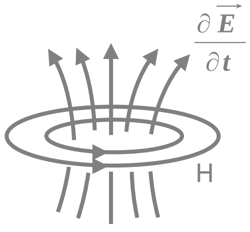
Chapter 7 Electromagnetic Wave

7.1 Displacement Current

7.2 The Generation & Travel of EMW

7.3 The Fundamental Properties of EMW

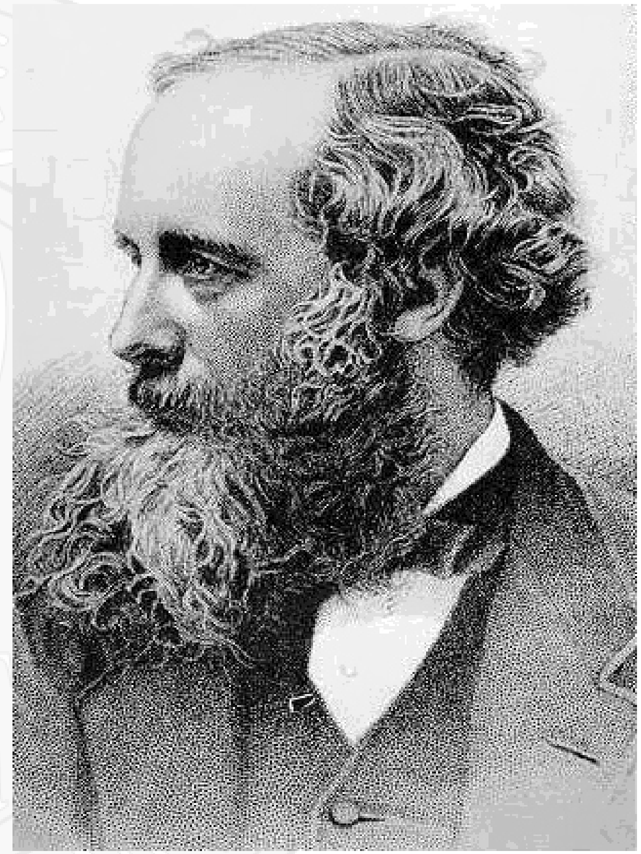
7.4 Electromagnetic Spectrum



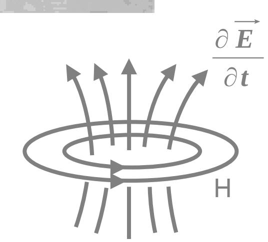
7.1 Displacement Current

Maxwell, English physicist

James Clerk Maxwell: Scottish physicist best known for his formulation of electromagnetic theory. He is regarded by most modern physicists as the scientist of the 19th century who had the greatest influence on 20th-century physics, ...



Maxwell



7.1 Displacement Current

◈ Maxwell's Equations

Integral Form

$$\oiint_{(S)} \vec{D} \cdot d\vec{S} = \sum_{(S内)} Q_0$$

$$\oint_{(L)} \vec{E} \cdot d\vec{l} = -\iint_{(S)} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oiint_{(S)} \vec{B} \cdot d\vec{S} = 0$$

$$\oint_{(L)} \vec{H} \cdot d\vec{l} = \sum_{(L内)} I_0$$

Differential Form

$$\nabla \cdot \vec{D} = \rho_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

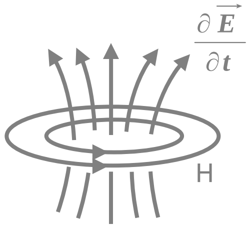
$$\nabla \times \vec{H} = \vec{J}_0$$

$$\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$$

$$\vec{B} = \mu_r \mu_0 \vec{H}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\oint_{(S)} \vec{J} \cdot d\vec{S} = -\frac{dq}{dt}$$



7.1 Displacement Current

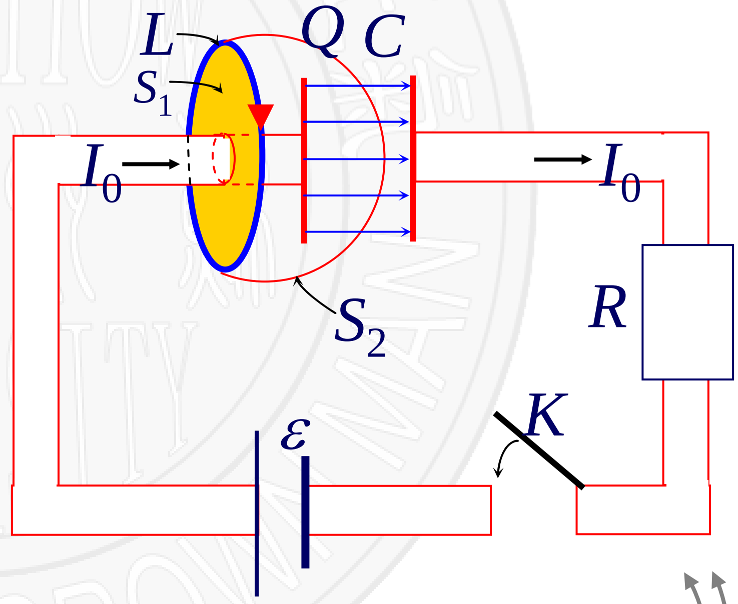
◈ Displacement Current

Are these Equations Complete ?

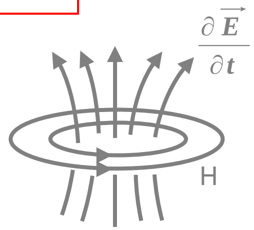
- Argument from integral form of circuital law for ***H***

  - Switch on
  - Charging

$$\oint_{(L)} \vec{H} \cdot \mathrm{d}\vec{l} = \sum_{(L内)} I_0$$



? Is it ok for any surface bounded by L?



7.1 Displacement Current

◈ Displacement Current

- For surface S_1: $\oint_{(L)} \vec{H} \cdot d\vec{l} = \sum_{(L内)} I_0$
- For surface S_2: $\oint_{(L)} \vec{H} \cdot d\vec{l} = 0$

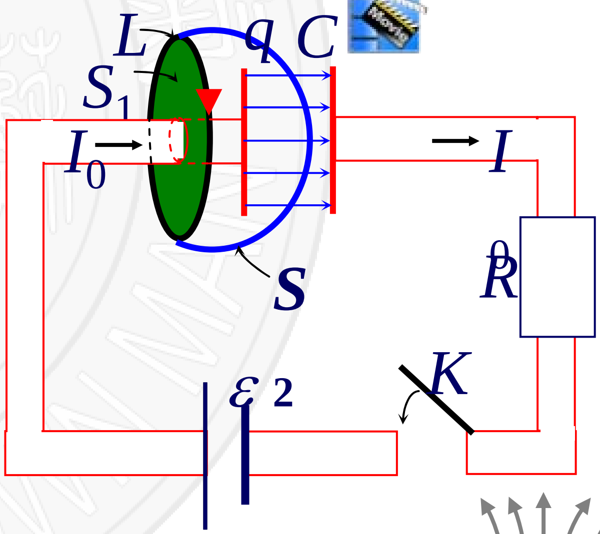
} Discrepancy !

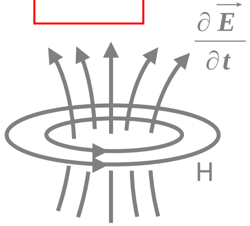
- No doubt, A is correct

How do we deal with B ?

Looking for general circuital Law

No charge flows across S_2, D in C changes with time when charging, because Q is changing.





7.1 Displacement Current

◈ Displacement Current

- Time-varying E(D)

Maxwell proposed time-varying E(D) can create B-field.

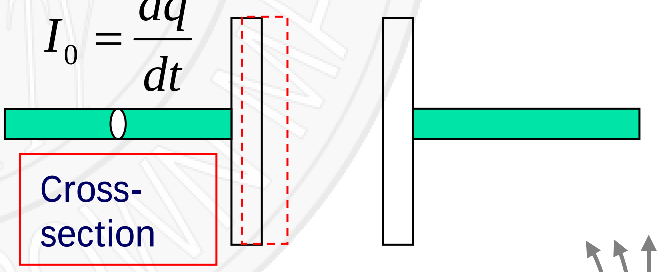
According to the definition of current $I_0 = \frac{dq}{dt}$

dq is the charge passing through a cross-section of the wire, it is also the change on the plate.

From Gauss's law

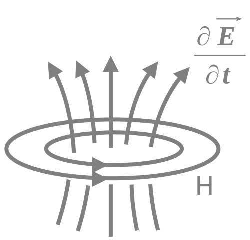
q

$I_0 = \frac{dq}{dt}$



$$\oiint_{(S)} \vec{D} \cdot \mathrm{d}\vec{S} = \sum_{(S内)} Q_0 \qquad DS = q$$

$$I_0 = \frac{dq}{dt} = \frac{dD}{dt} S$$



7.1 Displacement Current

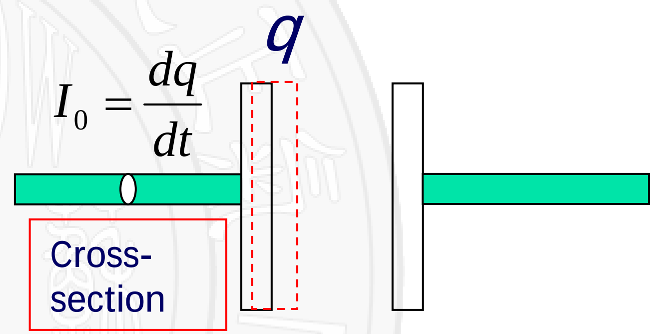
- Displacement current:

$$I_d = \frac{dD}{dt} S$$

- Displacement current density:

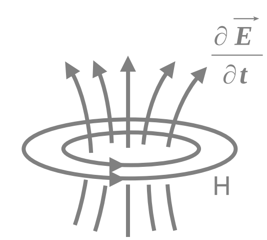
$$J_d = \frac{dD}{dt}$$

- Total current: $I_{Total} = I_0 + I_d$



◇ The general form of circuital law

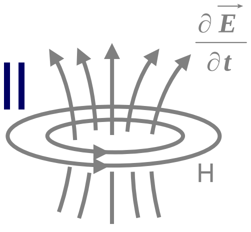
$$\oint_{(L)} \vec{H} \cdot d\vec{l} = I_0 + I_d$$



7.1 Displacement Current

◈ Displacement Current

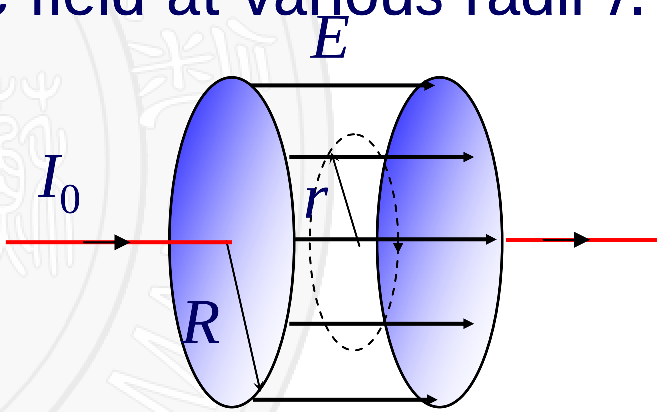
- Difference between I_c & I_D
  - Displacement current is induced by time-varying electric field
  - Conduction current is induced by the motion of charge
  - Conduction current can produce thermal energy dissipation
  - Displacement current doesn't produce thermal energy dissipation
  - Displacement current and conduction current all create magnetic field



7.1 Displacement Current

◈ Displacement Current

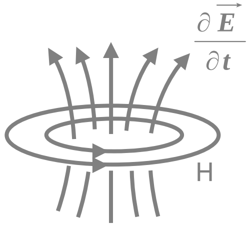
Example 7.1 A parallel-plate capacitor with circular plates whose radius is *R* is being charged as the figure shown Suppose the time-varying rate of electric field is dE/dt . Derive an expression for the induced magnetic field at various radii *r*. Consider both *r*<*R* and *r* >*R*



Solution: According to Ampere's circuital law, we have

$$\oint_{(L)} \vec{H} \cdot \mathrm{d}\vec{l} = I_0 + I_d = I_d$$

$$I_d = \frac{dD}{dt} S \qquad J_d = \frac{dD}{dt} = \varepsilon_0 \frac{dE}{dt}$$



7.1 Displacement Current

Displacement Current

r<R

$$\oint_{(L)} \vec{H} \cdot \mathrm{d}\vec{l} = 2\pi r H \qquad I_d = \frac{dD}{dt} S = \varepsilon_0 \frac{dE}{dt} \pi r^2$$

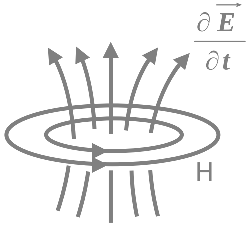
$$H = \frac{1}{2} \varepsilon_0 r \frac{dE}{dt} \qquad B = \mu_0 H = \frac{1}{2} \varepsilon_0 \mu_0 r \frac{dE}{dt}$$

r>R

$$I_d = \frac{dD}{dt} S = \varepsilon_0 \frac{dE}{dt} \pi R^2$$

$$H = \frac{1}{2} \varepsilon_0 \frac{R^2}{r} \frac{dE}{dt}$$

$$B = \frac{1}{2} \varepsilon_0 \mu_0 \frac{R^2}{r} \frac{dE}{dt}$$



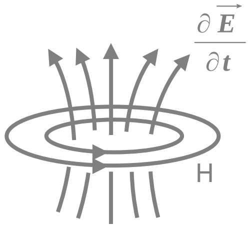
7.1 Displacement Current

- Fundamental part of displacement current
  - Pay attention to displacement current I_d
  - Electric field is the fundamental part of displacement current

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$J_d = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t}$$

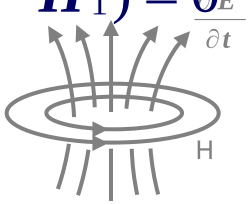
  - In vacuum, P=0, if there is time-varying electric field E,$J_d \neq 0$
  - Completion of Maxwell's Equations



7.1 Displacement Current

- Finally

Maxwell's Equations	Field Quantities	Boundary condition
$\oiint_{(S)} \vec{D}\cdot \mathrm{d}\vec{S} = \sum_{(S内)} Q_0$	$\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$	$\vec{n}\cdot(\vec{D}_2 - \vec{D}_1) = 0$
$\oint_{(L)} \vec{E}\cdot \mathrm{d}\vec{l} = -\iint_{(S)} \frac{\partial \vec{B}}{\partial t}\cdot \mathrm{d}\vec{S}$	$\vec{B} = \mu_r \mu_0 \vec{H}$	$\vec{n}\times(\vec{E}_2 - \vec{E}_1) = 0$
$\oiint_{(S)} \vec{B}\cdot \mathrm{d}\vec{S} = 0$	$\vec{J} = \sigma(\vec{E} + \vec{E}_n)$	$\vec{n}\cdot(\vec{B}_2 - \vec{B}_1) = 0$
$\oint_{(L)} \vec{H}\cdot \mathrm{d}\vec{l} = I_0 + \varepsilon_0 \iint_{(S)} \frac{\partial \vec{E}}{\partial t}\cdot d\vec{S}$	$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$	$\vec{n}\times(\vec{H}_2 - \vec{H}_1) = 0$
	$\oint_{(S)} \vec{J}\cdot \mathrm{d}\vec{S} = -\frac{\mathrm{d}q}{\mathrm{d}t}$	



7.1 Displacement Current