Chapter 4 Magnetic Field

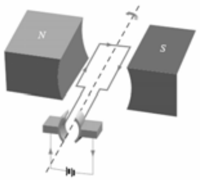
4.1 The Magnetic Field

4.2 The Biot - Savart Law

4.3 The Gauss's Law & Ampere's Circuital Law

4.4 The Magnetic Forces on Current Conductors

4.5 The Motion of Charge in Magnetic Field



4.4 Magnetic Force on Current

Magnetic Force on a Current

- Electric Force on Charge

$$\vec{F} = q\vec{E}$$

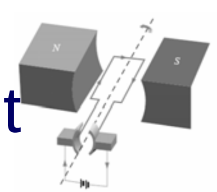
- Lorentz Force on Moving Charge

$$\vec{F} = q\vec{v} \times \vec{B}$$

- Magnetic Force on a Current

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

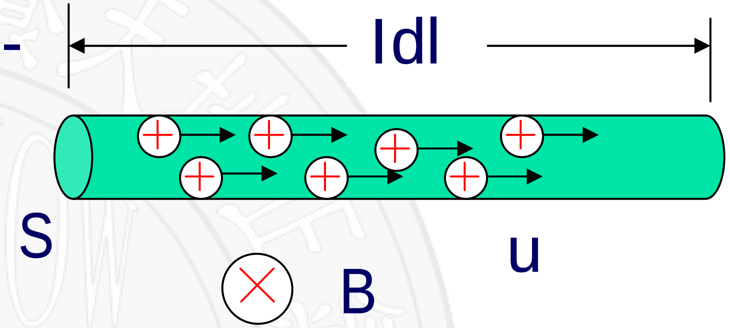
• Magnetic force on a current equal to the total forces on the moving charges in the element current



4.4 Magnetic Force on Current

◈ Magnetic Force on a Current

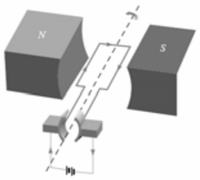
- n: Number density of charge-carrying particle
- q: charge on a particle
- dl: length of a current
- I: current



- The relationship between Lorentz force and Ampere’s force
  - Force on moving charge

$$\vec{F} = q\vec{u} \times \vec{B}$$

$$F = quB \qquad \text{(upward)}$$



4.4 Magnetic Force on Current

◈ Ampere's force & Lorentz Force

Show Ampere's Force

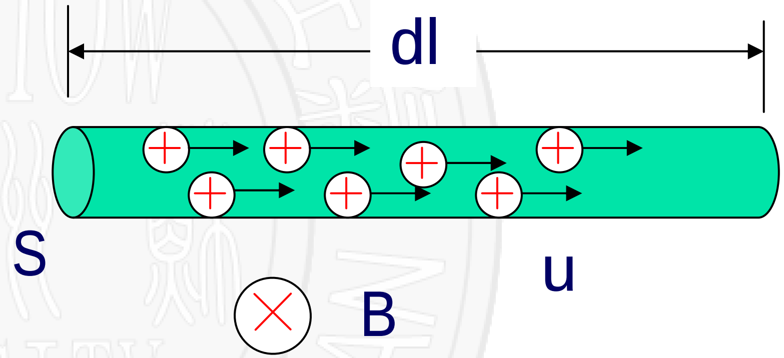
- Total Force on all moving charges

$$dF = dN \cdot quB = nSdl \cdot quB = (qunS)dlB$$

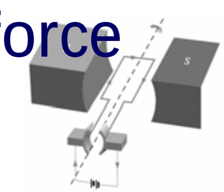
$$I = \frac{dq}{dt} = \frac{qnSdl}{dt} = qunS$$

$$dF = (qunS)dlB = IdlB$$

$$d\vec{F} = Id\vec{l} \times \vec{B}$$



- Ampere's force is the macroscopic show of Lorentz force, and Lorentz force is the microscopic view of Ampere's force



4.4 Magnetic Force on Current

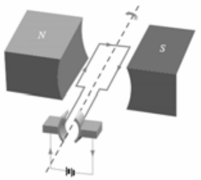
$$\vec{F} = \int d\vec{F} = \int I d\vec{l} \times \vec{B}$$

$$F_x = \int dF_x$$

$$F_y = \int dF_y$$

$$F_z = \int dF_z$$

Application of Ampere’s Force-Railgun



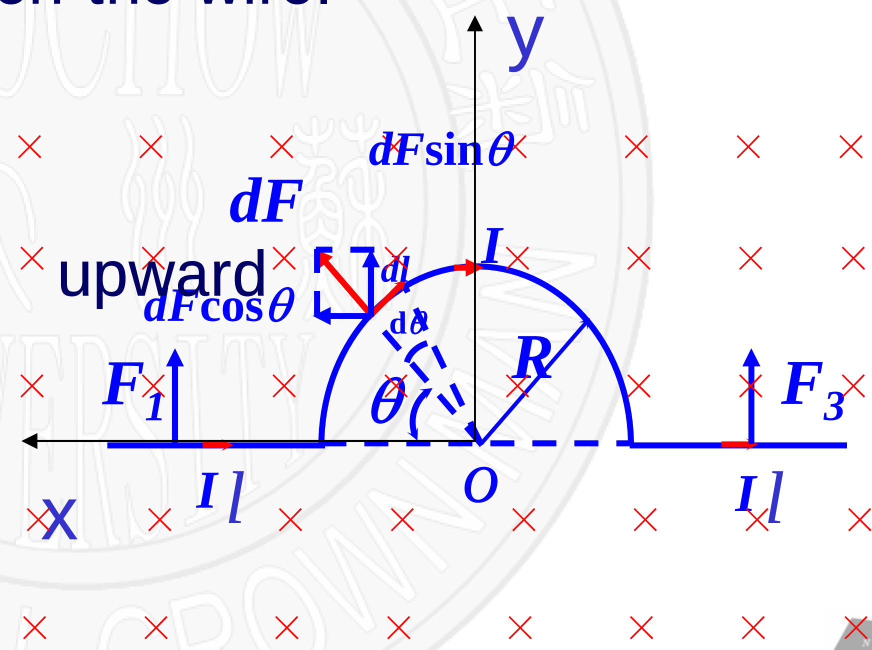
4.4 Magnetic Force on Current

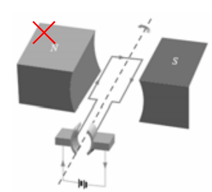
Example 4.6 A wire bent as shown in the figure carries a current and is placed in a uniform magnetic field ***B*** that emerges from the plane of the figure. Calculate the force acting on the wire.

Solution:

The forces on S_1 and S_3 are the same.

$$F_1 = F_3 = BIl$$

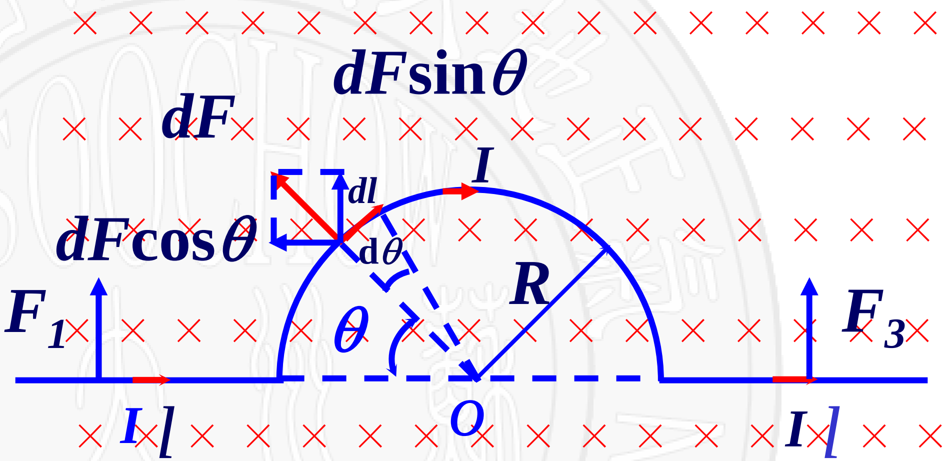




4.4 Magnetic Force on Current

From Ampere's law, the force on a element current in segment 2 is

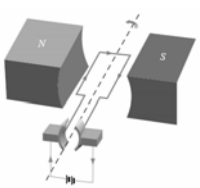
$$d\vec{F} = Id\vec{l} \times \vec{B}$$



$$dF = IBdl$$

$$dF_x = IBdl\cos\theta$$

$$F_x = \int dF_x = \int_0^{\pi} IBdl\ \cos\theta = \int_0^{\pi} IB\cos\theta R d\theta = 0$$



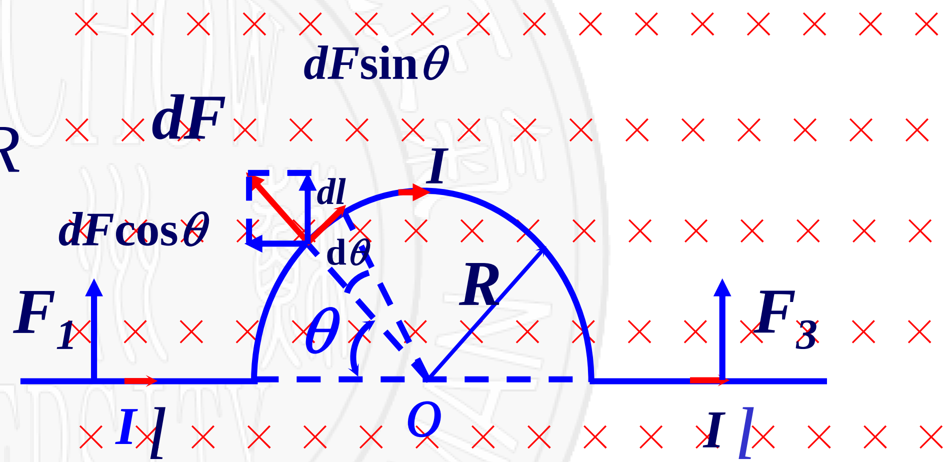
4.4 Magnetic Force on Current

$$dF_y = IBdl\sin\theta$$

$$F_y = \int dF_y = \int_0^{\pi} IBdl\sin\theta$$

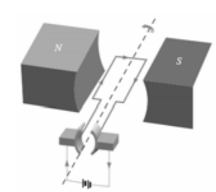
$$F_y = \int_0^{\pi} IB\sin\theta R d\theta = 2BIR$$

$$F_y = BI\ (2R\ )$$ upward



The total force is

$$F = 2F_1 + F_y = 2BIl + 2BIR$$ upward



4.4 Magnetic Force on Current

The Definition of Ampere

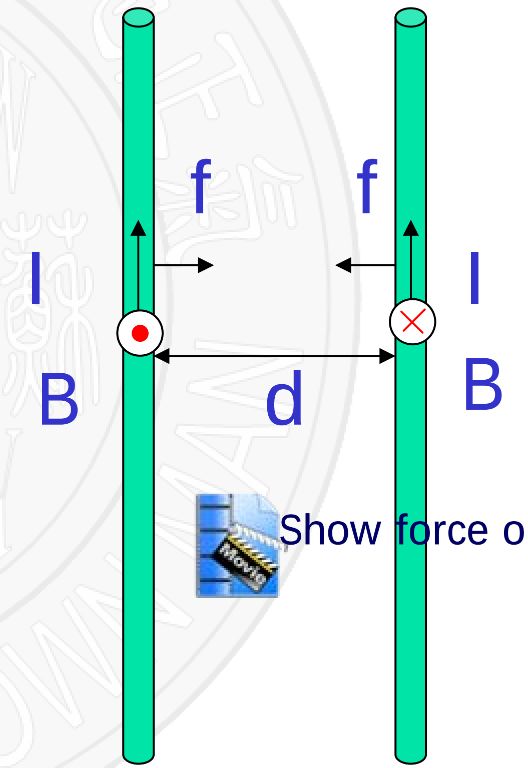
- The force on two parallel current-carrying conductor
  - The field due to a current

$$B = \frac{\mu_0 I}{2\pi d}$$

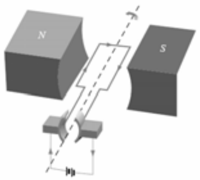
  - The force on an element current

$$dF = BIdl = \frac{\mu_0 I}{2\pi d} Idl$$

$$f = BI = \frac{\mu_0 I^2}{2\pi d}$$



Show force on current



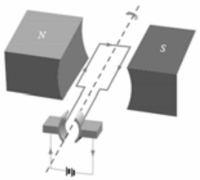
4.4 Magnetic Force on Current

◈ The Definition of Ampere

- Consider the force on two unit length current-carrying conductors they are parallel

$$f = BI = \frac{\mu_0 I^2}{2\pi d}$$

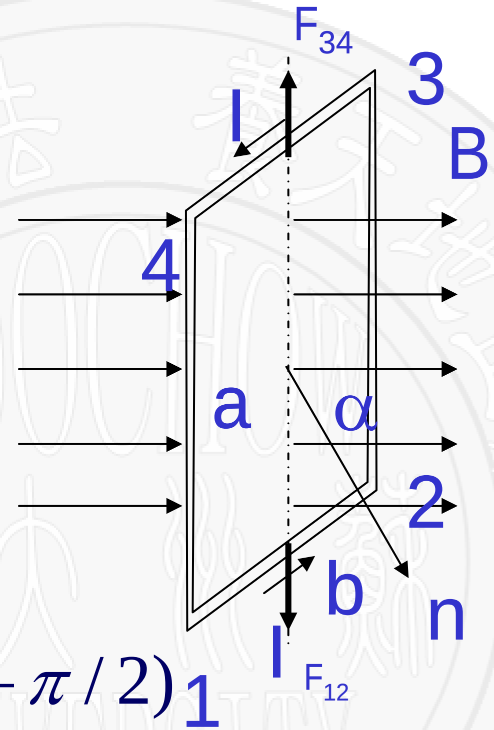
When they are 1meter apart, if the force is equal to 2×10^{-7}N, at the time the current is exactly 1 Ampere.

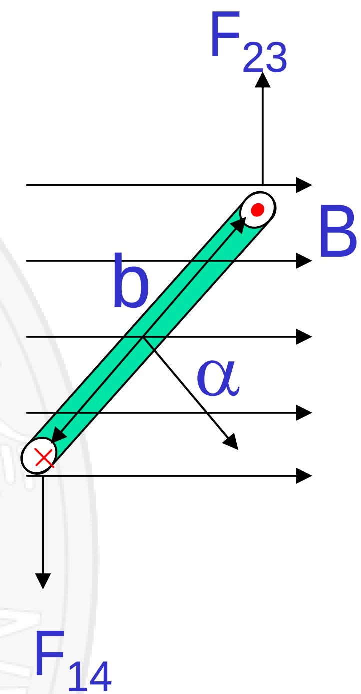


4.4 Magnetic Force on Current

Electric Motor

A Current-carrying Loop is placed in a uniform field B





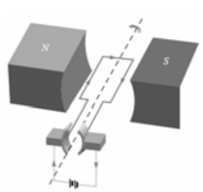
The forces on the edges are:

$$F_{12} = F_{34} = BIb\sin(\alpha + \pi/2)$$

$$F_{14} = F_{23} = BIa$$

The resultant force is equal to 0

F=0



4.4 Magnetic Force on Current

Electric Motor

$$L_{23} = BIa \cdot \frac{b}{2} \sin\alpha \qquad \text{up}$$

$$L_{14} = BIa \cdot \frac{b}{2} \sin\alpha \qquad \text{up}$$

$$L_{12} = L_{34} = 0$$

Show motor working

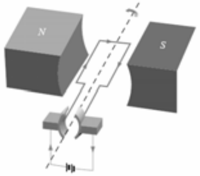
From www. youtube. com

The resultant torque is

$$L = BIab\sin\alpha = BIS\sin\alpha = Bm\sin\alpha$$

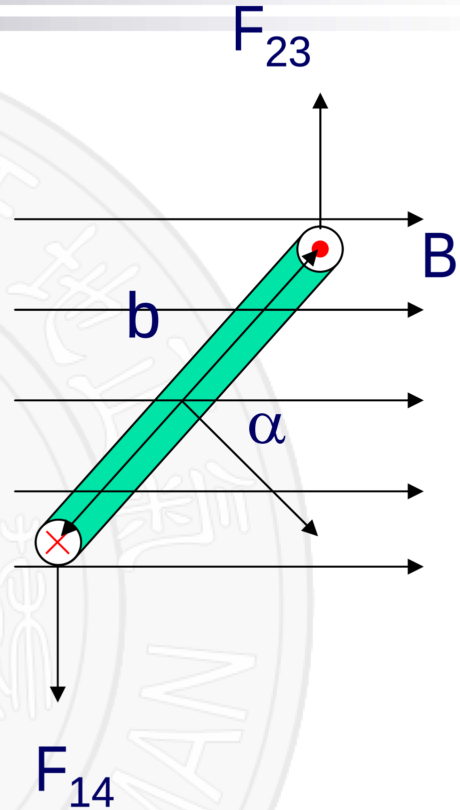
$$\vec{L} = \vec{m} \times \vec{B}$$

It holds true for any shaped loop.



4.4 Magnetic Force on Current

Electric Motor



$$dW = L d\alpha = Bm \sin\alpha \, d\alpha$$

$$W = \int_{\pi/2}^{\alpha} dW = \int_{\pi/2}^{\alpha} L \, d\alpha$$

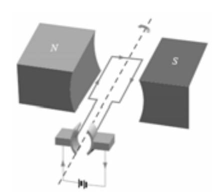
$$= \int_{\pi/2}^{\alpha} Bm \sin\alpha \, d\alpha = -mB\cos\alpha$$

$$W = U = -\vec{m} \cdot \vec{B}$$

$\alpha=0$, $U=-mB$. The state of the lowest energy

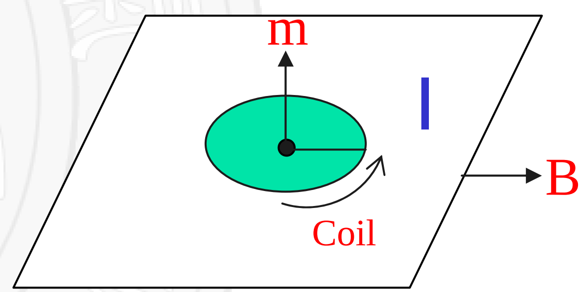
$\alpha=\pi/2$, $U=0$.

$\alpha=\pi$, $U=mB$. The state of the highest energy



4.4 Magnetic Force on Current

Example 4.9 A circular coil of wire 0.05 m in radius, having 30 turns, lies in a horizontal plane, as shown in the figure. It carries a current of 5A, in a counterclockwise sense when *viewed* from above. The coil is in a magnetic field directed toward the right, with magnitude 1.2T. Find the magnetic moment and the torque on the coil.

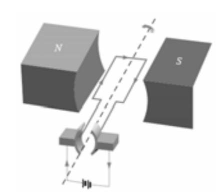


Solution:

$$S = \pi r^2 = \pi(0.05)^2 = 7.85 \times 10^{-3}\ (\mathrm{m}^2)$$

$$m = N\,IS = 30 \times 5 \times 7.85 \times 10^{-3} = 1.18(\mathrm{A \cdot m^2})$$

$$L = Bm\sin\alpha = (1.2 \times 1.18 \times \sin 90^\circ) = 1.41\mathrm{N \bullet m},$$



4.4 Magnetic Force on Current

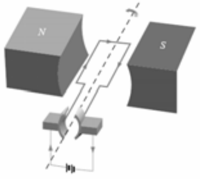
Example 4.10 If the coil in the last Example rotates from its initial position to a position where its magnetic moment is parallel to ***B***, what is the change in potential energy?

Solution:

$$U_1 = -1.18 \times 1.2\cos 90^\circ = 0,$$

$$U_2 = -1.18 \times 1.2\cos 0^\circ = -1.41\text{J}.$$

$$\Delta U = U_2 - U_1 = -1.41\text{J}$$



4.4 Magnetic Force on Current

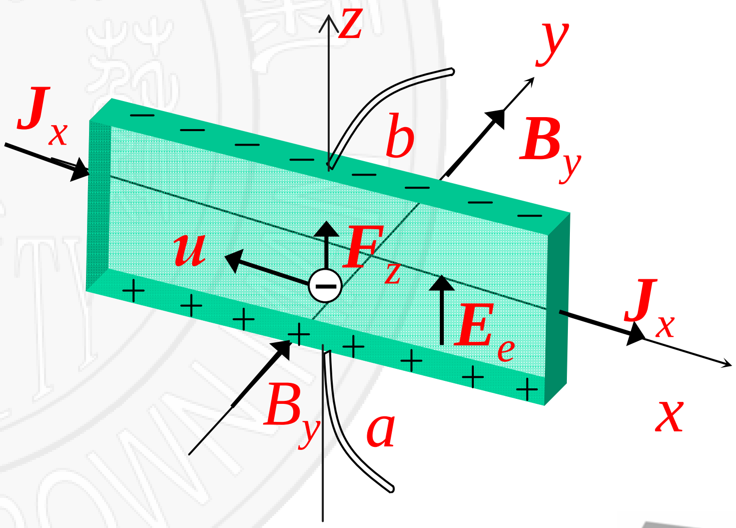
◈ The Hall Effect

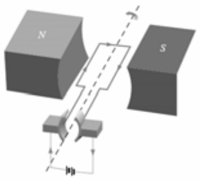
More infor. about Hall Effect

- Current flowing in a bulky conductor in magnetic field

- Potential difference (Hall Voltage) appears between upside and downside.

- Explanation
  - Current flowing along x-axis
  - Lorentz force
  - Charge carriers are negative
  - -q accumulated upside, q appears down by induction





4.4 Magnetic Force on Current

◈ The Hall Effect

• E points up, the potential of bottom higher, the top lower, conversely

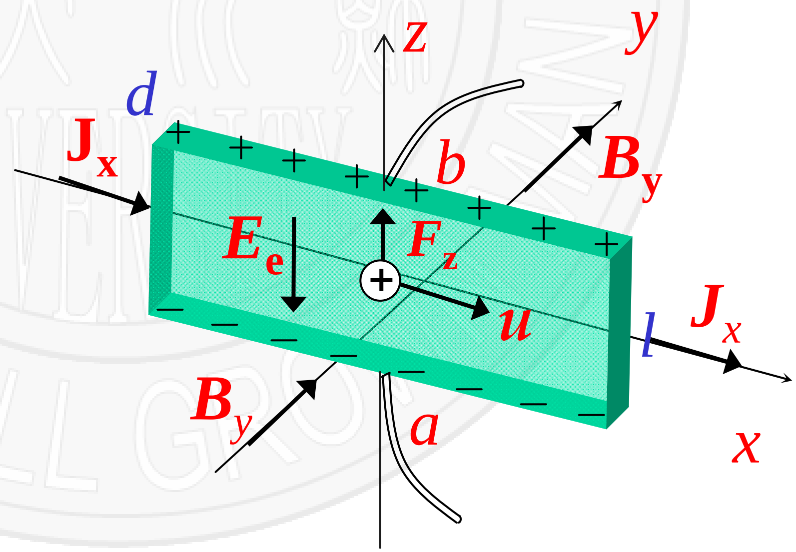
- Charge carriers are positive
- +q accumulated upside, -q appears down by induction
- E points down, the potential of bottom lower, the top higher

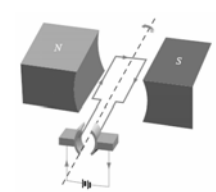
$$qE_z = qu_xB_y$$

$$E_z = uB_y = \frac{V_H}{l}$$

$$I = qunS$$

$$V_H = uBl = \frac{IB}{qnd}$$



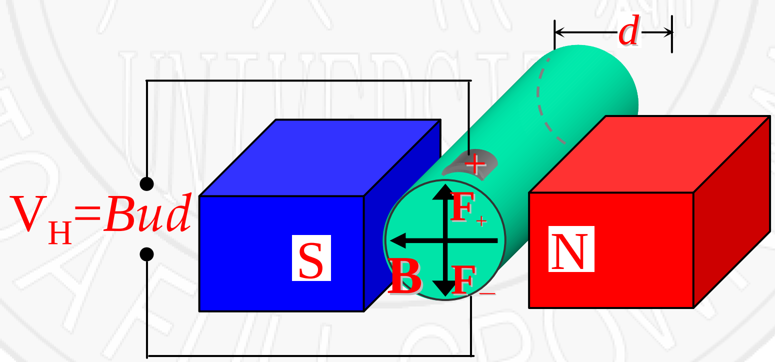


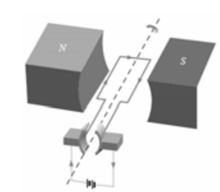
4.4 Magnetic Force on Current

◈ The Hall Effect

In experiment, If V_H, I, B, d, q is known,n can be measured, V_H, I, B, d, n is known,q can be measured, so on.

Another application of the Hall effect is to measure fluid flow in any fluid that has free charges. A magnetic field applied perpendicular to the flow direction produces a Hall Voltage V_H as shown in the figure





4.4 Magnetic Force on Current

◈ The Hall Effect

The magnitude of the Hall Voltage is $V_H = Bud$, where d is fluid diameter, so that the average velocity u can be determined from providing the other factors are known.