Chapter 1 Electrostatic Field

1.1 Charge and Matter

1.2 Electric Field and Intensity

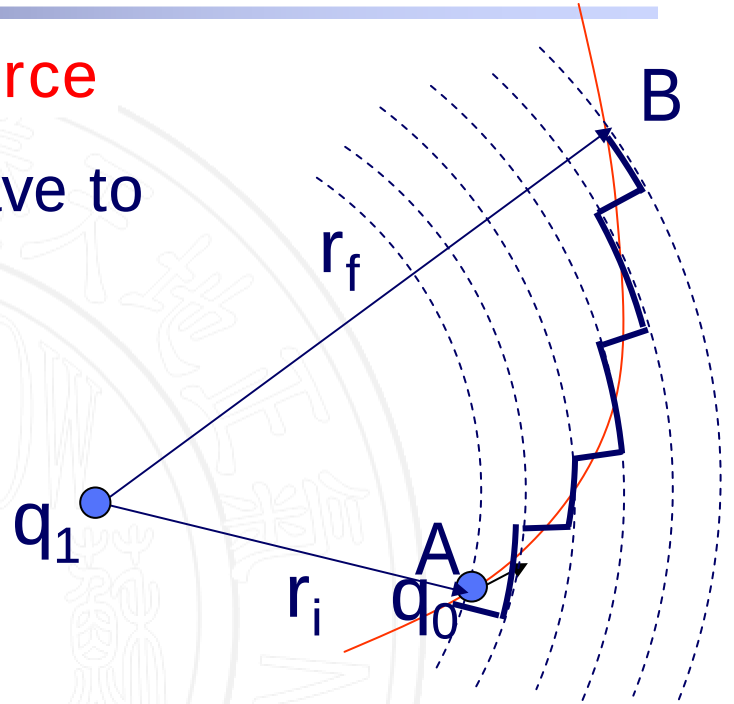
1.3 The Gauss's Law For **E**

1.4 Electric Potential

1.4 Electric Potential

◈ The Work Due to Electric Force

- How much work does $F_{coulomb}$ have to do to move q_0 from r_i to r_f



$$dA = \vec{F}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r^2}\hat{r}\cdot \mathrm{d}\vec{l}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r^2}\mathrm{d}l\cos\theta$$

$$A = \int_{r_i}^{r_f}\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r^2}dl\cos\theta = \int_{r_i}^{r_f}\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r^2}dr$$

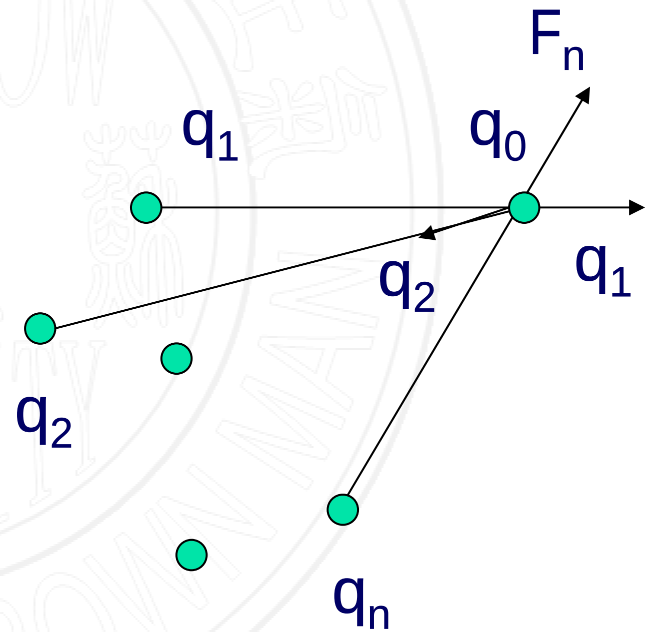
$$= \frac{q_1 q_0}{4\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{1}{r^2}dr = \frac{q_1 q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_i}-\frac{1}{r_f}\right)$$

1.4 Electric Potential

◈ The Work Due to Electric Force

- *The work done by the force action on q_0 due a point charge q is path independent, and only depends on the initial and final position.*

- If we have a group of charges, q_1, q_2, … ,q_n how much work does F_c have to do ?



- Superposition Principle

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$$

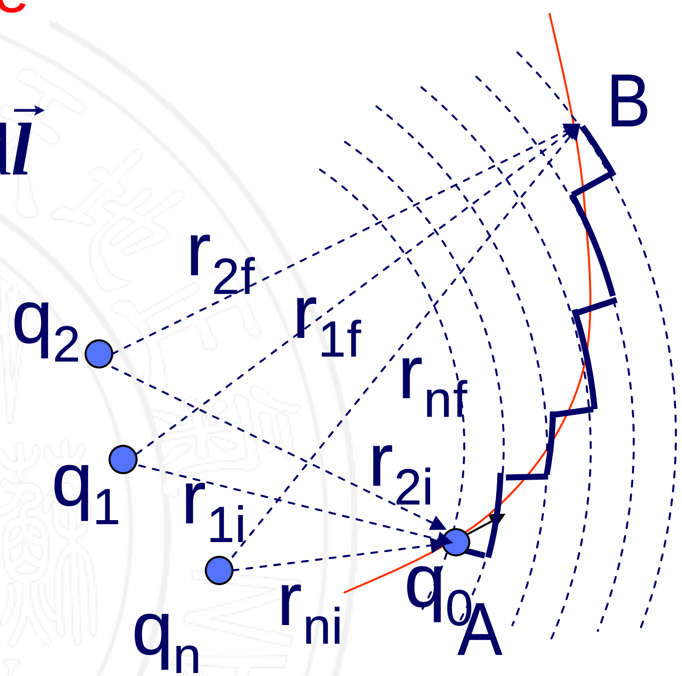
1.4 Electric Potential

◈ The Work Due to Electric Force

$$dA = \vec{F}\cdot d\vec{l} = (\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n)\cdot d\vec{l}$$

$$= \int_{r_{1i}}^{r_{1f}} \vec{F}_1 \cdot d\vec{l} + \int_{r_{2i}}^{r_{2f}} \vec{F}_2 \cdot d\vec{l}$$

$$+ \cdots + \int_{r_{ni}}^{r_{nf}} \vec{F}_n \cdot d\vec{l}$$



$$= \frac{q_1 q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_{1i}} - \frac{1}{r_{1f}}\right) + \frac{q_2 q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_{2i}} - \frac{1}{r_{2f}}\right) + \frac{q_n q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_{ni}} - \frac{1}{r_{nf}}\right)$$

1.4 Electric Potential

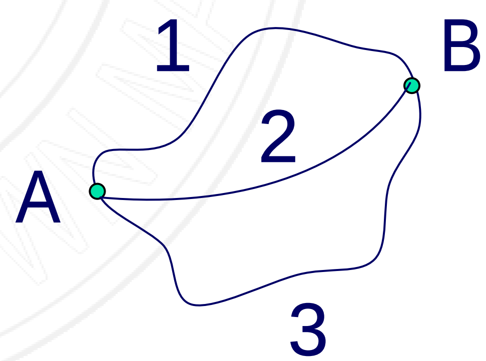
The Work Due to Electric Force

- *The work done by the force of electric field due to any distribution of charges is path independent, and only depends on the initial and final position.*

- *The work done by electric force to move a charge from A to B along the different path is the same.*

$$A_{AB} = \int_{path1} \vec{F} \cdot d\vec{l} = \int_{path\ 2} \vec{F} \cdot d\vec{l}$$

$$= \int_{path\ 3} \vec{F} \cdot d\vec{l}$$



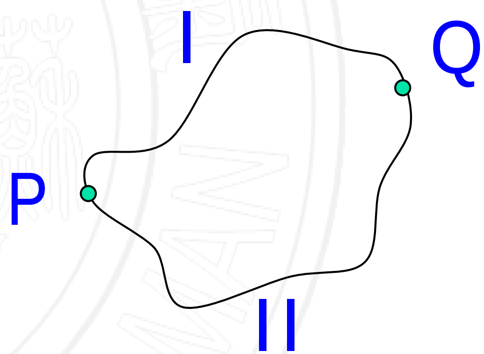
1.4 Electric Potential

Loop Theorem of Electric Field

- The work done to move a charge on a closed path

$$A_{PP}=\oint_{any}\vec{F}\cdot d\vec{l}=\int_{(I)P}^{Q}\vec{F}\cdot d\vec{l}+\int_{(II)Q}^{P}\vec{F}\cdot d\vec{l}$$

$$=\int_{(I)P}^{Q}\vec{F}\cdot d\vec{l}-\int_{(II)P}^{Q}\vec{F}\cdot d\vec{l}=0$$



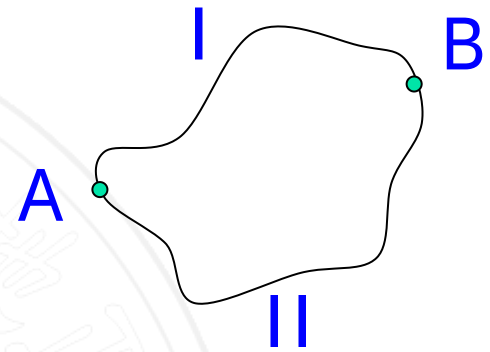
$$\oint_{any} q\vec{E}\cdot d\vec{l}=0$$

$$\oint_{any}\vec{E}\cdot d\vec{l}=0$$

1.4 Electric Potential

Loop Theorem of Electric Field

$$\oint_{any} \vec{E} \cdot d\vec{l} = 0$$



- The work done to move a charge on a closed path is zero.
- The integration of Electric Field along any loop is zero.
- It expresses that electrostatic force is conservative.

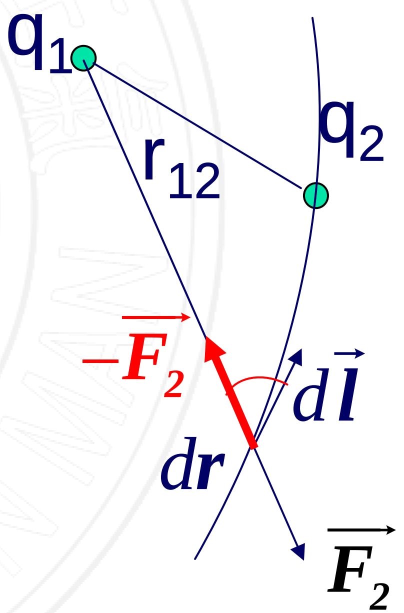
Electrostatic field is conservative.

To describe conservative field by the concept of potential!

1.4 Electric Potential

Energy of a system of charges

- How much work does agent force do to assemble a certain configuration of charges?
  - Move q_1 , $W_1 = 0$, $F=0$
  - Move q_2 ,q_1 presents



$$dA_2 = -\vec{F}_2 \cdot d\vec{l} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}dr$$

$$A_2 = -\int_{\infty}^{r_{12}} \vec{F}_2 \cdot d\vec{l} = -\int_{\infty}^{r_{12}} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}dr$$

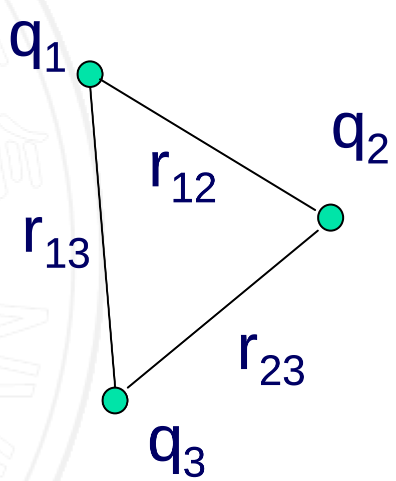
$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

1.4 Electric Potential

Energy of a system of charges

How much work does agent force do to assemble a certain configuration of charges?

- Move q_3 ,q_1 ， q_2 present



$$A_3 = A_{13} + A_{23} = -\int_{\infty}^{r_{12}} \vec{F}_{13} \cdot d\vec{l} + -\int_{\infty}^{r_{12}} \vec{F}_{23} \cdot d\vec{l}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

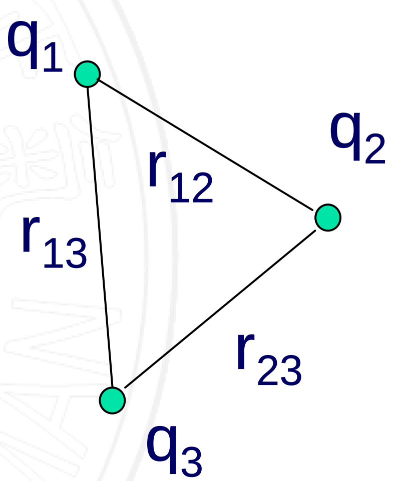
$$A = A_1 + A_2 + A_3 \ldots = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

1.4 Electric Potential

Energy of a system of charges

How much work does agent force do to assemble a certain configuration of charges?

$$A = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$



- Where does the work go?
- It is stored in the electric field.

$$W = \frac{1}{2}\sum_{(i=1,j=1,i\neq j)}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$

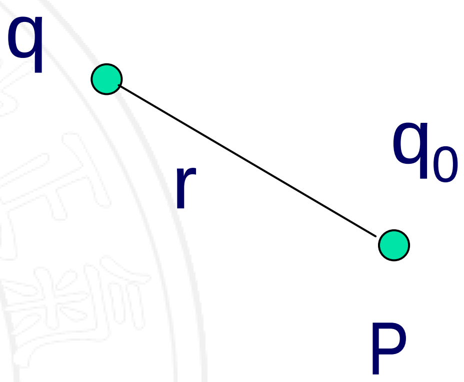
- The Combination Energy of molecule or atom

1.4 Electric Potential

◈ Electric Potential Difference, Electric Potential

- Electric Potential Energy

$$W = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{r}$$



- Difference of Electric Potential Energy

$$\Delta W = W_B - W_A = \frac{q_0 q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

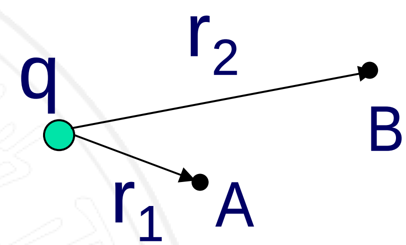
- ΔW the work done by electric force to move q_0(test charge) from A to B= Difference of Energy

- Electric potential energy at P is defined the work done by electric force to move q from P to infinite.

1.4 Electric Potential

◈ Electric Potential Difference, Electric Potential

$$V_{AB} = \frac{W_{A-B}}{q_0} = \int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



- Difference V_{AB} of Electric potential is defined the work done by electric force to move unit +charge from A to B.

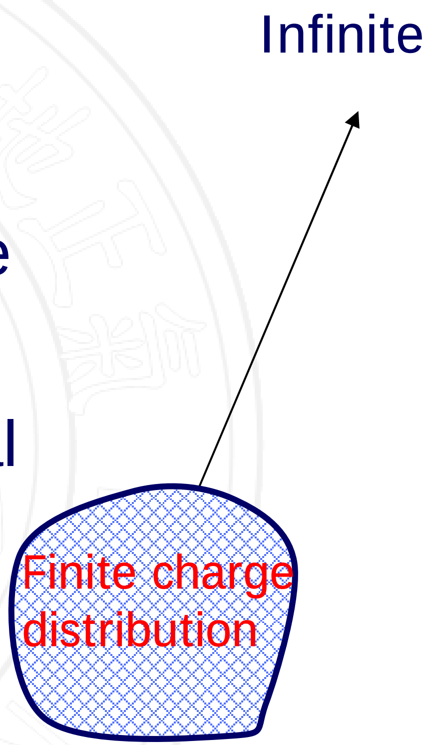
- Electric Potential is defined the work done by electric force to move unit +charge from P to infinite.

$$V_P = \frac{W_{P-\infty}}{q_0} = \int_P^\infty \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

1.4 Electric Potential

◈ Electric Potential Difference, Electric Potential

$$V_P = \frac{W_{P-\infty}}{q_0} = \int_P^{\infty} \vec{E} \cdot d\vec{l}$$



- Infinite is the reference point, if the charge distribution is finite, inversely.

- Find potential energy from potential

$$W_p = qV_P$$

- Find difference of potential energy from potential difference

$$W_A - W_B = q(V_A - V_B)$$

1.4 Electric Potential

◈ The Calculation of Electric Potential

- Two methods to find V_P
  - Definition Method (E known or easy to find)

$$V_P = \int_P^R \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}} \qquad (1.43)$$

  - Superposition Principle (E unknown or hard to find)
- Examples for definition method

Math trick: chose the easiest path for integration

1.4 Electric Potential

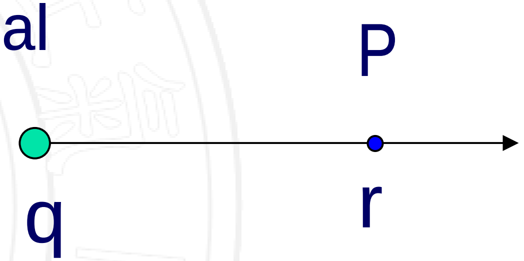
◈ The Calculation of Electric Potential

Example 1.12 Find electric potential due to a point charge.

Solution:

According to the definition of potential

$$V_P = \int_P^{\infty} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$$



and E of point charge, we get

$$V_P = \int_P^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{\vec{\boldsymbol{r}}} \cdot d\vec{\boldsymbol{l}}$$ (choose $d\vec{\boldsymbol{l}} = d\vec{\boldsymbol{r}}$)

$$V_P = \int_P^{\infty} \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dr = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

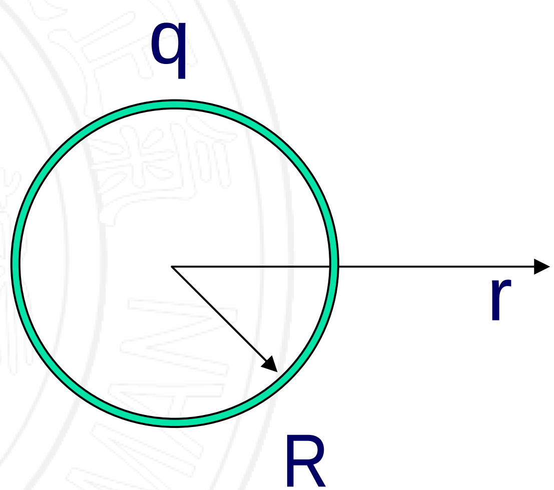
1.4 Electric Potential

◈ The Calculation of Electric Potential

Example 1.13 Find electric potential due to a charged sphere shell with charge q and radius R.

Solution:

According to the definition of potential



$$V_P = \int_P^{\infty} \vec{E} \cdot d\vec{l}$$

and E of the charged sphere shell,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(r>R)}$$

Easy to find by Gauss's law

$$E=0 \quad \text{(r<R)}$$

1.4 Electric Potential

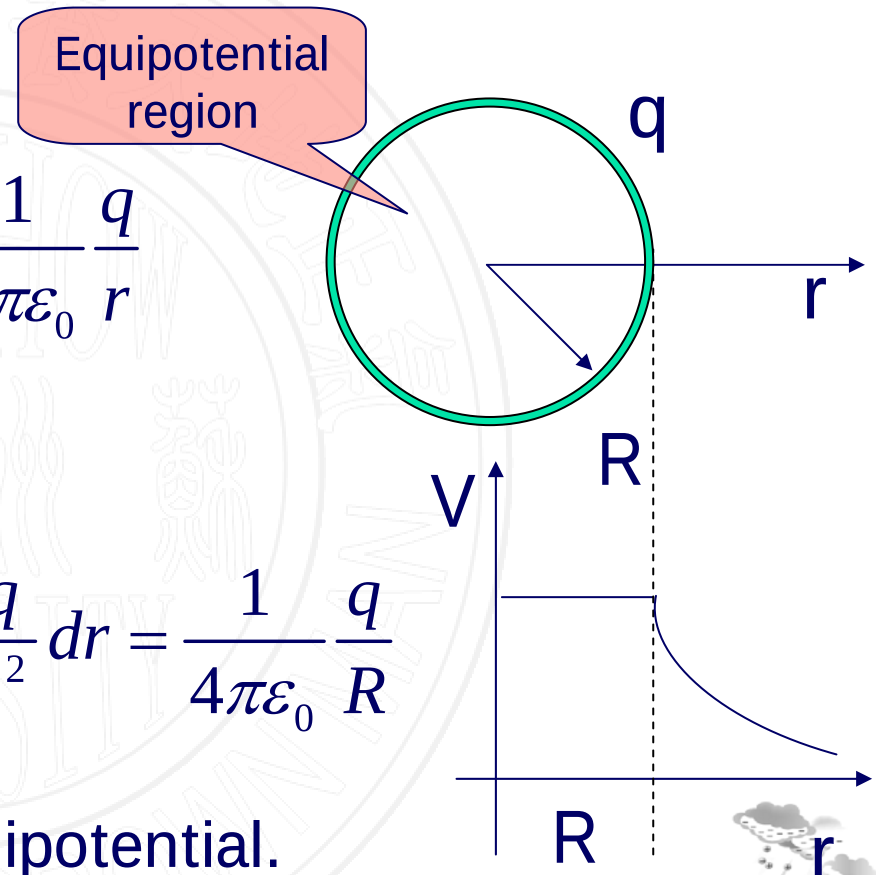
◈ The Calculation of Electric Potential

If (r>R), Out

$$V=\int_r^{\text{Ref.}}\vec{E}\cdot d\vec{l}=\int_r^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

If (r<R) In

$$V=\int_r^{\text{ref.}}\vec{E}\cdot d\vec{l}=\int_r^{R}0dr+\int_R^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$



- Any concentric sphere is equipotential.
- Is the potential continuous at R?

1.4 Electric Potential

◈ The Calculation of Electric Potential

- Superposition principle for
  - a set of charges

$$\vec{E} = \sum \vec{E}_i \qquad V_P = \int_P^{\infty} \vec{E} \cdot d\vec{l}$$

$$V_P = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$$ (Algebraic sum)

  - continuous distribution of charges

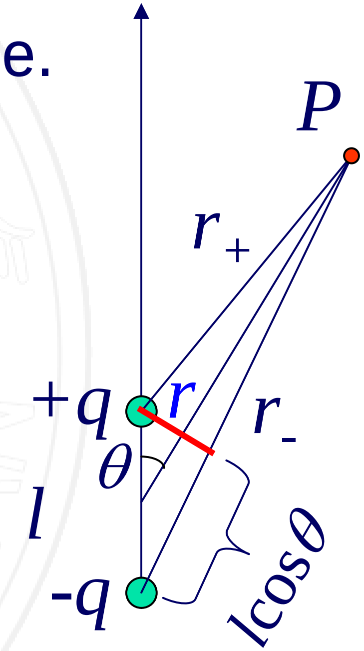
$$V_P = \int \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$$ (Integral operation)

1.4 Electric Potential

◈ The Calculation of Electric Potential

✸ Superposition principle for point charges

Example 1.14 Find potential due to a dipole.



Solution: potential of point charge
superposition principle

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r_+}+\frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+}-\frac{1}{r_-}\right)$$

$$=\frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right)$$

$$r_- = r+\frac{l}{2}\cos\theta \qquad r_+ = r-\frac{l}{2}\cos\theta$$

1.4 Electric Potential

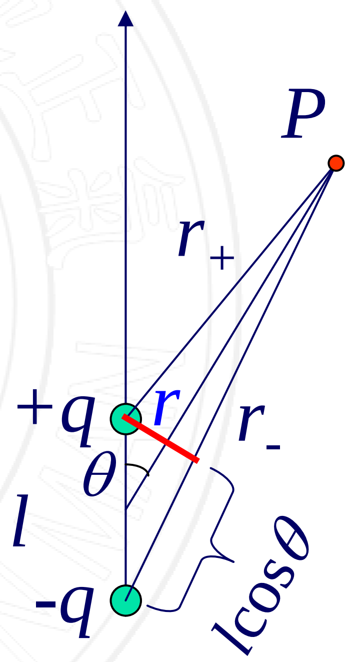
◈ The Calculation of Electric Potential

- Superposition principle for point charges

$$V = \frac{q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2 - \frac{1}{4}l^2\cos^2\theta}$$

- If $r >> l$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$



1.4 Electric Potential

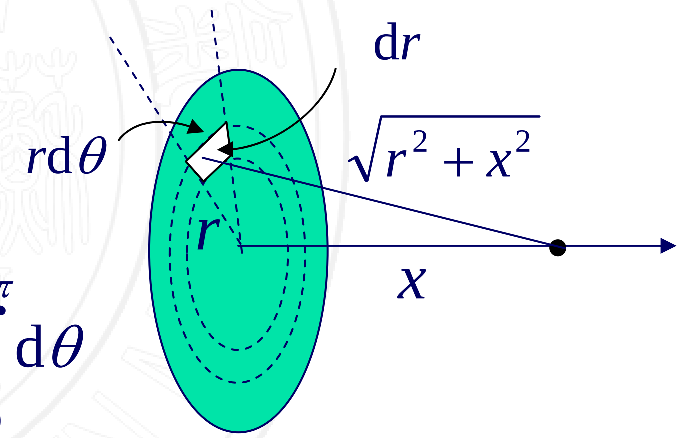
◈ The Calculation of Electric Potential

- Superposition principle for Cont. distribution charges

Example 1.15 Find the electric potential on the axis of charged disk, charge density is σ

Solution: Differential charge dq

$$dq = \sigma\, ds = \sigma\, r d\theta dr$$



$$V_P = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{r^2+x^2}} = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{\sigma r dr}{\sqrt{r^2+x^2}}\int_0^{2\pi} d\theta$$

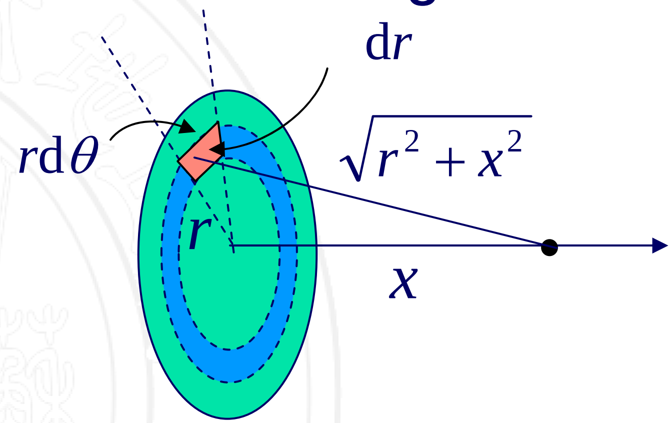
$$= \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+x^2}\,\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+x^2} - x\right)$$

1.4 Electric Potential

◈ The Calculation of Electric Potential

- Superposition principle for Cont. distribution charges

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + x^2} - x\right)$$



- Math trick, Tylor's expansion

If $x >> R$

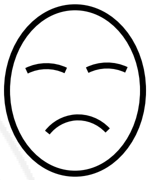
$$\sqrt{x^2 + R^2} = x\left(1 + \frac{R^2}{x^2}\right)^{1/2} = x\left(1 + \frac{1}{2}\frac{R^2}{x^2} + \cdots\right) \approx x + \frac{R^2}{2x}$$

$$V = \frac{\sigma}{2\varepsilon_0}\frac{R^2}{2x} = \frac{\sigma}{2\varepsilon_0}\frac{\pi R^2}{2\pi x} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x}$$

1.4 Electric Potential

◈ The Calculation of E from V

- We have two ways to find E:



  Superposition Principle, difficult

  Gauss's Law, easy but strict

- To describe electric field by **E** or V, To find V from **E.** Can we find **E** from V ?

- Yes. The Gradient!

  Equipotential surface

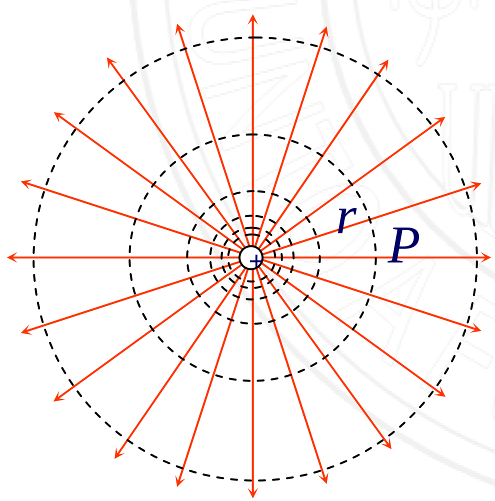
  Have you ever heard contour line?

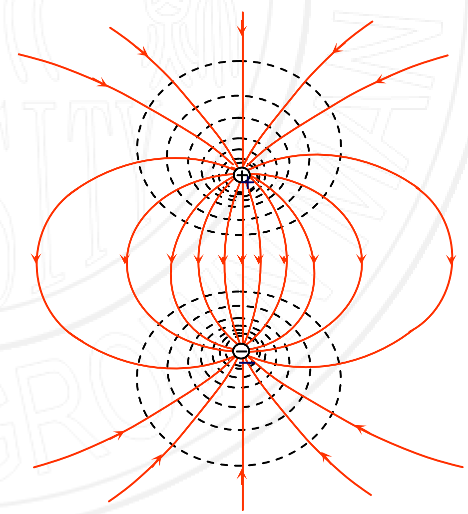
1.4 Electric Potential

- Equipotential surface: same potential everywhere

  Characteristics of EPS:

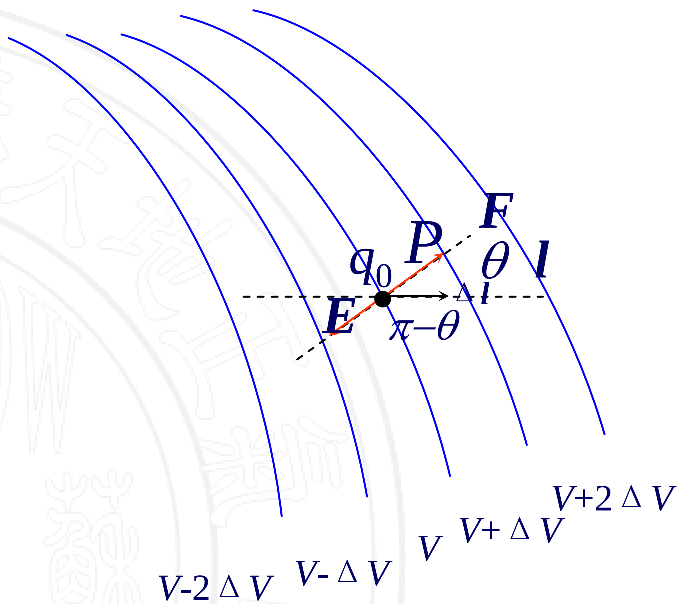
  - E ⊥ equipotential surface.
  - The denser equipotential surface, the stronger the field.
  - Equipotential surface never cross.
  - Equipotential surface can be closed.





1.4 Electric Potential

- Gradient, Find E from V
  - How much work done by agent force to make q_0 displace Δl ?



$$\Delta W = \vec{F} \cdot \Delta\vec{l} = q_0 \Delta V$$

$$\vec{F} \cdot \Delta\vec{l} = -q_0 \vec{E} \cdot \Delta\vec{l} =$$

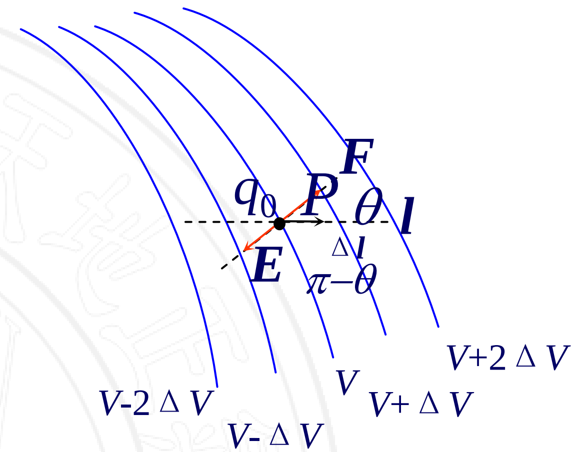
$$-q_0 E \Delta l \cos(\pi - \theta) = q_0 E \Delta l \cos\theta$$

$$q_0 \Delta V = q_0 E \Delta l \cos\theta$$

$$\frac{\Delta V}{\Delta l} = E\cos\theta$$

1.4 Electric Potential

- Gradient, Find E from V



$$\frac{\Delta V}{\Delta l} = E\cos\theta$$

$$E_l = -\frac{\Delta V}{\Delta l}$$

Different direction, different differential coefficient V.

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

1.4 Electric Potential

- Gradient, Find E from V

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

- V=V(x,y,z) Rectangular coordinate

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

- V=V (r,θ,ϕ) Spherical coordinate

$$\nabla = \vec{e}_r\frac{\partial}{\partial r} + \vec{e}_\theta\frac{1}{r}\frac{\partial}{\partial \theta} + \vec{e}_\phi\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

- V=V(r,θ,z) Cylindrical coordinate

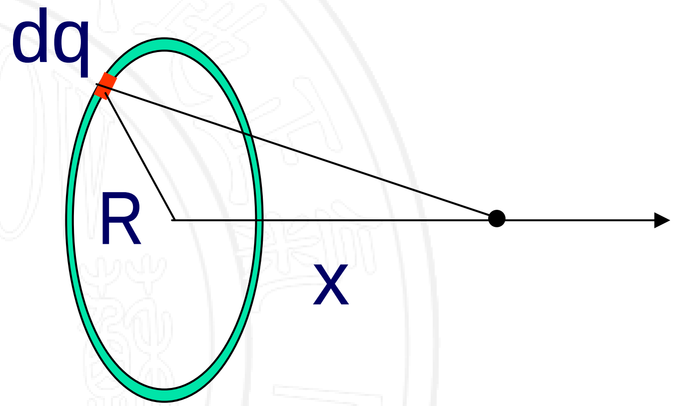
$$\nabla = \vec{e}_r\frac{\partial}{\partial r} + \vec{e}_\theta\frac{1}{r}\frac{\partial}{\partial \theta} + \vec{e}_z\frac{\partial}{\partial z}$$

1.4 Electric Potential

- Gradient, Find E from V

Example 1.15 Find E on the axis of charged ring

Solution:



From superposition principle,

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

We get

$$V = \oint \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2 + x^2}}$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2 + x^2)^{3/2}}$$

$$E_y = E_z = 0$$