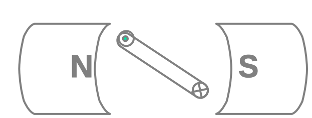
Chapter 5 Faraday's Law Transient Process

5.1 Faraday's Law of Induction

5.2 Two Kinds of Electromotance

5.3 Mutual Inductance and Self-Inductance

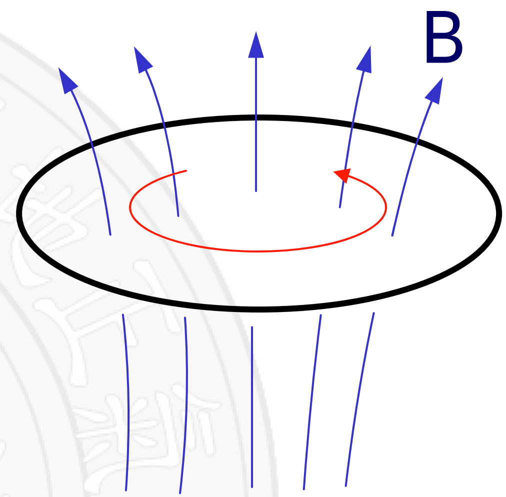
5.4 Transient Process



5.2 Two Kinds of Electromotive Force

Last time...

◈ Faraday's Law of induction



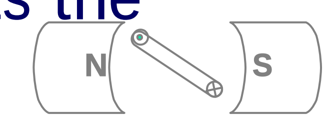
✸ The induced emf ε in a circuit is equal to the rate of B flux.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

✸ Lenz's Law

- The field of induced current always opposes the change of the field producing induction current

- The effect of induced current always resists the reason producing induction current.



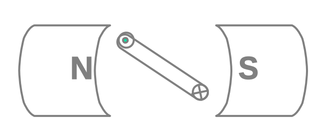
5.2 Two Kinds of Electromotive Force

We have two ways to produce electromotive force.

- Conductors moving in magnetic field
- The magnetic field around conductors changing.

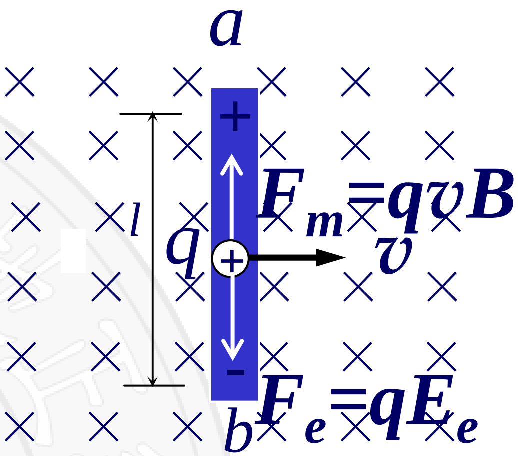
We can divide the electromotive forces into two kinds

- Motional electromotive force, relative motion.
- Transformer electromotive force, B changing



5.2 Two Kinds of Electromotive Force

◈ Motional electromotive force



Conductor moving, v

Lorentz Force --Non-electrostatic force

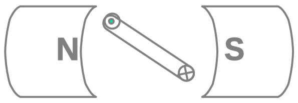
$$\vec{\mathbf{F}}_n = q\vec{v}\times\vec{B}$$

Non-electrostatic force per unit positive charge

$$\vec{\mathbf{E}}_n = \vec{v}\times\vec{B}$$

Charges accumulated at the ends,and produce E field

When $\vec{\mathbf{E}} = \vec{\mathbf{E}}_n$ Charge accumulation is over



5.2 Two Kinds of Electromotive Force

- According to the definition of emf

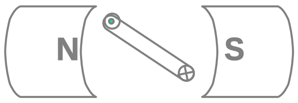
$$\varepsilon = \int_a^b \vec{E}_n \cdot d\vec{l} = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= \int_a^b vB \cdot dl = Blv$$

- The direction of ε is from a to b

- In general, emf can be calculated by

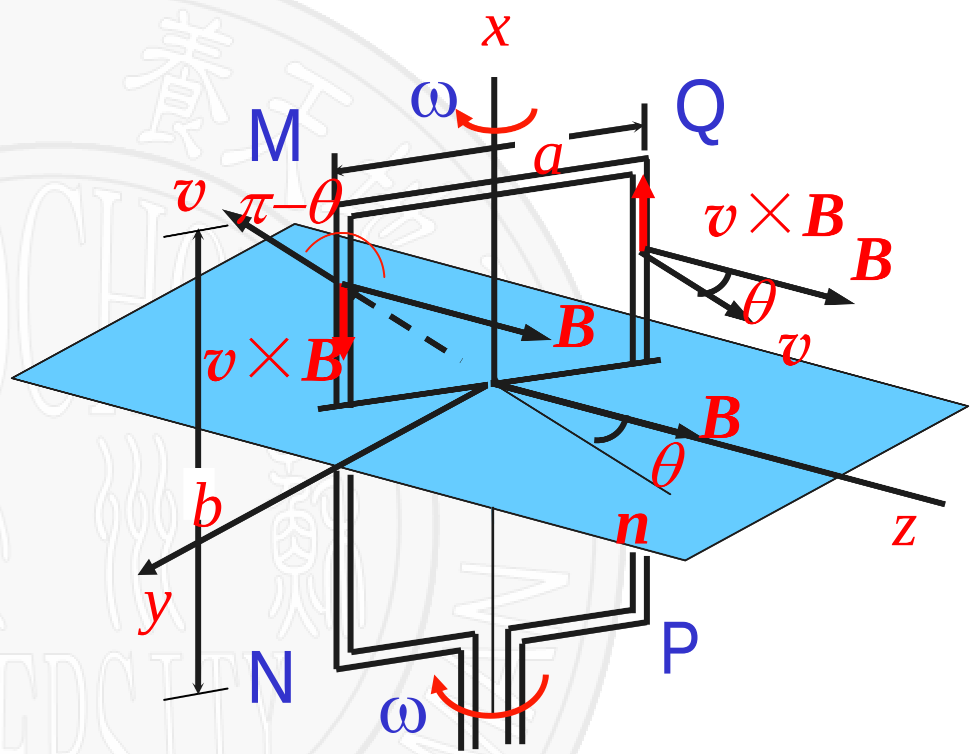
$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

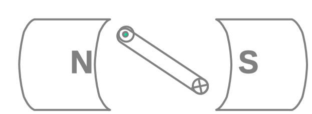


5.2 Two Kinds of Electromotive Force

- Alternating current generator

Example 5.2. Alternating current generator The rectangular loop of length a and width b, is rotating with uniform angular velocity ω about the x-axis. The entire loop lies in a uniform, constant **B** field, parallel to the z-axis. Calculate the induced emf in the loop.

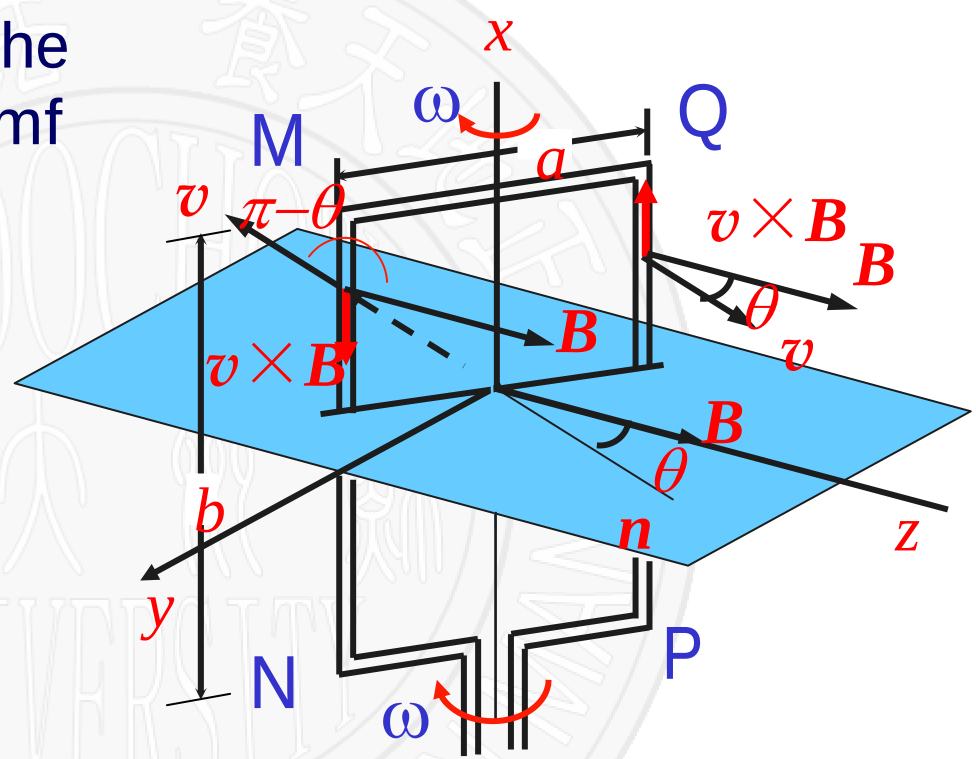




5.2 Two Kinds of Electromotive Force

- Alternating current generator

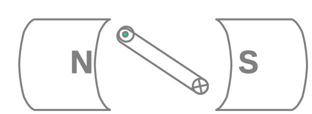
Solution: According to the definition of motional emf



$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$= \int_P^Q (\vec{v} \times \vec{B}) \cdot d\vec{l} + 0$$

$$+ \int_M^N (\vec{v} \times \vec{B}) \cdot d\vec{l} + 0$$



5.2 Two Kinds of Electromotive Force

◈ Alternating current generator

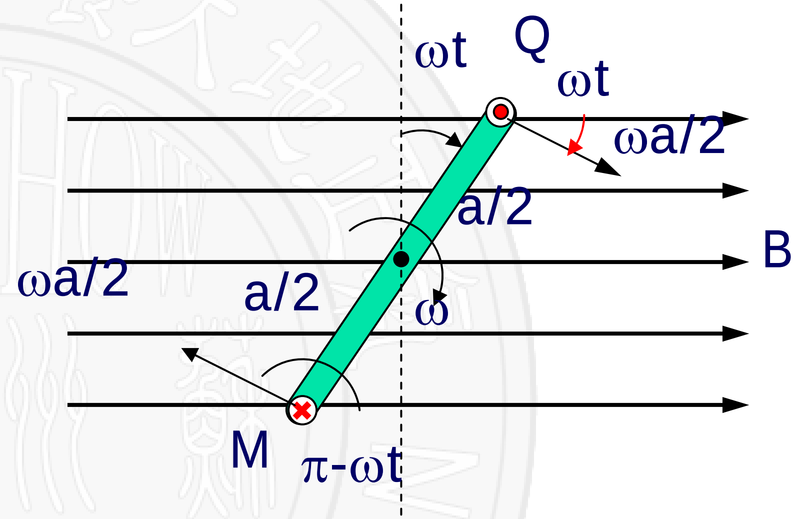
$$\varepsilon = \int_P^Q (\vec{v}\times\vec{\boldsymbol{B}})\cdot d\vec{\boldsymbol{l}} + \int_M^N (\vec{v}\times\vec{\boldsymbol{B}})\cdot d\vec{\boldsymbol{l}}$$

$$= vB\sin\theta b + vB\sin(\pi-\theta)b$$

$v=\omega a/2$, $\theta = \omega t$

$$\varepsilon = \omega abB\sin\omega t$$

ε is alternative.



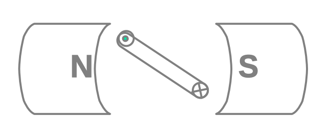
We can calculate ε by faraday's law

$$\phi_B = Bab\cos\omega t$$

$$\varepsilon = -\frac{d\phi_B}{dt} = \omega abB\ \sin\ \omega t$$

Show Generator

Show principle of generator
Run flash.exe first



Motional emf

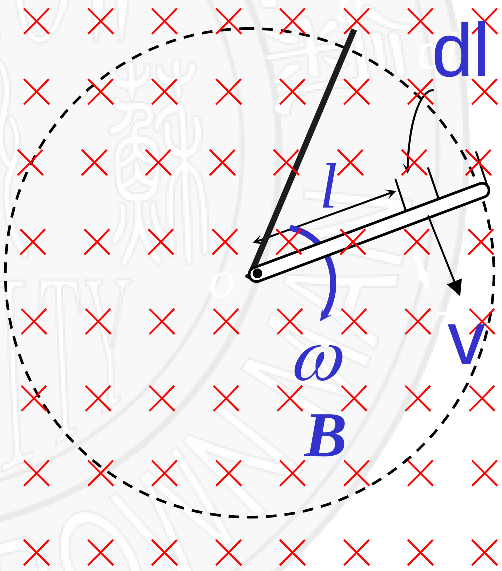
Example 5.3 A copper rod of length *L* rotates at angular frequency ω in a uniform magnetic field ***B*** as shown in the figure.Find the emf ε developed between the two ends of the rod.

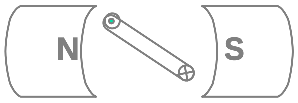
Solution: According to the definition of motional emf

$$d\varepsilon = (\vec{v} \times \vec{\mathbf{B}}) \cdot d\vec{\mathbf{l}}$$

$$= vBdl = \omega lBdl$$

$$\varepsilon = \int_0^L \omega lBdl = \frac{1}{2}\omega BL^2$$





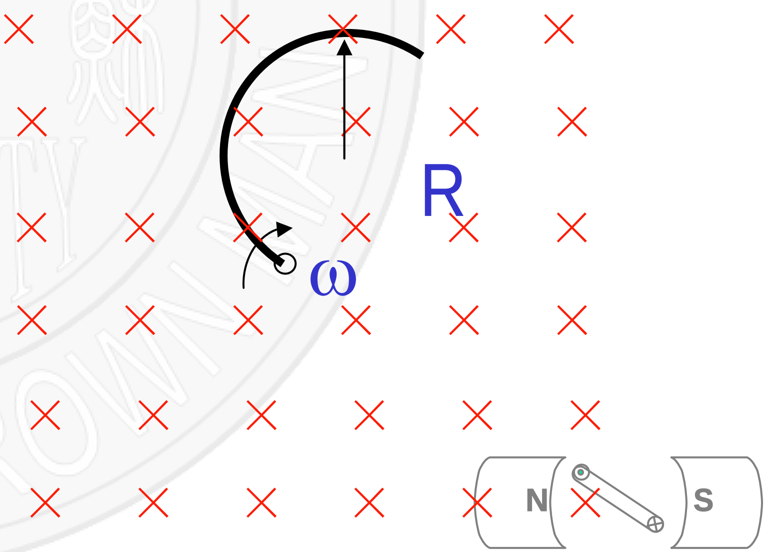
5.2 Two Kinds of Electromotive Force

Can we find ε by Faraday's law?

To complement part conductors and form a closed loop.

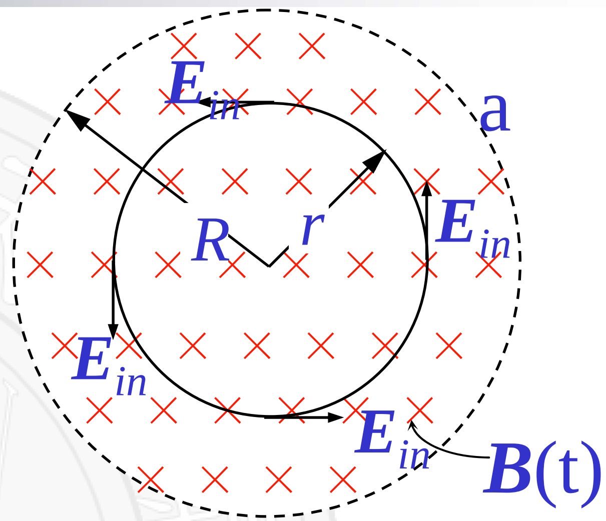
$$\varepsilon = \frac{d\phi_B}{dt} = \frac{BdS}{dt} = B\frac{\frac{1}{2}L^2\omega dt}{dt} = \frac{1}{2}B\omega L^2$$

Find emf in the semicircle

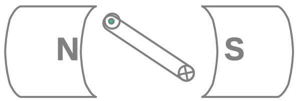


5.2 Two Kinds of Electromotive Force

- Transformer electromotive force
  - No physical motion
  - Magnetic field variation with time
  - Who drive particle to move ?
  - Maxwell——Induced Electric Field， produced by Variation of Magnetic Field



In section of solenoid, B is increasing.
Put a conductor loop,Electric Current Appears



5.2 Two Kinds of Electromotive Force

- Induced Electric Field

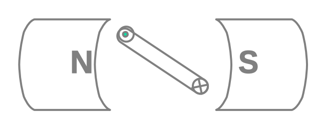
Maxwell thought The variation of magnetic field can create an electric field (Induced Electric Field E_{in}).

Transformer Electromotive Force

$$\varepsilon = \oint \vec{E}_{in} \cdot d\vec{l}$$

We can calculate emf by Faraday's law

$$\varepsilon = -\frac{d\phi_B}{dt} = \oint \vec{E}_{in} \cdot d\vec{l}$$

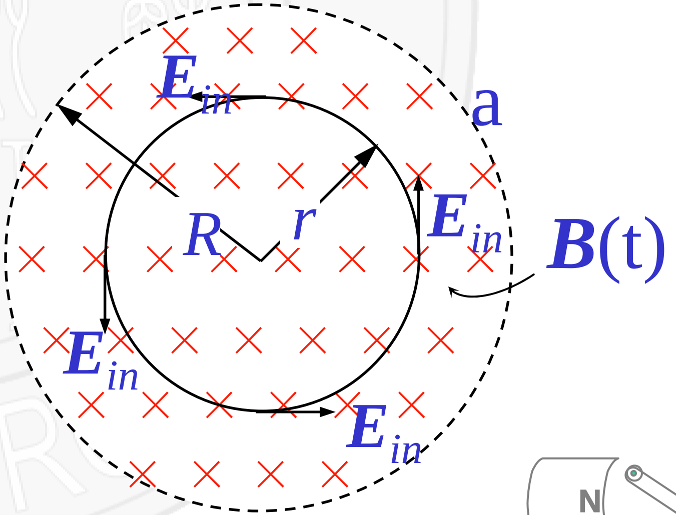


5.2 Two Kinds of Electromotive Force

Example 5.4 Let $\boldsymbol{B}$ in the figure be increasing at the rate dB/dt. Let R be the radius of the cylindrical region in which the magnetic field is assumed to exist. What is the magnitude of the electric field $\boldsymbol{E}_{in}$ at any radius r?

Solution: (a) For $r < R$, the flux ϕ_B through the loop is

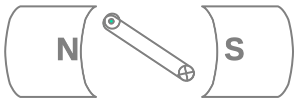
$$\phi_B = \pi r^2 B$$



Substituting into Faraday's law

$$\oint \boldsymbol{E}_{in} \cdot d\boldsymbol{l} = -\frac{d\phi_B}{dt}$$

$$\oint \boldsymbol{E}_{in} \cdot d\boldsymbol{l} = E_{in} 2\pi r$$



5.2 Two Kinds of Electromotive Force

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d(B\pi r^2)}{dt} = -\pi r^2 \frac{dB}{dt}$$

$$E_{in} 2\pi r = -\pi r^2 \frac{dB}{dt}$$

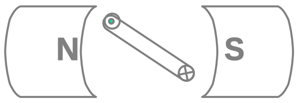
$$E_{in} = -\frac{1}{2} r \frac{dB}{dt}$$

(b) For $r > R$, the flux ϕ_B through the loop is

$$\phi_B = \pi R^2 B$$

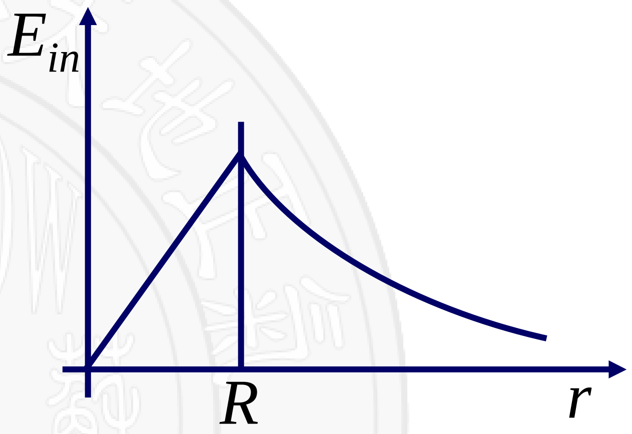
$$E_{in} 2\pi r = -\pi R^2 \frac{dB}{dt}$$

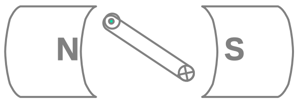
$$E_{in} = -\frac{1}{2} \frac{R^2}{r} \frac{dB}{dt}$$



5.2 Two Kinds of Electromotive Force

Although the magnetic field distributes inside the solenoid, the induced electric field can be exist outside of the solenoid

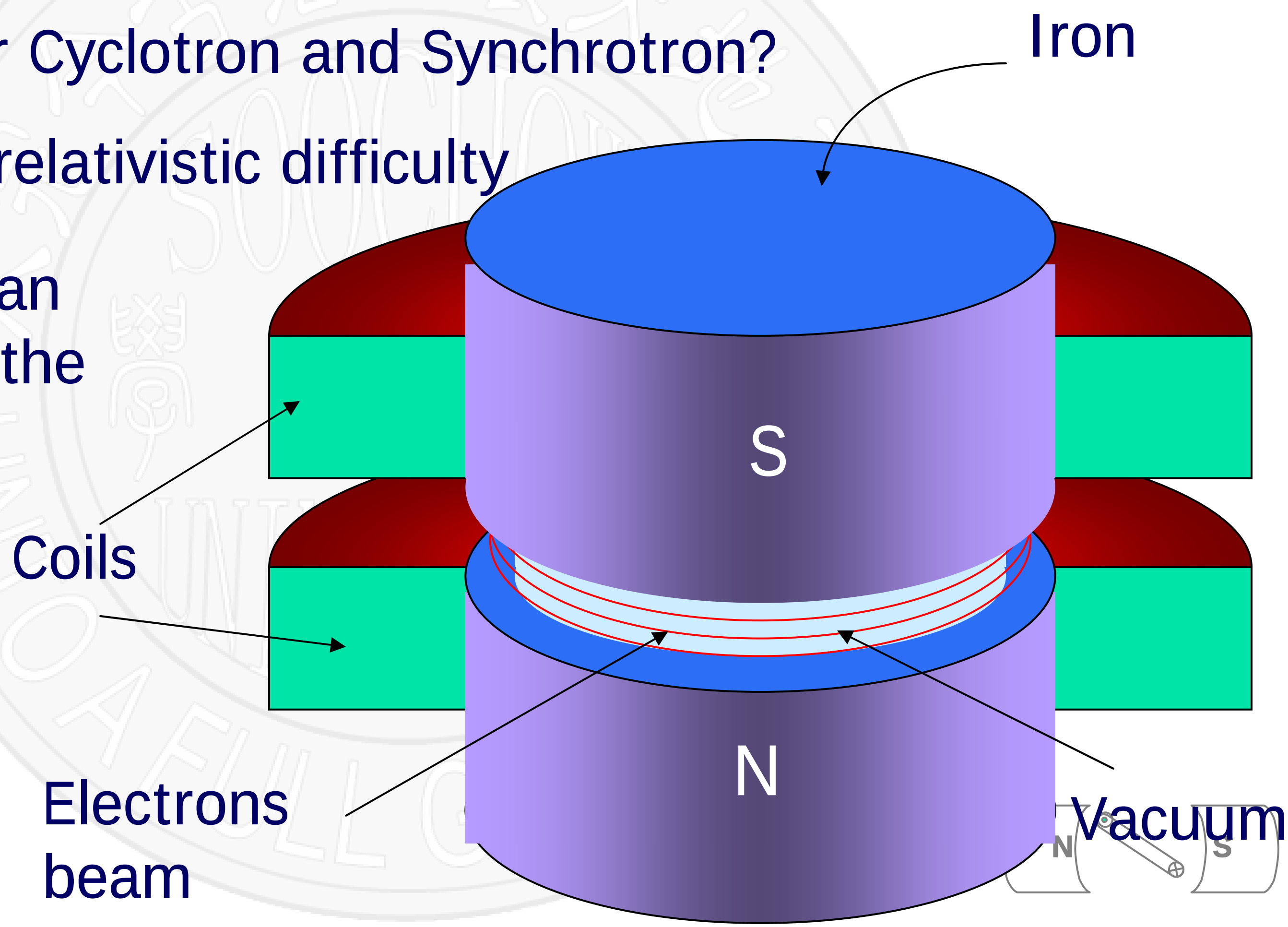




5.2 Two Kinds of Electromotive Force

- Betatron
  - Betatron can accelerate a charged particle
  - Remember Cyclotron and Synchrotron?
  - They met relativistic difficulty

Betatron can overcome the difficulty



5.2 Two Kinds of Electromotive Force

- Acceleration
  - First quarter period

$$\frac{dB}{dt} > 0$$

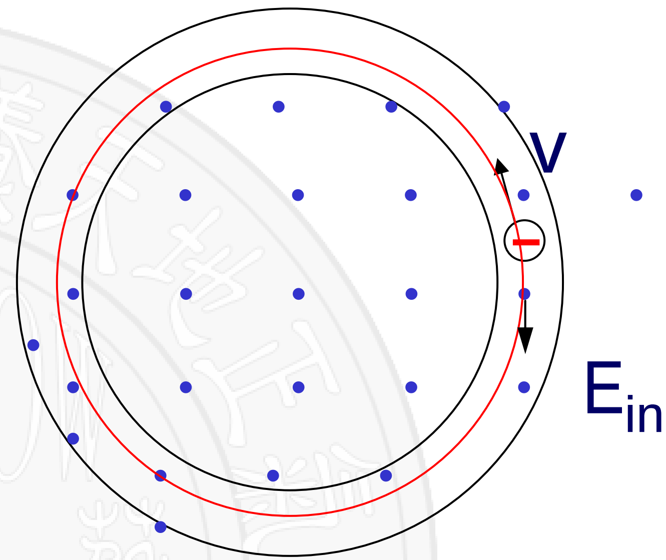
E_{in} clockwise

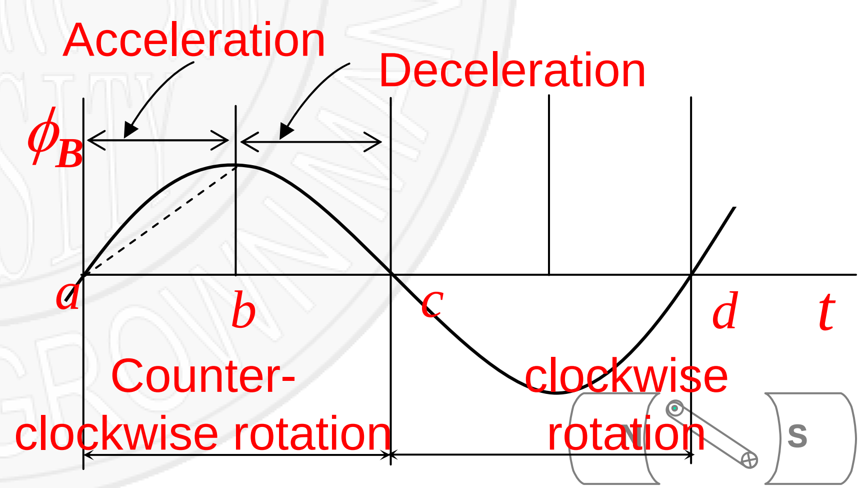
Fire electron upward

The electron can be accelerated and move in a circle.

Second quarter period

Deceleration





5.2 Two Kinds of Electromotive Force

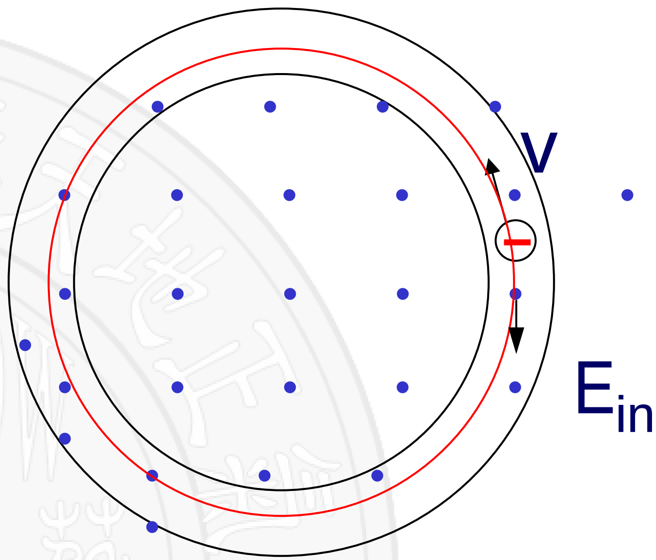
- Acceleration

  Third quarter period

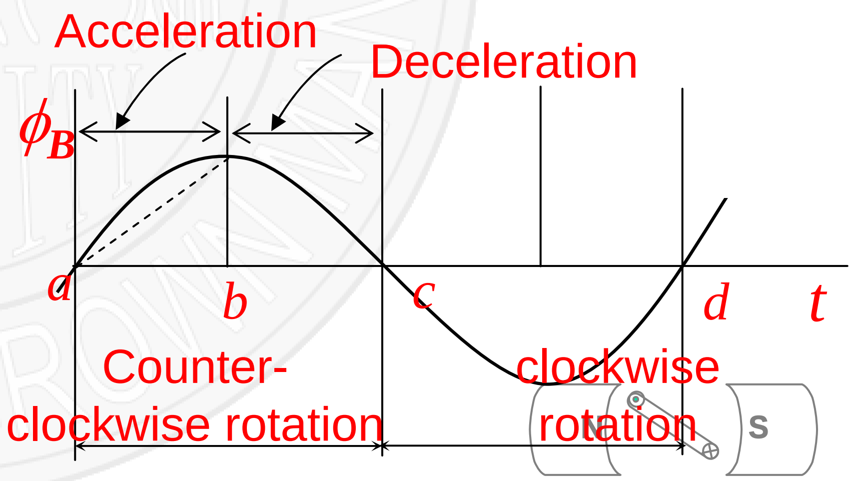
  Deceleration

  Fourth quarter period

  Can be accelerated but can not move in a circle. No centripetal force.



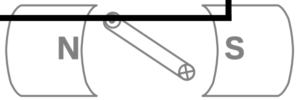
- Only first quarter period



5.2 Two Kinds of Electromotive Force

Similarities and Differences Between Vortex and static field

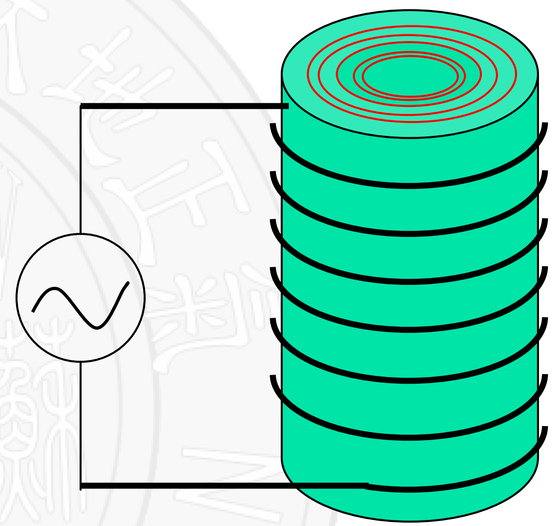
Electrostatic field	Vortex Electric field
Created by stationary charge	Created by variation of *B* with *t*
Field lines not closed	Field lines closed
Conservative field	Nonconservative field
Force on charge	Force on charge



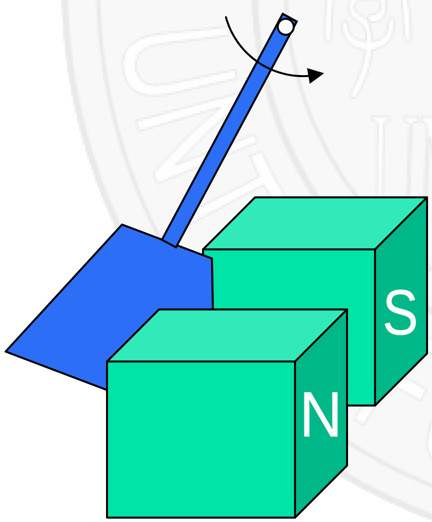
5.2 Two Kinds of Electromotive Force

- Applications of Electromagnetic induction
  - Eddy current

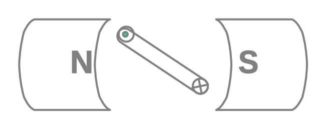
    high frequency induction furnace
  - Magnetic Damping



Show hardening



Show damped pendulum



Induced Emf and Reference Frames

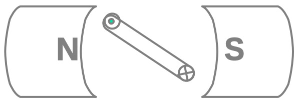
- The general equation of motional emf

$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

where v is the velocity of the length element dl of the moving conductor.

- Induced emf associated with the non-conservative electric field:

$$\varepsilon = \oint \vec{E}_{in} \cdot d\vec{l}$$



However, whether an object is moving or stationary actually depends on the reference frame.

- A bar magnet is approaching a conducting loop.

An electric field E_{nc} is induced to drive the current around the loop

$$F = qE_{nc}$$

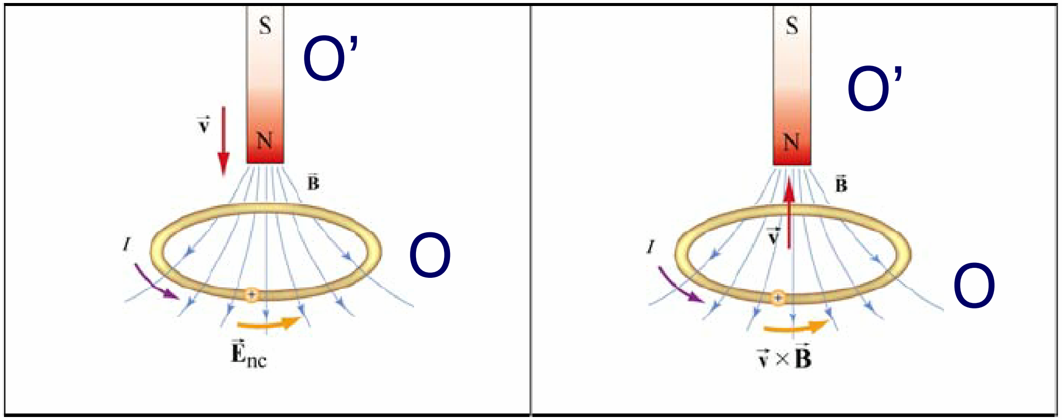
- An observer O' sees the loop moving toward the magnet.

$$F_B = qv \times B$$

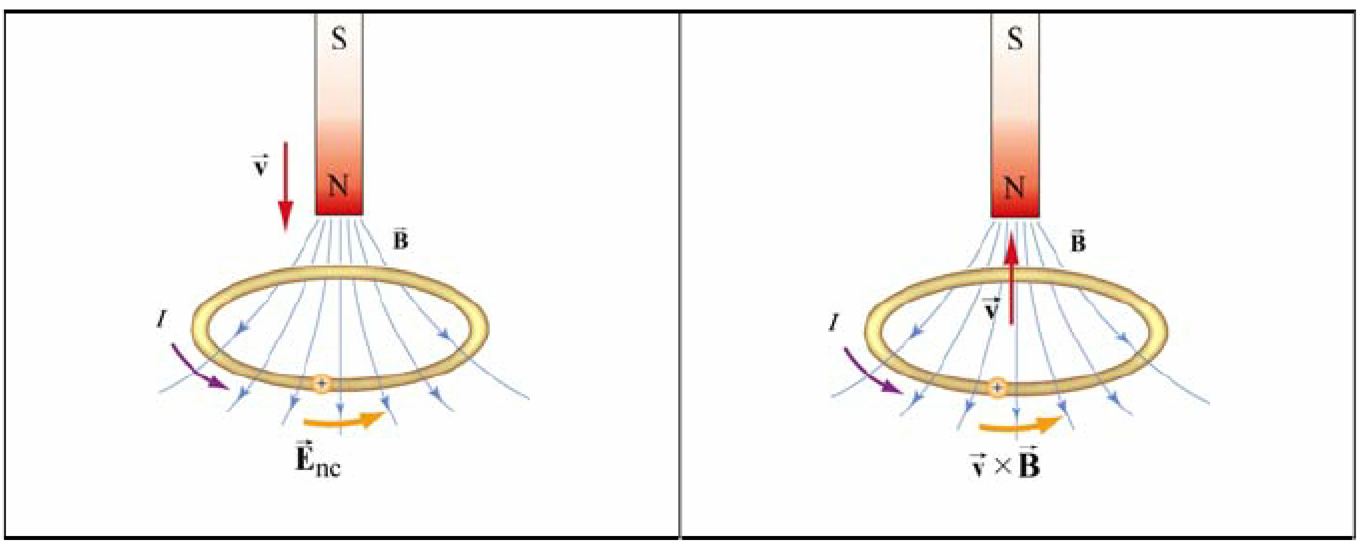
Same event

$$F_e = F_B$$

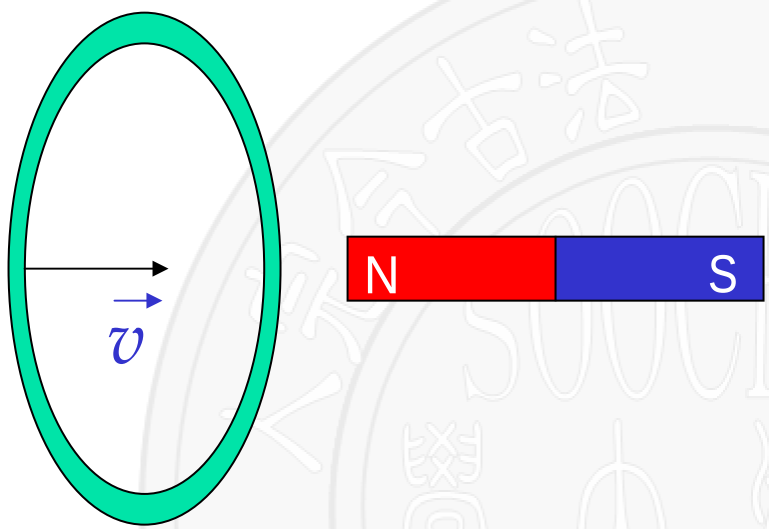
$$E_{nc} = v \times B$$



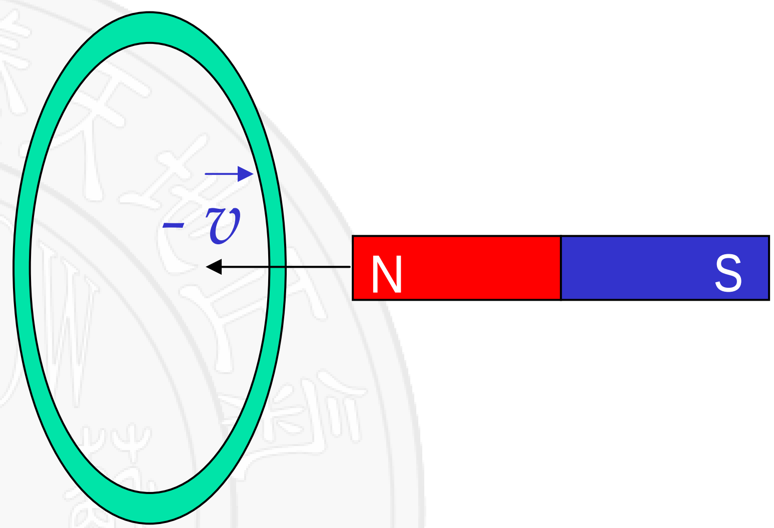
In general, as a consequence of relativity, an electric phenomenon observed in a reference frame O may appear to be a magnetic phenomenon in a frame O' that moves at a speed v relative to O.



5.2 Two Kinds of Electromotive Force



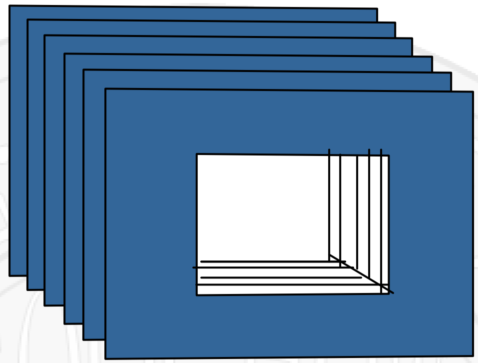
Motional emf

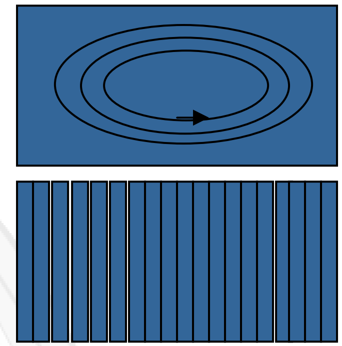


Induced emf

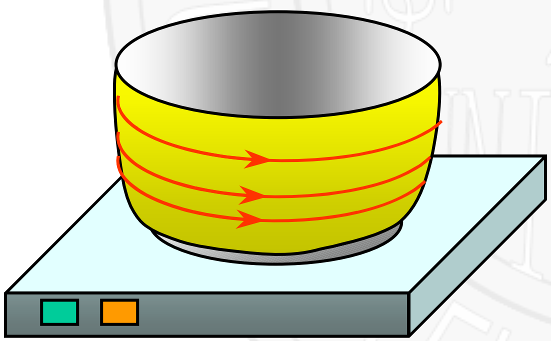
5.2 Two Kinds of Electromotive Force

- Transformer

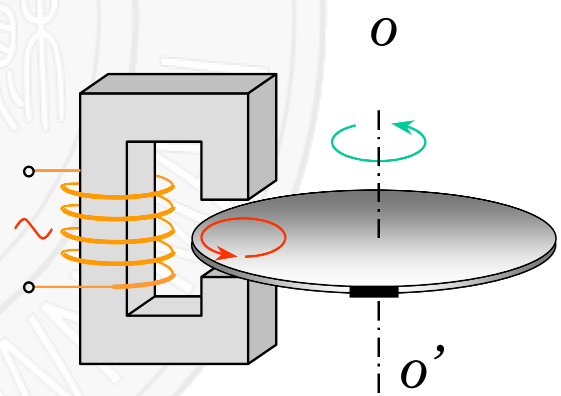


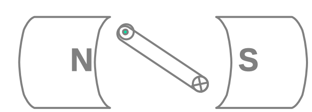


- Electromagnetic furnace



电磁炉





5.2 Two Kinds of Electromotive Force

- Skin effect

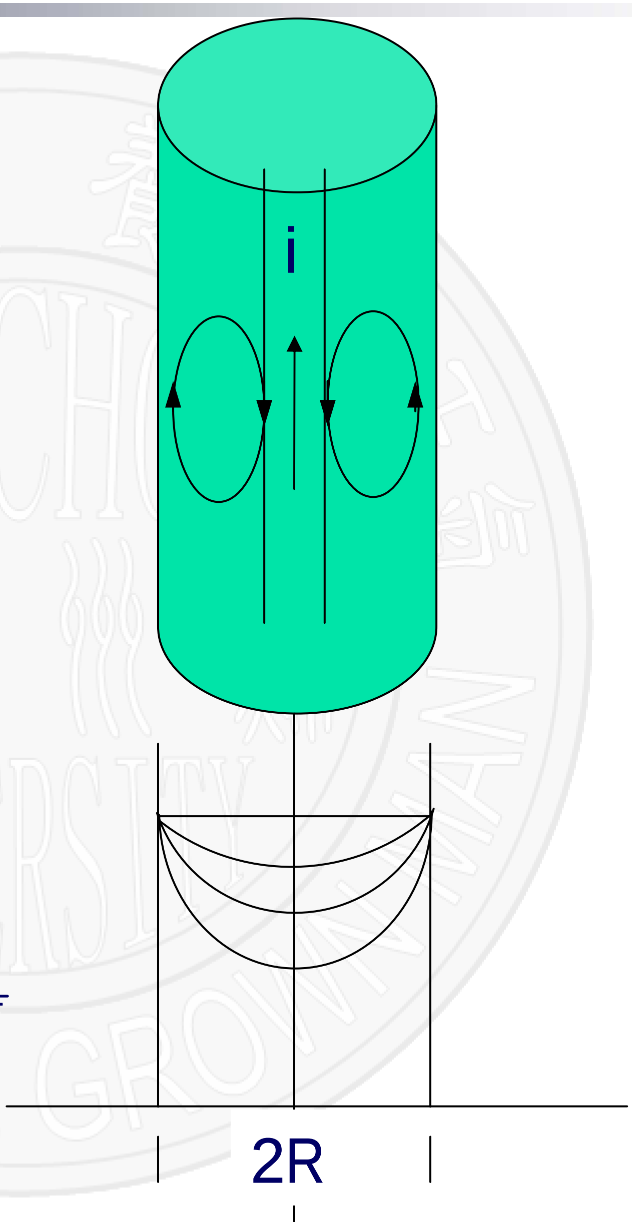
$i = i_0 \cos\omega t$

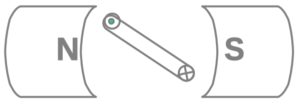
$$j = j_0 e^{-\frac{d}{d_s}}$$

d_s: Skin depth

$j \to j_0/e$

$$d_s = \sqrt{\frac{2}{\omega\mu\mu_0\sigma}} = \frac{503}{\sqrt{f\,\mu\sigma}}$$





5.2 Two Kinds of Electromotive Force

Lorentz Force Doesn't do work ?

$$\vec{f} = \text{-e}(\vec{v} \times \vec{B})$$

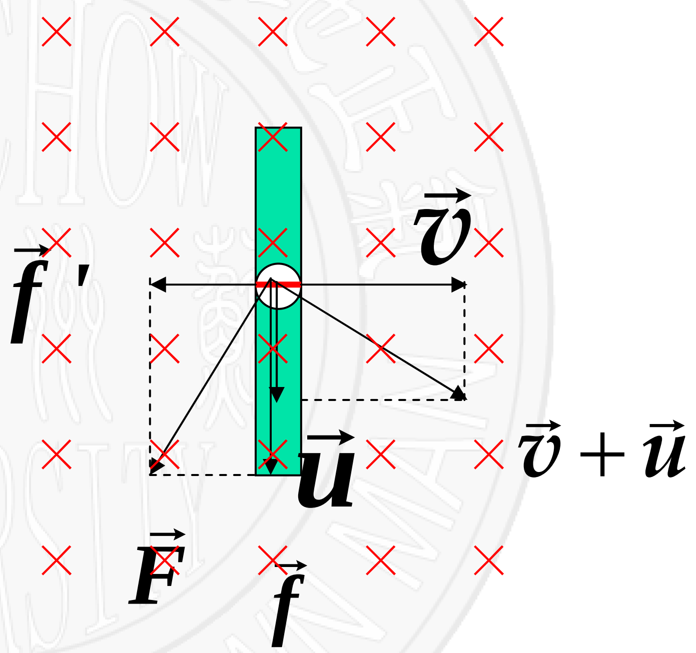
$$\vec{f}\,' = \text{-e}(\vec{u} \times \vec{B})$$

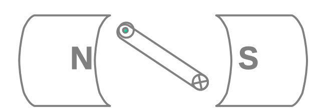
$$\vec{F} = \vec{f}\,' + \vec{f}$$

$$= \text{-e}(\vec{v} + \vec{u}) \times \vec{B}$$

$$\vec{f} \cdot \vec{u} = \text{-e}(\vec{v} \times \vec{B}) \cdot \vec{u}$$

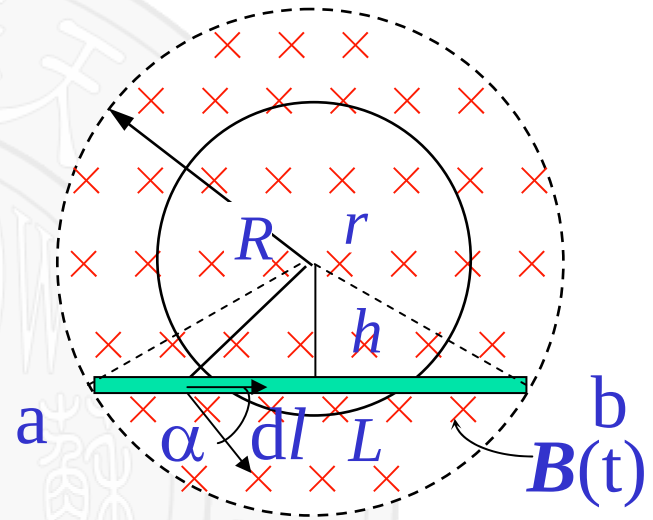
$$\vec{f}\,' \cdot \vec{v} = \text{-e}(\vec{u} \times \vec{B}) \cdot \vec{v}$$





5.2 Two Kinds of Electromotive Force

Example 5.5 A uniform magnetic field ***B*** fills a cylindrical volume of radius *R*. A metal rod of length L is placed as shown in the figure. If ***B*** is changing at the rate d*B*/d*t*, Find that the emf that is produced by changing magnetic field



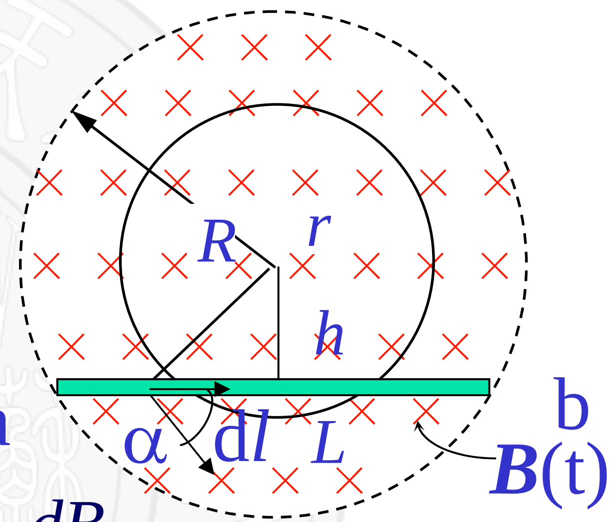
Solution: According to the Definition of induced emf

$$\varepsilon = \oint \vec{E}_{in} \cdot d\vec{l}$$

5.2 Two Kinds of Electromotive Force

The induced electric field inside the solenoid

$$E_{in} = -\frac{1}{2} r \frac{dB}{dt}$$



$$\varepsilon = \int_a^b \vec{E}_{in} \cdot d\vec{l} = \int_a^b E_{in} dl \cos\alpha$$

$$= \int_a^b E_{in} dl \cos\alpha = \int_a^b \frac{1}{2}\frac{dB}{dt} r dl \frac{h}{r} = \frac{1}{2} hL \frac{dB}{dt}$$

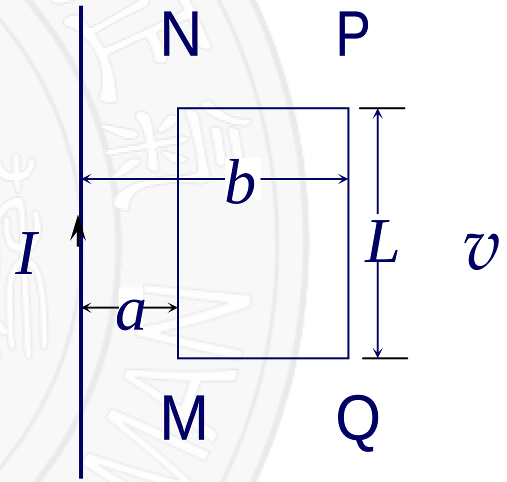
another way

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d(B\frac{1}{2}Lh)}{dt} = -\frac{1}{2} hL \frac{dB}{dt}$$

5.2 Two Kinds of Electromotive Force

Example 5.6 A rectangular loop moves away an infinite wire with current I, the speed is v, the other configurations as shown in the figure, find emf in the loop.

Solution:



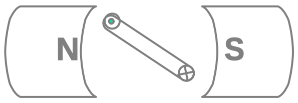
$$\varepsilon_{MN} = B_1 L v \qquad B_1 = \frac{\mu_0 I}{2\pi a}$$

$$\varepsilon_{QP} = B_2 L v \qquad B_2 = \frac{\mu_0 I}{2\pi b}$$

$$\varepsilon_{NP} = \varepsilon_{QM} = 0$$

$$\varepsilon = \varepsilon_{MN} - \varepsilon_{PQ} = \frac{\mu_0 I L v}{2\pi a} - \frac{\mu_0 I L v}{2\pi b}$$

Direction: Clockwise



5.2 Two Kinds of Electromotive Force

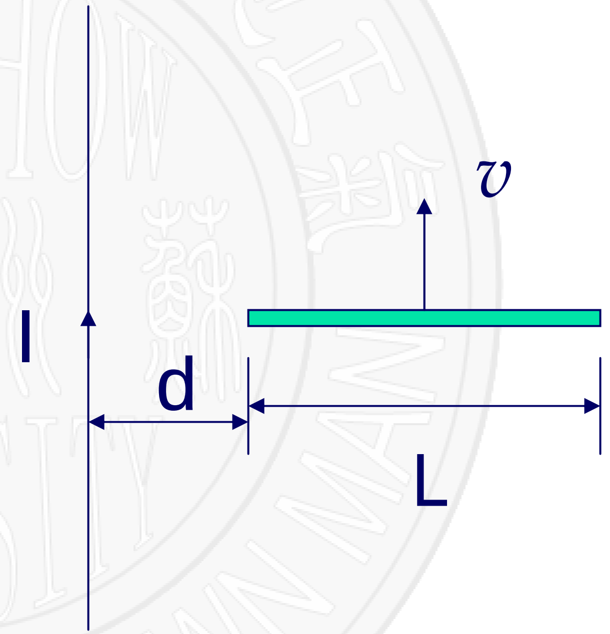
Example 5.7 A wire moves upward near an infinite wire with current I, the speed is v, the other configurations as shown in the figure, find emf in the wire.

Solution:

$$\varepsilon = \int_a^b (\vec{v} \times \vec{\boldsymbol{B}}) \cdot d\vec{\boldsymbol{l}}$$

$$\varepsilon = -\int_a^b vBdl = -\int_a^b v \frac{\mu_0 I}{2\pi r} dr$$

$$= -\frac{\mu_0 I v}{2\pi} \ln\left(\frac{d+L}{d}\right)$$



5.2 Two Kinds of Electromotive Force

Example 5.8 A wire is rotating with angular velocity ω as shown in the figure in magnetic field B, find emf in the wire.

Solution:

$$\varepsilon = \int_a^b (\vec{v}\times\vec{\mathbf{B}})\cdot d\vec{\mathbf{l}}$$

$$\varepsilon = \int_o^A \omega r B dl \cos\alpha$$

$$\varepsilon = \int_o^A \omega r B dr = \int_0^{L\sin\theta} \omega B r dr$$

$$= \frac{1}{2}\omega B (L\sin\theta)^2$$