Chapter 4 Magnetic Field

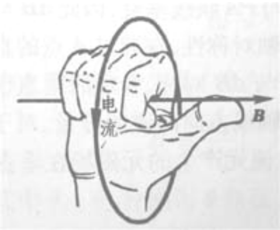
4.1 The Magnetic Field

4.2 The Biot - Savart Law

4.3 The Gauss's Law & Ampere's Circuital Law

4.4 The Magnetic Forces on Current Conductors

4.5 The Motion of Charge in Magnetic Field



4.2 Biot - Savart Law

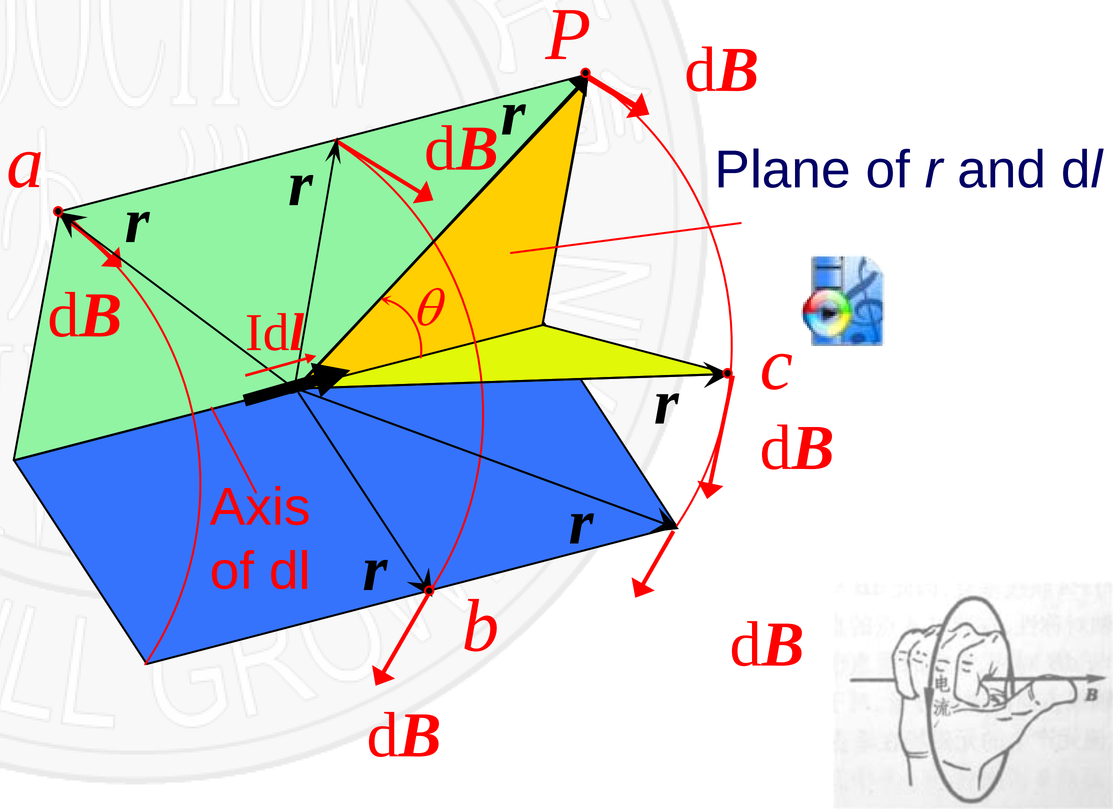
Biot -Savart Law

- Superposition Principle For E.
- We can also compute B by Biot-Savart Law.

$$\mathrm{d}\vec{B} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\vec{l}\times\hat{r}}{r^2}$$

$$\mathrm{d}B = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}l\sin\theta}{r^2}$$

$$\vec{B} = \int \mathrm{d}\vec{B} = \int \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\vec{l}\times\hat{r}}{r^2}$$



4.2 Biot - Savart Law

◈ Biot -Savart Law

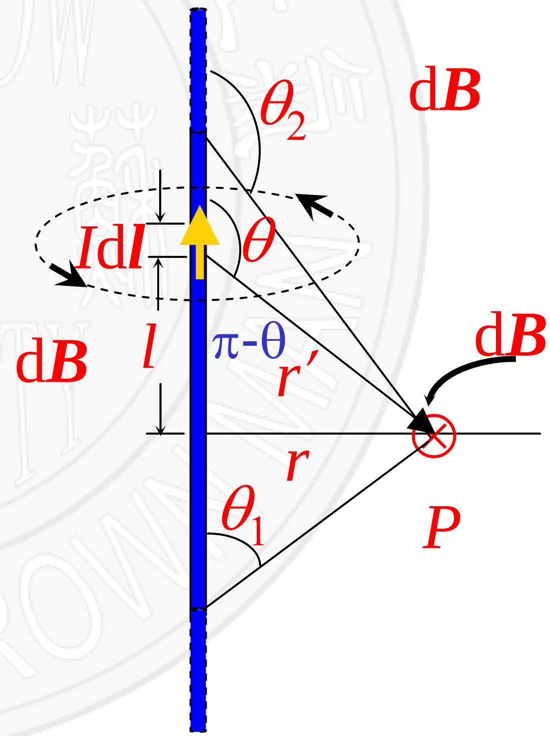
✹ Applications of Biot-Savart Law

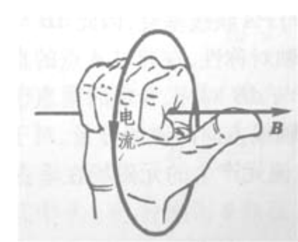
Find the magnetic field of a straight wire with current *I*.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$B = \int dB = \frac{\mu_0 I}{4\pi} \int \frac{\sin\theta dl}{r'^2}$$

$$\sin(\pi - \theta) = r/r'$$





4.2 Biot - Savart Law

Biot -Savart Law

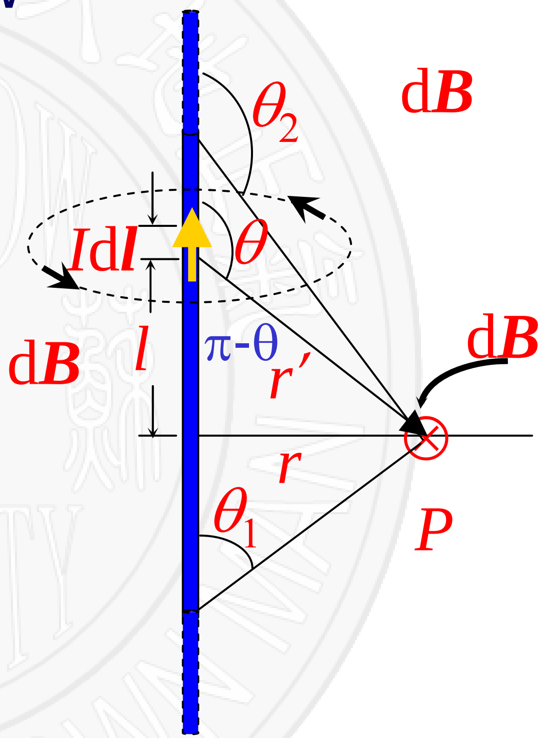
Applications of Biot-Savart Law

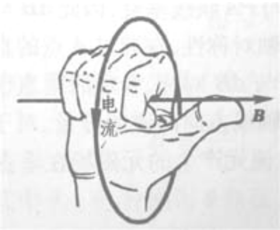
$$r/l = \tan(\pi - \theta)$$

$$\mathrm{d}l = \frac{r\,\mathrm{d}\theta}{\sin^2\theta}$$

$$B = \frac{\mu_0 I}{4\pi r}\int_{\theta_1}^{\theta_2} \sin\theta\,\mathrm{d}\theta$$

$$B = \frac{\mu_0 I}{4\pi r}(\cos\theta_1 - \cos\theta_2)$$





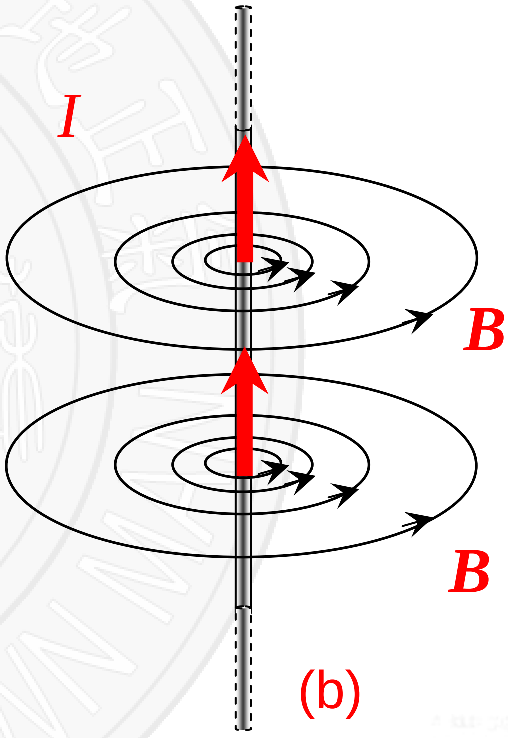
4.2 Biot - Savart Law

◈ Biot -Savart Law

- ✷ The magnetic field of a Infinite current *I*

If the line is infinite, $\theta_1=0, \theta_2=\pi$

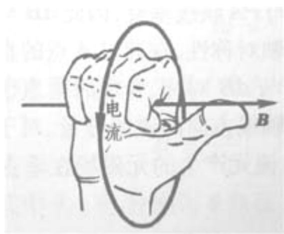
$$B = \frac{\mu_0 I}{2\pi r}$$



If the line is semi-infinite,

$\theta_1=0, \theta_2=\pi/2$ or $\theta_1=,\pi/2, \theta_2= \pi$

$$B = \frac{\mu_0 I}{4\pi r}$$



4.2 Biot - Savart Law

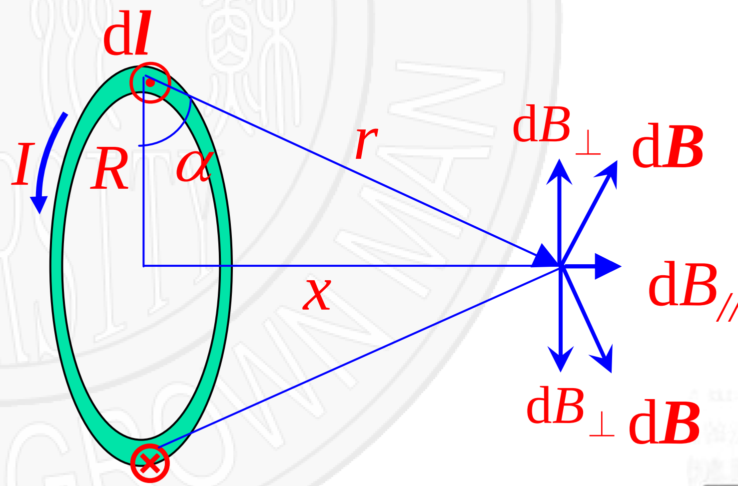
- Applications of Biot-Savart Law

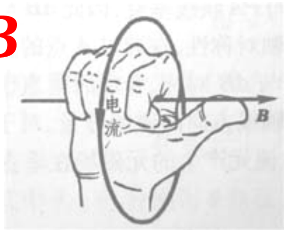
Example 4.3 A circular current loop. The figure below shows a circular loop of radius R carrying a current I. Calculate B for points on the axis.

Solution: According Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

$$dB = \frac{\mu_0 I dl \sin 90^\circ}{4\pi r^2}$$

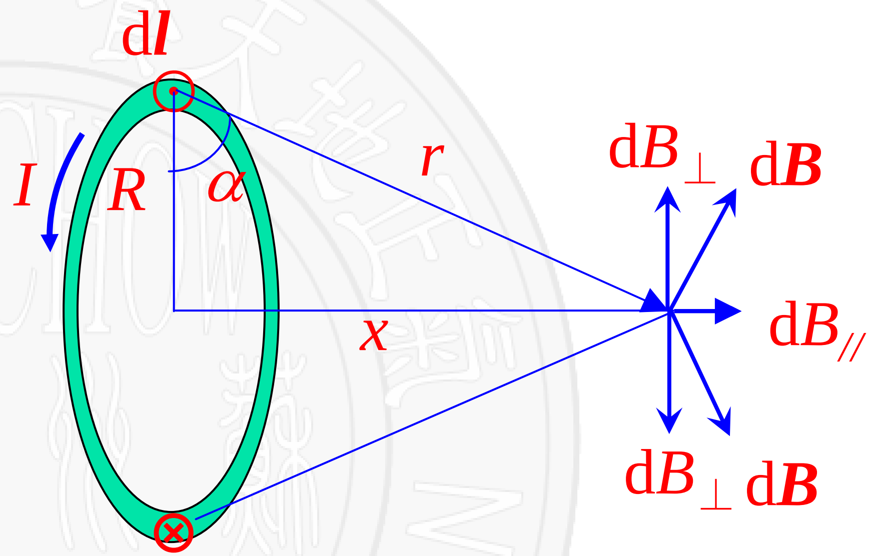




4.2 Biot - Savart Law

- Applications of Biot-Savart Law

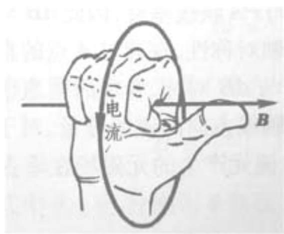
$$dB = \frac{\mu_0 I dl \sin 90^\circ}{4\pi r^2}$$



$$dB_{//} = dB\cos\alpha$$

$$B = \int dB_{//}$$

$$dB_{//} = \frac{\mu_0 I\cos\alpha dl}{4\pi r^2} \quad \begin{cases} r = \sqrt{x^2 + R^2} \\ \cos\alpha = \dfrac{R}{r} = \dfrac{R}{\sqrt{R^2 + x^2}} \end{cases}$$



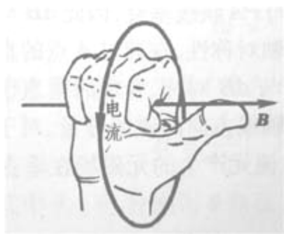
- Applications of Biot-Savart Law

$$\mathrm{d}B_{//} = \frac{\mu_0 I R}{4\pi(R^2 + x^2)^{\frac{3}{2}}}\mathrm{d}l$$

$$\mathrm{B} = \oint \mathrm{d}B_{//} = \oint \frac{\mu_0 I R}{4\pi(R^2 + x^2)^{\frac{3}{2}}}\mathrm{d}l = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$$

If x=0, the center of loop

$$B = \frac{\mu_0 I}{2R}$$



4.2 Biot - Savart Law

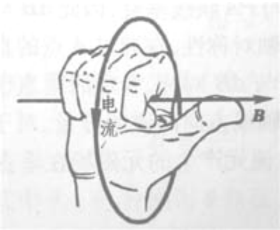
- Applications of Biot-Savart Law

If x>>R

$$B = \frac{\mu_0 I R^2}{2 x^3}$$

$$B = \frac{\mu_0}{2\pi} \frac{(NIS)}{x^3} = \frac{\mu_0}{2\pi} \frac{m}{x^3}$$

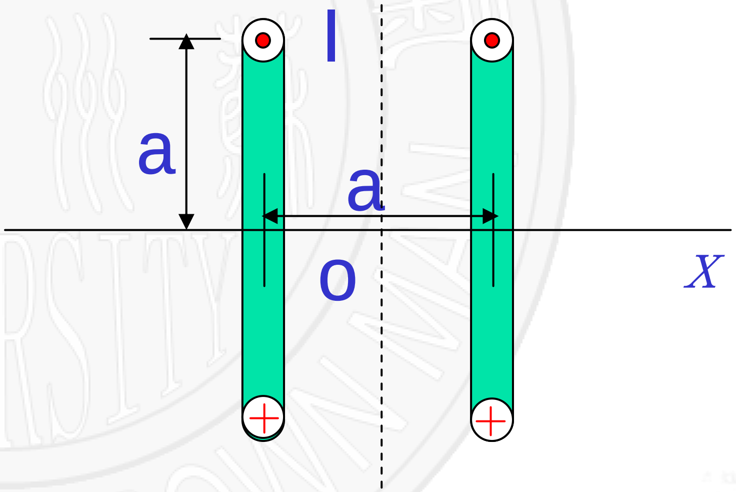
Magnetic dipole

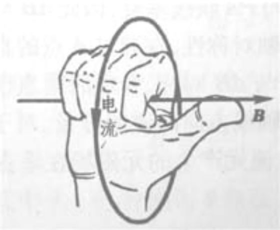


4.2 Biot - Savart Law

- Applications of Biot-Savart Law

Helmholtz coils consisting of two circular conductors, each of radius a, are placed parallel to each other with axes coinciding, a distance d apart, as shown the figure. Each loop carries current I in the same sense of circulation.

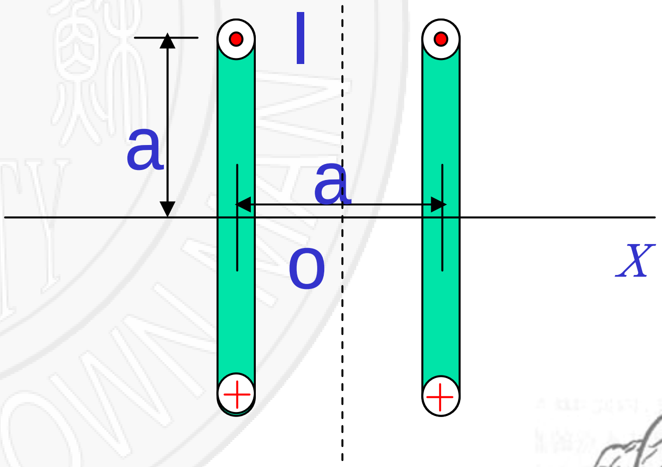


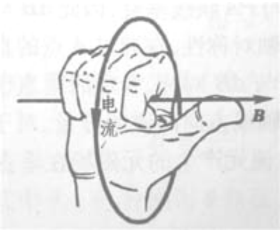


4.2 Biot - Savart Law

Example 4.4 Helmholtz coils： (a) Sketch in the field lines you expect for a cross-sectional plane containing the common axis(the x-y plane). (b) Calculate the magnetic field B on the common axis at the midpoint between the coils. (c) Demonstrate that this field of the Helmholtz coils is relatively uniform at the midpoint by show that

$$\left.\frac{dB_x}{dx}\right|_{x=0} = 0 \qquad \left.\frac{d^2B_x}{dx^2}\right|_{x=0} = 0$$

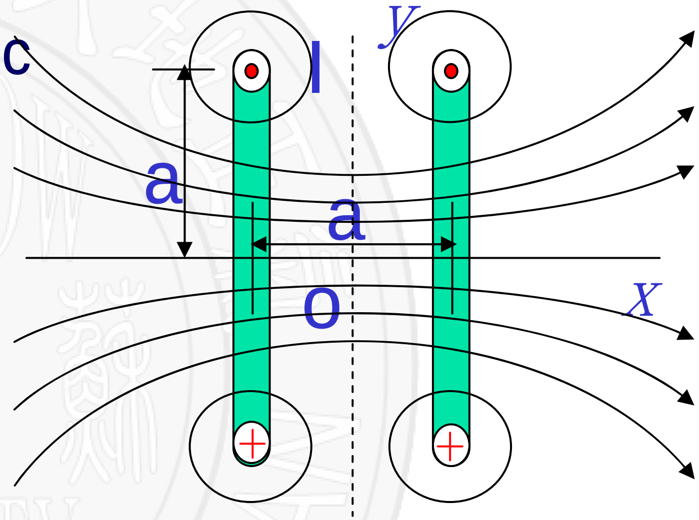




4.2 Biot - Savart Law

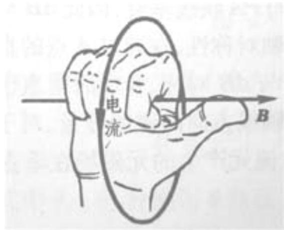
Solution: (a) The lines is shown in the figure

(b)From the equation of magnetic field due to current ring



$$B = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

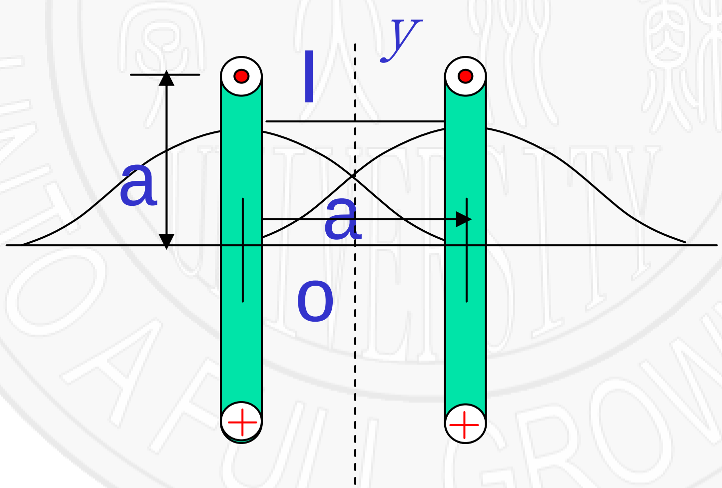
Obviously, B have the max Value at the center. At the region between coils along the axis, the total field is the sum of fields due to the coils.

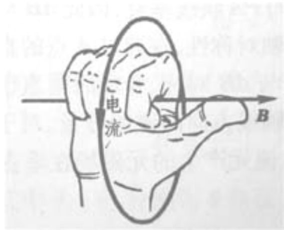


4.2 Biot - Savart Law

Helmholtz coils

$$B=\frac{\mu_0 I a^2}{2(a^2+(x+a/2)^2)^{\frac{3}{2}}}+\frac{\mu_0 I a^2}{2(a^2+(x-a/2)^2)^{\frac{3}{2}}}$$





4.2 Biot - Savart Law

Helmholtz coils

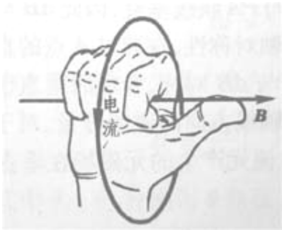
$$\frac{dB}{dx}=\frac{3\mu_0 I a^2}{2}\left[\frac{x+a/2}{(a^2+(x+a/2)^2)^{\frac{5}{2}}}+\frac{x-a/2}{(a^2+(x-a/2)^2)^{\frac{5}{2}}}\right]$$

If x=0, dB/dx=0

$$\frac{d^2B}{dx^2}=\frac{3\mu_0 I a^2}{2}\left[\frac{4(x+a/2)^2-a^2}{(a^2+(x+a/2)^2)^{\frac{7}{2}}}+\frac{4(x-a/2)^2-a^2}{(a^2+(x-a/2)^2)^{\frac{7}{2}}}\right]$$

If x=0, $d^2B/dx^2=0$

So, B is relatively uniform near the midpoint

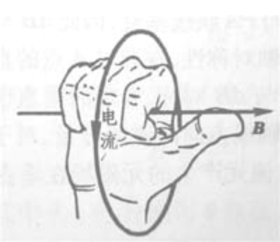


4.2 Biot - Savart Law

Example 4.5 In the Bohr model of the hydrogen atom the electron circulated around the nucleus in a path of radium 5.1×10^{-11}m at a frequency $\nu=6.8\times10^{15}$Hz.(a) What value of B is set up at the center of the orbit; (b) What is the equivalent magnetic dipole moment?

Solution:

(a) The current is the rate at which charge passes any point on the orbit and is given



4.2 Biot - Savart Law

$$I = \frac{q}{T} = e\nu = 1.6\times10^{-19}\times6.8\times10^{15} = 1.1\times10^{-3}(A)$$

$$\mathrm{B} = \frac{\mu_0 \mathrm{I}}{2R} = \frac{4\pi\times10^{-7}\times1.1\times10^{-3}}{2\times5.1\times10^{-11}} = 14\mathrm{T} = 14\, wb/m^2$$

$$m = \mathrm{NI}S = 1\times1.1\times10^{-3}\times\pi\times(5.1\times10^{-11}m)^2$$

$$m = 9.0\times10^{-24}(\mathrm{A}\cdot m^2)$$