



Tutorial for Chapter 4

★ The Calculation of Magnetic field

(1) Biot-savart Law

$$\mathbf{B} = \int d\mathbf{B} = \int \frac{\mu_0 I}{4\pi} \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

(2) Ampere's Law

$$\oint_{(L)} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

(3) Vector Potential \mathbf{A}

$$\mathbf{B} = \nabla \times \mathbf{A}$$

★ The Fundamental Properties of Static Field

(1) Gauss's Law



Tutorial for Chapter 4

$$\oiint_{(S)} \mathbf{B} \cdot d\mathbf{S} = 0$$

The Magnetic Field is a Non-divergent Field.

Ampere's Circuital Law

$$\oint_{(L)} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

The Magnetic Field is a Rotational Field.

✧ Magnetic Force

Ampere's Force

$$\mathbf{F} = \int I d\mathbf{l} \times \mathbf{B}$$



Tutorial for Chapter 4

Torque on Current-carrying Loop

$$\mathbf{L} = \mathbf{m} \times \mathbf{B}$$

The Potential of a Magnetic Dipole in a Magnetic Field

$$U = -\mathbf{m} \cdot \mathbf{B}.$$



Tutorial for Chapter 4

✧ The Motion of Charged Particle in a Magnetic Field

Circulating Charge

$$R = \frac{mv}{qB}$$

$$v = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

Helical Motion

$$R = \frac{mv \sin \theta}{qB}$$

$$h = v_{\parallel} T = \frac{2\pi m v \cos \theta}{qB}$$



Tutorial for Chapter 4

✧ Some important results

The Magnetic Field of Finite Straight Line with Current I

$$B = \frac{\mu_0 I}{4\pi r} (\cos \theta_1 - \cos \theta_2)$$

The Magnetic Field of Infinite Straight Line with Current I

$$B = \frac{\mu_0 I}{2\pi r}$$

The Magnetic Field of a *Circular Current Loop*



Tutorial for Chapter 4

✧ Some important results

The Magnetic Field of a *Circular Current Loop*

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

$$B_{center} = \frac{\mu_0 I}{2R}$$

The Magnetic Field of a *Solenoid*

$$B = \mu_0 n I$$

Tutorial for Chapter 4

4.2.1. A long "hairpin" is formed by bending a piece of wire as shown in Fig.4.6. If a 10-A current is set up, what are the direction and magnitude of \mathbf{B} at point a ? at point b ? Take $R=0.50$ cm. what's the circulation of \mathbf{B} for loop I,II,III

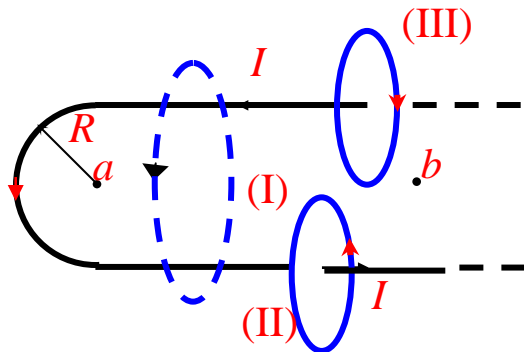


Fig.4.6 Problem 4.2.1

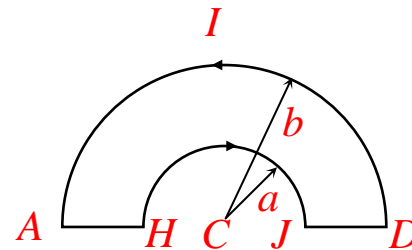


Fig.4.7 Problem 4.2.2

4.2.2. Use the Biot-Savart law to calculate the magnetic field \mathbf{B} at C , the common center of the semicircular arcs AD and HJ , of radii b and a , respectively, forming part of the circuit $ADJHA$ carrying current I , as shown in Fig. 4.7.



Tutorial for Chapter 4

4.2.10. A plastic disk of radius R has a charge q uniformly distributed over its surface. If the disk is rotated at an angular frequency ω about its axis, show that (a) the magnetic field at the center of the disk is

$$B = \frac{\mu_0 \omega q}{2\pi R}$$

and (b) the magnetic dipole moment of the disk is

$$\mu = \frac{\omega q R^2}{4}$$

(c) If the disc is placed by a sphere, do the same as (a) & (b)



Tutorial for Chapter 4

4.2.15. (a) A wire in the form of a regular polygon of n sides is just enclosed by a circle of radius a . If the current in this wire is I , show that the magnetic field B at the center of the circle is given in magnitude by

$$B = \frac{\mu_0 n I}{2\pi a} \tan(\pi/n)$$

4.3.2. A long solid cylindrical copper wire of radius R carries a current I distributed uniformly over the cross section of the wire. Sketch roughly the magnitude of the magnetic field B as a function of the distance r from the axis of the wire for (a) $r < R$ and (b) $r > R$.

Tutorial for Chapter 4

4.3.4. A long coaxial cable consists of two concentric conductors with the dimensions shown in Fig. 4.23. There are equal and opposite currents I in the conductors, (a) Find the magnetic field B at r within the inner conductor ($r < a$), (b) Find B between the two conductors ($a < r < b$). (c) Find B within the outer conductor ($b < r < c$). (d) Find B outside the cable ($r > c$).

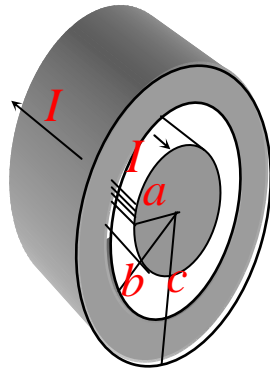


Fig.4.23 Problem4.3.4

Tutorial for Chapter 4

4.3.8.A conductor consists of an infinite number of adjacent wires each infinitely long and carrying a current I . Show that the lines of \mathbf{B} will be as represented in Fig. 4.27 and that \mathbf{B} for all points in front of the infinite current sheet will be given by

$$\mathbf{B} = \frac{1}{2} \mu_0 n i$$

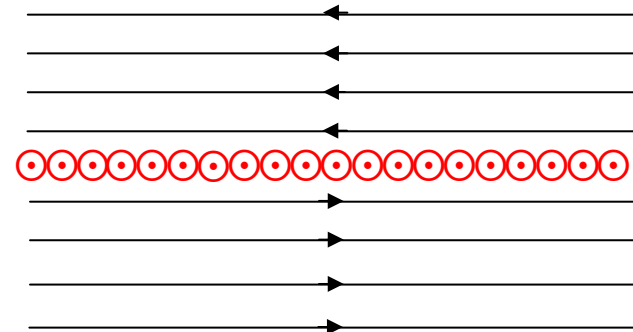


Fig.4.27 Problem 4.3.8

Tutorial for Chapter 4

An equilateral triangle loop with current I_2 is placed near an infinite wire with current I_1 , they share the same plane. One side of the triangle is parallel to the wire. Other configurations are shown in the figure. Find the forces on each side of the triangle due to current I_1

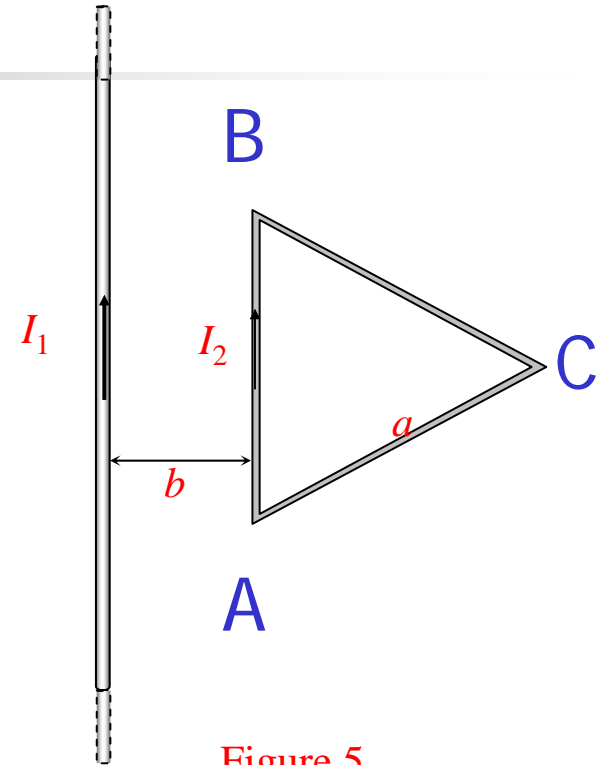


Figure 5

$$F_{BCx} = \frac{\mu_0 I_1 I_2 \tan 30^\circ}{2\pi} \ln \frac{b + \frac{\sqrt{3}}{2} a}{b} = F_{ACx}$$

$$F_{ABx} = \frac{\mu_0 I_1 I_2}{2\pi b} a$$

$$F_{BCy} = \frac{\mu_0 I_1 I_2}{2\pi} \ln \frac{b + \frac{\sqrt{3}}{2} a}{b} = F_{ACy}$$

Tutorial for Chapter 4

4.4.9. Figure 4.39 shows a wire of arbitrary shape carrying a current i between points a and b . The wire lies in a plane at right angles to a uniform magnetic field \mathbf{B} . Prove that the force on the wire is the same as that on a straight wire carrying a current i directly from a to b . (Hint: Replace the wire by a series of "steps" parallel and perpendicular to the straight line joining a and b .)

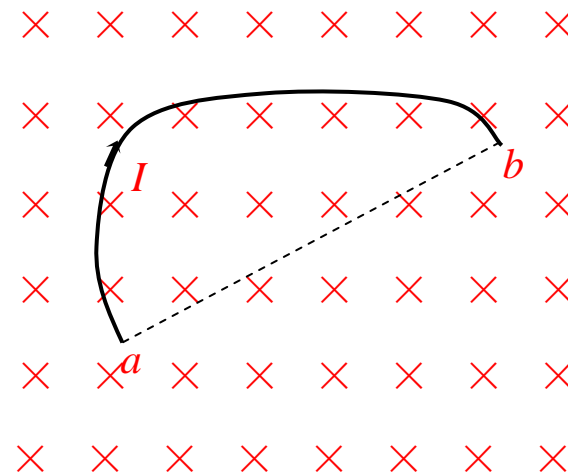


Fig.4.39 Problem 4.4.9

Tutorial for Chapter 4

4.4.8. A U-shaped wire of mass m and length l is immersed with its two ends in mercury (Fig. 4.38). The wire is in a homogeneous magnetic field \mathbf{B} . If a charge, that is, a current pulse $q = \int I dt$, is sent through the wire, the wire will jump up. Calculate, from the height h that the wire reaches, the size of the charge or current pulse, assuming that the time of the current pulse is very small in comparison with the time of flight. Make use of the fact that impulse of force equals $\int F dt$, which equals mv .

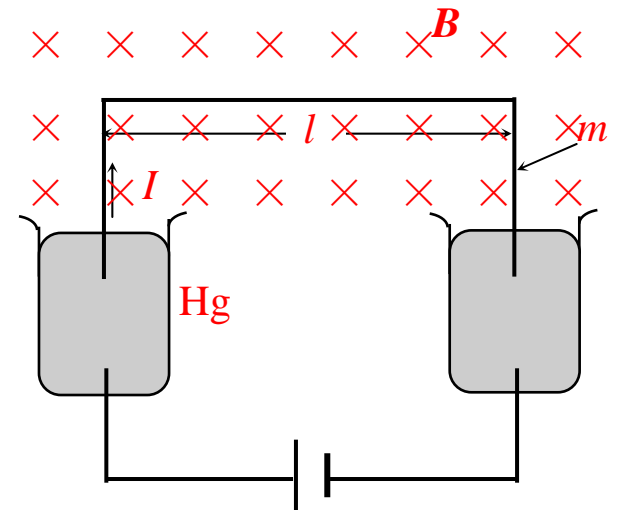
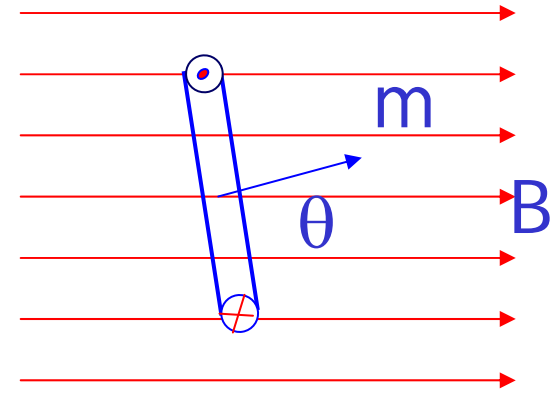


Fig.4.38 Problem 4.4.8

Tutorial for Chapter 4

A circle loop of radius R , mass m and current I is placed in a uniform magnetic field B on stable state initially. (a) If the loop is rotated a small angle ($\theta < 5^\circ$) from its equilibrium state, and released. Demonstrate the motion of the loop will be harmonic motion. (b) If the loop is rotated from its stable state to the unstable state, what is the work done by the external torque.

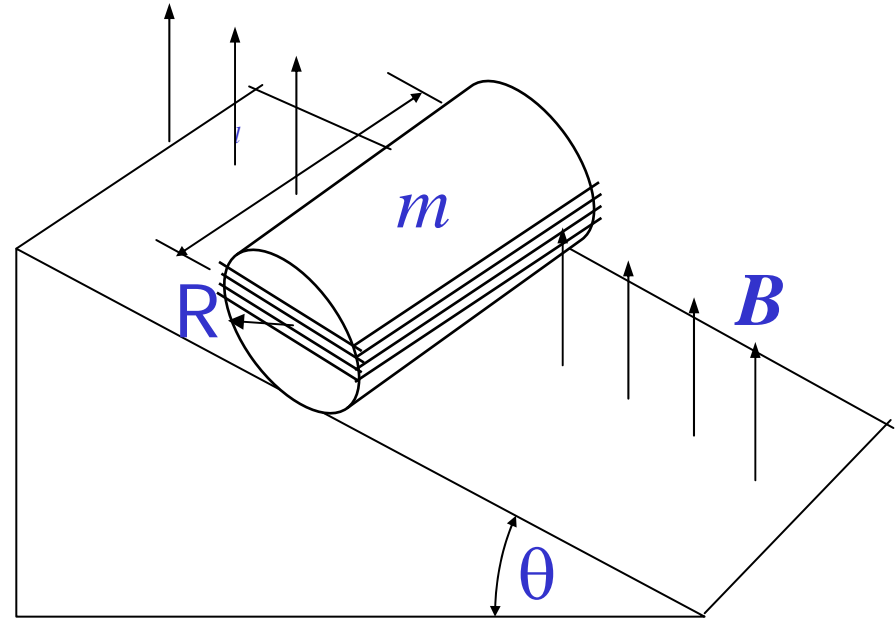




Tutorial for Chapter 4

4.4.11. Figure 4.41 shows a wooden cylinder with a mass m of 0.25 kg, a radius R , and a length l of 0.1 m with N equal to 10 turns of wire wrapped around it longitudinally, so that the plane of the wire loop contains the axis of the cylinder. What is the least current through the loop that will prevent the cylinder from rolling down an inclined plane whose surface is inclined at an angle θ to the horizontal, in the presence of a vertical field of magnetic induction 0.5 T, if the plane of the windings is parallel to the inclined plane?

Tutorial for Chapter 4



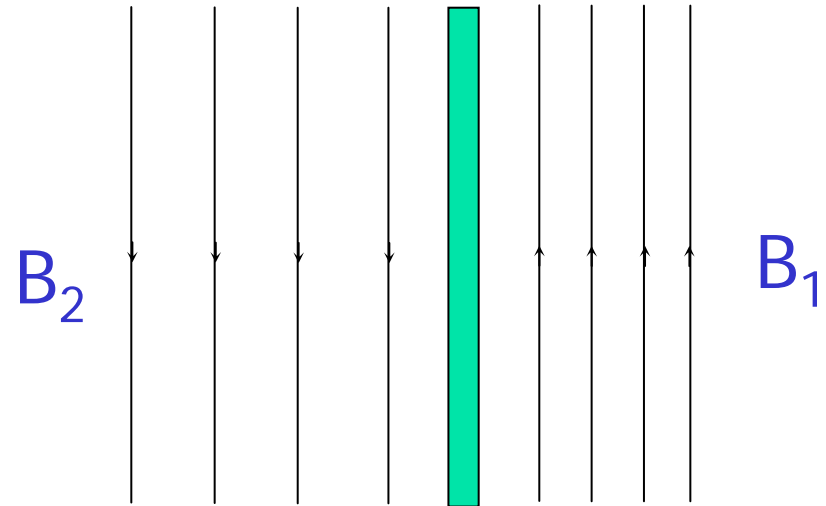


Tutorial for Chapter 4

4.3.1. In the region without current, if the lines of induction \mathbf{B} are parallel, is the magnetic field uniform? How about in electrostatic situations?

Tutorial for Chapter 4

There is a uniform field B_1 on the left of a sheet with current, a uniform field B_2 on the right of a sheet with current.



(a) How about the current distributed on the sheet?

(b) Find the force on a unit area of the sheet