Tutorial for chapter 3

- Electric current I:

$$I = \frac{dq}{dt}$$

- Electric current density j

$$j = \frac{\mathrm{d}I}{\mathrm{d}S_{\perp}}$$

- Continuity equation:

$$\oiint_{(S)} \vec{\boldsymbol{j}} \cdot \mathrm{d}\vec{\boldsymbol{S}} = -\frac{dq}{dt}$$

Tutorial for chapter 3

- Steady-state condition:

$$\oiint_{(S)} \vec{j} \cdot \mathrm{d}\vec{S} = 0$$

- Ohm's law:

Macroscopic: $I = \frac{V}{R}$

Microscopic: $\vec{J} = \sigma \vec{E}$

R=resistance $R = \int \rho \frac{\mathrm{d}l}{S}$

Tutorial for chapter 3

- Resistors in series

$$R = R_1 + R_2 + \ldots + R_3$$

- Resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

- The Condition of Maximum Output

$$R = r$$

- Charging and Discharging

$$V = \varepsilon + Ir$$

$$V = \varepsilon - Ir$$

Tutorial for chapter 3

- Kirchhoff 's first Law

Point rule (current law for node or junction theorem) The algebraic sum of the currents toward any node is zero

$$\sum(\pm I) = 0$$

- Kirchhoff 's Second Law

Loop rule(voltage law) The algebraic sum of the voltage drop in any loop, including those associated with emfs and those of resistive elements, must equal zero:

$$\sum(\pm\varepsilon)+\sum(\pm IR)=0.$$

Tutorial for chapter 3

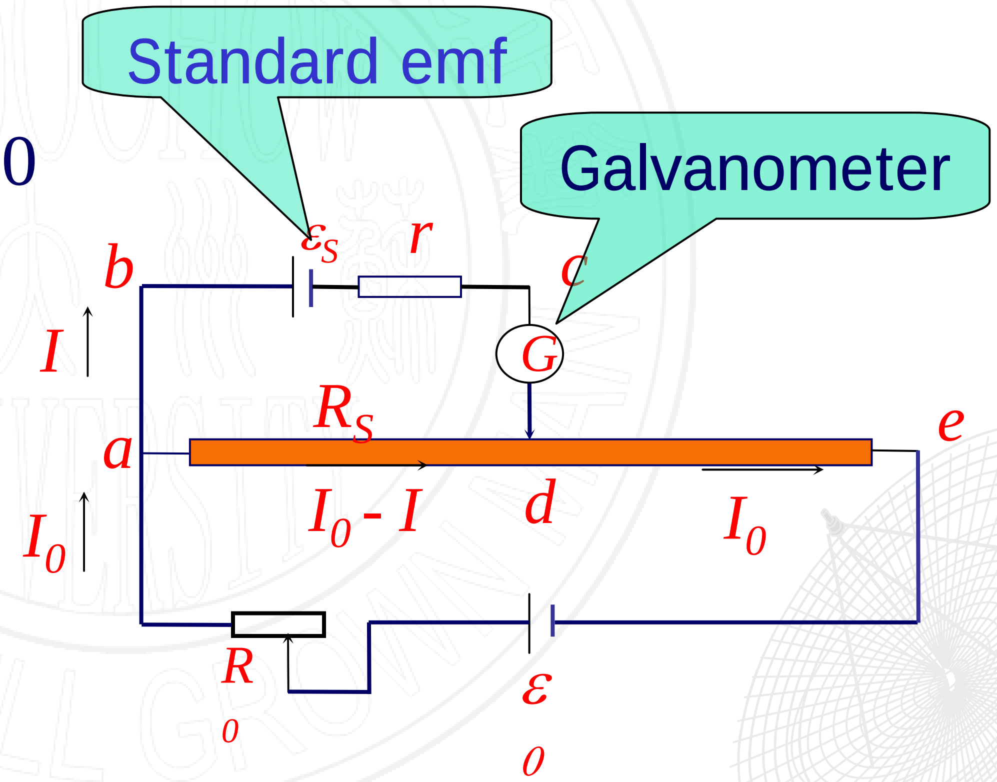
Example 3.4. The potentiometer The Figure shows the rudiments of a potentiometer, which is a device for measuring an unknown emf ε_x. The currents and emfs are marked as shown. Applying the loop theorem to loop *abed* yields

$$\varepsilon_S + I(r + R_G) - (I_0 - I)R_S = 0$$

$$I = \frac{I_0 R_S - \varepsilon_S}{R_S + r + R_G}$$

Adjust R_0 to make $I=0$

$I_0 R_S = \varepsilon_S$



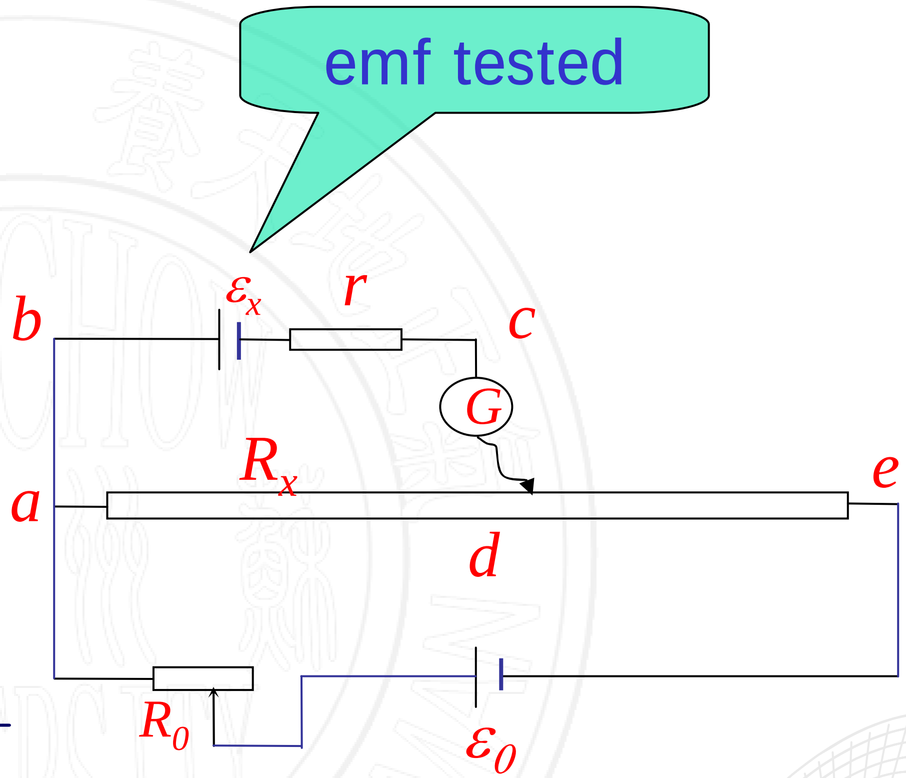
Tutorial for chapter 3

$$\varepsilon_x + I(r + R_G) - (I_0 - I)R_x = 0$$

$$I = \frac{I_0 R_x - \varepsilon_x}{R + r + R_G}$$

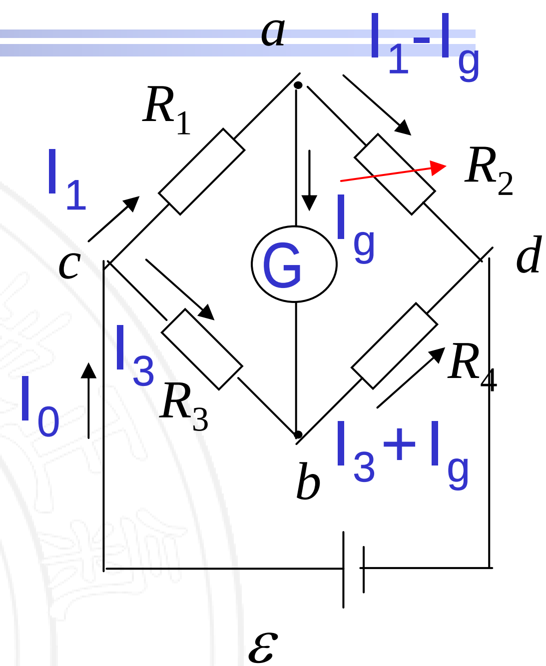
Adjust R_x to make I=0

$$\left.\begin{aligned} I_0 R_S &= \varepsilon_S \\ I_0 R_x &= \varepsilon_x \end{aligned}\right\} \quad \varepsilon_x = \varepsilon_s \frac{R_x}{R_s}$$



Tutorial for chapter 3

The Wheatstone bridge. In figure on the right , R_2 is to be adjusted in value until points *a* and *b* are brought to exactly the same potential. (One tests for this these points are at the same potential, the meter will not deflect——balanced.) Show that when this adjustment is made, the following relation holds:



$$R_2R_3=R_1R_4$$

Solution: According to Kirchhoff's Law, We get

$$I_0=I_1+I_3 \qquad I_1R_1+I_gR_g-I_3R_3=0$$

$$I_3R_3+(I_3+I_g)R_4-\varepsilon=0 \qquad (I_1-I_g)R_2-(I_3+I_g)R_4-I_gR_g=0$$

Tutorial for chapter 3

$$I_g = \frac{(R_2R_3 - R_1R_4)R_3\varepsilon}{R_1(R_3+R_4)(R_gR_4+R_2R_3+R_3R_4+R_3R_g)+(R_2R_3-R_1R_4)(R_3R_4+R_gR_3+R_gR_4)}$$

If $(R_2R_3-R_1R_4)=0$,then, $I_g=0$,No current passes through the Galvanometer, the bridge is in the balance.

The condition for balance of bridge: $R_2R_3=R_1R_4$

When a bridge is in the balance, The equivalent resistance between c and d can be calculated as follow

$$R_{cd} = \frac{(R_1+R_2)(R_3+R_4)}{(R_1+R_2+R_3+R_4)}$$ $(R_1+R_2)//(R_3+R_4)$ Cut circuit

|| ?

or $$R_{cd} = \frac{R_1R_3}{R_1+R_3} + \frac{R_2R_4}{R_2+R_4}$$ $(R_1//R_3)+(R_2//R_4)$ short circuit

Tutorial for chapter 3

3.2.13 When a metal rod is heated, not only its resistance but also its length and its cross-sectional area change. The relation $R = \rho L/A$ suggests that all three factors should be taken into account in measuring ρ at various temperatures. (a) If the temperature changes by 1.0C°, what percent changes in R, L, and A occur for a copper conductor? (b) What conclusion do you draw? The coefficient of linear expansion is $\gamma=1.7\times10^{-5}$C°.

Solution: From $\rho=\rho_0(1+\alpha\Delta t)$

$$\frac{\rho-\rho_0}{\rho_0}=\alpha\Delta t=3.9\times10^{-3}\times1=0.39\%$$

From $l=l_0(1+\gamma\Delta t)$

$$\frac{l-l_0}{l_0}=\gamma\Delta t=1.7\times10^{-5}\times1=0.0017\%$$

Tutorial for chapter 3

$$\frac{A - A_0}{A_0} = 2\gamma\Delta t = 2\times 1.7\times 10^{-5}\times 1 = 0.0034\%$$

$$R = \rho\frac{l}{A}$$

The logarithm of the equation above is

$$\ln R = \ln\rho + \ln l - \ln A$$

Differential to the equation is

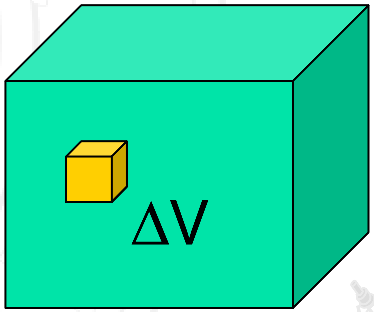
$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A} = 0.39\% + 0.0017\% - 0.0034\% = 0.3883\%$$

So the change of ρ with temperature is mainly part.

Tutorial for chapter 3

3.2.19.(a) Derive the formulas $p = j^2\rho$ and $p = E^2/\rho$ where p = power per unit volume in a resistor, (b) A cylindrical resistor of radius 0.50cm and length 2.0cm has a resistivity of $3.5\times10^{-5}\Omega$.m. What are the current density and potential difference when the power dissipation is 1.0W?

Solution: Consider a element volume ΔV



$P=i^2R$ $\quad$ $i=j\Delta S, R=\rho\Delta l/\Delta S$

$P=i^2R=j^2\,\Delta S^2(\rho\Delta l/\Delta S)$

$= j^2\rho\,\Delta l\Delta S= j^2\rho\,\Delta V$

$p= P/\,\Delta V= j^2\rho$

$p= j^2\rho= \rho E^2/\,\rho^2= E^2/\rho$

Tutorial for chapter 3

$$R = \rho \frac{l}{A} = \frac{3.5\times10^{-5}\times2\times10^{-2}}{3.14\times(0.5\times10^{-2})^2} = 8.9\times10^{-3}\Omega$$

$P=i^2R$

$$i = \sqrt{\frac{P}{R}} = \sqrt{\frac{1}{8.9\times10^{-3}}} = 1.06A$$

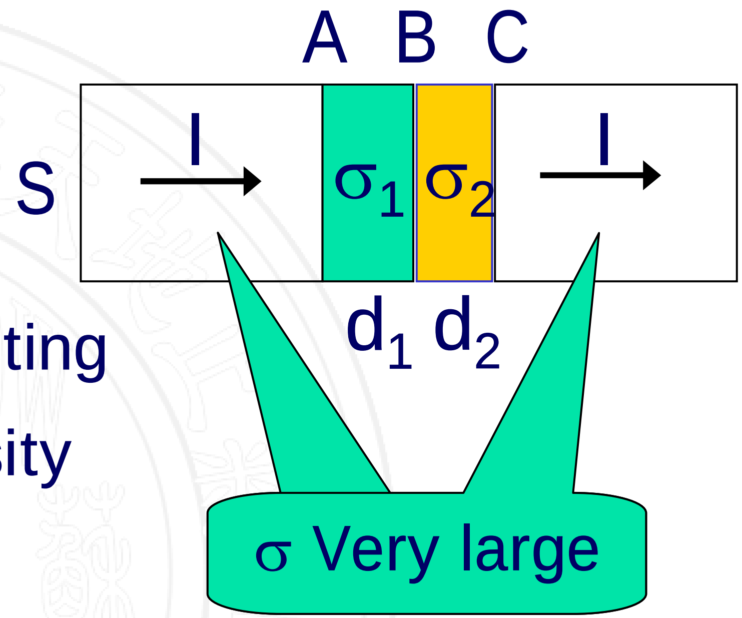
$$j = \frac{i}{S} = \frac{1.06}{3.14\times(0.5\times10^{-2})^2} = 1.35\times10^4 A/m^2$$

$V=iR=1.06\times8.9\times10^{-3}=9.434\times10^{-3}V$

Tutorial for chapter 3

Find (a) the electric fields in the materials 1 and 2 ,$E_1$and E_2

(b) Potential differences V_{AB},V_{BC}



Solution: The materials are conducting

From the definition of current density

$$j = \frac{I}{S}$$

According to differential form of Ohm's Law: j=σE, we can get

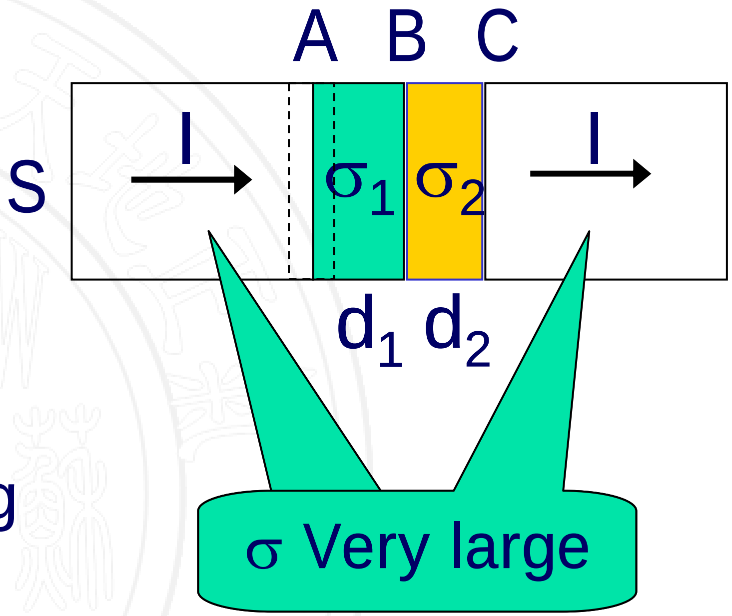
$$E_1 = \frac{j}{\sigma_1} = \frac{I}{\sigma_1 S} \qquad E_2 = \frac{I}{\sigma_2 S}$$

$$V_{AB} = E_1 d_1 = \frac{Id_1}{\sigma_1 S} \qquad V_{BC} = E_2 d_2 = \frac{Id_2}{\sigma_2 S}$$

Tutorial for chapter 3

(c)Find the charges on the surfaces A,B,C.

Because σ is very large, the electric fields in the conductors, right and left, are zero



$$E = \frac{j}{\sigma}$$

Choose a Gaussian surface enclosing surface A

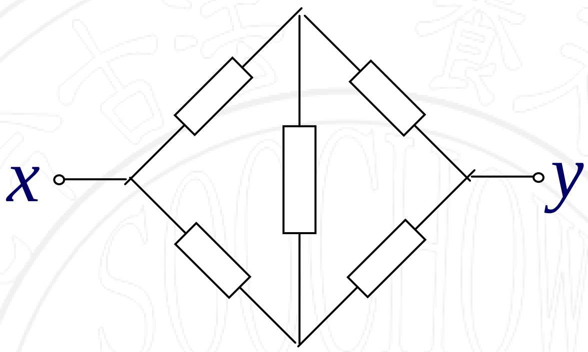
$$\oiint_{(S)} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E_1 S = \frac{I}{\sigma_1} = q_A / \varepsilon_0$$

$$q_A = \frac{\varepsilon_0 I}{\sigma_1} \qquad q_C = -\frac{\varepsilon_0 I}{\sigma_2} \qquad q_B = \frac{\varepsilon_0 I}{\sigma_2} - \frac{\varepsilon_0 I}{\sigma_1}$$

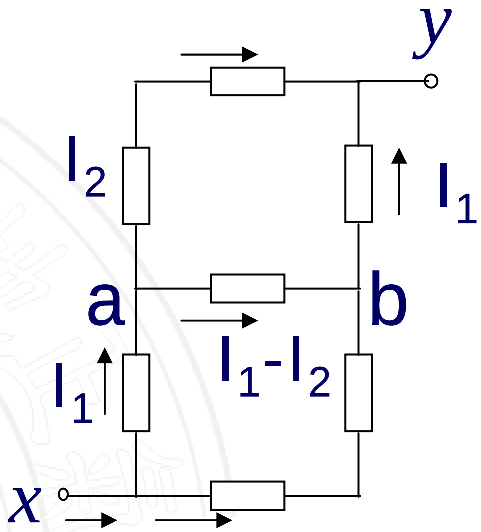
Tutorial for chapter 3

$$R_a = \frac{R}{2} + \frac{R}{2} = R$$

$$R_a = 2R // 2R = R$$



(a)



(b)

$V_{xy} = I_1R + I_2(2R)$

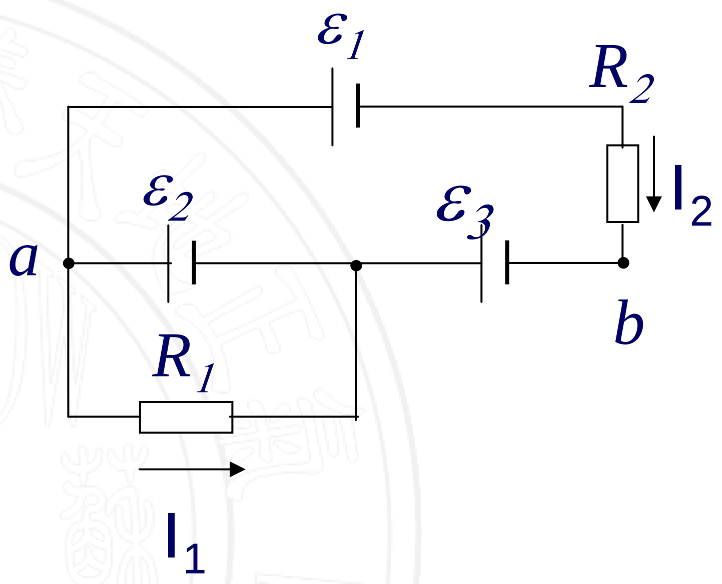
$I = I_1 + I_2$

$$R_{xy} = \frac{V_{xy}}{I} = \frac{I_1R + 2I_2R}{I_1 + I_2} = \frac{R + 2R\frac{I_2}{I_1}}{1 + \frac{I_2}{I_1}} = \frac{R + 2R \times \frac{2}{3}}{1 + \frac{2}{3}} = \frac{7}{5}R$$

Loop xabx: $I_1R + (I_1 - I_2)R - 2I_2R = 0$ $\quad$ $I_2/I_1 = 2/3$

Tutorial for chapter 3

Find the current in each resistor and the potential difference between *a* and *b*. Put ε_1 = 6.0V, ε_2 = 5.0V, ε_3 = 4.0V, R_1 = 100Ω, and R_2 =50 Ω.



Solution:

$V_{ab}=\varepsilon_2+\varepsilon_3=9V$

$I_1R_1-\varepsilon_2=0$

$I_2R_2+\varepsilon_1-\varepsilon_2-\varepsilon_3=0$

$I_1=\varepsilon_2/R_1=50mA$

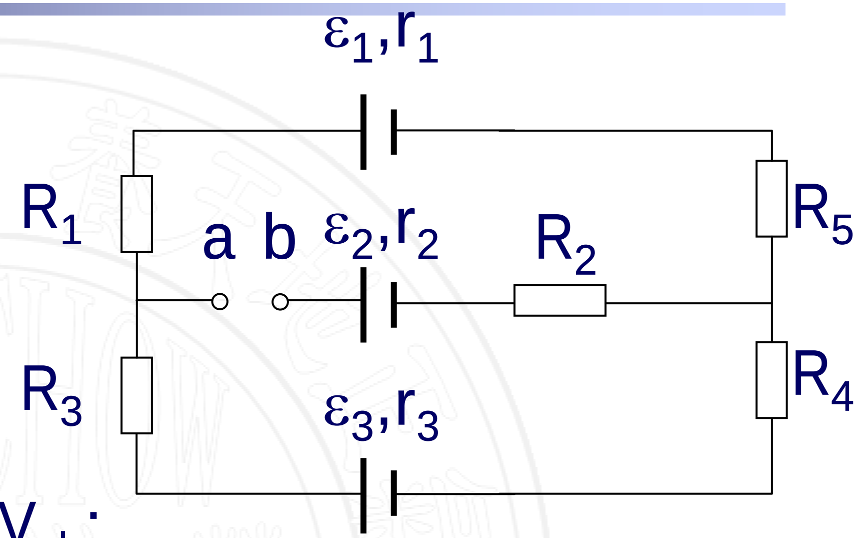
$I_2=(\varepsilon_2+\varepsilon_3-\varepsilon_1)/R_2=60mA$

Tutorial for chapter 3

ε_1=12V, ε_2=9V, ε_3= 8V

$r_1=r_2=r_3=1\Omega$ $R_2=3\Omega$

$R_1=R_3=R_4=R_5=2\Omega$



(a)ab is not connected ,find V_{ab};

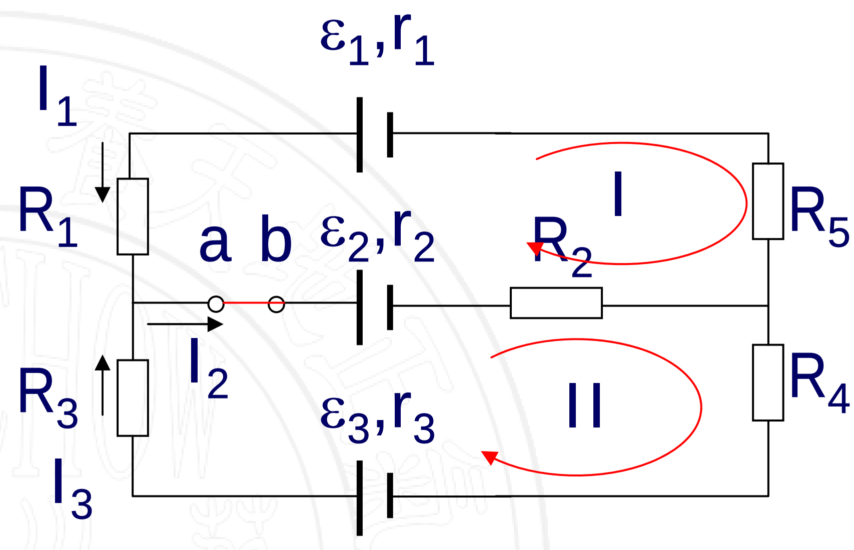
(b)ab is connected ,find V_{ab} and the current flowing R_2

Solution:

$\varepsilon_3-\varepsilon_1+I(R_1+R_3+R_4+R_5+r_1+r_2)=0$

$$I=\frac{\varepsilon_1-\varepsilon_3}{(R_1+R_2+R_4+R_5+r_1+r_3)}=0.4A$$

Tutorial for chapter 3



$V_{ab}=\varepsilon_3-\varepsilon_2+I(R_3+R_4+r_3)$

$=8-9+0.4\times5=1V$

(b) When ab is connected, from Kirchhoff's Law

$I_1+I_3=I_2$

I $\quad \varepsilon_1-\varepsilon_2-I_1(r_1+R_1+R_5)-I_2(r_2+R_2)=0$

II $\quad \varepsilon_2-\varepsilon_3+I_3(r_3+R_3+R_4)+I_2(r_2+R_2)=0$