Tutorial for chapter 5

- Faraday's Law of Induction

The induced emf ε in a circuit is equal to the rate at which the flux linkages of B through the circuit is changing.

$$\varepsilon = -\frac{\mathrm{d}\,\Psi_B}{dt}$$

- Lenz's Law

  - The field of induced current always opposes the change of the field of producing induction current

  - The effect of induced current always resists the reason of producing induction current.

Tutorial for chapter 5

- Two Kinds of Electromotive Force
  - Motional electromotive force

$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = -\frac{\mathrm{d}\,\Psi_B}{dt}$$

  - Transformer electromotive force

$$\varepsilon = \oint \vec{E}_{in} \cdot d\vec{l} = -\frac{\mathrm{d}\,\Psi_B}{dt}$$

Tutorial for chapter 5

- Mutual Inductance and Self-Inductance

- Mutual Inductance

$$M_{21} = \frac{\Psi_{21}}{I_1} = \frac{N_2 \phi_{21}}{I_1}$$

$$M_{21} = \frac{\varepsilon_2}{-\frac{\mathrm{d}I_1}{\mathrm{d}t}}$$

- Self-Inductance

$$L = \frac{\Psi}{I} = \frac{N\phi}{I}$$

$$L = \frac{\varepsilon_L}{-\frac{\mathrm{d}I}{\mathrm{d}t}}$$

Tutorial for chapter 5

- Transient Process
  - The R-L Circuit
    - Switch on with source

$$i = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t}) = I_0(1 - e^{-\frac{R}{L}t})$$

Time constant $\tau = \frac{L}{R}$

    - Switch on without source

$$i = I_0 e^{-\frac{R}{L}t} = \frac{\varepsilon}{R} e^{-\frac{R}{L}t}$$

Tutorial for chapter 5

- The R-C Circuit
  - charging

$$q = \varepsilon C(1 - e^{-\frac{t}{RC}}) = Q_f(1 - e^{-\frac{t}{RC}})$$

$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$

Time constant $\tau = RC$

  - discharging

$$q = Q_f e^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = -\frac{Q_f}{RC} e^{-\frac{t}{RC}} = -I_0 e^{-\frac{t}{RC}}$$

Tutorial for chapter 5

Energy in an Inductor

$$W = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 S l I^2$$

Energy density of Magnetic Field

$$w = \frac{1}{2} \mu_0 n^2 I^2 = \frac{B^2}{2\mu_0}$$

Tutorial for chapter 5

- Combinations of coils

  - Combination in order

$$L = L_1 + L_2 + 2M$$

  - Combination in back

$$L = L_1 + L_2 - 2M$$

Extremely tight wound

$$M = \sqrt{L_1 L_2}$$

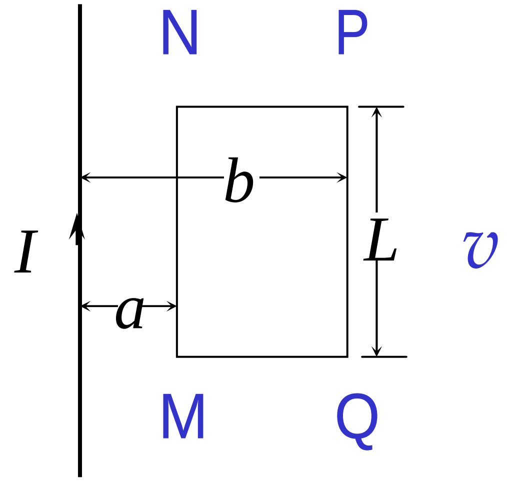
  - The energy in combinations of coils

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$ Combination in order

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2$$ Combination in back

Tutorial for chapter 5

Example 5.1



$$\varepsilon_{MN} = B_1 L v \qquad B_1 = \frac{\mu_0 I}{2\pi a}$$

$$\varepsilon_{QP} = B_2 L v \qquad B_2 = \frac{\mu_0 I}{2\pi b}$$

$\varepsilon_{NP} = \varepsilon_{QM} = 0$

$$\varepsilon = \varepsilon_{MN} - \varepsilon_{PQ} = \frac{\mu_0 I L v}{2\pi a} - \frac{\mu_0 I L v}{2\pi b}$$

Direction: Clockwise

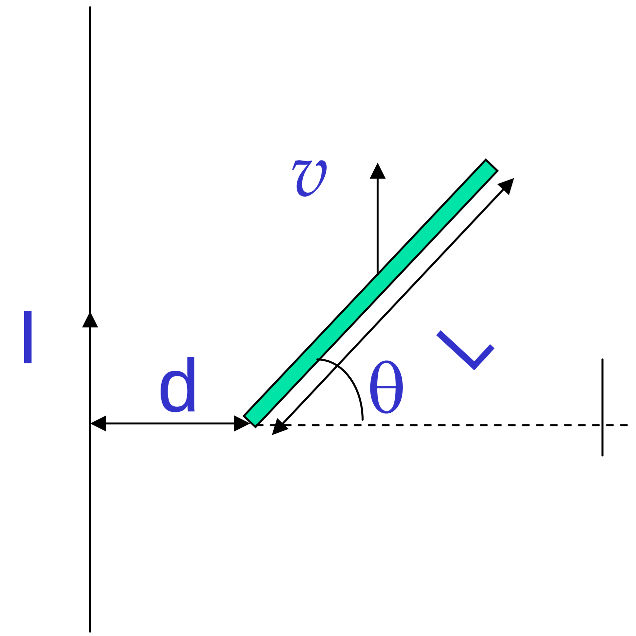
Tutorial for chapter 5

Example 5.2

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$\varepsilon = -\int_a^b vBdl\cos\theta = -\int_a^b v\frac{\mu_0 I}{2\pi r}dr$$

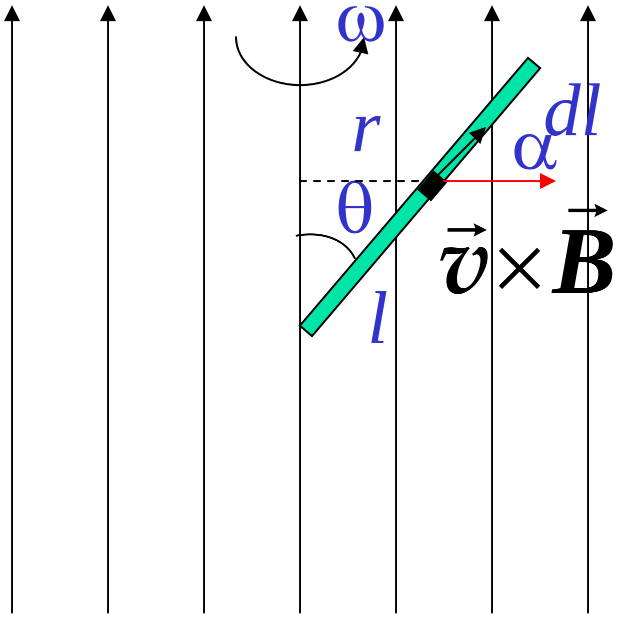
$$= -\frac{\mu_0 I v}{2\pi}\ln\left(\frac{d + L\cos\theta}{d}\right)$$



Tutorial for chapter 5

Example 5.8

$$\varepsilon = \int_a^b (\vec{v} \times \vec{\boldsymbol{B}}) \cdot d\vec{\boldsymbol{l}}$$

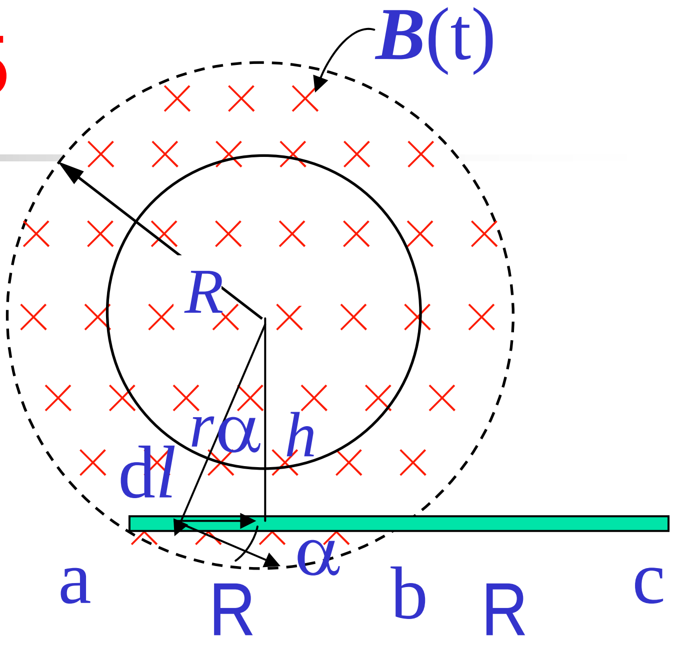


$$\varepsilon = \int_o^A \omega r B dl \cos\alpha$$

$$\varepsilon = \int_o^A \omega r B dr = \int_0^{L\sin\theta} \omega B r dr$$

$$= \frac{1}{2}\omega B (L\sin\theta)^2$$

Tutorial for chapter 5



$$E_{\text{in}} = -\frac{1}{2} r \frac{dB}{dt}$$ inside

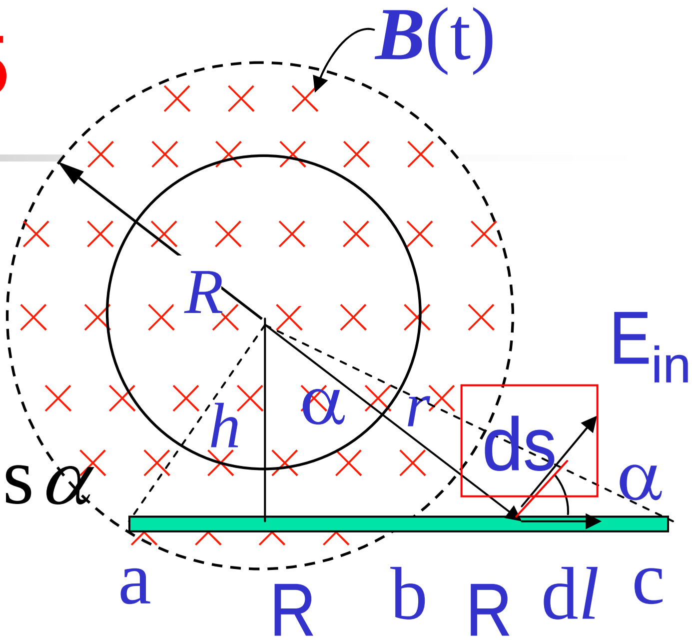
$$\varepsilon_{ab} = \int_a^b \vec{E}_{\text{in}} \cdot d\vec{l} = \int_a^b E_{\text{in}} dl \cos\alpha$$

$$= \int_a^b E_{\text{in}} dl \cos\alpha = \int_a^b \frac{1}{2}\frac{dB}{dt} r dl \frac{h}{r} = \frac{1}{2} h R \frac{dB}{dt}$$

$$h = \frac{\sqrt{3}}{2} R$$

$$\varepsilon_{bc} = \int_b^c \vec{E}_{in} \cdot d\vec{l} = \int_b^c E_{\text{in}} dl \cos\alpha$$ outside

Tutorial for chapter 5



$$E_{\text{induced}} = -\frac{R^2}{2r}\frac{dB}{dt}$$

$$\varepsilon_{bc} = \int_b^c \vec{\boldsymbol{E}}_{induced} \cdot d\vec{\boldsymbol{l}} = \int_b^c E_{induced}\, dl \cos\alpha$$

$$\varepsilon_{bc} = \int_b^c \frac{R^2}{2r}\frac{dB}{dt} dl \cos\alpha$$

$$\varepsilon_{bc} = \frac{R^2}{2}\frac{dB}{dt}\int_b^c \frac{1}{r} dl \cos\alpha = \frac{R^2}{2}\frac{dB}{dt}\int_b^c \frac{ds}{r}$$

$$\varepsilon_{bc} = \frac{R^2}{2}\frac{dB}{dt}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d\alpha = \frac{\pi R^2}{12}\frac{dB}{dt}$$

Tutorial for chapter 5

$$\varepsilon = \varepsilon_{ab} + \varepsilon_{bc} = \frac{\sqrt{3}}{4} R^2 \frac{dB}{dt} + \frac{\pi}{12} R^2 \frac{dB}{dt}$$

$$\varepsilon = \oint \vec{\boldsymbol{E}}_{in} \cdot d\vec{\boldsymbol{l}} = -\frac{\mathrm{d}\,\boldsymbol{\Psi}_{\boldsymbol{B}}}{dt}$$

$$\phi = BS = B[(\frac{1}{2} R \frac{\sqrt{3}}{2} R) + \frac{1}{2} R^2 \frac{\pi}{6}]$$

$$\varepsilon = \frac{d\phi}{dt} = (\frac{\sqrt{3}}{4} R^2 + \frac{\pi}{12} R^2) \frac{dB}{dt}$$

Tutorial for chapter 5

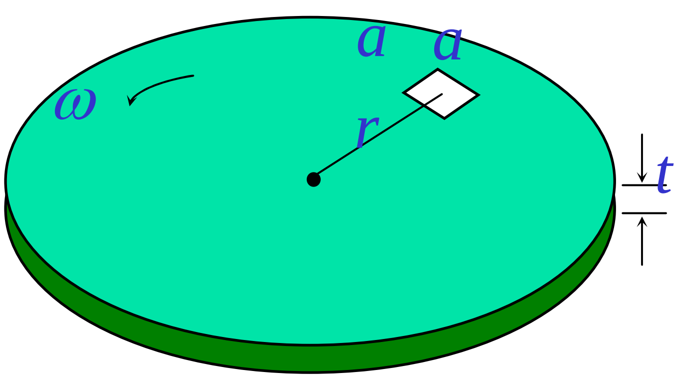
5.2.6. An electromagnetic "eddy current" brake consists of a disk of conductivity s and thickness *t* rotating about an axis through its center with a magnetic field ***B*** applied perpendicular to the plane of the disk over a small area a^2 (see Fig.5.30). If the area a^2 is at a distance *r* from the axis, find an approximate expression for the torque tending to slow down the disk at the instant its angular velocity equals ω.

Solution:

$\varepsilon = Bav = B\omega ar$

$$i = \frac{\varepsilon}{R} = \frac{B\omega ar}{R}$$

$$\tau = Fr = Biar = Bar\frac{B\omega ar}{R}$$



Tutorial for chapter 5

$$\tau = Fr = Biar = Bar\frac{B\omega ar}{R}$$

$$R = \frac{a}{\sigma at}$$

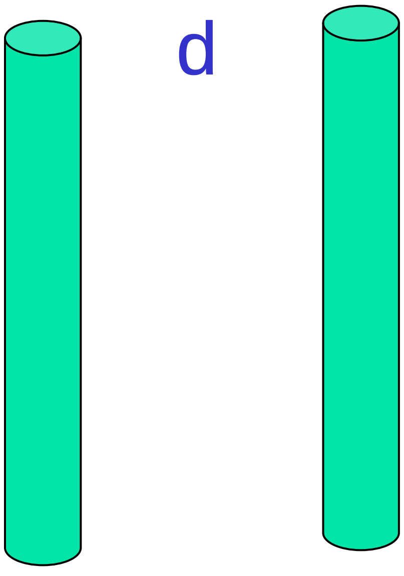
$$\tau = \frac{B^2a^2r^2\omega}{\frac{1}{\sigma t}} = B^2a^2r^2\omega\sigma t$$

Tutorial for chapter 5

5.3.12.Two long parallel wires whose centers are a distance *d* apart carry equal currents in opposite directions. Show that, neglecting the flux within the wires themselves, the inductance of a length *l* of such a pair of wires is given by

L=where *a* is the wire radius.

$$\frac{\mu_0 l}{\pi}\ln\frac{d-a}{a}$$



Tutorial for chapter 5

5.3.11.Inductors in Parallel Two inductors L_1 and L_2 an connected in parallel and separated by a large distance, (a) Show that the equivalent inductance L is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

(b) Why must their separation be large for this relationship to hold?

5.2.14.A square wire of length *l*, mass *m*, and resistance *R* slides without friction down parallel conducting rails of negligible resistance, as in Fig. 5.35. The rails are connected to each other at the bottom by a resistanceless rail parallel to the wire, so that the wire and rails form a closed rectangular conducting loop. The plane of the rails makes an angle θ with the horizontal, and a uniform vertical magnetic field ***B*** exists throughout the region, (a) Show that the wire acquires a steady-state velocity of magnitude.

$$v = \frac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$$

(b) Prove that this result is consistent with the conservation of energy principle,

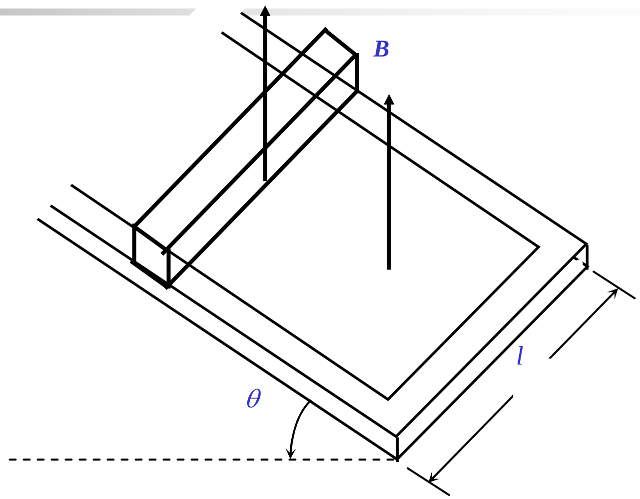


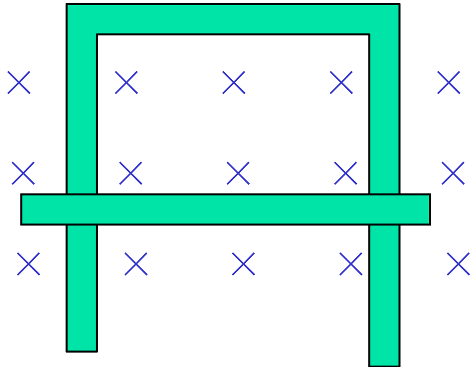
Fig.5.35 Problem 5.2.14

5.3.1.A solenoid of length 10 cm and radius 2 cm is wound uniformly with 1000 turns. A second coil of 50 turns is wound around the solenoid at its center. What is the mutual inductance of the two coils?

A thin wire of mass m and long l has a resistance R. The wire can slide freely, with no friction(摩擦),long twin vertical tails(of negligible resistance)as shown in the Figure 11. A horizontal, uniform magnetic field of magnitude B is perpendicular to the rails and wire. Neglect the magnetic field of the Earth. The wire falls under the action of the gravitational force near the surface of the Earth.

Find the velocity of the wire at any time;

Show that the falling wire attains a terminal speed equal to



5.3.4.A toroidal solenoid has two coils with N_1 and N_2 turns, respectively; it has radius r and cross-sectional area S.

(a) Derive an expression for the self-inductance L_1 when only the first coil is used, and that for L_2 when only the second soil is used.

(b) Derive an expression for the mutual inductance of the two coils.

(c) Show that $M_2 = L_1L_2$. This result is valid whenever all the flux linked by one coil is also linked by the other.

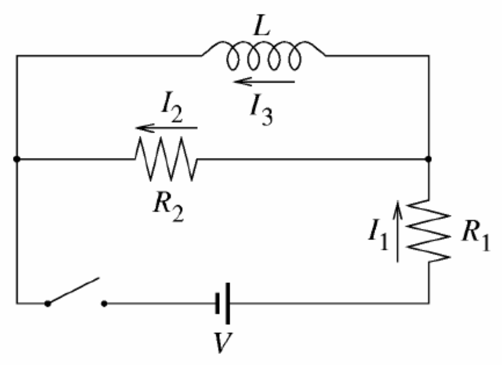
5.3.9. Inductor in Series Two inductors L_1 and L_2 and connected in series and are separated by a large distance, (a) Show that the equivalent inductance L is $L_1 + L_2$. (b) Why must their separation be large?

5.3.12. Two long parallel wires whose centers are a distance d apart carry equal currents in opposite directions. Show that, neglecting the flux within the wires themselves, the inductance of a length l of such a pair of wires is given by

$$L = \frac{\mu_0 l}{\pi} \ln \frac{d-a}{a}$$

5.4.2.How many “times constants” must we wait for the current in an *L-R* circuit to build up to within 0.10 percent of its equilibrium value?

The switch in the circuit below has been open for a long, long time. Determine the currents I_1, I_2, I_3 in tl resistors and in the self- inductor at moment



a. the switch is closed.

b. a long time after the switch is closed.

The internal resistance of the battery is negligibly small. Express your answers ONLY in terms of V, R_1, R_2 and L.

A conducting bar of mass m and resistance R slides down two frictionless conducting rails which make an angle θ with the horizontal , and are separated by a distance *l*, as shown in Figure 10.11.2. In addition, a uniform magnetic field B is applied vertically downward. The bar is released from rest and slides down.

(a) Find the induced current in the bar. Which way does the current flow, from a to b or b to a?

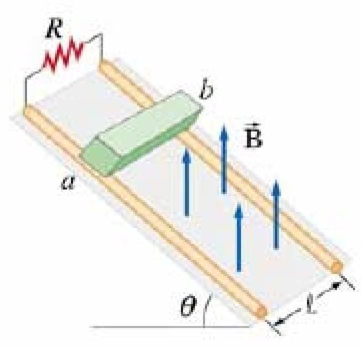
(b) Find the terminal speed v_t of the bar.

After the terminal speed has been reached,

(c) What is the induced current in the bar?

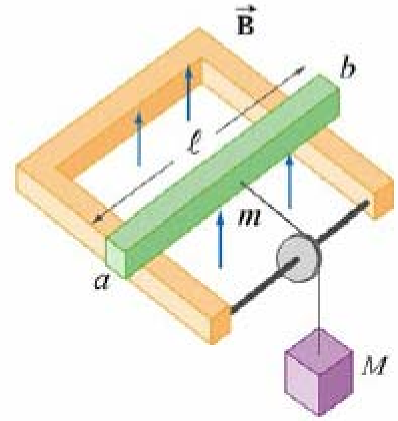
(d) What is the rate at which electrical energy has been dissipated through the resistor?

(e) What is the rate of work done by gravity on the bar?



A conducting bar of mass m and resistance R is pulled in the horizontal direction across two frictionless parallel rails a distance l apart by a massless string which passes over a frictionless pulley and is connected to a block of mass M, as shown in Figure 10.11.4. A uniform magnetic field is applied vertically upward. The bar is released from rest.

(a) Let the speed of the bar at some instant be v. Find an expression for the induced current . Which direction does it flow, from a to b or b to a? You may ignore the friction Between the bar and the rails.



(b) Solve the differential equation and find the speed of the bar as a function of time.