Tutorial for chapter 1

Summery for Chapter One

- Coulomb's Law

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^{\ 2}}\hat{\boldsymbol{r}}_{12}$$

- Superposition Principle

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} \cdots \vec{F}_{i1}$$

- Electric Field Intensity

$$\vec{E} = \frac{\vec{F}}{q_0}$$

- The Calculation of Electric Field Intensity E

Tutorial for chapter 1

Summery for Chapter One

The electric field of a point charge

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$$

- Superposition Principle

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \cdots + \vec{E}_n$$

- The electric field of a group of point charges

$$\vec{E} = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r^2}\hat{\boldsymbol{r}}$$

- The electric field of continuous charges

$$\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\boldsymbol{r}}$$

Tutorial for chapter 1

Summery for Chapter One

- The Calculation of Electric Field Intensity E

•The Method of Gauss's Law

$$\oiint_{(S)} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{S}} = \frac{1}{\varepsilon_0} \sum_{(S内)} q_i$$

•The Method of Gradient of V

$$\vec{\boldsymbol{E}} = -\nabla V = -\left(\frac{\partial V}{\partial x}\vec{\boldsymbol{i}} + \frac{\partial V}{\partial y}\vec{\boldsymbol{j}} + \frac{\partial V}{\partial z}\vec{\boldsymbol{k}}\right)$$

$$\vec{\boldsymbol{E}} = -\nabla V = -\left(\frac{\partial V}{\partial r}\overrightarrow{\boldsymbol{e}_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\overrightarrow{\boldsymbol{e}_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\overrightarrow{\boldsymbol{e}_\phi}\right)$$

$$\vec{\boldsymbol{E}} = -\nabla V = -\left(\frac{\partial V}{\partial r}\overrightarrow{\boldsymbol{e}_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\overrightarrow{\boldsymbol{e}_\theta} + \frac{\partial V}{\partial z}\overrightarrow{\boldsymbol{e}_z}\right)$$

Tutorial for chapter 1

Summery for Chapter One

- Electric Potential

Definition $$V_P = \int_P^R \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$$

- The Calculation of Electric Potential

•The Method by Definition

$$V_P = \int_P^R \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$$

• Superposition Principle

$$\boldsymbol{V} = \boldsymbol{V}_1 + \boldsymbol{V}_2 \cdots + \boldsymbol{V}_n$$

Tutorial for chapter 1

Summery for Chapter One

- The Calculation of Electric Potential
  - The electric potential for a point charge

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

  - a group of point charges

$$V_P = \sum_{i=1}^{N}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

  - continuous distribution of charges

$$V_P = \int\frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

Tutorial for chapter 1

Summery for Chapter One

- Electric Potential Difference

$$V_{AB} = V_A - V_B = \int_A^B \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$$

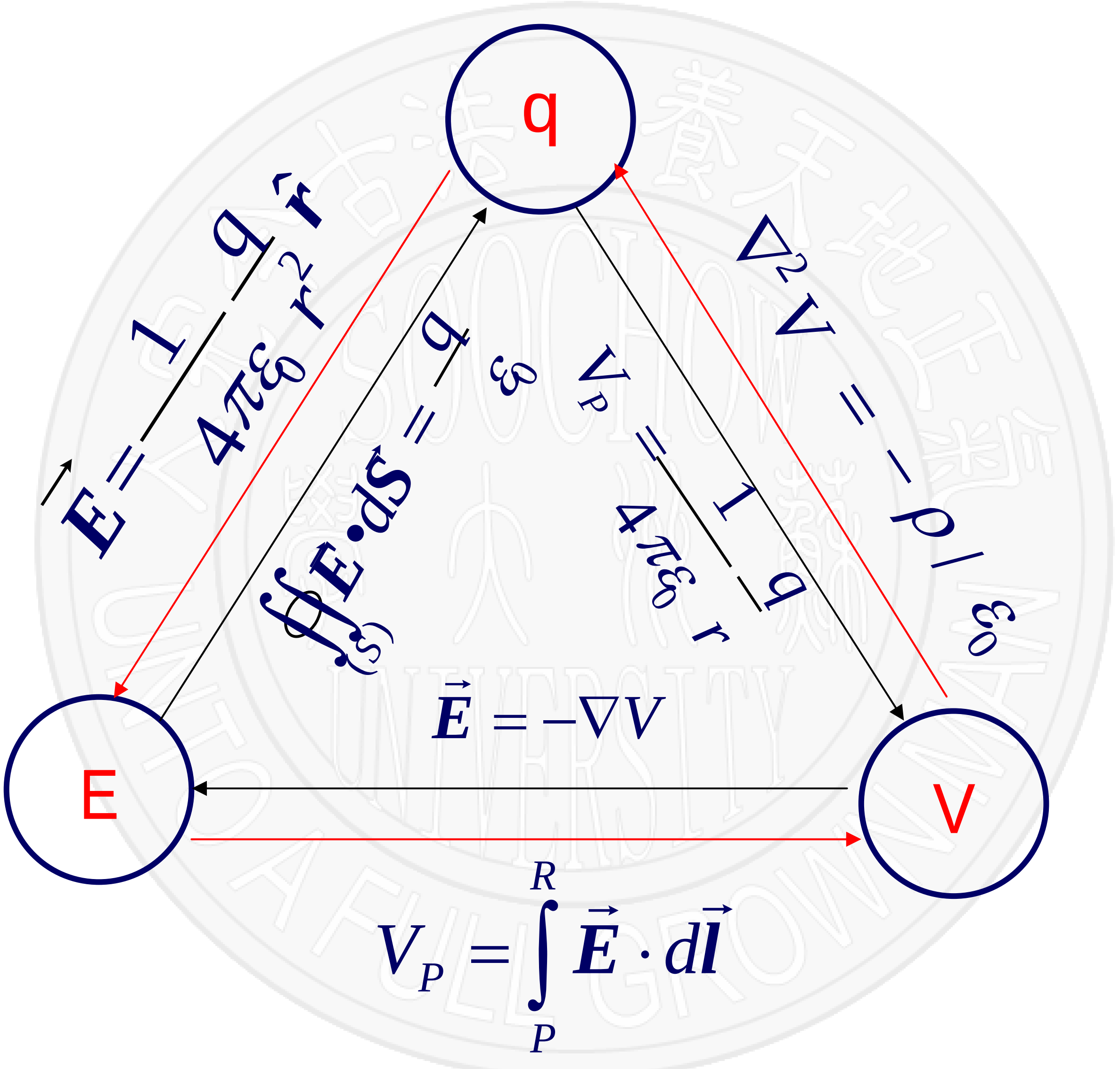
- Energy of a system of charges

$$W = \frac{1}{2} \sum_{(\mathrm{i=1,j=1,i\neq j})}^{\mathrm{n}} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$

- Electric Potential Energy

$$W = q_0 V_P$$

Tutorial for chapter 1

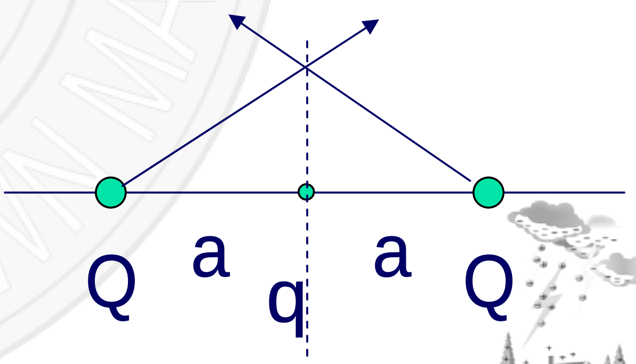


Tutorial for chapter 1

1.1.7.Two equal positive charges, Q, are fixed at a distance $2a$ apart. The force on a small positive test charge q midway between them is zero. If the test charge is displaced a short distance either (a) toward one of the charges or (b) at right angles to the line joining the charges, find the direction of the force on it. Is the equilibrium stable or unstable in each case?

Solution: (a) stable; (b)Unstable.

If test charge is negative (a)Unstable
(b)stable



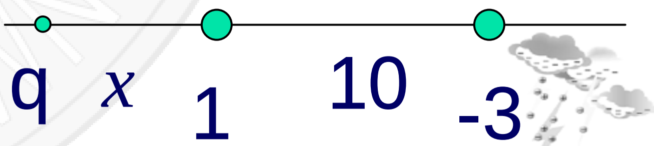
Tutorial for chapter 1

1.1.2. Two fixed charges, $+1.0\times10^{-6}$ C and -3.0×10^{-6} C, are 10 cm apart, (a) Where may a third charge be located so that no force acts on it? (b) Is the equilibrium of third charge stable or unstable?

$$F_1=\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} \qquad F_2=\frac{1}{4\pi\varepsilon_0}\frac{-3q}{(x+0.1)^2}$$

$F_1=F_2$ $\qquad x=0.137\text{cm}$

$$V_P=\frac{1}{4\pi\varepsilon_0}\frac{q}{x}-\frac{1}{4\pi\varepsilon_0}\frac{3q}{(x+0.1)}$$



Tutorial for chapter 1

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{x} - \frac{1}{4\pi\varepsilon_0}\frac{3q}{(x+0.1)}$$

$$\frac{dV_P}{dx} = 0 \qquad X = 0.137\text{cm}$$

$$\frac{dV_P^2}{dx^2}\bigg|_{x=0.137} > 0 \quad \text{Min potential}$$

q>0,stable;q<0,unstable

$$\frac{dV_P^2}{dx^2}\bigg|_{x=0.137} < 0 \quad \text{Max potential}$$

q>0, unstable;q<0, stable

Tutorial for chapter 1

A certain charge *Q* is to be divided into two parts, *q* and *Q*— *q*. What is the relationship of *Q* to *q* if the two parts, placed a given distance apart, are to have a maximum Coulomb repulsion?

Solution:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{x^2}$$

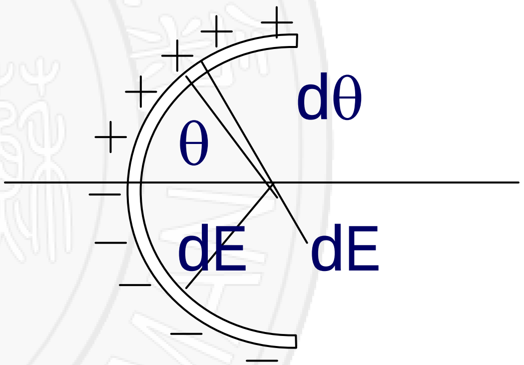
$$\frac{\mathrm{d}F}{dq} = \frac{1}{4\pi\varepsilon_0}\frac{Q-2q}{x^2} = 0$$

q=Q/2

Tutorial for chapter 1

1.2.16.A thin glass rod is bent into a semicircle of radius *R*. A charge +*Q* is uniformly distributed along the upper half and charge –*Q* is uniformly distributed along the lower half, as shown in Fig.1.27.Compute the electric field *E* at *P*, the center of the semicircle.

Solution:



$$\mathrm{dE} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathrm{R}^2}$$

$$\mathrm{dE_y} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\mathrm{d}\theta}{\mathrm{R}^2}\sin\theta$$

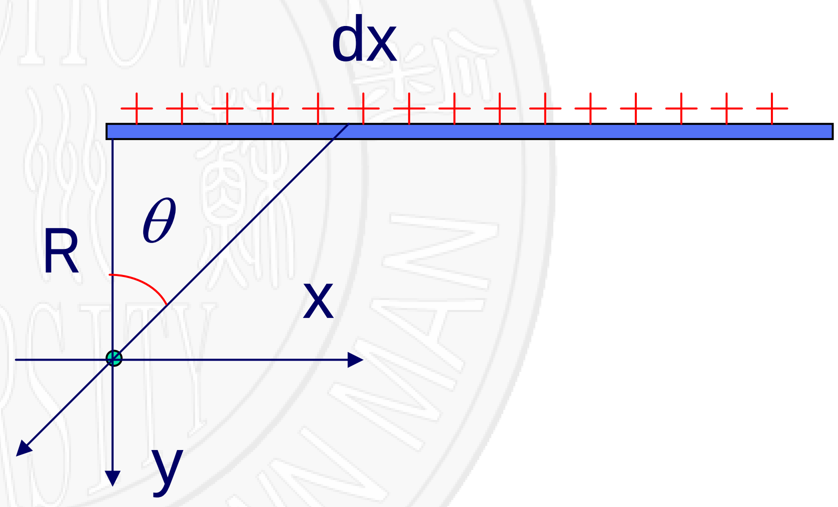
$$\mathrm{E_y} = 2\int_0^{\pi/2}\frac{1}{4\pi\varepsilon_0}\frac{\lambda R\mathrm{d}\theta}{\mathrm{R}^2}\sin\theta = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\mathrm{R}} = \frac{1}{\pi^2\varepsilon_0}\frac{Q}{\mathrm{R}^2}$$

Tutorial for chapter 1

A "semi-infinite" insulating rod (Fig.1.28) carries a constant charge per unit length of λ. Show that the electric field at the point *P* makes an angle of 45° with the rod and that this result is independent of the distance *R*.

Solution:

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{R^2 + x^2}$$

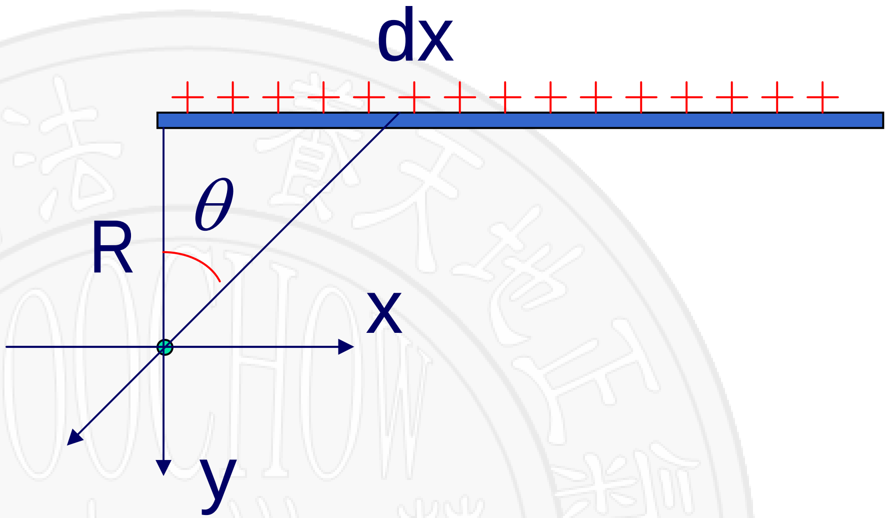


$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{R^2 + x^2}\sin\theta$$

$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{R^2 + x^2}\cos\theta$$

Tutorial for chapter 1

$$\tan\theta = \frac{x}{R}$$



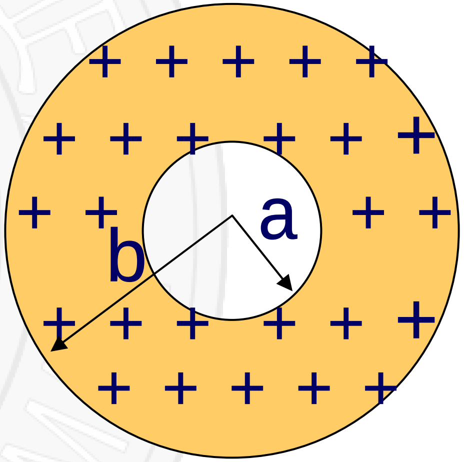
$$dx = \frac{R\,d\theta}{\cos^2\theta}$$

$$E_x = -\int_0^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{R^2 + x^2} \sin\theta = -\frac{\lambda}{4\pi\varepsilon_0 R} \int_0^{\pi/2} \sin\theta d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

$$E_y = \int_0^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{R^2 + x^2} \cos\theta = \frac{\lambda}{4\pi\varepsilon_0 R} \int_0^{\pi/2} \cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}$$

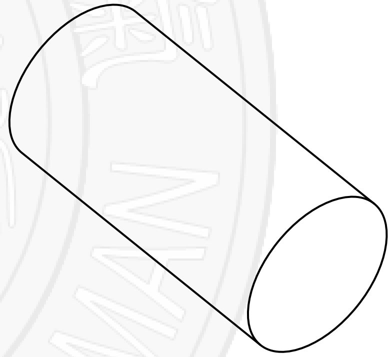
Tutorial for chapter 1

1.3.9.Figure 1.44 shows a spherical nonconducting shell of charge of uniform density ρ (C/m^3). Find the distribution of the electric field from r =0 to $r \to \infty$.



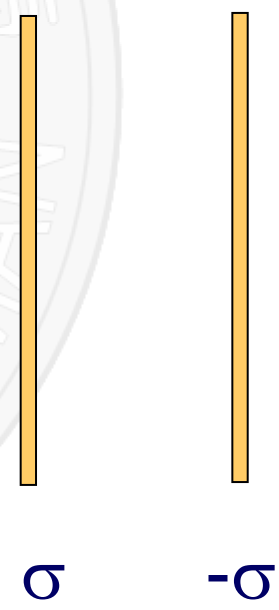
Tutorial for chapter 1

1.3.13. Figure 1.47 shows a section through a long, thin-walled metal tube of radius R, carrying a charge per unit length λ on its surface. Derive expressions for E for various distances r from the tube axis, considering both (a) $r > R$ and (b) $r < R$.



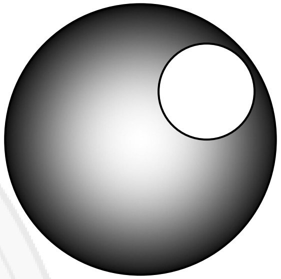
Tutorial for chapter 1

1.49 and carry charges with surface charge density $+\sigma$ and $-\sigma$, respectively, What is *E* at points (a) to the left of the sheets, (b) between them, and (c) to the right of the sheets, consider only points not near the edges whose distance from the sheets is small compared to the dimensions of the sheet.



Tutorial for chapter 1

A solid nonconducting sphere carries a uniform charge per unit volume ρ. Let $\boldsymbol{r}$ be the vector from the center of the sphere to a general point P within the sphere. (a) Show that the electric field at P is given by $\boldsymbol{E} = \rho \boldsymbol{r}/3\varepsilon_0$. (b)A spherical cavity is created in the above sphere, as shown in Fig.1.46.Using surperposition concepts show that the electric field at all points with the cavity is $\boldsymbol{E} = \rho \boldsymbol{a}/3\varepsilon_0$ (uniform field), where $\boldsymbol{a}$ is the vector connecting the center of the sphere with the center of the cavity. Note that both these results are independent of the radii of the sphere and the cavity.



Tutorial for chapter 1

1.3.19.Show that stable equilibrium under the action of electrostatic forces alone is impossible. (Hint: Assume that at a certain point P in an $\boldsymbol{E}$ field a charge $+q$ would be in stable equilibrium if it were placed there which it is not.
Draw a spherical Caussian surface about P, imagine how $\boldsymbol{E}$ must point on this surface, and apply Gauss's law.)

Tutorial for chapter 1

1.3.18 An infinite nonconducting plane slab of thickness d has a uniform volume charge density r. Find the magnitude of the electric field at all points in space both (a) inside and (b) outside the slab.

Tutorial for chapter 1

1.4.2 A charge q is distributed uniformly throughout a nonconducting spherical
volume of radius R. (a) Show that the potential a distance r from the center, where $r < R$, is given by

$$V = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_0 R^3}$$

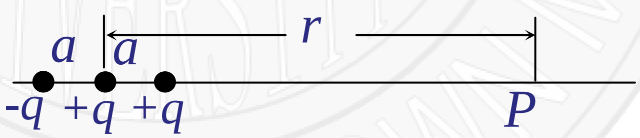
(b) is it reasonable that, according to this expression, V is not zero at the center of the sphere?

Tutorial for chapter 1

1.4.14.For the charge configuration of Fig.1.61, show that $V(r)$ for points on the horizontal axis, assuming $r \gg a$ is given

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2}\right)$$

Is this an expected result? (Hint: The charge configuration can be viewed as the sum of an isolated charge and a dipole.).



Tutorial for chapter 1

In the region without charge, if the lines of induction ***E*** are parallel, is the electric field uniform?

Tutorial for chapter 1