refused concept C, that is that all the relations corresponding to C must be eliminated from the query construction process. Once again, the graph can be used to determine the set of these relations.

5 CONCLUSION

This paper presents an extension of the smooth transition model for building graphs of conceptual neighborhoods among binary topological relations. The new model, the so called *movement model*, is defined generically for any pair of objects and defines the movements of one object with respect to the second one. The example of the graph for line-zone relations defined by the 9-intersection is provided. The prospective applications of these extended graphs are mainly oriented toward the conception of analyzers and human/machine dialogue engines that enter in the definition of new GIS interfaces.

ACKNOWLEDGMENT

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