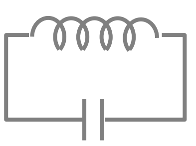
Chapter 5 Faraday's Law Transient Process

5.1 Faraday's Law of Induction

5.2 Two Kinds of Electromotance

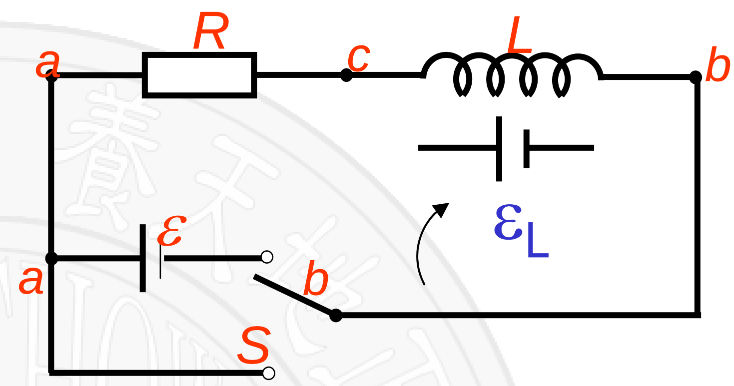
5.3 Mutual Inductance and Self-Inductance

5.4 Transient Process



5.4 Transient Process

◈ The R-L Circuit



✷ Switch up

- *i* increases, self-induction emf opposes the increase

Show RL-Circuit

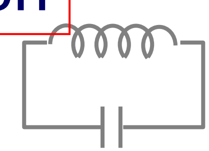
- According to Kirchhoff's Second Law, we get

$$iR + L\frac{di}{dt} - \varepsilon = 0$$

- A differential equation

$$\frac{\mathrm{d}i}{(\varepsilon / R) - i} = \frac{R}{L}\mathrm{d}t$$

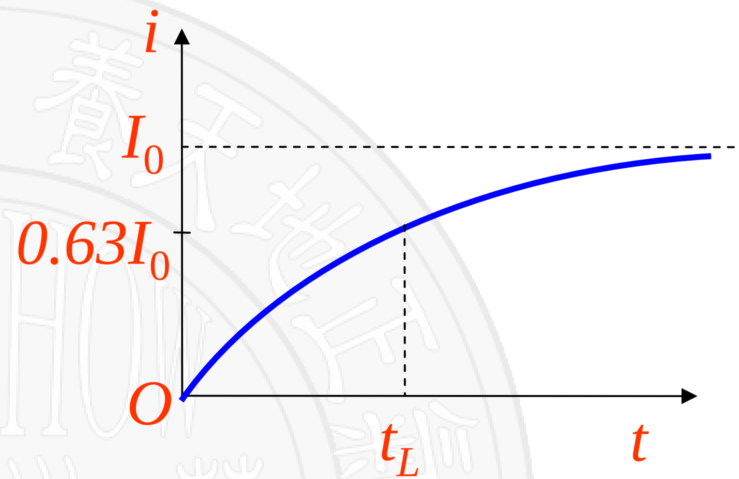
variables separation



5.4 Transient Process

The R-L Circuit

Switch up



$$\int_0^i \frac{\mathrm{d}i}{(\varepsilon / R) - i} = \int_0^t \frac{R}{L}\mathrm{d}t$$

$$\frac{\varepsilon}{R} - i = \frac{\varepsilon}{R} e^{-\frac{R}{L}t}$$

$$i = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t}) = I_0(1 - e^{-\frac{R}{L}t})$$

- Time constant $\tau = \frac{L}{R}$

decay constant or relaxation time

At a time *t* equal to *L/R* the current has risen to (1-1/*e*) or about 63% of its final value.

5.4 Transient Process

The R-L Circuit

- Half-life time
  - The time required for the current to reach half its final value

$$e^{-(R/L)t} = \frac{1}{2}$$

$$T_{\frac{1}{2}} = \frac{L}{R}\ln 2$$

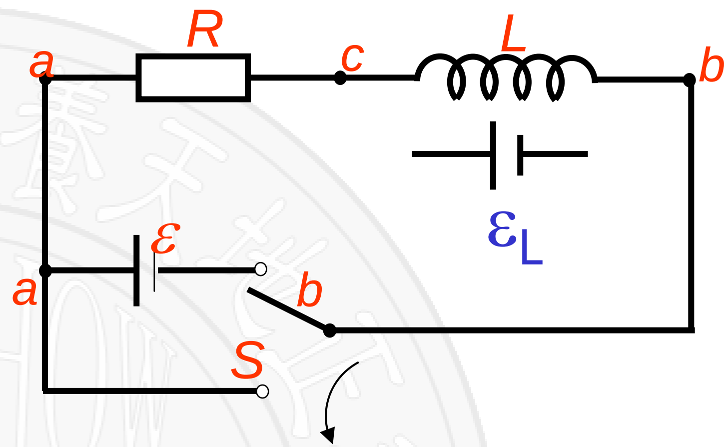
  - time unit

$$\tau = \frac{L}{R}$$

L:H, 1H=1Ω·s, τ:s

5.4 Transient Process

◇ The R-L Circuit



- Switch down
  - *i* decreases, self-induction emf opposes the decrease
  - According to Kirchhoff's Second Law, we get

$$iR + L\frac{di}{dt} = 0$$

  - A differential equation

5.4 Transient Process

The R-L Circuit

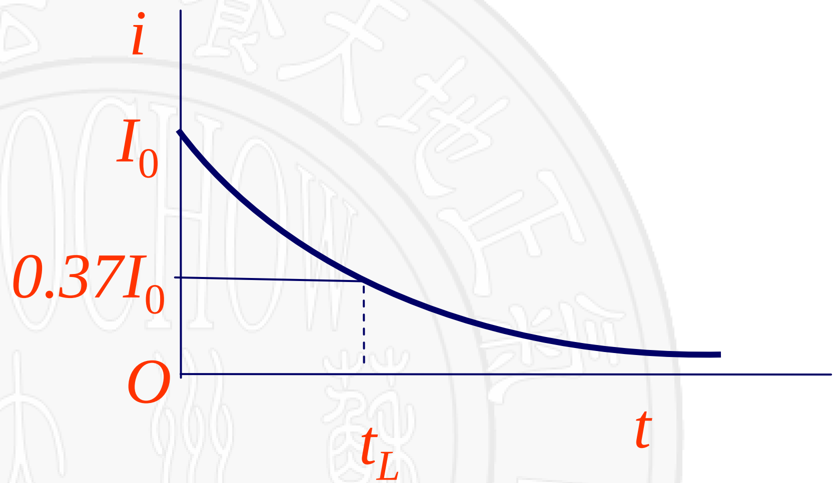
- Switch down

$$\frac{\mathrm{d}i}{i} = -\frac{R}{L}\mathrm{d}t$$

$$\int_{I_0}^{i} \frac{\mathrm{d}i}{i} = \int_{0}^{t} -\frac{R}{L}\mathrm{d}t$$

$$\ln\left(\frac{\mathrm{i}}{\mathrm{I}_0}\right) = -\frac{R}{L}t$$

$$i = I_0 e^{-\frac{R}{L}t} = \frac{\varepsilon}{R} e^{-\frac{R}{L}t}$$



When t=L/R, i=37% of I_0

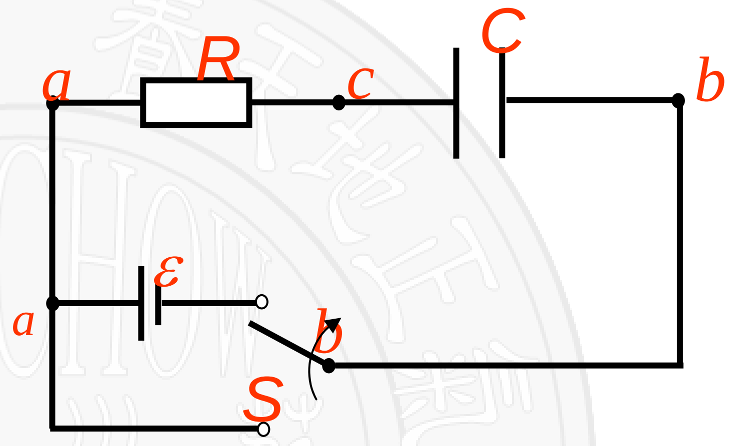
◈ The R-C Circuit

✷ Switch Up: charging

According to Kirchhoff's Second Law, we get

$$iR + \frac{q}{C} - \varepsilon = 0$$

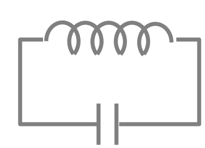
$$i = \frac{dq}{dt}$$



Show RC-Circuit

differential equation

$$\frac{\mathrm{d}\, q}{\mathrm{d}\, t} = \frac{\varepsilon}{R} - \frac{q}{RC}$$



◈ The R-C Circuit

- Switch Up: charging

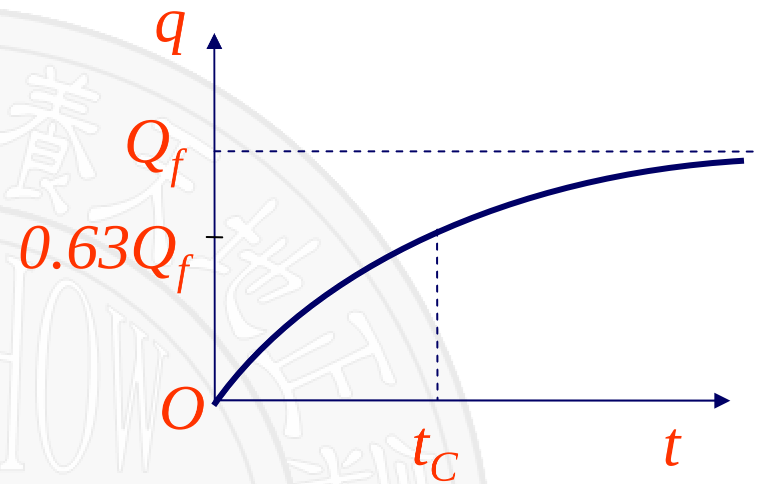
$$\frac{\mathrm{d}\,q}{\varepsilon C - q} = \frac{\mathrm{d}\,t}{RC}$$

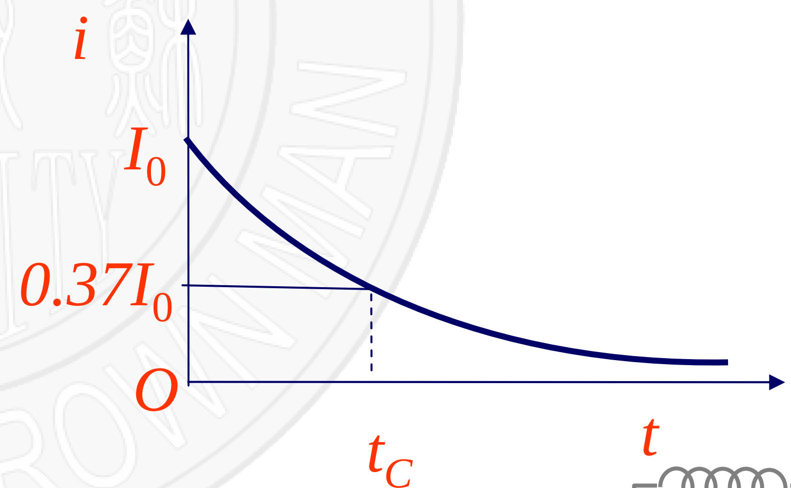
$$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC}$$

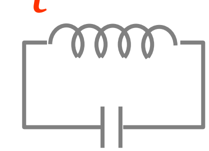
$$-\ln\frac{\varepsilon C - q}{\varepsilon C} = \frac{t}{RC}$$

$$q = \varepsilon C\,(1 - e^{-\frac{t}{RC}}) = Q_f\,(1 - e^{-\frac{t}{RC}})$$

$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$





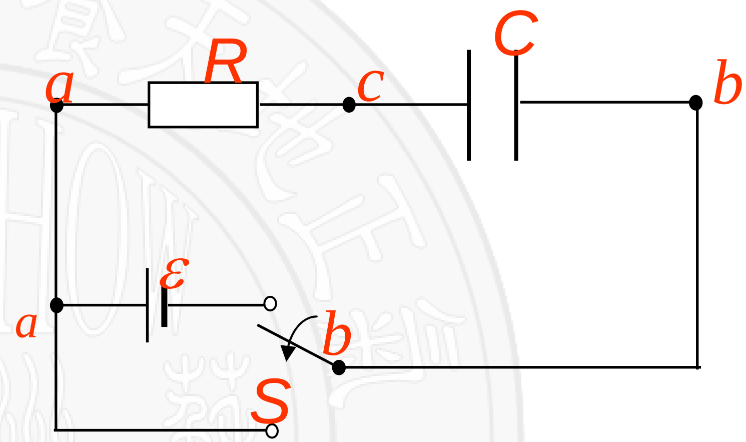


5.4 Transient Process

The R-C Circuit

- time constant

  τ=RC　　RC： time unit



- Switch Down: Discharging

  According to Kirchhoff's Second Law, we get

$$iR + \frac{q}{C} = 0$$

Show Capacitor Charging &Discharging

From www. Youtube .com

$$i = \frac{dq}{dt} \qquad \frac{\mathrm{d}\,q}{\mathrm{d}\,t} = -\frac{q}{RC}$$

differential equation

5.4 Transient Process

The R-C Circuit

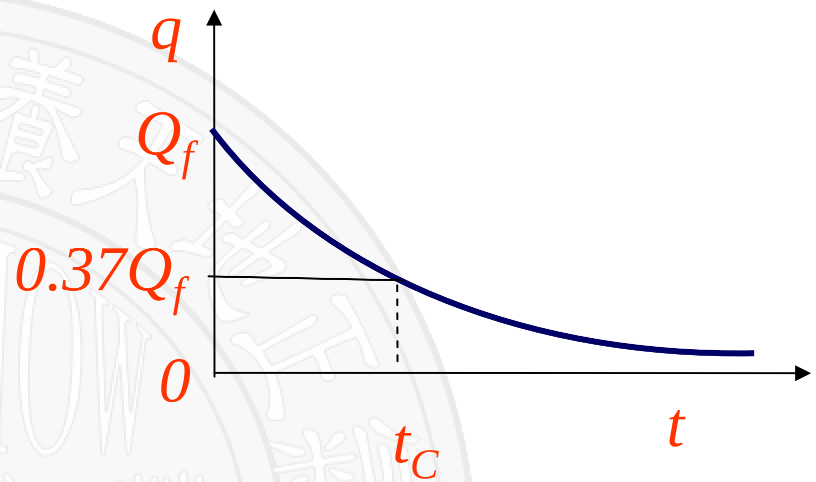
- Switch Down: Discharging

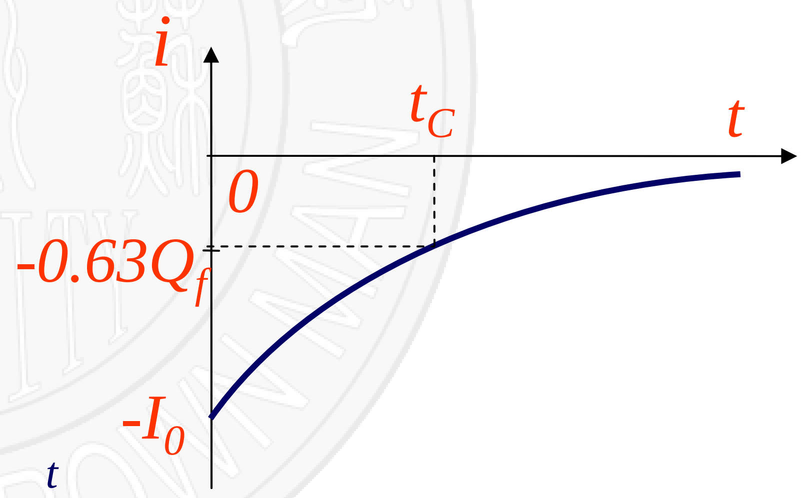
$$\int_{Q_f}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

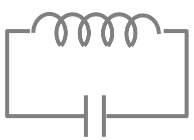
$$\ln\left(\frac{q}{Q_f}\right) = -\frac{t}{RC}$$

$$q = Q_f e^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = -\frac{Q_f}{RC} e^{-\frac{t}{RC}} = -I_0 e^{-\frac{t}{RC}}$$







The L-C Circuit

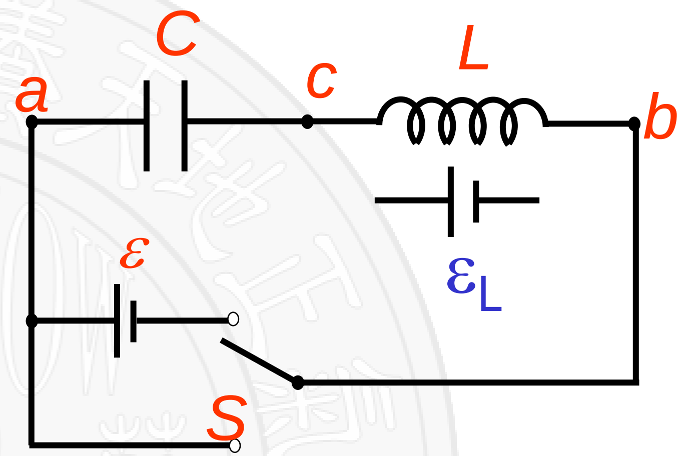
- Switch up

From Kirchhoff's Second Law, we get

$$\frac{q}{C}+L\frac{di}{dt}-\varepsilon=0$$

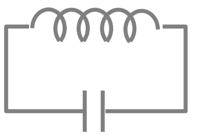
$$\frac{di}{dt}=\frac{d^2q}{dt^2}$$

$$L\frac{d^2q}{dt^2}+\frac{q}{C}-\varepsilon=0$$



L-C Oscillation Circuit

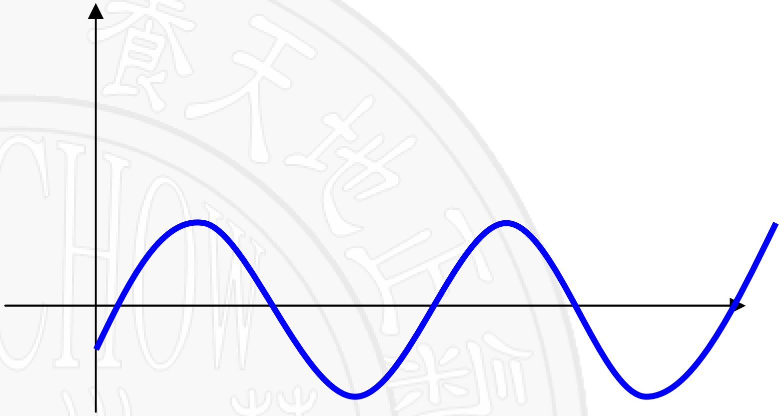
Show RLC-Circuit



5.4 Transient Process

The L-C Circuit

$$L\frac{d^2q}{dt^2}+\frac{q}{C}-\varepsilon=0$$



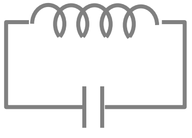
- General solution:

$$Q=C\varepsilon\cos\left(\frac{1}{\sqrt{LC}}t+\phi\right)$$

- Special solution:

$$\bar{q}=C\varepsilon$$

$$q=C\varepsilon\cos\left(\frac{1}{\sqrt{LC}}t+\phi\right)+C\varepsilon$$



The L-C Circuit

- Switch Down

From Kirchhoff's Second Law, we get

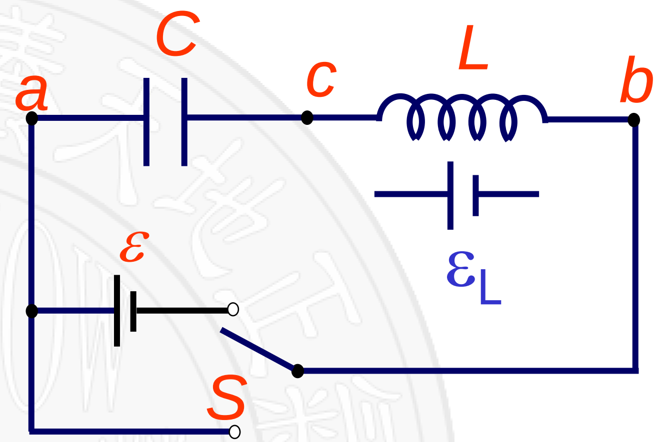
$$\frac{q}{C}+L\frac{di}{dt}=0$$

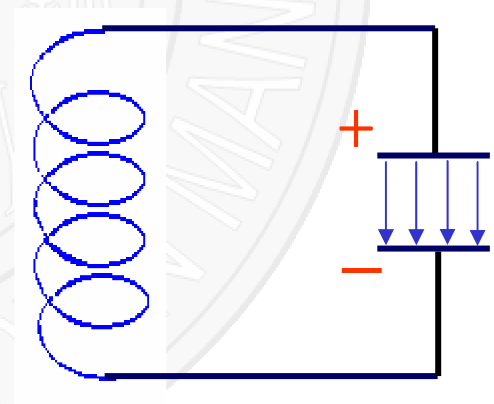
$$L\frac{d^2q}{dt^2}+\frac{q}{C}=0$$

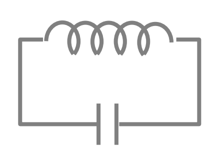
$$q=C\varepsilon\cos(\frac{1}{\sqrt{LC}}t+\phi)$$

$\phi=0$

$$i=\frac{dq}{dt}=-\varepsilon\sqrt{\frac{C}{L}}\sin(\frac{1}{\sqrt{LC}}t)$$

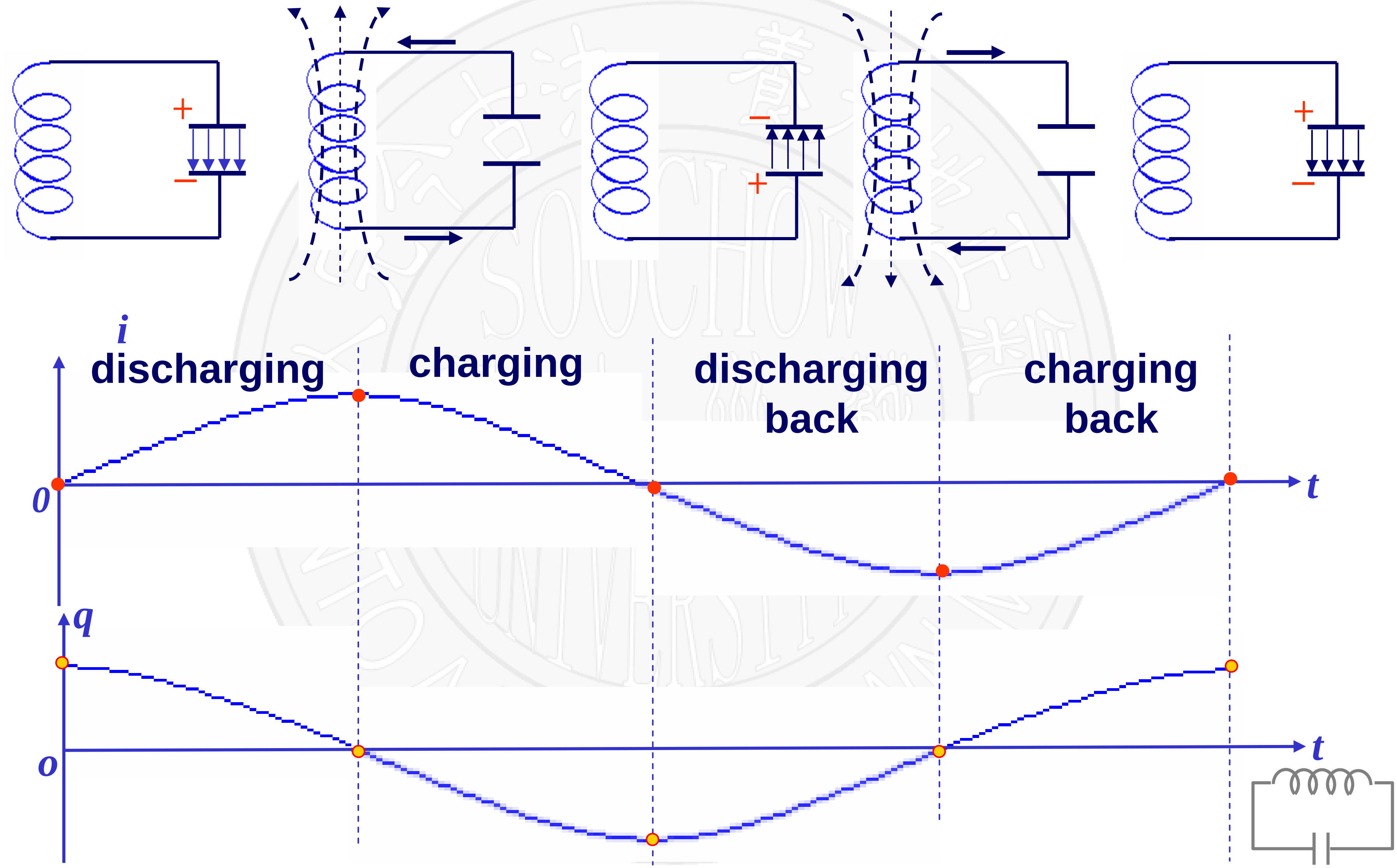






5.4 Transient Process

◈ The L-C Circuit



5.4 Transient Process

The R-L-C Circuit

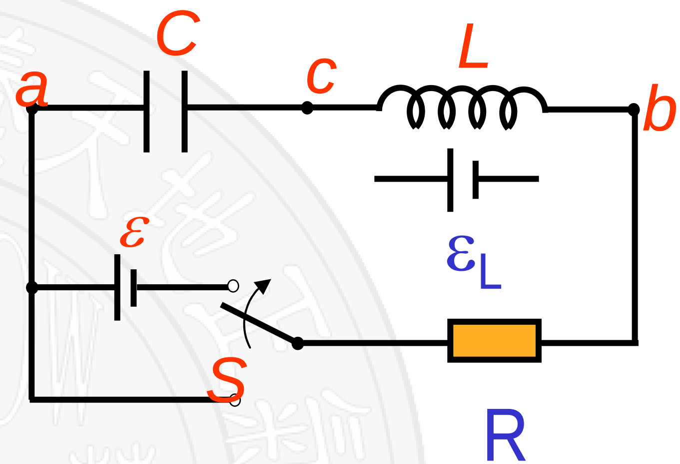
- Switch up

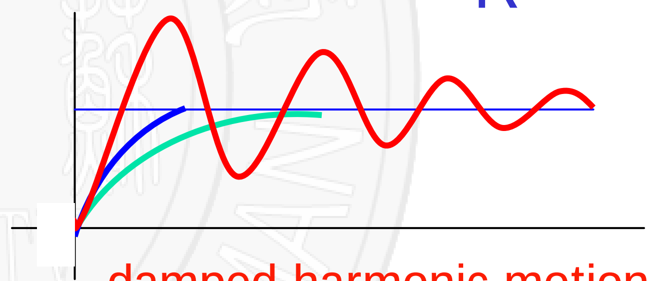
From Kirchhoff's Second Law, we get

$$L\frac{di}{dt}+iR+\frac{q}{C}-\varepsilon=0$$

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=\varepsilon$$

resistance is to dissipate the electromagnetic energy



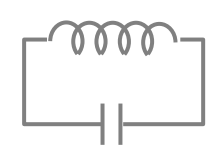


damped harmonic motion

Overdamped

Critical damped

$$\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} \qquad \nu=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

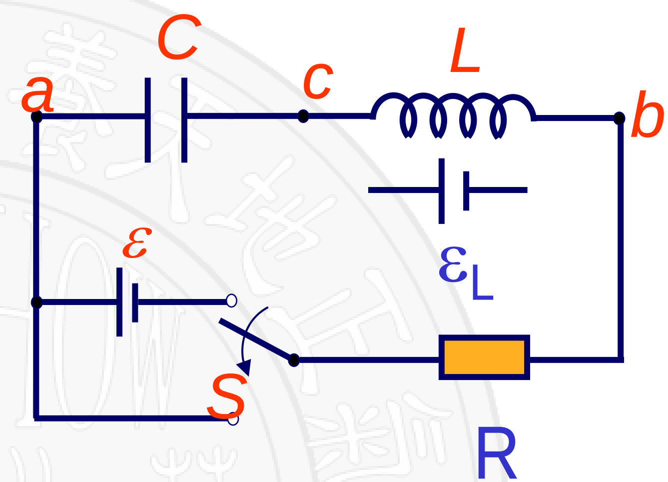


5.4 Transient Process

The R-L-C Circuit

- Switch down

From Kirchhoff's Second Law, we get



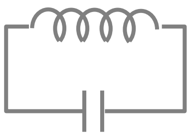
$$L\frac{di}{dt}+iR+\frac{q}{C}=0$$

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0$$

$$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{q}{LC}=0$$

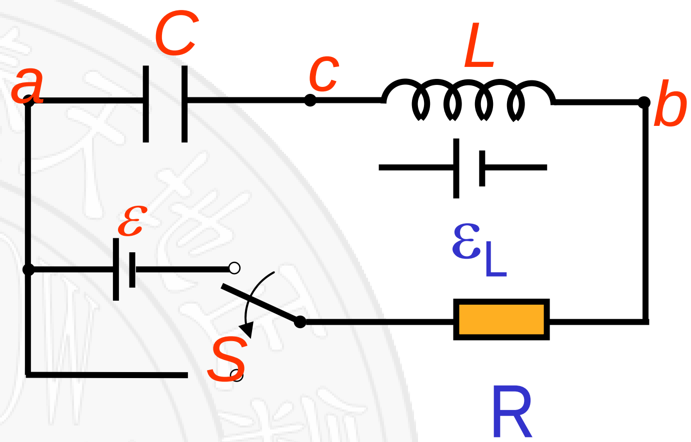
$$\omega_0^2=\frac{1}{LC} \qquad \beta=\frac{R}{2L}$$

$$\frac{d^2q}{dt^2}+2\beta\frac{dq}{dt}+\omega_0^2 q=0$$



5.4 Transient Process

◈ The R-L-C Circuit



- Switch down

If $\beta < \omega_0$

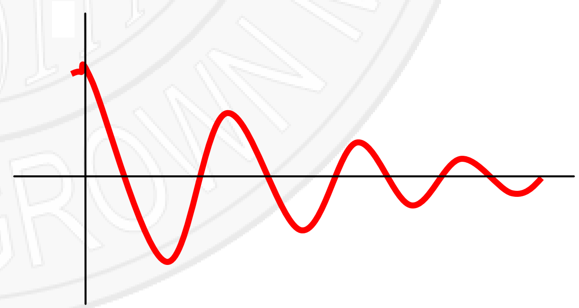
$$\beta = \frac{R}{2L}$$

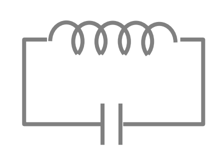
$$\frac{R}{2L} < \sqrt{\frac{1}{LC}}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$q = \varepsilon C e^{-(R/2L)t} \cos\left(\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\, t + \phi\right)$$

underdamped or damped harmonic motion



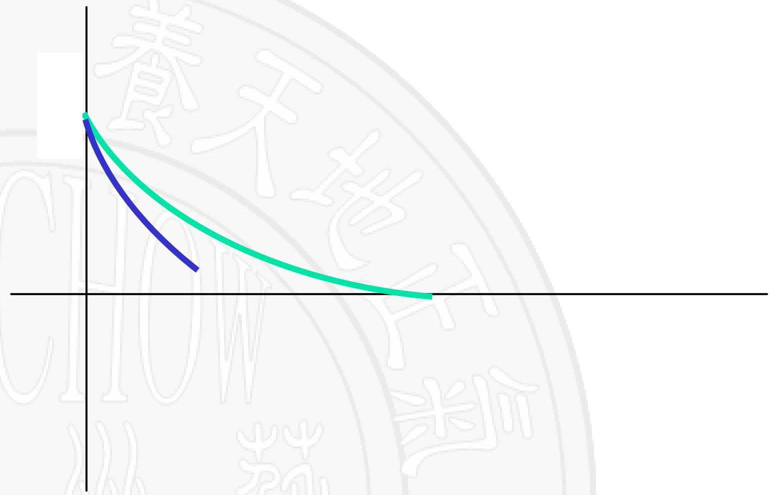


5.4 Transient Process

The R-L-C Circuit

- *Critical damped* motion

If $\frac{R}{2L} = \sqrt{\frac{1}{LC}}$



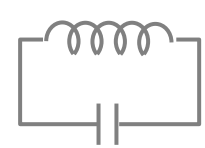
Critical damped motion

$$q = (c\varepsilon + c' t)\,e^{-\frac{R}{2L}t}$$

- *Over damped* motion

If $\frac{R}{2L} > \sqrt{\frac{1}{LC}}$

$$q = \frac{1}{2} C\varepsilon \left(e^{-\left[\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}\right]t} + e^{-\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}\right]t}\right)$$



5.4 Transient Process

The R-L-C Circuit

Example 5.9 resistor of resistance R = 10 MΩ is connected in series with a capacitor of capacitance 1μF Find the time constant τ.

Solution:

From formula of the time constant, we get

$$\tau = RC = (10 \times 10^{6} \times 10^{-6}) = 10\ (\mathrm{s}),$$

the half-life is

$$t_{h} = (10 \ln 2) = 6.9(\mathrm{s}).$$

On the other hand, if R =10Ω, the time constant is only 10×10^{-6} s, or 10 μs.