Chapter 4 Magnetic Field

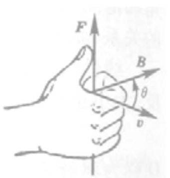
4.1 The Magnetic Field

4.2 The Biot - Savart Law

4.3 The Gauss’s Law & Ampere’s Circuital Law

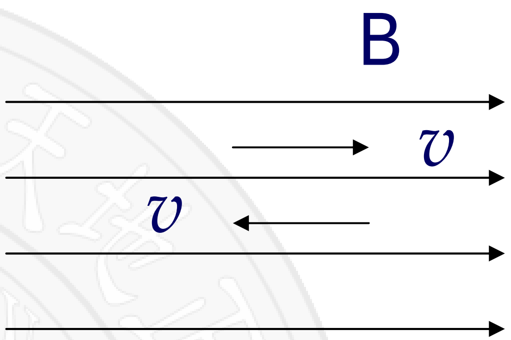
4.4 The Magnetic Forces on Current Conductors

4.5 The Motion of Charge in Magnetic Field



4.5 Charge Motion in Magnetic Field

Circulating Charge

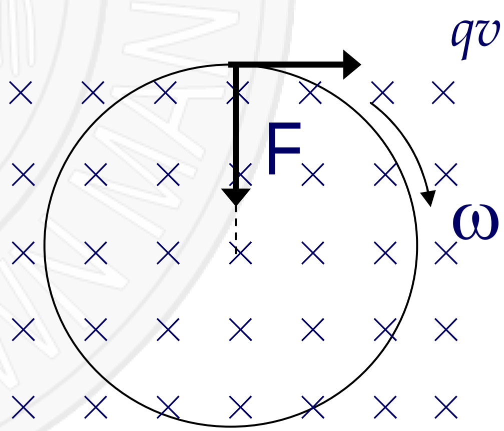


- Lorentz Force on Moving Charge

$$\vec{F} = q\vec{v} \times \vec{B}$$

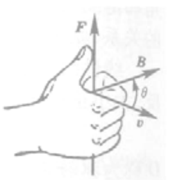
$$F = qvB\sin\theta$$

- θ=0,no force on the charge, move along a straight line
- θ=π/2, maximum force on the charge, circulating



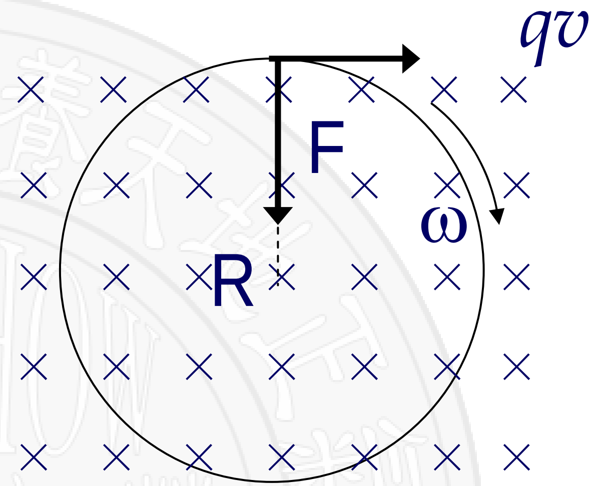
If q<0, going clockwise

If q>0, going counterclockwise



4.5 Charge Motion in Magnetic Field

- Circulating Radius

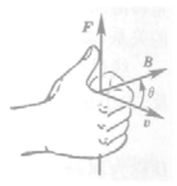


$$qvB = m\frac{v^2}{R}$$

$$R = \frac{mv}{qB}$$

v goes up, R increases, inversely

? If we have two same charges fired at the same time but different speeds, one fast, one slowly. which one arrive at the starting point first?



4.5 Charge Motion in Magnetic Field

- Circulating Frequency
  - Angular Frequency

$$\omega = \frac{v}{R} = \frac{qB}{m}$$

  - Frequency

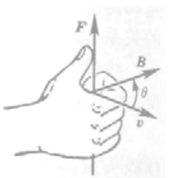
$$\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

- Circulating Period

$$T = \frac{1}{\nu} = \frac{2\pi m}{qB}$$

Show Circulating particle

Show Lorentz Force



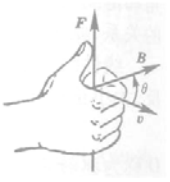
4.5 Charge Motion in Magnetic Field

Example 4.11 10-eV electron is circulating in a plane at right angles to a uniform magnetic held of 1.0×10^{-4}T (=1.0gauss)(a) What is its orbit radius? (b) What is the cyclotron frequency? (c) What is the period of revolution *T.*

Solution:

we can get the particle speed from the following equation

$$v = \sqrt{\frac{2E_k}{m}}$$



4.5 Charge Motion in Magnetic Field

So the radius is

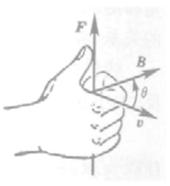
$$R = \frac{mv}{qB} = \frac{9.1\times10^{-31}\times1.9\times10^{6}}{1.6\times10^{-19}\times1.0\times10^{-4}} = 0.11m = 11cm$$

the frequency is

$$\nu = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times1.0^{-4}}{2\pi\times9.1\times10^{-31}} = 2.8\times10^{6}(Hz)$$

the period is

$$T = \frac{1}{\nu} = 3.6\times10^{-7}(s)$$



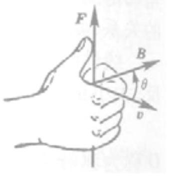
4.5 Charge Motion in Magnetic Field

Cyclotron

- According to Lorentz Force, F⊥V, Lorentz Force cannot change the energy of charged particle.

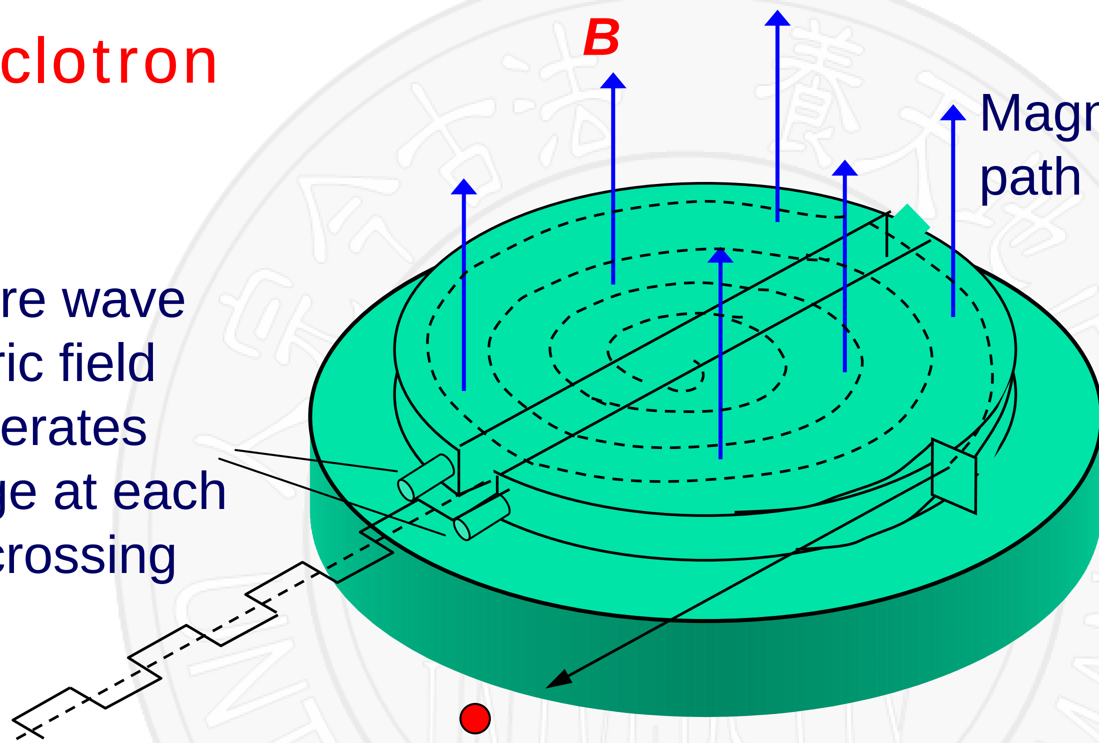
- To obtain high energy charged particle, we have to use electric field to accelerate the particle.

- If the path of accelerating particle is straight, distance is limited, but if we combine E&B, when circulating, sometimes in Magnetic field, sometimes in electric field, it can be accelerated continuously.



4.5 Charge Motion in Magnetic Field

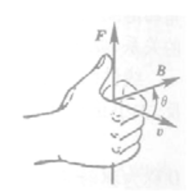
◈ Cyclotron



$$\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

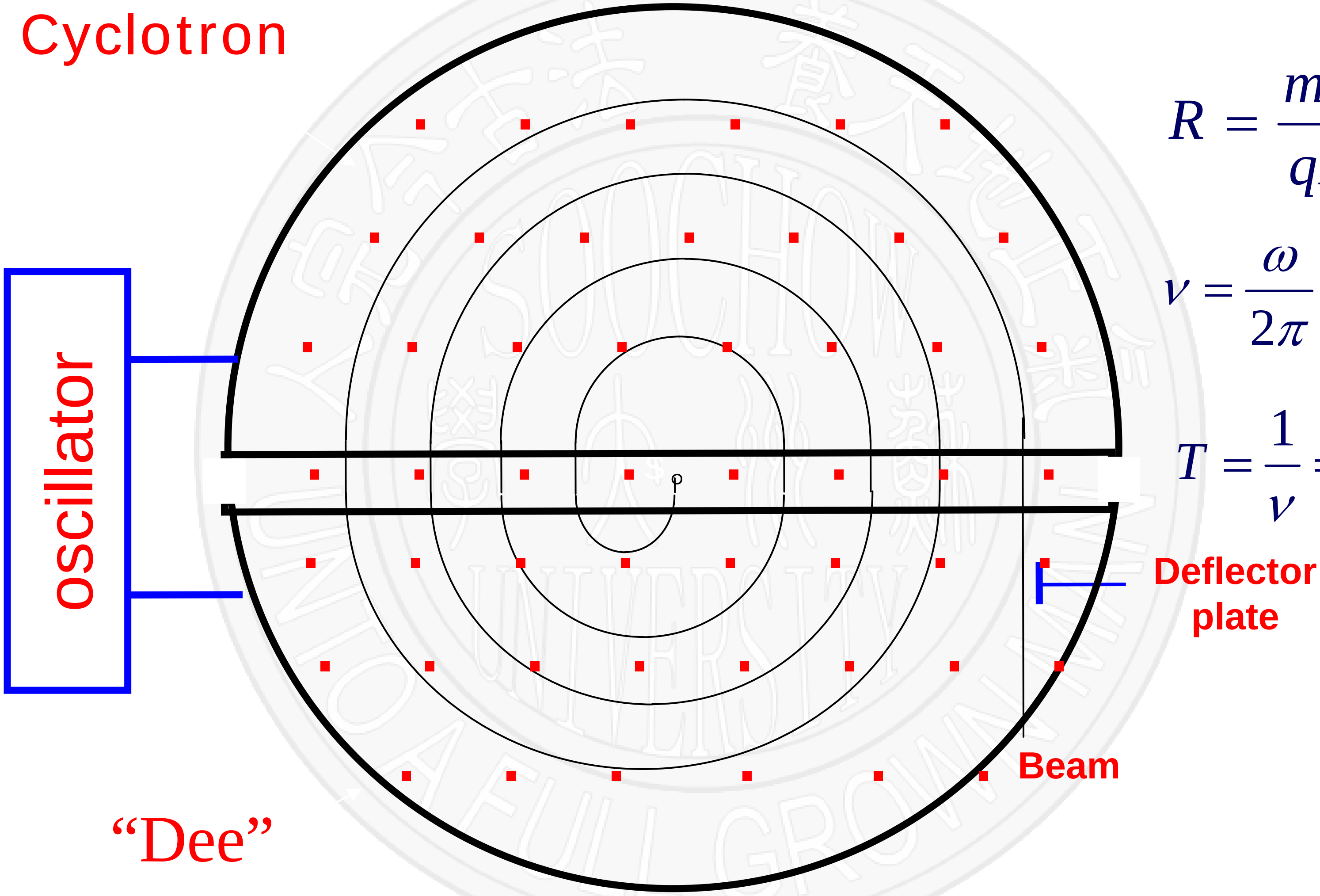
$$T = \frac{1}{\nu} = \frac{2\pi m}{qB}$$

The principle of the cyclotron



4.5 Charge Motion in Magnetic Field

◈ Cyclotron



$$R = \frac{mv}{qB}$$

$$\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

$$T = \frac{1}{\nu} = \frac{2\pi m}{qB}$$

4.5 Charge Motion in Magnetic Field

Synchrotron

$$\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

More information about Synchrotron

Show Synchrotron

Show Synchrotron

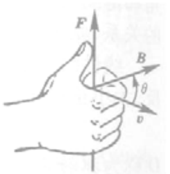
Show Synchrotron

From www. youtube. com

Relative effect

$$m = \frac{m_0}{\sqrt{1-(\frac{v}{c})^2}}$$

v↑,m↑,ν↓, we design distribution of B, make ν keep constant——synchrotron



4.5 Charge Motion in Magnetic Field

◈ Synchrotron

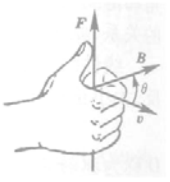
Adjust the frequency of the oscillator,

$$\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi m_0}\sqrt{1-(\frac{v}{c})^2}$$

Initial Energy(1931,lawrence):0.08MeV

The Energy(2002,lawrence):5×10^5MeV

It is suitable to accelerate heavy particle, the relative effect is not very obvious. But for electron, it is very obvious.$m_0 \rightarrow 5m_0$



4.5 Charge Motion in Magnetic Field

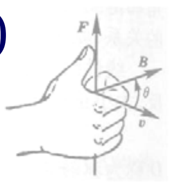
Example 4.11 The University of Pittsburgh cyclotron had an oscillator frequency of 12×10⁶Hz and a dee radius of 53cm, (a)What value of *B* is needed to accelerate deuterons? (b) What deuteron energy results?

Solution:

(a)
$$B=\frac{2\pi v_0 m}{q}=\frac{2\pi\times 12\times 10^{6}\times 3.3\times 10^{-27}}{1.6\times 10^{-19}}=1.6T$$

(b)
$$R=\frac{mv}{qB} \qquad mv=qBR$$

$$E_k=\frac{1}{2}mv^2=\frac{(mv)^2}{2m}=\frac{1}{2m}(qBR)^2=\frac{1.6\times 10^{-19}\times 1.6\times 0.53}{2\times 3.3\times 10^{-27}}=17(MeV)$$



4.5 Charge Motion in Magnetic Field

Discovery of electron

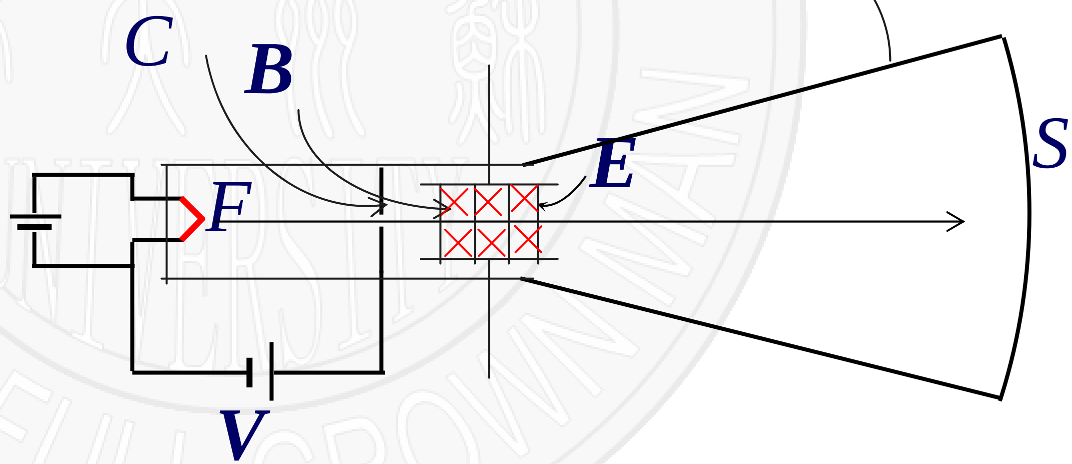
In 1897 J. J. Thomson measured the ratio of charge e to mass m of the electron by the following experiment.

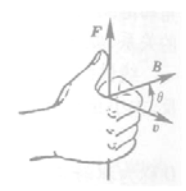
Modernized version Thomson apparatus

Glass envelope

Emitted electron from F.

Accelerated by Voltage V





4.5 Charge Motion in Magnetic Field

◈ Discovery of electron

Entered velocity selector

E⊥B

Resultant force

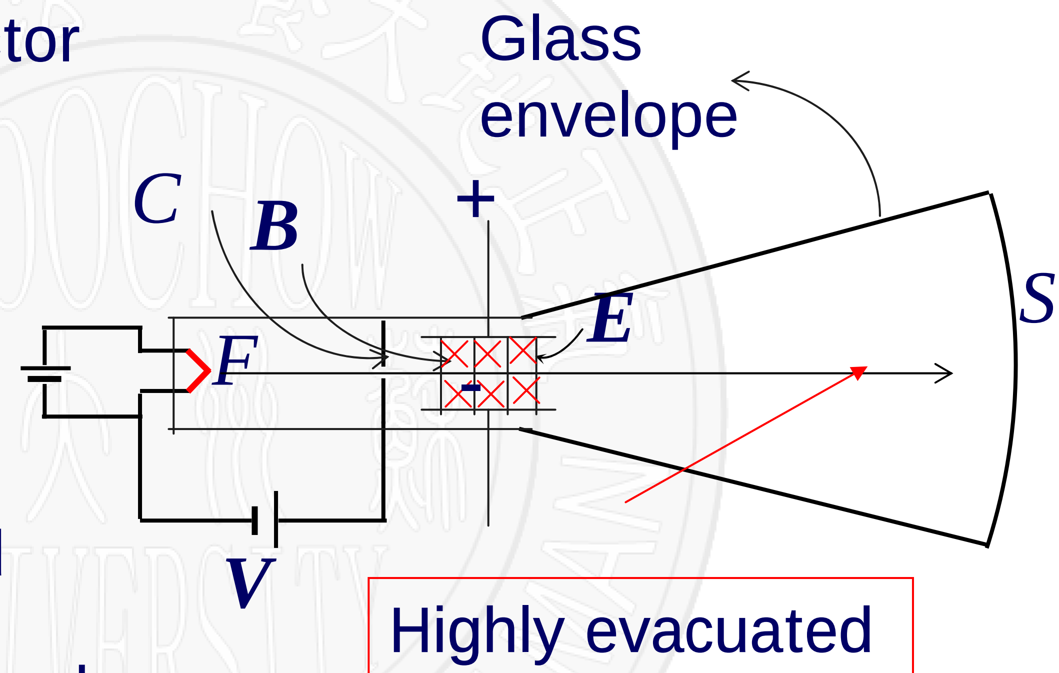
$$\vec{F} = q_0 \vec{E} + q_0 \vec{v} \times \vec{B}$$

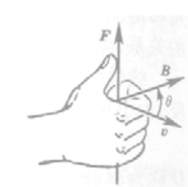
E deflecting e upward

B deflecting e downward

If they are to cancel

$eE = evB \qquad E = vB$





4.5 Charge Motion in Magnetic Field

Discovery of electron

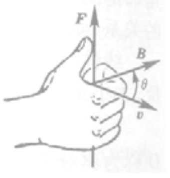
To adjust *E* or *B, no deflecting*

Thomson's procedure :

(a) To note the position of the undeflected beam spot, with ***E*** and ***B*** both equal to zero;

(b) To apply a fixed electric field ***E***, measuring on the fluorescent screen the deflection;

(c) To apply a magnetic field and adjust its value until the beam deflection is restored to zero.



4.5 Charge Motion in Magnetic Field

◈ Discovery of electron

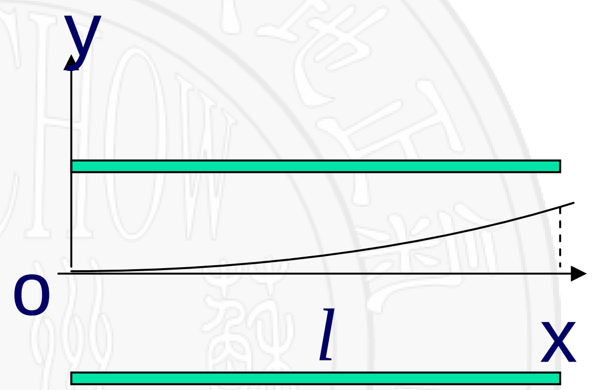
Only electric field is present, to measure the deflection y

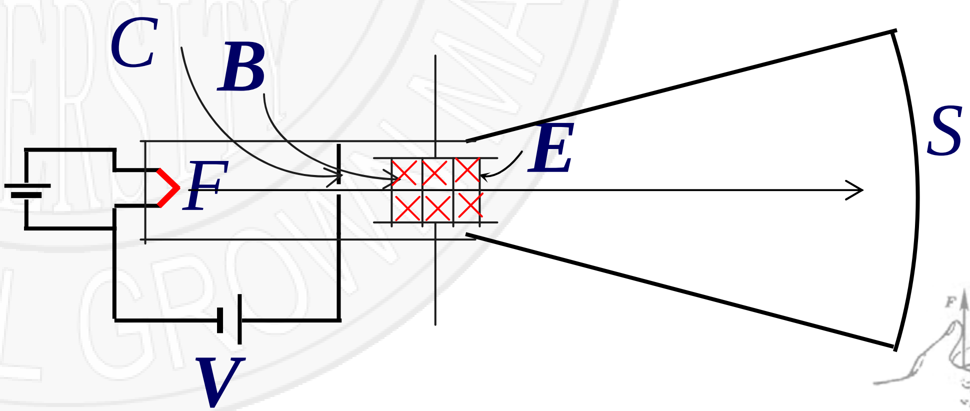
$$x = vt = l$$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}t^2$$

$$= \frac{1}{2}\frac{eE}{m}\left(\frac{l}{v}\right)^2$$

$$y = \frac{eEl^2}{2mv^2}$$





4.5 Charge Motion in Magnetic Field

- Discovery of electron

where *v* is the electron speed

l is the length of the deflecting plates

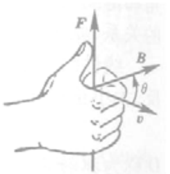
y measured displacement of the spot on the screen

e/*m* can be calculated, if v is known

How can we find v? Step (c)

Magnetic field B is applied and increased up to no deflecting.

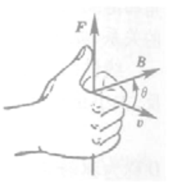
$$v=E/B \qquad y=\frac{eEl^2}{2mv^2} \qquad \frac{e}{m}=\frac{2yE}{B^2l^2}$$



4.5 Charge Motion in Magnetic Field

Discovery of electron

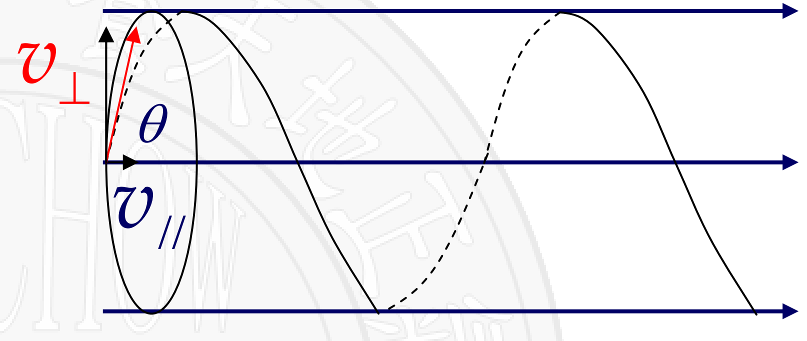
Thomson's value for e/m was 1.7×10^{11} C/kg, in full agreement with the 1977 value of 1.758805×10^{11} C/kg.



4.5 Charge Motion in Magnetic Field

Magnetic Focus

A beam of electrons fired



- Parallel Component

$$v_{//} = v\cos\theta$$

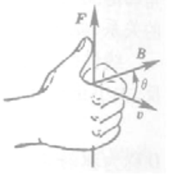
- perpendicular Component

$$v_{\perp} = v\sin\theta$$

- Helix motion

Helix pitch $$h = v_{//}T = v_{//}\frac{2\pi m}{qB} = \frac{2\pi m v\cos\theta}{qB}$$

radius $$R = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$$

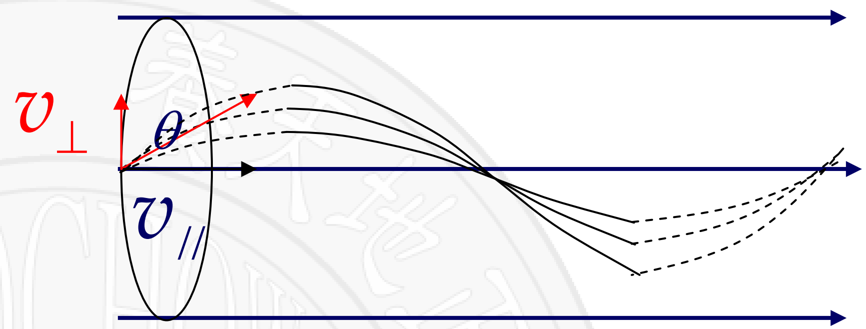


4.5 Charge Motion in Magnetic Field

◈ Magnetic Focus

$$v_{//} = v\cos\theta$$

$$v_{\perp} = v\sin\theta$$



- θ very small, but different
- Same speed

Show Magnetic focusing

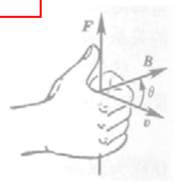
Show charge motion in non-uni field

Show aylon radiation

different radius $R = \dfrac{mv_{\perp}}{qB} = \dfrac{mv\theta}{qB}$

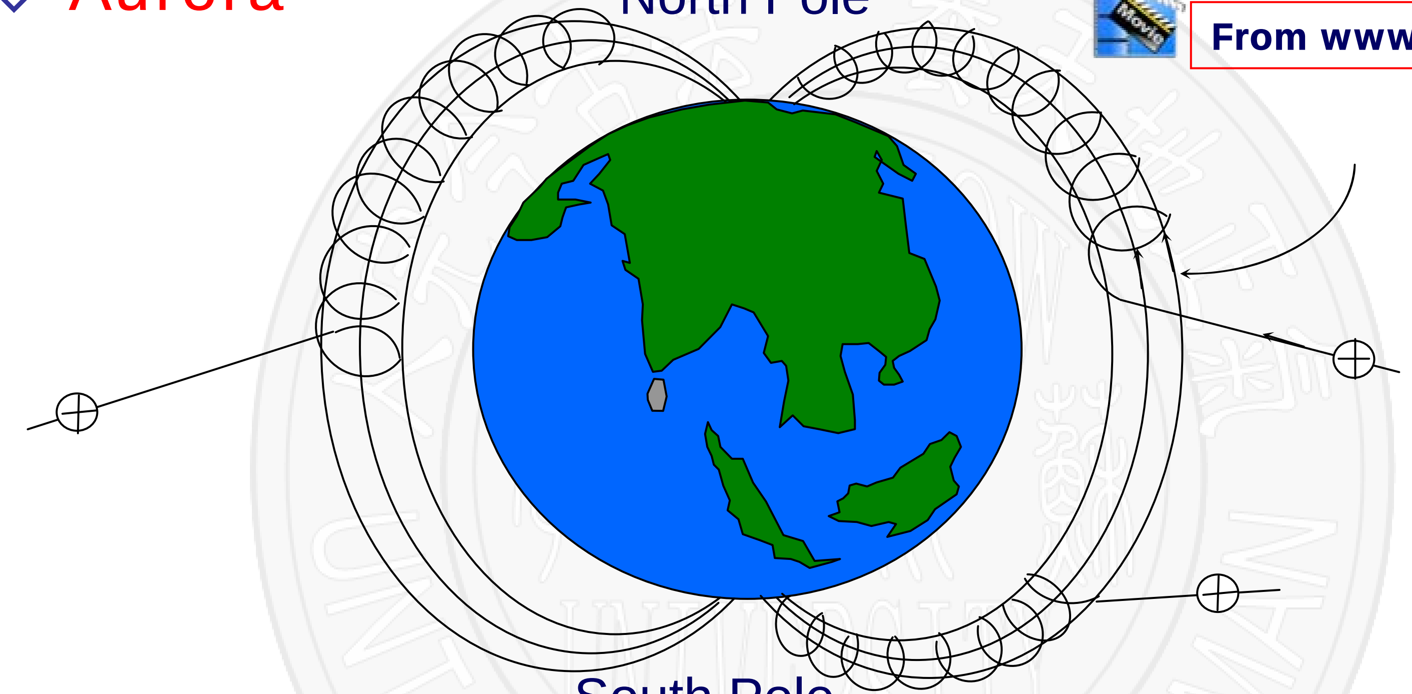
same pitch $h = \dfrac{2\pi mv\cos\theta}{qB} = \dfrac{2\pi mv}{qB}$

Magnetic Focus



4.5 Charge Motion in Magnetic Field

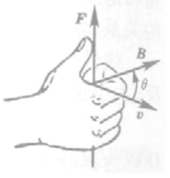
- Aurora



Aurora Borealis show

From www. youtube. com

Charged particles enter the earth's atmosphere from the solar wind

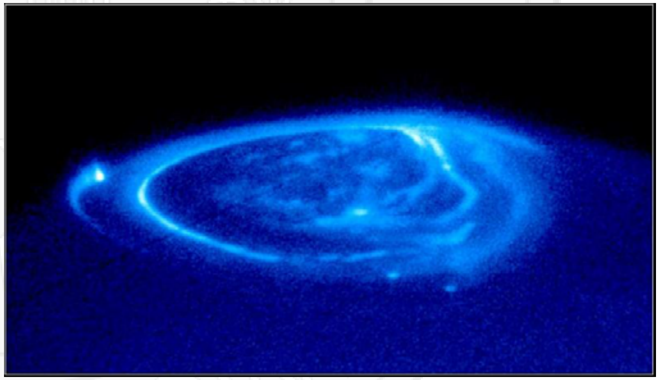


4.5 Charge Motion in Magnetic Field

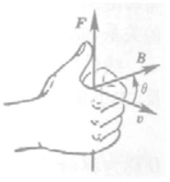
Aurora

Like the aurora near the Earth's poles, the glowing display near Jupiter's poles comes from the interaction of charged particleswith the planet's magnetic field, which is more intense near the poles. Jupiter's aurora

Figure show the distinct magnetic footprints of three of Jupiter's larger moons: Io, Europa and Ganymede.



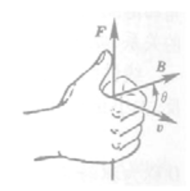
Jupiter Aurora



4.5 Charge Motion in Magnetic Field

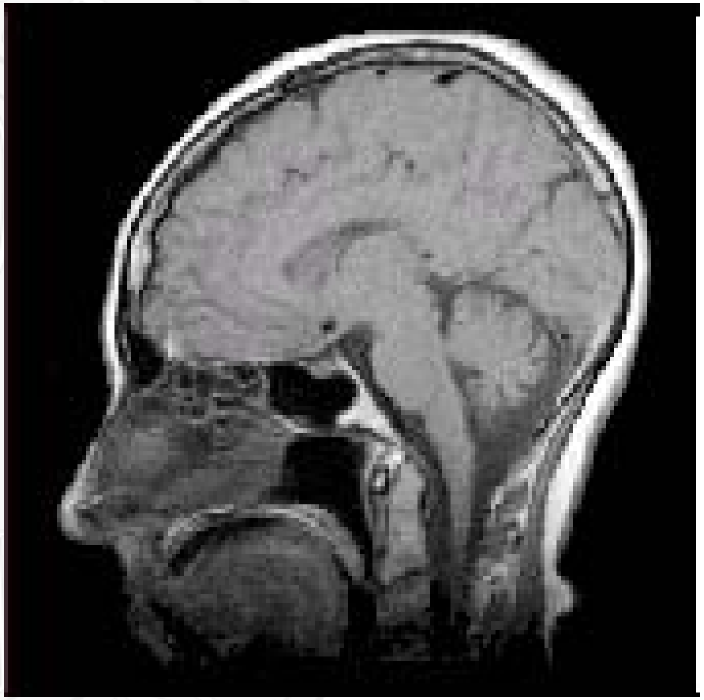
◈ More applications of magnetism

MRI ： Magnetic Resonance Imaging Magnetic resonance imaging is one of the most useful medical imaging tools. It produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x rays. MRI is based feet called nuclear magnetic resonance (NMR) in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons) These certain nuclei have magnetic fields, *similar to* those of electrons and the current loops discussed in this chapter.



4.5 Charge Motion in Magnetic Field

- More applications of magnetism (MRI)



Introduction to MRI

Show MRI

From www. youtube. com