第三节　有导体时电磁波的传播

张子珍

山西大同大学物理与电子科学学院

2013.11.13

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重点：趋肤效应
难点：复介电常数，衰减波

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故导体中的能量主要是磁场能。

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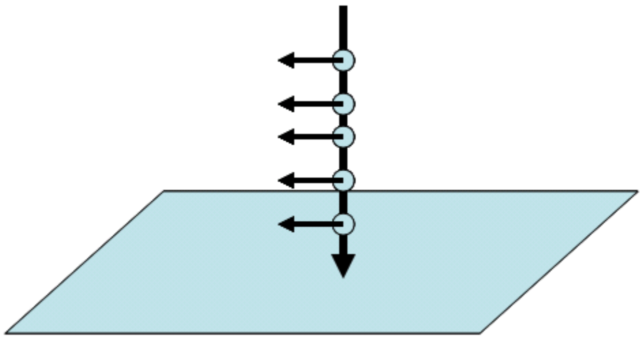

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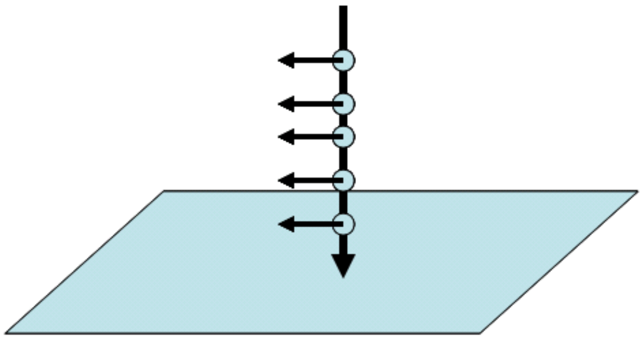

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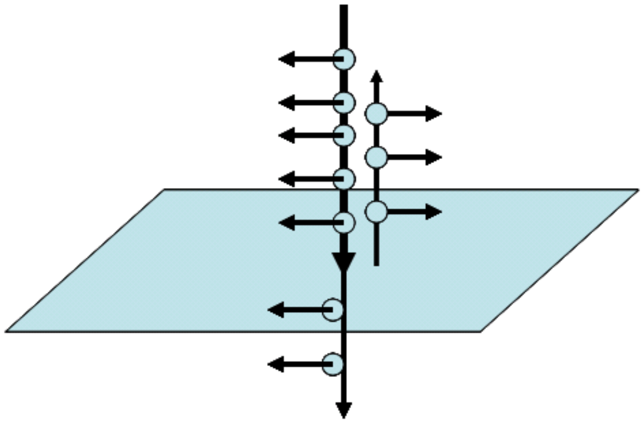

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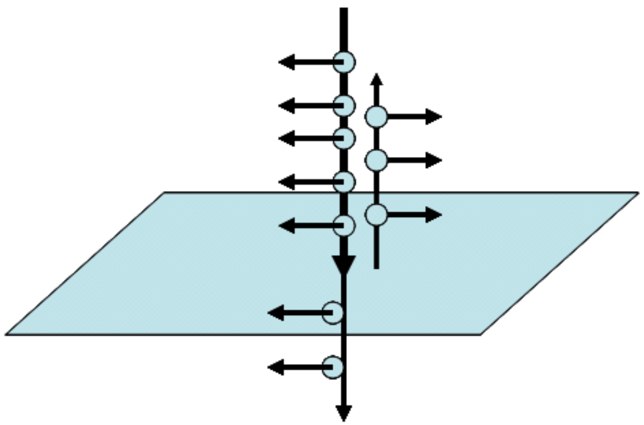

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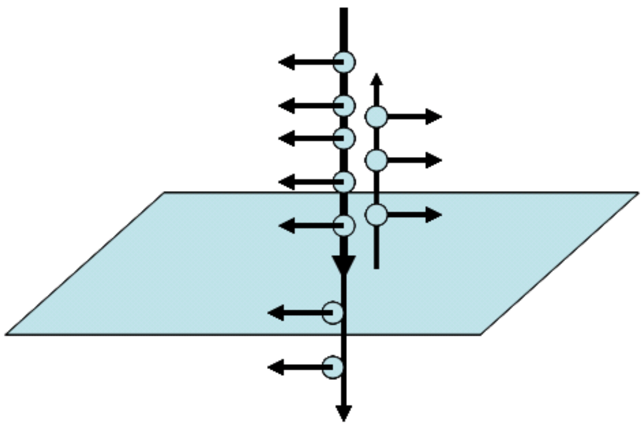

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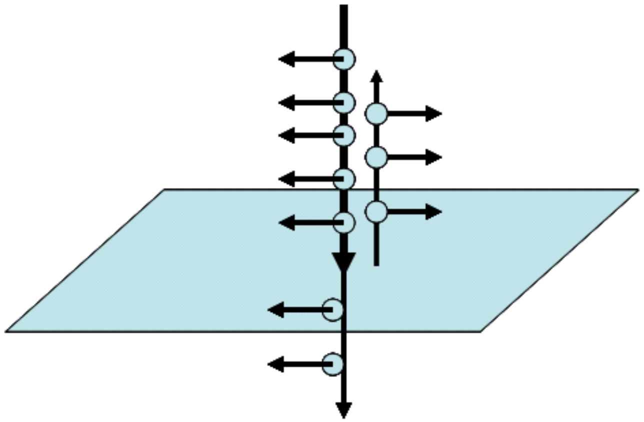

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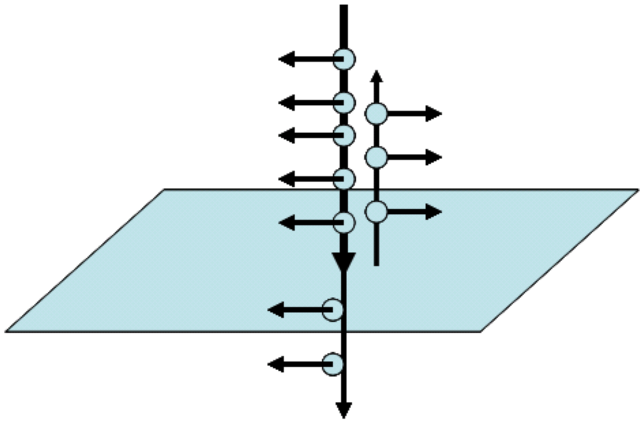

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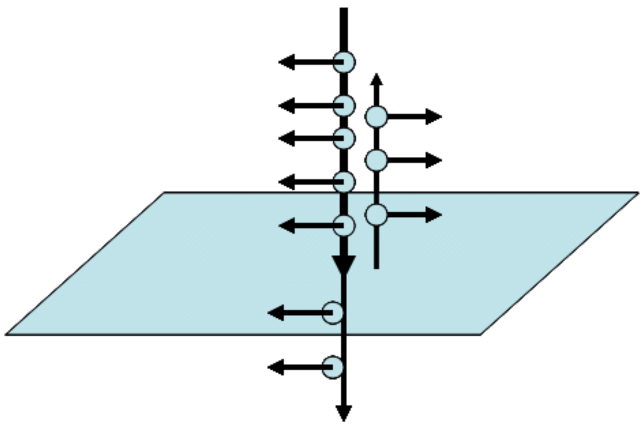

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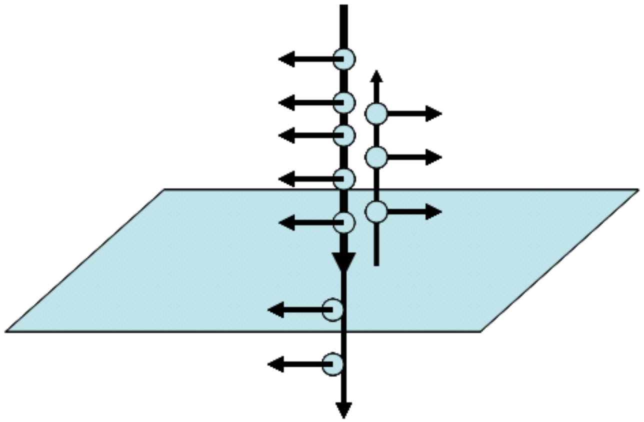

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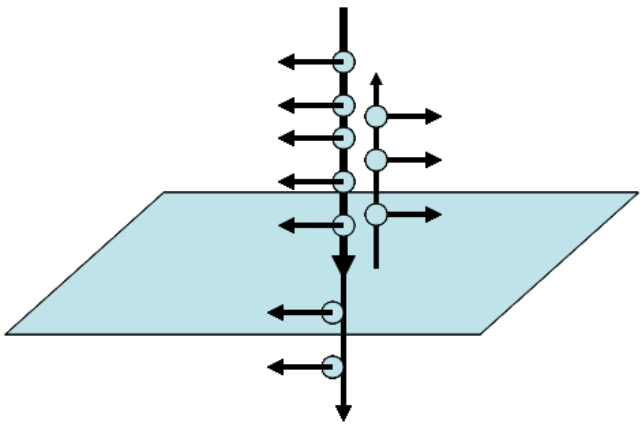

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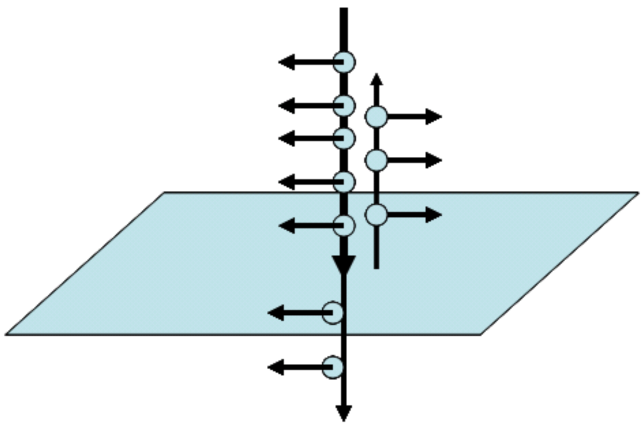

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