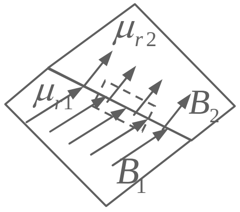
Chapter 6 Magnetic Materials

6.1 Magnetization **M** & Magnetization Current

6.2 Ferromagnetism

6.3* The Fundamental Magnetic Properties of Superconductors

6.4 Magnetic Circuit Theorem

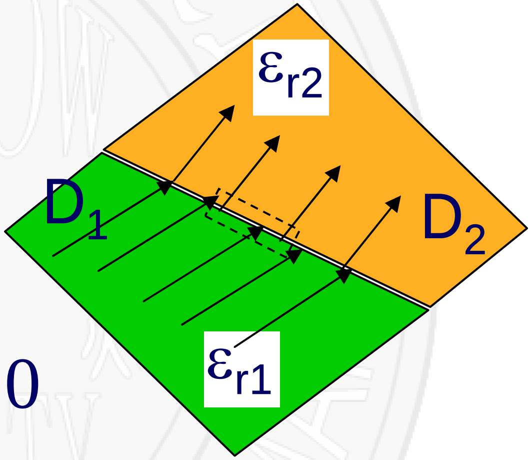


6.4 Magnetic Circuit Theorem

◈ Boundary Condition of Dielectrics

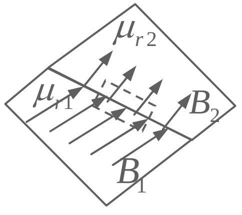
At the Boundary of the Dielectrics,
no free charges

$$\oiint_{(S)} \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} = \sum_{insideS} q_0 = 0$$



$$D_2 \Delta S \cos\theta_2 - D_1 \Delta S \cos\theta_1 = 0$$

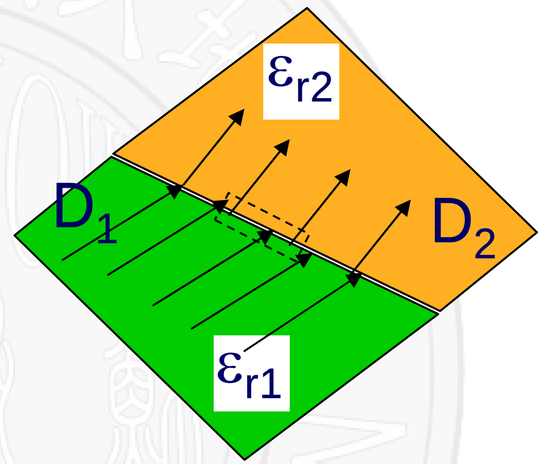
$$D_{2n} = D_{1n}$$



6.4 Magnetic Circuit Theorem

Boundary Condition of Dielectrics

The normal component of Displacement is continuous

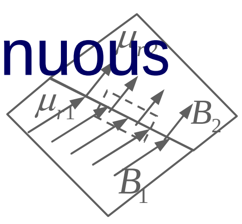


$$D = \varepsilon_r \varepsilon_0 E$$

$$\varepsilon_{r1}\varepsilon_0 E_{1n} = \varepsilon_{r2}\varepsilon_0 E_{2n}$$

$$\varepsilon_{r1} E_{1n} = \varepsilon_{r2} E_{2n}, E_{1n} \neq E_{2n}$$

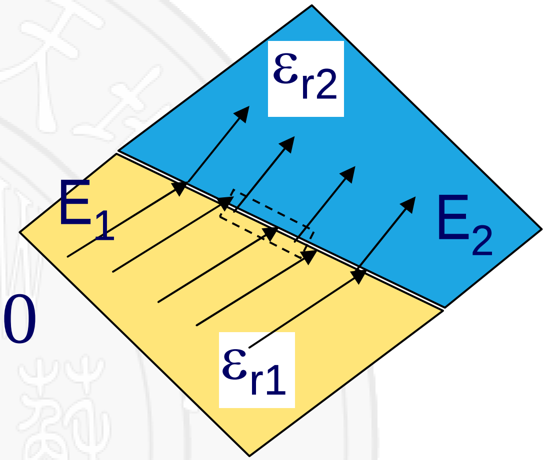
The normal component of Electric field is not continuous



6.4 Magnetic Circuit Theorem

◈ Boundary Condition of Dielectrics

Consider the tangent component



$$\oint_{(L)} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = 0$$

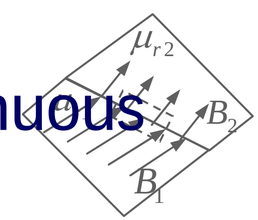
$$E_2 \Delta l \cos\alpha_2 - E_1 \Delta l \cos\alpha_1 = 0$$

$$E_{2t} = E_{1t}$$

The tangent component of Electric field is continuous

$$E = \frac{D}{\varepsilon_r \varepsilon_0} \qquad \frac{D_{1t}}{\varepsilon_{r1}\varepsilon_0} = \frac{D_{2t}}{\varepsilon_{r2}\varepsilon_0} \qquad D_{1t} \neq D_{2t}$$

The tangent component of Displacement is not continuous

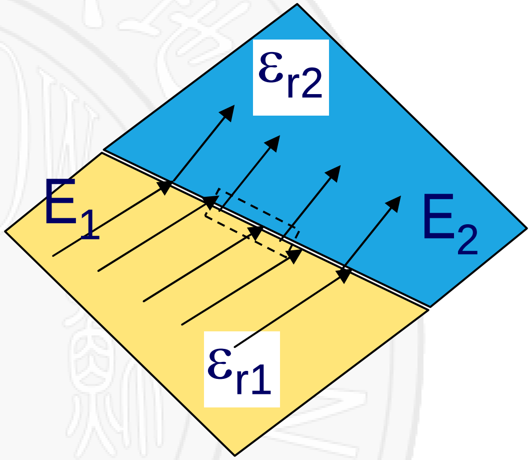


6.4 Magnetic Circuit Theorem

◈ Boundary Condition of Dielectrics

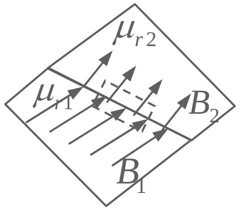
Consider the tangent component

$$\oint_{(L)} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = 0$$



$$E_2 \Delta l \cos\alpha_2 - E_1 \Delta l \cos\alpha_1 = 0$$

$$E_{2t} = E_{1t}$$



6.4 Magnetic Circuit Theorem

◈ Boundary Condition of Magnetic Medium

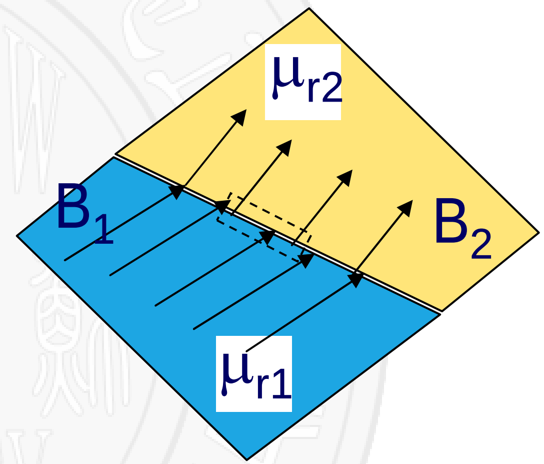
The normal component of Magnetic Induction B is continuous

$$B = \mu_r \mu_0 H$$

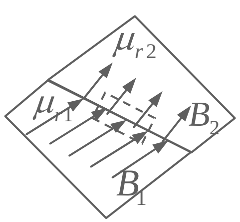
$$B_{1n} = B_{2n}$$

$$\mu_{r1}\mu_0 H_{1n} = \mu_{r2}\mu_0 H_{2n}$$

$$\mu_{r1} H_{1n} = \mu_{r2} H_{2n}, H_{1n} \neq H_{2n}$$



The normal component of Magnetic field H is not continuous

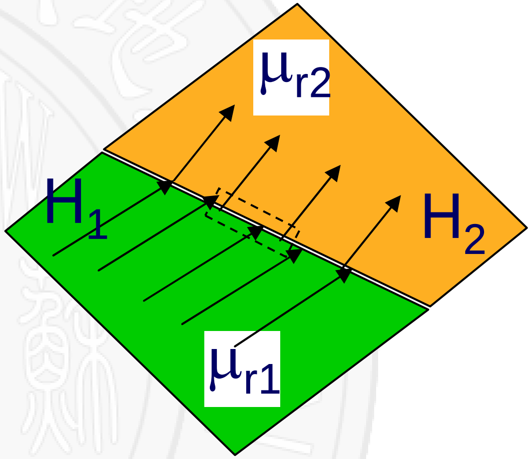


6.4 Magnetic Circuit Theorem

◈ Boundary Condition of Magnetic Medium

Consider the tangent component

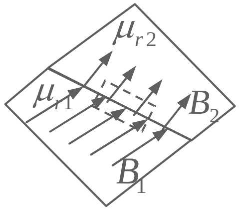
$$\oint_{(L)} \vec{H} \cdot d\vec{\mathbf{l}} = \sum I_0 = 0$$



$$H_2 \Delta l \cos\alpha_2 - H_1 \Delta l \cos\alpha_1 = 0$$

$$H_{2t} = H_{1t}$$

The tangent component of H is continuous



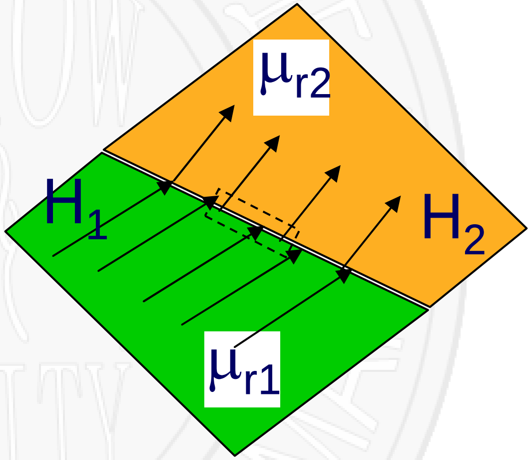
6.4 Magnetic Circuit Theorem

◈ Boundary Condition of Magnetic Medium

Consider the tangent component of B

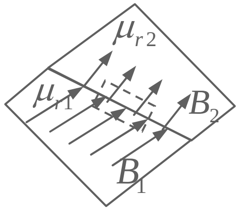
$$H = \frac{B}{\mu_r \mu_0}$$

$$\frac{B_{1t}}{\mu_{r1}\mu_0} = \frac{B_{2t}}{\mu_{r2}\mu_0}$$



$$B_{1t} \neq B_{2t}$$

The tangent component of Magnetic Induction B is not continuous



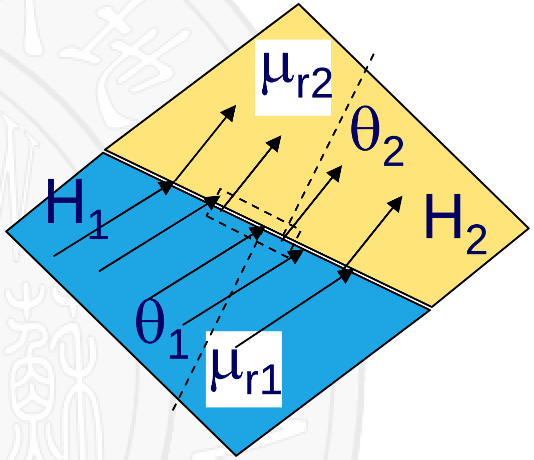
6.4 Magnetic Circuit Theorem

Boundary Condition of Magnetic Medium

The reflection of Induction Lines on the Boundary

$$H_{2t} = H_{1t}$$

$$B_{2n} = B_{1n}$$

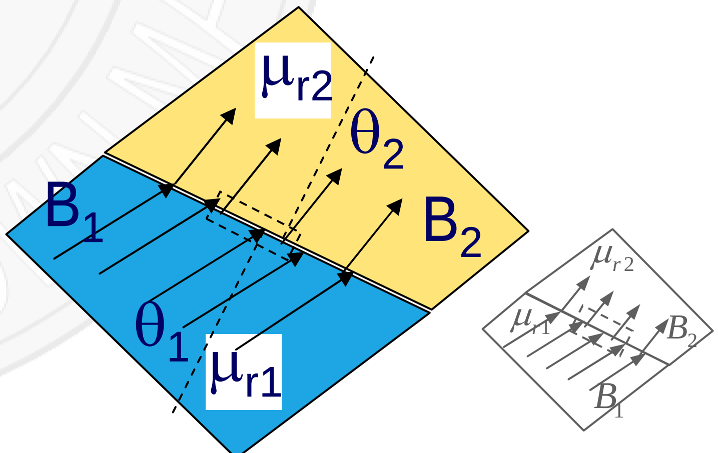


The ratio

$$\frac{H_{2t}}{B_{2n}} = \frac{H_{1t}}{B_{1n}}$$

$$\frac{H_2 \sin\theta_2}{B_2 \cos\theta_2} = \frac{H_1 \sin\theta_1}{B_1 \cos\theta_1}$$

$$\frac{1}{\mu_{r2}\mu_0}\tan\theta_2 = \frac{1}{\mu_{r1}\mu_0}\tan\theta_1$$

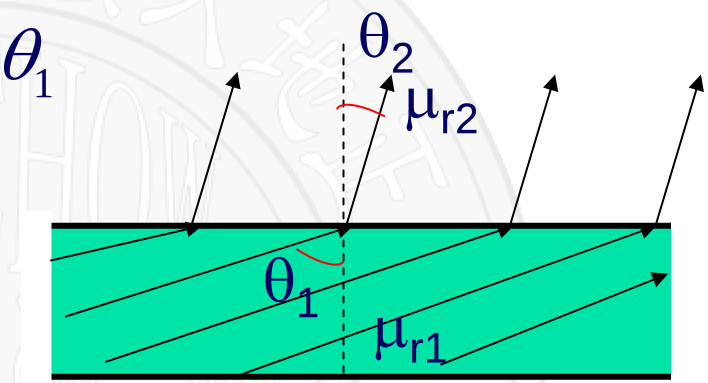


6.4 Magnetic Circuit Theorem

Boundary Condition of Magnetic Medium

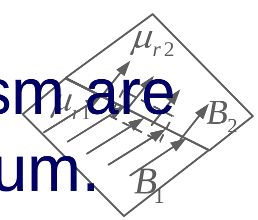
$$\frac{1}{\mu_{r2}\mu_0}\tan\theta_2 = \frac{1}{\mu_{r1}\mu_0}\tan\theta_1$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r1}}{\mu_{r2}}$$



If medium 2 is vacuum or not Ferromagnetism, $\mu_{r2}=1$, and medium 1 is Ferromagnetism, μ_{r1} is very large. So θ_2 is very small and θ_1 is approximately right angle.

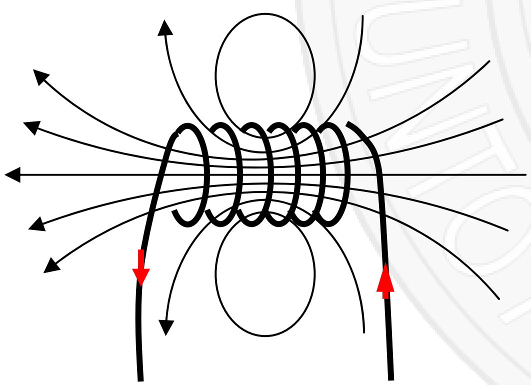
It means that the lines of induction in Ferromagnetism are parallel to the surface, few lines get out of the medium.

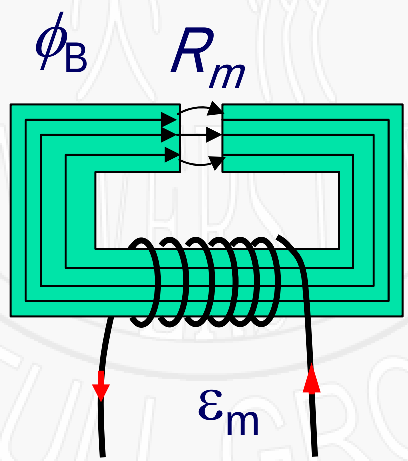


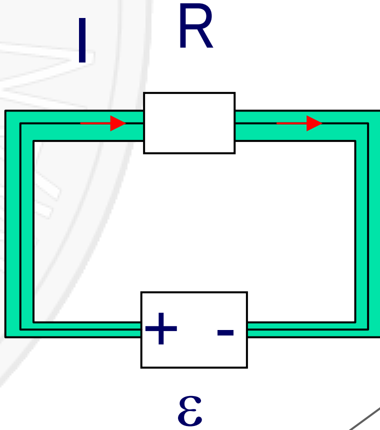
6.4 Magnetic Circuit Theorem

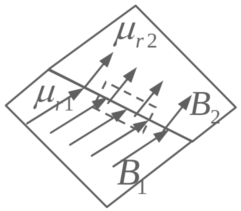
◈ Magnetic Circuit Theorem

According to the reflection of induction lines, the induction lines due to a current-carrying coil without iron will go everywhere. But if there is a iron inside the coil, the induction lines will concentrate inside the iron and go along the iron, Induction tube, electric current tube, like electric circuit, Magnetic Circuit .



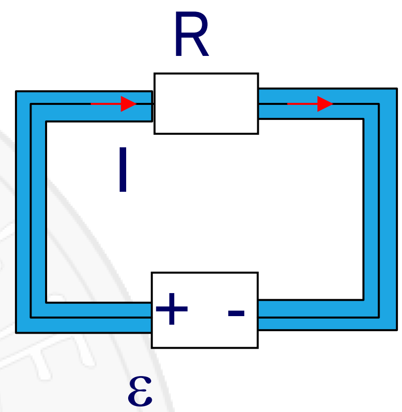






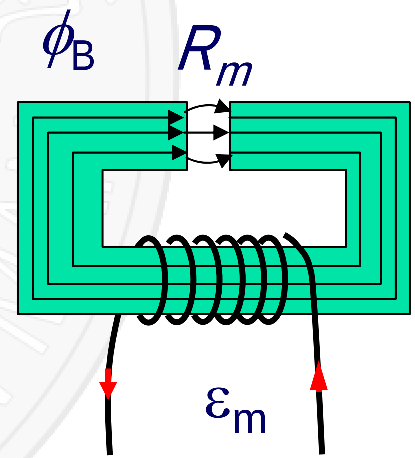
6.4 Magnetic Circuit Theorem

◇ Magnetic Circuit Theorem



In the electric circuit with combinations of resistors in series, the currents through every resistor are same.

In the magnetic circuit with irons, the fluxes ϕ_B through every cross section are same.

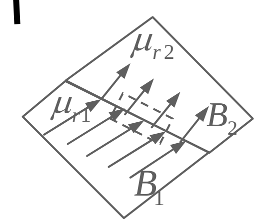


For electric circuit:

$$\varepsilon = \sum_i IR_i = I\sum_i R_i = I\sum_i \frac{l_i}{\sigma_i S_i}$$

For magnetic circuit:

$$NI_0 = \oint_{(L)} \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = \sum_{(L)} H_i l_i = \sum_{(L)} \frac{B_i}{\mu_i \mu_0} l_i = \sum_{(L)} \frac{\phi_{Bi}}{\mu_i \mu_0 S_i} l_i$$

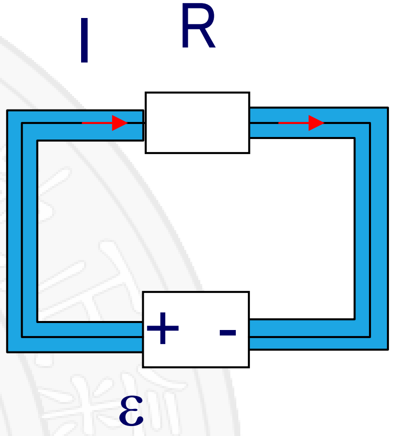


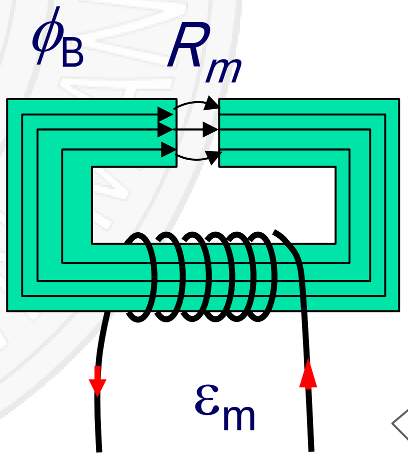
6.4 Magnetic Circuit Theorem

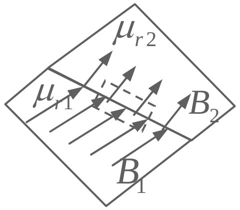
◈ Magnetic Circuit Theorem

For the right magnetic circuit, the flux ϕ_B is the same for every segment

$$NI_0 = \sum_{(L)} \frac{\phi_{Bi}}{\mu_i \mu_0 S_i} l_i = \phi_B \sum_{(L)} \frac{l_i}{\mu_i \mu_0 S_i}$$







6.4 Magnetic Circuit Theorem

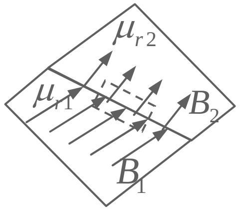
◈ Magnetic Circuit Theorem

Electric circuit	emf ε	Current I	Conductivity σ	Resistance $R_i=l_i/(\sigma_i S_i)$	Electric potential drop (IR_i)
Magnetic circuit	mmf $\varepsilon=NI_0$	flux of B ϕ_B	Permeability $\mu_i\mu_0$	Magnetic Resistance $R_{mi}=l_i/(\mu_i\mu_0 S_i)$	Magnetic potential drop $(\phi_B R_{mi})$

mmf: $\varepsilon_m=NI_0$

Magnetic Resistance: $R_{mi}=l_i/(\mu_i\mu_0)$

Magnetic potential drop: $H_i l_i=\phi_B \Sigma R_{mi}$



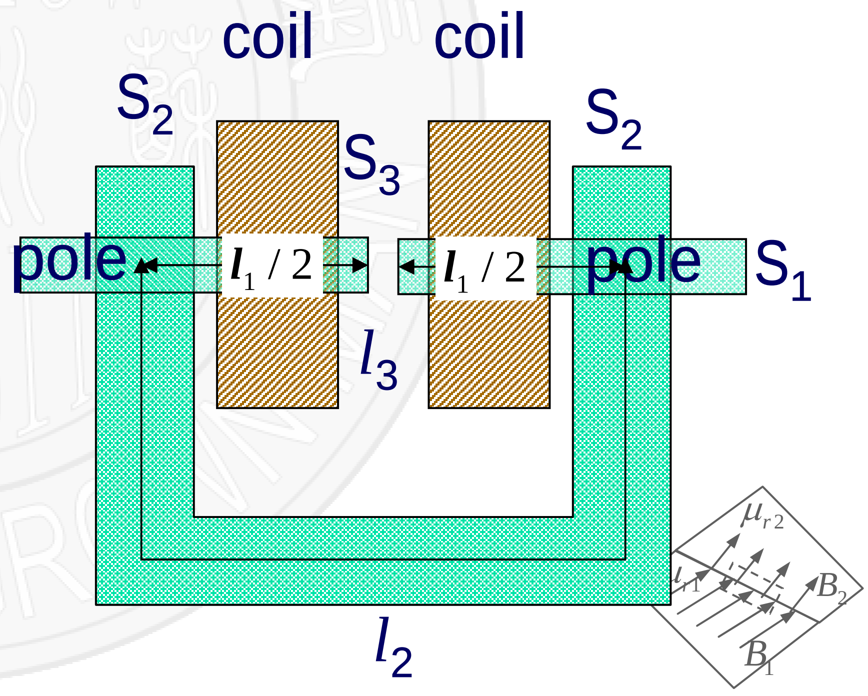
6.4 Magnetic Circuit Theorem

◈ Magnetic Circuit Theorem

$\varepsilon_m = H_i l_i = \phi_B \Sigma R_{mi}$

The magneto motive force in a closed magnetic circuit is equal to the algebraic sum of drop of magnetic potential.

Example U-shaped circuit, $s_1=0.01m^2$, $l_1=0.6m$, $\mu_1=6000$, $s_2=0.02m^2$, $l_2=1.4m$, $\mu_2=700$; air gap l_3 is variable between 0-0.05m. If the turns is N=5000, maximum current $I_0=4A$.

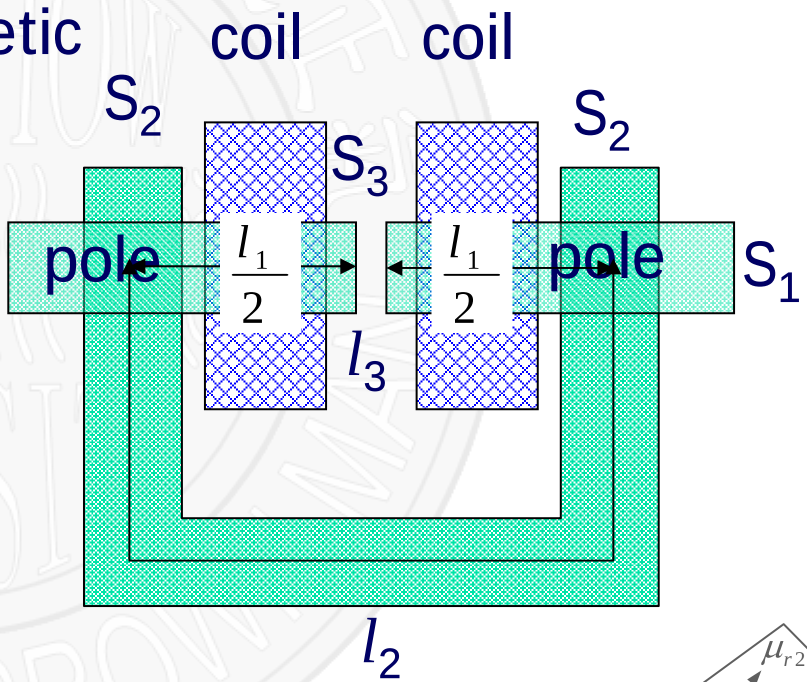


6.4 Magnetic Circuit Theorem

◇ Magnetic Circuit Theorem

Find the maximum magnetic field intensity when l_3=0.05m and 0.01m respectively.

Solution: According to Magnetic Circuit Theorem

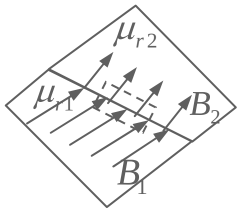


$$\phi_B = \frac{NI_0}{\dfrac{l_1}{\mu_{r1}\mu_0 S_1} + \dfrac{l_2}{\mu_{r2}\mu_0 S_2} + \dfrac{l_3}{\mu_0 S_3}}$$

$$\phi_B = \mu_0 H S_3$$

$$H = \frac{NI_0 / S_3}{\dfrac{l_1}{\mu_{r1} S_1} + \dfrac{l_2}{\mu_{r2} S_2} + \dfrac{l_3}{S_3}}$$

$S_3 \approx S_1$



6.4 Magnetic Circuit Theorem

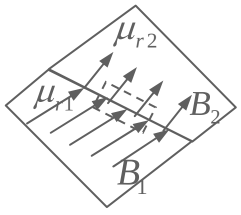
◈ Magnetic Circuit Theorem

L_3=0.05m, H=3.93×10^5A/m=4.9×10^3 Oersted

L_1=0.01m, H=1.8×10^6A/m=2.5×10^4 Oersted

Another unit of H is Oersted

$$1\text{Oersted} = \frac{10^3}{4\pi} A/m$$



6.4 Magnetic Circuit Theorem

Energy of Magnetic Field

- The Energy of Electric Field

$$W_e = \iiint_{(V)} w_e dV = \iiint_{(V)} \frac{1}{2}\vec{\mathbf{D}}\cdot\vec{\mathbf{E}}dV$$

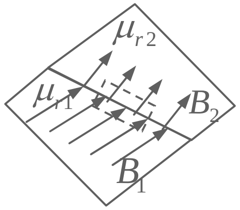
Where

$$w_e = \frac{1}{2}\vec{\mathbf{D}}\cdot\vec{\mathbf{E}}$$

- The energy stored in Inductor with current I and self-inductance L

$$W_m = \frac{1}{2}LI^2$$

For a solenoid with area S and length *l* filled with magnetic materials μ.



6.4 Magnetic Circuit Theorem

Energy of Magnetic Field

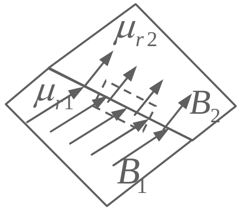
$$L = \mu n^2 S l$$

$$W_m = \frac{1}{2} L I^2 = \frac{1}{2} \mu n^2 S l I^2 = \frac{1}{2} \mu n^2 I^2 S l$$

$$= \frac{1}{2} \mu n I \cdot n I S l = \frac{1}{2} \vec{\mathbf{B}} \cdot \vec{\mathbf{H}} \cdot S l$$

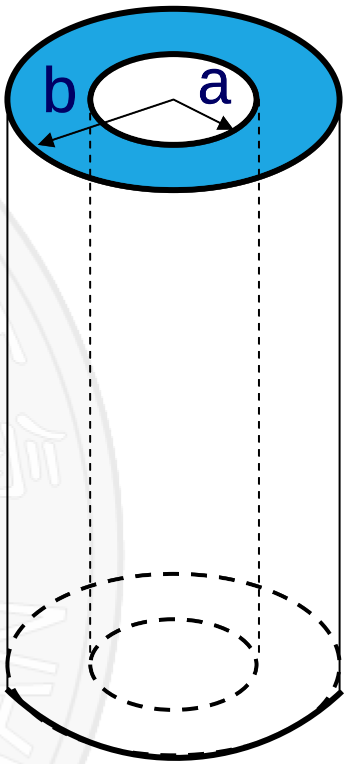
$$w_m = \frac{W_m}{V} = \frac{1}{2} \vec{\mathbf{B}} \cdot \vec{\mathbf{H}}$$

$$W_m = \iiint_{(V)} w_m dV = \iiint_{(V)} \frac{1}{2} \vec{\mathbf{B}} \cdot \vec{\mathbf{H}} dV$$



6.4 Magnetic Circuit Theorem

◈ Energy of Magnetic Field



Example A cable with radii a and b and length l. If magnetic material (relative permeability μ_r) is filled in, find the self-inductance of unit length.

Solution:

$$\oint_{(L)} \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} = \sum I_0$$

$$H = \frac{I}{2\pi r}$$

$$B = \frac{\mu_r \mu_0 I}{2}$$

6.4 Magnetic Circuit Theorem

◈ Energy of Magnetic Field

$$W_m = \iiint_{(V)} \frac{1}{2}\vec{\mathbf{B}}\cdot\vec{\mathbf{H}}dV = \int_a^b \frac{1}{2}\frac{I}{2\pi r}\frac{\mu_r\mu_0 I}{2\pi r}2\pi r dr$$

$$= \frac{\mu_r\mu_0 I^2}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_r\mu_0 I^2}{4\pi}\ln\frac{b}{a}$$

$$= \frac{1}{2}LI^2$$

$$L = \frac{\mu_r\mu_0}{2\pi}\ln\frac{b}{a}$$